

## Chapter 25

# Introduction

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This part, like all others in this book, consists of a mix of classic papers that have defined the area and modern ones illustrating important current issues. These texts provide rich fare, and they defy simple labels summarizing their content. Moreover, a look at the list of authors reveals a mixture of different academic cultures, from philosophy to computer science. One might also add that the texts do not all agree: they span a landscape with many positions and perspectives.

Epistemic logic as the systematic study of reasoning with knowledge and belief started with Jaakko Hintikka's classic book *Knowledge and Belief: An Introduction to Logic of the Two Notions*, which set the agenda for many subsequent lines of research and debate. It interpreted knowledge as what is true in some current range of epistemically accessible worlds, while doing something similar for belief and doxastic accessibility. Thus, general methods from modal logic became available for studying knowledge, and the resulting axiomatic systems have shaped many philosophical discussions for or against principles of 'omniscience', 'closure', and positive and negative 'introspection'. One general interest behind such specific issues has been the search for satisfactory definitions of knowledge, an interest with a long epistemological history running from Plato's "justified true belief" to post-Gettier strengthenings involving forms of robustness of true beliefs under new information, under new relevant considerations, or across counterfactual variations

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of the actual world – and in a related spirit, in various forms of faithful tracking during a history of investigation. The past half-century of formal studies has even produced further perspectives, such as the availability of proof or evidence in some precise sense, or convergence in the limit in some process of inquiry.

The paper by Dretske makes the original Hintikka semantics a more dynamic process, showing how knowledge claims are always based on some current range of relevant worlds, which can change under the pressure of legitimate new considerations. In a related vein, Lewis provides systematic principles guiding this process of selecting relevant worlds. Following a further intuition, Nozick proposes a counterfactual idea of knowledge as a true belief that would stay attuned to the facts in non-actual worlds close to ours. While these approaches are semantic, Artemov's 'justification logic' brings explicit proof and evidence into epistemic logic, allowing us to syntactically manipulate reasons for our beliefs. Finally, Kelly discusses the learning-theoretic view of temporal convergence for knowledge of complete histories of the world.

Since this part is about logics of knowledge and belief, many readers will be interested not just in formal languages and semantics, but also in complete calculi for reasoning capturing the operational proof-theoretic aspects of reasoning with knowledge or belief. Stalnaker's paper presents a broad logical view of possible modal systems and defensible epistemic and doxastic principles, and in another perspective, so does Artemov. Interestingly, not all notions of knowledge proposed in the 'robustness' tradition have been studied in this systematic manner, and many questions remain open, though actively pursued by some young philosophical logicians.

Another running theme is the issue of which epistemic attitudes form a natural family that requires scrutiny in its entirety. Knowledge and belief need not be enough, and for instance, Stalnaker's survey of doxastic and epistemic logics proposes a new notion of 'safe belief' that will survive true new information, as an intermediate between logic and belief simpliciter. Parikh even suggests an algebraic framework that ties together a wide abstract range of knowledge, belief, and action in general. Specializing general action again to informational action, we arrive at what has been Hintikka's guiding interest throughout: the combination of knowledge as based on our current semantic information with acts that systematically change that information, such as questions, and games of inquiry over time.

It is only a short step then to a dynamic focus on learning rather than the mere statics of knowledge. Kelly's article looks at this dynamics from the viewpoint of learning theory, and investigates which truths can be acquired in the limit, i.e., which processes of inquiry will reliably converge to stable true belief about the answer to the main question at stake. In a somewhat related mode, Williamson focuses on the scope of purely operational definitions of knowledge, and shows that knowledge-based epistemology remains indispensable. And these are just two dynamic or computational aspects of knowledge and belief. There is more to information dynamics when one begins to study effects of specific local acts of

knowledge update or belief revision as an agent navigates the world. Many of these topics will be addressed in the next section on interactive epistemology, since much information flow that is crucial to humans involves more than one party: be it a group of agents, or just one agent interacting with Nature.

There are many further themes to ponder when reading these articles. Does the semantics describe internal first-person views of epistemic agents, or the theorist's external view of their situation? Do different epistemic attitudes correlate with different sorts of information? How does knowledge of propositions tie in with knowledge of objects, "that" or "whether" versus "which", and why not then also discuss knowledge "how" and "why"? And finally, what is the status of all these logical theories? Are they normative prescriptions, or do they represent some existing cognitive practice, if only idealized and at higher levels of abstraction? Reading the papers in this section will not necessarily answer all these questions, but it will make readers much better equipped to pursue these issues for themselves.

## Suggested Further Reading

Starting with a classical trailblazer, J. Hintikka, *Knowledge and Belief: An Introduction to the Logic of the Two Notions*, Cornell University Press 1962 and King's College Publications 2005, set the whole subject on its course. A series of later books broadened the paradigm to a general view of information and inquiry, as represented in J. Hintikka, *Logic, Language-Games and Information: Kantian Themes in the Philosophy of Logic*, Clarendon Press Oxford, 1973. Putting inquiry at center stage in epistemology has also been the persistent theme of Robert Stalnaker's work in the field, with *Inquiry*, The MIT Press, 1987, as a classic source. Meanwhile, a richer view of possible epistemic and doxastic attitudes suggested by natural language was investigated in W. Lenzen, "Recent Work in Epistemic Logic", *Acta Philosophica Fennica* 30 (1978): 1–219, which also discusses links with probability. Also still in the 1970s, epistemic and doxastic logic were rediscovered in the foundations of game theory, but references for this will be found in another part of these readings. But arguably the major development invigorating epistemic logic has been its crossing into computer science, in the study of information-driven agency. Two major books demonstrating the resulting outburst of new research are R. Fagin, J. Y. Halpern, Y. Moses & M. Vardi, *Reasoning about Knowledge*, The MIT Press, 1995, and W. van der Hoek & J-J Meijer, *Epistemic Logic for AI and Computer Science*, Cambridge University Press, 1995. An even more radically computational algorithmic understanding has been that of formal learning theory, inspired by learning methods for infinite structures like human languages or the process of scientific inquiry. The classic source for this is K. Kelly, *The Logic of Reliable Inquiry*, Oxford University Press, 1996. We conclude with two other angles on knowledge that bring in further mathematical paradigms. One is the verificationist perspective on knowledge through proof and evidence, for which a classic text is M. Dummett, *Truth and Other Enigmas*, Harvard University Press, 1978. Finally, while these publications are concerned with knowledge and belief, another broad stream has taken information flow through Shannon-type channels to be the basic underlying notion, following Dretske's classic *Knowledge and the Flow of Information*, The MIT Press, 1981. An innovative logical framework taking this road much further is J. Barwise & J. Seligman, *Information Flow*, Cambridge University Press, 1995.