Chapter 7
Philosophy of Language

Luc Bergmans, John Burgess, Amitabha Das Gupta and Harrie de Swart

Abstract This chapter aims to be an introduction to the philosophy of language and presents some major topics belonging to this field: the difference between use and mention, Frege's notions of Sinn (sense) and Bedeutung (reference), Mannoury's significs, speech acts, definite descriptions, Berry's and Grelling's paradox, the theory of direct reference, Kant's notions of analytic versus synthetic, logicism, logical positivism, presuppositions, Wittgenstein on meaning, syntax - semantics - pragmatics, conversational implicature, conditionals, Leibniz, de dicto - de re distinction, and grammars. It is fair to say that the Dutch mathematician Gerrit Mannoury (1867 - 1956) invented the notion of speech act long before Austin, Searle and others used this notion. In the subsection on Logicism we explain that - contrary to what many philosophers of science claim even nowadays - Kant was right in asserting that mathematical statements are not analytic, but synthetic.

7.1 Use and Mention

If we want to say something **about** an object, we use the name of that object. We are used to doing so when the object is a person, but one frequently gets confused when the object is a linguistic one. Names of linguistic objects can be formed by enclosing the linguistic object in single (or double) quotation marks. For instance, in the proposition

John is a teacher

we make a statement **about** a person using the name of that person; and, similarly, in the proposition

'man' is monosyllabic

we make a statement **about** the word (linguistic object) *man*, using the name of that word. Using the terminology of W.V. Quine, we say that in

Man is a rational animal

the word *man* is used, but not mentioned; and that in

H. de Swart, *Philosophical and Mathematical Logic*, Springer Undergraduate Texts in Philosophy, https://doi.org/10.1007/978-3-030-03255-5_7

'Man' is monosyllabic

the word *man* is mentioned, but not used.

In practice the quotation marks are frequently suppressed, causing an equivocacy which is often convenient and harmless on the condition that one realizes what one is doing. So, instead of

'man' is monosyllabic

one may come across

man is monosyllabic.

Adopting Carnap's terminology, we say that the word *man* in the latter expression is *used autonymously*, i.e., as the name of that same word. So, in

man is monosyllabic

the word *man* is both mentioned and used, though used in an anomalous manner, namely, autonymously. Some more examples: in

The English translation of the French word *homme* has three letters

the word *man* is mentioned, but not used. In

The second letter of man is a vowel, and in
Man is a noun with a irregular plural,

the word *man* is mentioned and used autonymously.

The equivocacy, resulting from using the same word, *man*, both as a proper name of a linguistic expression and as a common name of certain mammals, may be removed by the use of added words in the sentence, or by the use of quotation marks, or of italics, as in

The word man is monosyllabic,
'Man' is monosyllabic,
Man is monosyllabic.

The latter device has been used above several times. The reader is advised to do Exercise 7.1. The examples above are from Church [10].

Exercise 7.1. (B. Mates, *Elementary Logic,* 1972, pp. 40-41) Not using words autonymously, which of the following sentences are true?

1. 'The Iliad' is written in English.
2. 'The Iliad' is an epic poem.
3. 'The Morning Star' and 'The Evening Star' denote the same planet.
4. The Morning Star is the same as the Evening Star.
5. '7 + 5' = '12'
6. The expression ' 'The Campanile' ' begins with a quotation mark.
7. The expression ' 'der Haifisch' ' is suitable as the subject of an English sentence.
8. Saul is another name of Paul.
9. 'Mark Twain' was a pseudonym of Samuel Clemens.
10. 2 + 2 = 4 is synthetic.
11. Although 'x' is the 24th letter of a familiar alphabet, some authors have said x is the unknown.
12. We are using capital Roman letters 'A', 'B', 'C', ... to stand for any formulas.

7.2 Frege's Sinn und Bedeutung (Sense and Reference)

In his Begriffsschrift of 1879 Frege made a distinction between '-*A*' for 'the proposition that *A*' and '$\vdash A$' for 'it is a fact that *A*'. In '-*A*' and in '$\vdash A$' Frege calls '*A*' the conceptual content (begrifflichen Inhalt). Thus if '$\vdash A$' is an abbreviation for the statement 'unlike magnetic poles attract each other', '-*A*' is to convey only the thought of mutual attraction between unlike magnetic poles, without any judgment of the correctness of that thought (G. Frege [14], Section 2).

In section 8 of his Begriffsschrift Frege introduces $\vdash a \equiv b$ as meaning: the sign *a* and the sign *b* have the same conceptual content so that *a* can always be replaced by *b* and conversely. However, if we consider

$$\vdash \text{the morning star} \equiv \text{the evening star}$$

it becomes clear that the definition just given must be wrong for the following two reasons: i) The expressions 'the morning star' and 'the evening star' have different conceptual contents, and ii) 'The morning star is identical with the evening star' has a meaning quite different from 'The morning star is identical with the morning star'. To verify the truth of the first sentence, astronomical observation is needed, but it is not necessary for the second one.

