Chapter 2
Introduction

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There are various possible ways of articulating what Bayesian epistemology is and how it relates to other branches of formal and mainstream epistemology. Following the steps of Ramsey, Richard Jeffrey outlines in his article "Probable Knowledge" a possible way of constructing an epistemology grounded on Bayesian theory. While knowledge is a central notion in traditional epistemology (and in various branches of formal epistemology) Jeffrey suggests an epistemology where knowledge does not have the importance generally attributed to it. The idea is "[...] to try to make the concept of belief do the work that philosophers have generally assigned to the grander concept" (knowledge). Moreover the notion of belief is pragmatically analyzed along the lines proposed by Ramsey: "the kind of measurement of belief with which probability is concerned is .... a measurement of belief qua basis of action". The result of this move is to conceive the logic of partial belief as a branch of decision theory. So, the first two essays in this section are also quite relevant for the section of decision theory presented below (Ramsey's essay contains the first axiomatic presentation of decision theory). Both Jeffrey and Ramsey present the foundations of an epistemology which is deeply intertwined with a theory of action.

Horacio Arló-Costa was deceased at the time of publication.

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H. Arló-Costa et al. (eds.), *Readings in Formal Epistemology*, Springer Graduate Texts in Philosophy 1, DOI 10.1007/978-3-319-20451-2_2

This move has a behaviorist pedigree but perhaps the behavioral inspiration is not an essential ingredient of an interpretation of the formal theory that thus arises.

The notion of certainty or full belief does not play a central role in Jeffrey's epistemology either. According to him we are only certain of mathematical and logical truths and the truths related immediately to experience. The rest is the domain of probable knowledge. To be coherent with this view Jeffrey has to propose a modification of the classical notion of learning by conditioning on data (which occupies a central role in various versions of Bayesian Epistemology as used in the social sciences like economics or psychology). In fact, according to conditioning when one learns a new piece of evidence this information acquires measure one in spite of being based on perfectly fallible evidence. A modification of conditioning that permits to update on uncertain evidence is presented in "Probable Knowledge". The version of Jeffrey's article reprinted here contains as well comments by L. Hurwicz and P. Suppes and responses by Jeffrey. Some of Suppes' comments point in the direction of constructing a theory of rationality that is sensible to our cognitive limitations. The possibility of constructing a "bounded" theory of rationality only started with the seminal work of Herb Simon (Rational choice and the structure of the environment, *Psychological Review*, Vol. 63 No. 2, 129–138. 1956) and is today an active area of investigation in economics, psychology and philosophy.

The uses of Bayesianism in epistemology are usually dismissed by realist philosophers for delivering a subjective picture of rationality that is not sufficiently sensible to the way in which the behavior of rational agents is connected with the structure of the environment. Simon's work was certainly sensible nevertheless to ecological considerations. And Ramsey's essay ends with programmatic ideas differentiating what he called "the logic of consistency" from the "logic of truth". Even Bruno de Finetti who is usually presented as a precursor of Jeffrey's radical probabilism, had philosophical ideas about certainty that clashed with this view (he thought that certainty has to be assumed as a primitive alongside probability, and that we can be certain of more than mere tautologies). Moreover his mature philosophical work veered towards a more objective point of view. For example he dismissed the use of Dutch Book arguments and embraced the use of foundational arguments in terms of scoring rules, a methodological move favored today by many "objective" Bayesians (a presentation of de Finetti's mature views appear in: *Philosophical Lectures on Probability*: collected, edited, and annotated by Alberto Mura, Synthese Library, Springer, 2008).

Van Fraassen introduces in his essay a version of radical probabilism (the term was coined by Jeffrey) where the only epistemological primitive is a notion of conditional probability. Van Fraassen sees this notion as encoding a notion of supposition from which he derives a non-trivial notion of full belief. According to this view it is perfectly possible to be sure of the contingent propositions of science and everyday knowledge. One can see van Fraassen's theory as introducing paradox-free acceptance rules that link probability and belief (some of the usual acceptance rules of this type like high probability rules are known to be the victim of various forms of paradoxes, like the paradox of the lottery first proposed by Henry Kyburg (*Probability and the Logic of Rational Belief*, Wesleyan University

Press, 1961)). Jeffrey renounced to the use of any form of acceptance rules of this type and therefore proposed to eliminate any notion of qualitative belief without a probabilistic origin. Van Fraassen has exactly the opposite intention: namely to tend bridges between traditional and Bayesian epistemology via the use of novel acceptance rules.