It is probably for these reasons that Frege, in his *Ueber Sinn und Bedeutung* of 1892, abandoned his talk of conceptual content and introduced a distinction between *sense (Sinn)* and *reference (Bedeutung)* instead. 'The morning star is identical with the evening star' then means: a) the expressions 'the morning star' and 'the evening star' refer to the same object, i.e., the planet Venus, called the reference (die Bedeutung), but b) they do so in different ways, because they have a different cognitive meaning or sense (Sinn).

The *reference (Bedeutung)* of an expression is what it 'stands for'. In the case of a proper name (Plato, France, the Titanic), it is the thing named; in the case of a singular definite description (Plato's father, the president of the United States), the object that fits the description.

In addition to words and the things (references) they stand for, Frege also insisted on taking into account the sense or cognitive meaning of words, since it is *through* its sense that an expression refers to an object. The sense provides the 'mode of presentation' of the object and referring to a reference is always achieved by way of sense.

Frege's *sense (Sinn)* includes the information content (cognitive meaning) of an expression, but not such features as (1) associations (emotional, literary; like the difference between 'horse' and 'steed'), (2) level of speech (formal, colloquial, slang, dialect, obsolescent, obscene; like the difference between 'regurgitate' and 'puke'), (3) indications of speaker's attitude (like the difference between 'but' and 'and' in 'he is a politician, but relatively honest', or the difference between 'they (still) have not arrived' with or without the 'still').

In poetry these other features are important, and a translation which merely preserved Frege's sense and lost these other features of 'meaning' would be a poor one.

In dry, objective scientific prose, only the sense is important. But for the study of literature, these extra features are of great importance. They distinguish 'I am determined/you are stubborn/he is pig-headed', which have, except for the change in personal pronoun, more or less the same information content or Fregean sense.

Frege emphasized the abstract nature of senses – they do not belong to any particular language (words in different languages can have the same cognitive meaning) and do not consist of individual psychological reactions in speakers of a language, but are something common to *all* speakers.

Obviously it is possible that two names or descriptions stand for the same object without being synonyms: we can have two expressions *a* and *b* with the same reference but with different senses. (The opposite cannot happen as we shall see below.) Indeed, this is Frege's explanation of how a statement '$a = b$' can be informative. In 'the Morning Star is the same thing as the Evening Star', for example, the two expressions 'Morning Star' and 'Evening Star' *refer to the same reference* (the planet Venus), but they *express a different sense*. And in '$2 + 2 = 4$' the names '$2 + 2$' and '4' refer to the same number, but they express a different sense.

Names for Frege include both proper names but also singular definite descriptions. Other writers use *singular term* or *designator* or *denoting-phrase* for Frege's *name*, which is less misleading, since we definitely want to include more than proper names.

The reference of a name is called an *object* (*ein Gegenstand*) by Frege. In other words, objects are anything which can be referred to by a name. This includes not just people and physical objects, but also abstractions (the Equator, numbers, justice) and events (the battle of Hastings). Frege has no special term for the sense of names. Carnap has called them *individual concepts*, not to be confused with Frege's *concepts* discussed below.

The expressions 'the greatest natural number' and 'the present king of France' do have a sense, but do not have a reference, because these expressions refer to nothing.

An expression is said to *express* its *sense* and *refer* to its *reference*. Other philosophers use the words *connotation* or *meaning* or *intension* for *sense* and *connote* or *mean* for *express*. *Denotation*, *designation, extension* and *signification* have all been used for *reference*, and *denote, designate* and *signify* for *refer*.

Frege goes on to argue that besides names and descriptions, predicates (or *general terms*) and sentences have both sense and reference.

Frege has no special term for the sense of a predicate ('is bald', 'lives in Princeton'). Others have said the predicate expresses a *property*, *attribute* or *quality*.

According to Frege, the reference (Bedeutung) of a predicate is a *concept* (*ein Begriff*). However, the nature of concepts is rather obscure. In addition to concepts, Frege recognizes classes. These are simply collections of objects. The class corresponding to a predicate, e.g., the class of all bald people corresponding to the predicate 'is bald', Frege calls the *extension* of the predicate. Most philosophers who follow Frege on the whole discard his concepts and simply speak of the class, and do not distinguish reference from extension.

The two predicates 'has a heart' and 'has a liver' may be true of all the same things (*co-extensive*, as Frege says) without being synonymous: they can have the same reference and extension without having the same sense. (The opposite is impossible according to principle (i) below.) The *class* of all creatures with hearts may be exactly the same class as the *class* of all creatures with livers, but the *property* of having a heart is different from the *property* of having a liver: to say something has a heart does not mean the same as saying it has a liver.

In order to figure out what the *reference (Bedeutung)* of a sentence is, Frege seems to invoke two principles:

(i) expressions with the same sense have the same reference, and

(ii) (principle of *compositionality*:) the reference of a compound is entirely determined by the references of its parts. This implies that

(ii*) if we replace a name or description in a sentence by another name or description of the same object, the reference of the sentence is unchanged.

By the principles (i) and (ii*) all sentences below have the same reference:

Scott wrote Waverly;

Scott is the author of the 29 Waverly novels (i);

29 is the number of Waverly novels that Scott wrote (i);

29 is the number of counties in Utah (ii*);

Utah has 29 counties (i).

So, (i) and (ii) imply that seeming unrelated true statements 'Scott wrote Waverly' and 'Utah has 29 counties' have the same reference (and similarly for false sentences). Frege concludes that the *reference* of a sentence is just its truth value, either *true* or *false*. (The example is from Church, [10], pp. 24-25.)

According to the principle of compositionality, mentioned above, if a name or description has no reference, no sentence of which it is a part can have a reference (truth value). So, the sentence 'The king of France is bald' does not have a truth value (reference, Bedeutung), because its subject 'the king of France' has no reference. If there is no Pegasus, 'Pegasus is flying' can be neither true nor false.

Frege explained the *presuppositions* of a statement as those things which must be true if that statement is to have any truth value at all, and specifically stated that a statement involving a description like 'the present king of France' *presupposes* the existence of the thing satisfying this description. Later writers on presupposition (e.g., Strawson) take Frege as their starting point (see Section 7.11).

Frege calls the *sense* of a sentence a *proposition* or *thought* (*ein Gedanke*). Actually, as commentators on Frege have pointed out, sentences like 'it is raining', 'I have a headache' express different propositions (sometimes true, sometimes false) according to when, where, and by whom they are uttered, so that it is necessary to distinguish between the proposition expressed on any given occasion and the *meaning* of the sentence, which is always the same. (In mathematics words like 'I', 'here', 'now' seldom occur, and so this problem does not arise. Frege was mainly concerned with this area of discourse.)

Frege distinguishes between a *proposition* and an *assertion*. According to Frege, when I state, 'The door is open' or ask, 'Is the door open?' or request, 'Please open

the door' or sigh, 'If only the door were open!' or command, 'Open the door!' or make a compound statement, 'If the door is open, then there'll be a draft', the same *proposition* is *expressed* in every case, but is only *asserted* in the first case. In the other cases no *assertion* is made; rather there is a question, request, etc.

Other philosophers have called a *proposition* a *phrastic* (Hare) or *locution* (Austin) and the element that must be added to make an assertion, or a question, or a request, etc., a *neustic* (Hare) or *illocutionary force* (Austin). The distinction between propositions and assertions is the origin of *speech act theory*, developed by Austin, Searle, and others (see Section 7.4 on speech acts).

Embedded in the aspects of Frege's theory of sense and reference, which have been dealt with so far, is the following contradiction. Consider the sentences:

(1) Somebody wonders whether Amsterdam is the capital of the Netherlands, and

(2) Somebody wonders whether Amsterdam is Amsterdam.

While (1) is probably true, (2) is false. So, (1) and (2) are likely to have different references (truth values). However, since the reference of 'Amsterdam' is the same as the reference of 'the capital of the Netherlands', the principle of compositionality seems to imply that (1) and (2) have the same truth value (reference).

Frege was aware of this problem and adapted his theory as follows. He postulated that in intensional contexts, created by phrases such as 'wonder whether', 'know that', and so on, expressions have an *indirect* (or *oblique*) *reference* and *sense* instead of their *direct* (or *ordinary*) *reference* and *sense*. The *indirect reference* of an expression is its ordinary sense and its indirect sense is something else. Consequently, the expressions 'Amsterdam' and 'the capital of the Netherlands' in the sentences (1) and (2) above have a different (indirect) reference, because both occur in the context 'wonders whether'. For that reason the principle of compositionality cannot be applied in order to derive that (1) and (2) would have the same reference (truth value).

The following schema gives a summary of Frege's theory of sense (Sinn) and reference (Bedeutung).

	proper names and singular definite descriptions	predicates	sentences
Examples	morning star; evening star; $2+2$; 4 the present king of France	has a heart has a liver	$2+2=4$; the morning star = the evening star. Scott wrote Waverly; Utah has 29 counties
Sinn sense		others: property, attribute, quality	ein Gedanke proposition, thought
Bedeutung reference	ein Gegenstand object	ein Begriff, concept extension of a predicate: class	truth value

'The morning star' and 'the evening star' express a different sense, but refer to the same reference (object). 'Has a heart' and 'has a liver' express a different sense (property), but refer to the same reference (class). 'Scott wrote Waverly' and 'Utah has 29 counties' express a different sense (proposition), but refer to the same reference (truth value).

'The present king of France' does have a sense, but does not have a reference. Hence, the sentence 'The present king of France is bald' does not have a reference (truth value).

As noted by J.R. Searle, although the distinction between sense and reference seems to be quite natural for names (proper names and singular definite descriptions), its extension to predicates and sentences is less compelling.

> To my mind it loses the most brilliant insight of the original distinction, an insight which reveals the connection between reference and truth: namely that an expression refers to an object only because it conveys something true of that object. But a predicate does not convey something true of a concept nor does a sentence convey something true of a truth value. (Searle, [47], p. 3)

Reading list on Frege: Carnap [9]; Church [10], Introduction; Dummett [13]; Frege [15]; Frege [16]; Heijenoort [23]; Searle [47]; Strawson [52].