Most of the models of probability update considered above deal with synchronic or suppositional change. Is it possible to extend these models to cover cases of genuine changes of probabilistic belief? David Lewis, Bas van Fraassen and Paul Teller provided in the 1970s various dynamic coherence arguments showing that one should update diachronically via conditioning on pain of incoherence (see references in the section on "Further reading" below). If we denote by $P_t(B|A)$ the conditional probability of B given A at time t and by $P_t(.)$ the monadic probability P at time t, we can denote by $P_{t'}$ the monadic probability at time t' where the total evidence gathered between these two times is A. Then the idea that one should update diachronically via conditioning can be expressed formally by: $P_{t'}(B) = P_t(B|A)$. These arguments in favor of this diachronic principle are dynamic versions of the static Dutch Book arguments first proposed in Ramsey's essay. Unfortunately these arguments are considerably more controversial than the well know static Dutch Book argument. The article by Brian Skyrms summarizes almost 30 years of arguments pro and con dynamic book arguments and offers a temperate and more modest version of these arguments that he thinks is valid. This debate is nevertheless still open.

Levi's essay is embedded on his own version of Bayesian epistemology where the notion of full belief is taken as a primitive alongside probability. But the central goal of the article reprinted here is to study the case where probabilities are indeterminate, imprecise or vague. Levi also thinks that a theory of partial belief (precise or imprecise) should be conceived as a branch of decision theory and therefore proposes rules for deciding in conditions of uncertainty. Currently there is a fair amount of work in this area not only in philosophy but also in statistics, computer science and economics.

Gaifman's article focuses on characterizing the structure of higher order probability. In particular he investigates a form of the so-called Miller's principle by which a rational agent adjusts his or her probabilities in accordance to the probabilities of an expert. So, we have a principle of this form:

(Miller's Principle) $P_{you}(A \mid P_{expert}(A) = r) = r$[1]

van Fraassen proposed a diachronic version of this principle for a single agent:

(Reflection) $P_{now}(A \mid P_{later}(A) = r) = r$

[1] Actually Gaifman's formulation of the principle is formally cleaner. He defines $PR(A, r) = \{x \text{ in } W \mid p_x(A) = r\}$, where p_x is an expert function at world x, i.e. the distribution chosen by the expert at that world. Then he formulates the principle as follows: $P(A \mid PR(A, r)) = r$.

Arntzenius' article presents five puzzles showing that rational people can update their degrees of belief in manners that violate Bayesian conditioning and Reflection. But the article by M.J. Schervish, T. Seidenfeld and J. Kadane disputes that Arntzenius' examples impose any new restrictions or challenges to conditioning or Reflection beyond what is already familiar about these principles.

Suggested Further Reading

- An excellent introduction to Ramsey's philosophy in general and to the essay reprinted here in particular can be found in the corresponding chapters of: *The Philosophy of F.P. Ramsey*, by Nils-Eric Sahlin, Cambridge University Press, 2008. The classical introduction to Richard Jeffrey's decision theory is his: *The Logic of Decision*, University Of Chicago Press: 2nd edition (July 15, 1990). A detailed articulation of radical probabilism can be found in Probability and the Art of Judgment, Cambridge Studies in Probability, Induction and Decision Theory (Mar. 27, 1992). The theory of probability cores presented in van Fraassen's article has been slightly modified and extended in a paper by Horacio Arlo-Costa and Rohit Parikh: "Conditional Probability and Defeasible Inference," *Journal of Philosophical Logic* 34, 97-119, 2005. The best axiomatic presentation of primitive conditional probability is given by Lester E. Dubins in his article Finitely Additive Conditional Probabilities, Conglomerability and Disintegrations, *The Annals of Probability*, 3(1):89–99, 1975. Teddy Seidenfeld wrote an accessible note presenting recent results in this area in: Remarks on the theory of conditional probability: Some issues of finite versus countable additivity, *Probability Theory*, V.F. Hendricks et al. (eds.) 2001, pp. 167-178. Alan Hájek articulated a philosophical defense of the use of primitive conditional probability in: What Conditional Probability Could Not Be, *Synthese*, Vol. 137, No. 3, Dec., 2003. Finally there is an interesting article by David Makinson linking conditional probability and central issues in belief change: Conditional probability in the light of qualitative belief change, to appear in a 2011 issue of the *Journal of Philosophical Logic* marking 25 years of AGM. References to other classical articles in this area by Karl Popper, Alfred Renyi and Bruno de Finetti appear in the aforementioned articles.
- Brian Skyrms has also contributed to the theory of higher order probability. One accessible article is: "Higher Order Degrees of Belief," in D. H. Mellor (ed.), *Prospects for Pragmatism*. Cambridge: Cambridge University Press, 109–13. Isaac Levi has articulated his theory of indeterminate probabilities in various books and articles. One of the classical sources is: *The Enterprise of Knowledge*, MIT Press, Cambridge, 1983. More information about Levi's version of decision theory under uncertainty appears in section 7 on Decision Theory below.
- There are two classical sources for the formulation of dynamic Dutch books. One is: Teller, P. (1973), "Conditionalization and Observation", *Synthese* 26: 218-258. The other is: van Fraassen, Bas (1984), "Belief and the Will," *Journal of Philosophy* 81: 235–256. The second piece introduces also a theory of second order probability that complements the writings of Skyrms and Gaifman. Van Fraassen introduces there the Reflection Principle. The original formulation of some of the puzzles discussed by Arntzenius and Seidenfeld is a brief piece by Adam Elga: "Self-Locating Belief and the Sleeping Beauty problem, *Analysis*, 60(2): 143-147, 2000. More detailed reference to the work by Carnap on induction and confirmation can be found in the bibliography of Maher's paper. The so-called Raven's Paradox appeared for the first time in a seminal article by Carl Hempel: "Studies in the Logic of Confirmation (I.)," *Mind,* New Series, Vol. 54, No. 213 (Jan., 1945), pp. 1-26. Branden Fitelson and James Hawthorne offer an alternative and interesting Bayesian account of the paradox in: "How Bayesian Confirmation Theory Handles the Paradox of the Ravens," in E. Eels and J. Fetzer (eds.), *The Place of Probability in Science*, Chicago: Open Court. Further information about confirmation theory can be found in a classical book by John Earman: *Bayes or Bust? A Critical*