7.3 Mannoury (1867-1956), Significs

> The language, which is used by all people as a means of understanding, is full of unclean elements that poison society, such as contaminated water poisons the population of a whole city. For that reason it is immediately needed to show that the water supply and the sources from which the city receives its drinking water, is contaminated by germs, and that it is most urgent to first purify these sources. [F. van Eeden in: Brouwer, L. E. J., F. Van Eeden, J. Van Ginneken en G. Mannoury, Signifische dialogen. 1939; translated from Dutch.]

Gerrit Mannoury's writings are likely to be enriching and thought-provoking for any student or scholar who takes a genuine interest in the phenomenon of language. This great Dutch thinker made many piercing remarks on the essential functions of language, on the nature of formalism, and on the connectedness of language-types that are generally considered incompatible. His views on meaning and the methods of describing it tended to be stated with refreshing and liberating relativism.

Mannoury was one of the founding members of the International Institute of Philosophy in Amsterdam (1917), which in many ways prepared the activities of the later Dutch Significic Circle (1922-1926) (for a history of Dutch significs from 1892 to 1926, see H.W. Schmitz [44]) and he remained the witful explainer and propagator of the significic ideas long after the circle had been dissolved (see, for example, Mannoury [34]).

Among the most prominent features of significic thought, and of Mannoury's thought in particular, was the idea of the *intentional* nature of language. This be-

comes apparent from the way in which Mannoury characterized *communicative acts*, of which *linguistic acts* form a subcategory:

> We shall call *communicative act* any act by which living beings (say human beings to simplify matters) try to influence directly the behavior or activity of other living beings. (G. Mannoury [33], p. 13.)

For Mannoury and his fellow significians, language was in the first place an *expression of the will* or, to quote L.E.J. Brouwer, another authority in the field:

> all utterances in words are more or less developed verbal imperatives, ... hence addressing always comes down to commanding or threatening, and understanding always comes down to obeying. (L.E.J. Brouwer [7], p. 333).

A language shared by a group serves to regulate and coordinate individual will or, to cite Brouwer again:

> to keep the movement of the Will of separate persons on one track. (L.E.J. Brouwer [6], p. 38).

The volitional function, which is primary from a signific point of view, can be illustrated particularly well through what Karl Bücher regarded as the historically primitive forms of poetry and music, namely the singing that accompanied manual labor. Wilhelm Wundt commented on this type of language use in his *Völkerpsychologie* (1900):

> Whenever several people join in the same work, the sounds which accompany the cadenced movements ... automatically bring about a pattern of co-operation which allows every participant to make the movements to the same rhythm. The resulting multiplication of rhythmic sounds increases the awakening of ardour. If in addition the shared labor is oriented towards one and the same object, such as in the case of the rowing of a boat or the joint hoisting or hauling of loads, the regular utterance of sounds again naturally becomes an expedient which rhythmically orders the singular powers synchronically or according to the sequence in which they mesh with one another. (Wundt [58] (included in the bibliography of G. Mannoury [34]), pp. 263-264.)

Here all the characteristics of what the significians considered to be the most original forms of linguistic usage are united: language which accompanies activity; language of people who focus their attention on one and the same object, or who pursue one and the same goal; language as mutual imposition of the will. In less primitive forms of language use than the ones mentioned above, other functions, such as the indicative or declarative ones, become more prominent, or are perceived as such. It was the merit of the significians to point to purposeful will and roots in human activity even there.

To Mannoury the meaning of any communicative act was composed of emotional or volitional elements on the one hand, and indicative or declarative ones on the other. The essential task of significs he held to be the disentangling and connecting of both types of elements (G. Mannoury [31], p. 113). He displayed great skill in uncovering the volitional aspect of utterances that seem purely indicative:

> Strictly speaking one cannot reasonably ask: 'is it true what you say?', but only: 'what do you want from me when you say this to me?', and 'can I agree with your goal?'. But of course such general remarks should always be taken 'cum grano salis', and it will be clear that nobody would be able to say what is really aimed at, or which kind of will is expressed in a sentence such as *there is a running horse*, but still I am sure that if someone all of a sudden made this important announcement to you, and you could not remotely guess what made him draw your attention to the movement of the Rosinante, you would be astonished and you would ask even without the slightest philosophical reflection: 'what do you mean, what are you getting at?', or, putting it more philosophically, 'what is the cause of your judgement?, which motive made you create this combination of thoughts?'. (G. Mannoury and D. Vuysje, archives, university library of Amsterdam; text of the lecture in file 14, p. 7; published version of the lecture: Mannoury [30]).

It is clear that finding dictionary-meanings of words or unalterable definitions of terms was not the main concern of the significians. Instead they were interested in the *use* of words in a particular context by specific people. The meaning of a communicative act was characterized as follows by Mannoury:

> the associations which link this act to the psychic complexes determined by the participants involved. (Mannoury [33], p. 13)

These participants are 'the speaker' and 'the listener' (in a very general sense, because communicative acts can also be wars, smiles and paintings).

Two main methods of empirical significic research into the meaning of linguistic acts were presented in Mannoury [34] (see also Schmitz [45]). The first, called *method of exhaustion*, consists of finding the range of situations to which a person reacts in the same verbal way; the second, termed *method of transformation*, aims at collecting the verbal reactions of various people to one particular situation (Mannoury [35], p. 44). These two methods are especially well adapted to the study of non-technical language.

However, the scope of significic analysis is by no means reduced to everyday language. As most of the significians were active in some other scientific field (mathematics, law, psychology, biology, ...) their significic writings displayed a marked interest in the communicative acts of science. Here too their main concern was detecting the emotional or volitional and delineating it from the indicative. Mannoury felt that every logical or physical formalism, and even purely mathematical communicative acts, encompassed an *empirical content* (on the level of indication) and an element of *belief* (on the level of emotion and volition). In the case of mathematics the empirical content of the theorems or demonstrations consists of the *knowledge of preceding formalisms* which speaker and listener share. The element of belief is to be identified with the *esthetic* or *sportive* aspect of mathematics, which is the key to its deepest truth (Mannoury [33], p. 46).

This should be understood as follows: Two mathematicians in the course of a discussion try to find the solution to a problem and join their efforts in a project of which words and signs on paper are only the external marks of progress. They develop, through corroboration, which equally gifted or equally trained people give each other when they strive for the same end, a feeling of *certainty* or *beauty*, which is nothing other than approached truth.

Mannoury distinguished between *active / speaking* mathematics and *passive / listening* mathematics and pointed to the tension between the two:

> It is the old song: speaking mathematics and listening mathematics are at loggerheads. ... Speaking mathematics searches, supposes, conjectures, guesses right or wrong, enjoys and suffers, gets dizzy and hits some nails, but listening mathematics remains calm and hides behind ready-made definitions and has logarithmic tables printed with typesetting plates. And does not want to know its mother any more! It has risen so high that it forgets where it came from ... (Mannoury [31], p. 31).

One of Mannoury's merits as a significian was that he showed with many examples that formalisms should not be considered in isolation, but need to be studied in relation to the intuitive insights from which they sprang, and to the purpose which they are supposed to serve.

According to the Dutch signific group it is possible to develop a classification of language-types (Mannoury ([33], pp. 19-20), a graded scheme, in which each type of language is situated on a particular level, and words or expressions of a higher type can be interpreted or replaced by words or expressions of a lower one, but never vice versa. In such a scheme the symbolic language of sciences displaying advanced formalization, for example mathematical logic, occupy the highest degree; primitive forms of language, in which the immediate expression of emotions prevails, belong to the lowest degree; and the language of daily social intercourse is situated somewhere in between.

The principle of *linguistic gradation* makes clear how even the most abstract systems of language (which by virtue of their rigidly regulated syntax or their constant 'word-word-associations' (Mannoury [36], p. 161), to use one of Mannoury's own terms, give the impression of complete independence and perfect self-sufficiency) remain anchored in the living language of emotion and intuition. Disrooted abstractions lead to the creation of false problems (for example, with regard to the void or the actual infinity; Mannoury [33], p. 53) and therefore have to be dismissed. Any language that loses contact with life should be shed off as a snake's skin of dead formula.

The significian has a role to play in this process of sloughing and renewal. A prerequisite for the success of his undertaking is a thorough understanding of the field to which the language in question applies. It is only through familiarity with the objects and approach roads that he is able to detect the flaws and imperfections of the existing means of expression. The significian will then break through the language, which is like a passive crust that is moulded to fit the terrain as discovered thus far. He will proceed to an active and synthetic refinement that matches the new needs and allows him and his fellow explorers to draw nearer to the objects that required redefining.

Progress through refinement of a language that is starting to flounder was exemplified by David van Dantzig, a second generation significian:

> Inasfar as progress of science consists of the discovery of new regularities of the *formal system*, the preceding formalization will be very useful, but it may be (even if one is willing to replace the old formalism by a new one) an impediment to the discovery of such new

> properties of the objects under investigation, which require finer distinctions ('fine structure') of relations hitherto regarded (and formalized!) as 'identical'. It is to a large extent by such 'finer distinctions' and broader generalizations that progress of science proceeds, as numerous examples show. *After* they have been made, formalization may become useful again. Formalization therefore covers a small part of science only, in particular a part which to a certain extent is 'ready' or 'closed' at the moment, and therefore formalism is running *behind* actual science (van Dantzig [11], p. 515, quoted in Mannoury [36], p. 120).

The careful technical readjustment of formal or other language in order to serve modifying goals belongs to the synthetic activities of the significian.

Mannoury also tackled a synthetic project of a more general kind. In [33] (Polar Psychological Synthesis of Concepts) he developed a *unifying terminology* that aimed at bridging the sharp distinction between the *mathematical* way of thinking (characterized by formalism and specific objects of consideration) and the *ideological* way of thinking (characterized by metaphor and general points of view). In doing so Mannoury remained true to his relativist position, which allowed for polar opposition (in which each pole needs its opposite) rather than separated categories, and he avoided the false problems that arise from dualism.