Examination of Bayesian Confirmation Theory, MIT Press, 1992. Another classical source is: *Scientific Reasoning: The Bayesian Approach*, by Colin Howson and Peter Urbach, Open Court; 3rd edition, 2005. A interesting book touching a cluster of issues recently discussed in this area like coherence and the use of Bayesian networks in epistemology is: *Bayesian Epistemology* by Luc Bovens and Stephan Hartmann, Oxford University Press,. 2004.

- Another important formal epistemological issue is investigated by Timothy Williamson in his paper, "Conditionalizing on Knowledge", *British Journal for the Philosophy of Science* 49 (1), 1998: 89-121, which intends to integrate the theory of probability and probability kinematics, with other epistemological notions like the notion of knowledge. The theory of *evidential probability* that thus arises is based on two central ideas: (1) the evidential probability of a proposition is its probability conditional on the total evidence (or conditional on evidence propositions); (2) one's total evidence is one's total knowledge. The tools of epistemic logic are used in order to represent the relevant notion of knowledge.
- Jeffrey does not adopt (1) but according to his modified notion of updating once a proposition has evidential probability 1, it keeps it thereafter (monotony). This is a feature shared by Jeffrey's updating and the classical notion of updating. Williamson does embrace (1) but develops a model of updating that abandons monotony. This seems a very promising strategy given the limited applicability of a cumulative model of growth of knowledge. Similarly motivated models (that are nevertheless formally quite different) have been proposed by Isaac Levi, Peter Gärdenfors. Gärdenfors' model appears in his book *Knowledge in Flux* (see the corresponding reference in the bibliographical references of chapter 6). Levi presents his account in *The Enterprise of Knowledge* (the reference appears in the bibliographical section below). Both models appeal directly not only to qualitative belief but also to models of belief change (*contraction* and *revision* - see chapter 6).
- Philosophers of science have traditionally appealed to Bayesian theory in order to provide a Carnapian explication of the notoriously vague, elusive and paradox-prone notion of *confirmation* or *partial justification* in science. Patrick Maher revives in his article, "Probability Captures the Logic of Scientific Confirmation," in *Contemporary Debates in the Philosophy of Science*, ed. Christopher Hitchcock, Blackwell, 69–93, the Carnapian program of inductive inference in order to provide one of these explications. In contrast Clark Glymour and Kevin Kelly argue in their article, "Why Probability Does Not Capture the Logic of Scientific Justification", in Christopher Hitchcock, ed., *Contemporary Debates in the Philosophy of Science*, London: Blackwell, 2004, that Bayesian confirmation cannot deliver the right kind of account of the logic of scientific confirmation. One of the reasons for this skepticism is that they think that scientific justification should reflect how intrinsically difficult is to find the truth and how efficient one's methods are at finding it. So, their skepticism arises because they think that Bayesian confirmation captures neither aspect of scientific justification. While deploying their arguments the two articles discuss the well-known paradox of confirmation first proposed by Hempel, Carnap's research program on the philosophy of probability and induction and the possible application of learning theory in order to offer a non-Bayesian account of scientific justification. The article by Glymour and Kelly continues Glymour's earlier critique of the applications of Bayesianism in philosophy of science (also reprinted here). This earlier piece contains the original versions of some influential and much-discussed conundra engendered by Bayesian confirmation (like the problem of Old Evidence).