In this same light one should view Mannoury's scheme which distinguishes and combines two types of negation: a *negation of choice* and an *exclusive negation* (Mannoury [32], pp. 333-334). The *negation of choice* is used in contexts where two alternatives clearly present themselves to the speaker's mind (e.g. It is raining or it is *not* raining; if this is *not* a big town, it must be a small town; etc). The *exclusive negation* is based on a negative volition, a refusal without a clear alternative. In natural language the exclusive negation is often marked by words such as 'not ... at all', which indicate a stronger emotional involvement on the part of the speaker.

Mannoury shows that it is possible to combine these two negations, and gives the following example: 'What is not a small town could be a big town, but also something quite different' (ibidem). The small town/big town dichotomy is governed by the negation of choice. However, the words 'something quite different' illustrate the effects of the exclusive negation, namely drawing the attention away from the given alternatives (small town/big town) without proposing another possibility.

Double negation also plays a crucial role in intuitionistic mathematics and logic, as developed by L.E.J. Brouwer and his students. The question of inspiration and transmission of ideas between G. Mannoury and L.E.J. Brouwer, who were fellow significians, fellow mathematicians and intimate friends, is far from resolved (see Schmitz [43]). However, in this treatment of negation, as in many other cases, one is not surprised to find analogies in their thinking.

Both Brouwer and Mannoury seem to start from a language of dichotomy and clearly perceived entities. These are Mannoury's given alternatives or Brouwer's constructions in the mathematician's mind. Into this language they insert specific expressions involving two negations. These expressions hint at what extends beyond dichotomy and the clearly perceived. In other words, the inserts call up Mannoury's undefined alternatives or Brouwer's as yet unfulfilled goals of construction projects in the mathematician's mind.

The task of significs has been defined as showing the link between indication and volition/emotion, between what we think we have, and what we reach for, whenever

we communicate. Synthetic significs provides new language constructs that make explicit this connection. What Mannoury and the intuitionists do when combining and embedding the two negations, contributes to this explication. They make us see the link between the given and that what goes beyond, between specific objects of consideration and higher objectives, between mathematics and mysticism.

7.4 Speech Acts

According to J.L. Austin, any *speech act* comprises at least two, and typically three, sub-acts. These are what he calls the *locutionary*, the *illocutionary* and the *perlocutionary acts* involved in a total speech act.

The *locutionary act* includes the utterance of certain noises, the utterance of certain words in a certain construction, and the utterance of them with a certain meaning. *Locutionary acts* are acts of saying something and meaning it (and supplying a definite reference for any pronouns like 'this', 'he', etc.).

Most every time we say something and mean it – when we aren't just testing our voice, or acting in a play – we do in fact perform illocutionary acts.

Illocutionary acts are things we do *in* speaking like: requesting, welcoming, asking questions, demanding, inviting, giving orders, accusing, granting permission, asserting, promising, lying.

For more examples and a rough classification, see Austin [3] (*How to do things with words*, Lecture XII). Austin offers no precise definition of 'illocutionary act' (nor does anyone else for that matter), but one can pretty well agree how to extend the above list. The illocutionary act can be regarded as the force with which the sentence is employed. The distinction between locutionary and illocutionary acts recalls Frege's distinction between proposition (also called thought) and assertion (or question, command or whatever). As we have already had occasion to note in our discussion of Frege, the same proposition can be expressed in many different kinds of illocutionary acts:

Please come (request or invitation);
Will you come? (question);
You will come (prediction);

all having the same propositional contents.

Some illocutionary acts (greeting, resigning, condoling) do not involve expressing propositions.

If there is any distinction between locution and illocution it is this: while the meaning of what we say severely restricts the range of illocutionary acts we can be performing (e.g., 'Get in here this instant, you S.O.B.' cannot be a polite request; nor of course can it be a question, an assertion, a promise, etc.), it may not suffice to determine completely the illocutionary force of what we say (e.g., 'Come in here'

might, depending on the circumstances, be an invitation, a command, an official order, etc., 'I will come' might, depending on the context, be a promise, a statement of present intention, or a fatalistic prediction). Because the *conventional* meaning may not suffice to completely determine what illocutionary act is performed, it may happen that even when the hearer understands perfectly the meaning of speaker's words, there may be a gap between the speaker's *intentions* and how his utterance is *taken* by the hearer. (What is intended as a mere statement of present intention may be mistaken for a promise; what is intended as a polite request may be misinterpreted as a peremptory command.) The existence of a gap between locution and illocution means that the notion of illocutionary act belongs on the border between *semantics* (the theory of the meaning of words, what is conventional and common to all speakers independently of their particular circumstances) and *pragmatics* (the theory of the use of language by speakers taking into account not only the invariant meaning of the words but also aspects depending on the speaker's intentions and purposes in the particular speech situation).

Perlocutionary acts are things we can do *by* speaking like: persuading, perplexing, alarming, irritating, boring, convincing, deceiving, frightening.

In general it is possible to try and fail to perform a perlocutionary act (we can try to deceive someone but not succeed), whereas it hardly makes sense to speak of trying and failing in the case of an illocutionary act (like lying). Generally the illocutionary act is complete when we have spoken, so long as we have been understood, whereas the perlocutionary act requires our speech to have some kind of further *effect* on the hearer.

So, then 'I promise to come to dinner' will be the performance (1) of a locution – e.g., employing a certain grammatical construction, (2) of an illocution – that of making a promise, and (3) of a perlocution – e.g., cheering you up.

Rules. Illocutionary acts may be called a form of 'rule-governed behavior'. There are rules and procedures for how the act is to be performed, and rules saying what kind of further behavior on the part of the speaker and hearer is 'in order' once the act has been performed, and what kind of behavior is 'out of order'. (For example, a bigamist violates the procedural rules for getting married, one of which involves not being married already.) Breaking a promise, welcoming people and then treating them like unwanted intruders, etc., are violations of the rules about what is supposed to be done afterwards.

An important distinction between two kinds of rules has been made by J. Rawls and taken over by Searle and others: *regulative rules* prescribe how some form of behavior existing antecedently to and independently of the rules is to be carried out. Thus rules of table manners prescribe the manner in which people should eat, but they are going to eat anyhow whether or not anyone has thought up any rules of table manners. *Constitutive rules*, by contrast, create the very possibility of new forms of behavior which could not exist without the rules. The rules of a game like bridge or basketball constitute what it is to play bridge or basketball. Apart from the rules,

these games have no existence. The rules governing illocutionary acts belong in the *constitutive* category.

In [48] (*What is a Speech Act*) Searle distinguishes two (not necessarily separate) parts in a sentence used to perform an illocutionary act: the proposition-indicating element and the function-indicating device. The latter indicates the illocutionary act the speaker is performing in the utterance of the sentence. J.R. Searle gives the following two examples.

(1) I promise that I will come.
(2) I promise to come.

The function-indicating device and the proposition-indicating element are separate in (1), but not so in (2). As function-indicating devices Searle mentions, among others, word order, stress, punctuation and performative verbs such as 'apologize', 'warn', 'state', etc. See J.R. Searle [48], pp. 43-44.

7.5 Definite Descriptions

Both Russell (1872-1970) and Wittgenstein (1889-1951), for different sets of reasons, rejected Frege's distinction between sense and reference. In 'Russell's Rejection of Frege's Theory of Sense and Reference' J.R. Searle critically examines Russell's reasons for doing so.

Frege's analysis of a sentence like 'The king of France is bald' would be that this sentence lacks a truth value (reference), because the subject expression has no reference, but that the lack of a truth value does not render the sentence meaningless, since this sentence does have a sense. Now, how does Russell, having already rejected Frege's theory of sense and reference, explain how sentences like this one can be meaningful, while there is nothing for the proposition, expressed by the sentence, to be about. In *On Denoting* (1905) Russell claims that the sentence in question appears to be in subject-predicate form, but is not really so. Its grammatical form is misleading as to its logical form. Russell's analysis of

The king of France is bald

is as follows:
$\exists x[x \text{ is king of France } \wedge\ x \text{ is bald } \wedge\ \forall y[y \text{ is king of France} \rightarrow y = x]]$,
or equivalently, but shorter

$$\exists x[x \text{ is bald } \wedge\ \forall y[y \text{ is king of France} \rightleftarrows y = x]].$$

And since there is no king of France, this sentence is false.

Russell analyzed (say) 'The king of France is bald' as no simple subject-predicate statement but a far more complicated one, in which two different quantified variables occur. In Russell's theory, the deep structure of such statements is very different from what their surface grammar suggests.

So Russell does not give an explicit definition enabling one to replace a definite description by an equivalent wherever it appears, but a *contextual definition*, which

enables one to replace sentences containing definite descriptions by equivalent sentences not containing definite descriptions.

Russell used the following 'iota' -notation

$$\iota xA(x)\text{: the unique } x \text{ with property } A\text{, and}$$

$$C(\iota xA(x))\text{: the unique } x \text{ with property } A \text{ has property } C$$

as shorthand for

$$\exists x[A(x) \wedge C(x) \wedge \forall y[A(y) \rightarrow y = x]].$$

Where the condition C is complex, the iota notation is ambiguous. Russell's simple example is well known:

$$\neg B(\iota xF(x))\text{: The king of France is not bald.}$$

Here the ambiguity of the iota notation corresponds to an ambiguity in the English, between these two:

1. $\neg(B(\iota xF(x)))$, i.e., $\neg\exists x[F(x) \wedge B(x) \wedge \forall y[F(y) \rightarrow y = x]]$. There is no object x such that x is king of France and x is bald and x is the only king of France. And this happens to be true.
2. $(\neg B)(\iota xF(x))$, i.e., $\exists x[F(x) \wedge (\neg B)(x) \wedge \forall y[F(y) \rightarrow y = x]]$. There is some object x such that x is king of France and x is not bald and x is the only king of France. And this happens to be false; so we have $\neg((\neg B)(\iota xF(x)))$.

Note that this latter expression is not equivalent to $B(\iota xF(x))$, i.e., $\exists x[F(x) \wedge B(x) \wedge \forall y[F(y) \rightarrow y = x]]$ (the king of France is bald): $\neg((\neg B)(\iota xF(x)))$ is true, while $B(\iota xF(x))$ is false. In Russell's jargon, the definite description $\iota xF(x)$ has *narrow scope* in version 1 and *wide scope* in version 2.

A less confusing notation for definite descriptions would result by treating them as a kind of quantifier:

$$(Ix)(F(x), B(x)) \text{ instead of } B(\iota xF(x)).$$

Then the sentence in version 1, $\neg(B(\iota xF(x)))$, would be rendered by $\neg(Ix)(F(x), B(x))$, and the sentence in version 2, $(\neg B)(\iota xF(x))$, by $(Ix)(F(x), \neg B(x))$. While it was somewhat strange to have both,

$$\neg(B(\iota xF(x))) \text{ and } \neg((\neg B)(\iota xF(x)))$$

in the new notation this would become

$$\neg(Ix)(F(x), B(x)) \text{ and } \neg(Ix)(F(x), \neg B(x))$$

which looks similar to

$$\neg\forall x[A(x)] \text{ and } \neg\forall x[\neg A(x)]$$

which does not look like a contradiction at all.

7.6 Berry's and Grelling's Paradox

In Subsection 2.10.1 we discussed the antinomy of the liar. This paradox results from considering a sentence which says of itself that it is not true. By making a sharp distinction between object-language and meta-language we could avoid this paradox. In this subsection two other antinomies are presented, those of the librarian G.G. Berry and of Kurt Grelling (1908), which can be avoided in a similar way by making the distinction between language and meta-language. While the paradox of the Liar is on the level of sentences, Berry's paradox is on the level of names/definite descriptions and Grelling's antinomy is on the level of predicates.

Berry's Paradox Consider the following definite description: *The least natural number not specifiable in less than twenty-two syllables*. (*)

First of all we should verify that such a natural number exists. That this actually is the case follows from the following observations: (i) There are only finitely many (different) syllables. (ii) Consequently, there are only finitely many phrases of less than 22 syllables. (iii) There are infinitely many natural numbers.

From (ii) and (iii) it follows that there is a least natural number that is not specifiable in less than twenty-two syllables. However, counting the number of syllables in (*) we find that we have specified that particular number in 21 syllables. Therefore, here is *Berry's paradox*:

The least natural number not specifiable in less than twenty-two syllables is specifiable in 21 syllables.

In order to avoid this paradox, one should realize that the expression 'specifiable' does not have a clear meaning. It must be supposed that we are talking with reference to the resources of some particular language, say L_0. 'Specifiable in terms of (expressions of) L_0', abbreviated by 'specifiable$_0$', does have a clear meaning. However, the expression 'specifiable$_0$', which is short for 'specifiable in terms of L_0', does not belong to L_0 itself, but to the meta-language L_1 of L_0. Keeping this in mind, we easily see that Berry's paradox is the result of a very loose usage of words and of identifying object-language and meta-language. Expressing ourselves more precisely, what we have actually found is that

The least natural number not specifiable$_0$ in less than twenty-two syllables (of L_0) is specifiable$_1$ in 21 syllables (of L_1).

In its specification we have used the expression *specifiable$_0$* which does not belong to the object-language L_0, but to the meta-language L_1 of L_0. So, making a clear distinction between object-language and meta-language and expressing ourselves precisely, the paradox simply disappears.

We are perhaps not accustomed to thinking of a natural language such as English as a sequence English$_0$, English$_1$, English$_2$, ..., where for each natural number n, English$_{n+1}$ is a meta-language of English$_n$. However, the paradox of the Liar, Berry's paradox and others force us to conceive of English in such a way and after a while the distinction between object-language and meta-language seems to be self-evident.

That L_{n+1} is a meta-language of L_n means:
i) L_{n+1} contains L_n as a sublanguage ($L_n \subseteq L_{n+1}$), and
ii) L_{n+1} contains in addition means to talk **about** L_n.

Grelling's paradox Define the predicate 'autological' as 'being true of itself'. This predicate applies, for instance, to the adjectives 'short', 'English' and 'polysyllabic'. For example, the adjective 'short' is short and therefore, this adjective is autological.

Adjectives which are not autological are called heterological. The adjective 'long' is not long; the adjective 'German' is not German; and the adjective 'monosyllabic' is not monosyllabic. So, in Grelling's terminology, the adjectives 'long', 'German' and 'monosyllabic' are heterological.

Now consider the question whether the adjective 'heterological' is autological or not. If 'heterological' is autological, then it is true of itself, and hence it is heterological. Conversely, if 'heterological' is heterological, then it is true of itself and hence it is autological. So, this is *Grelling's paradox* (1908).

'heterological' is autological iff it is heterological (not autological).

This paradox is also the result of not making a sharp distinction between object-language and meta-language. Let 'true$_n$' belong to the language L_n. Then we can talk *about* expressions of language L_n, such as 'being not true$_n$ of itself (heterological$_n$)', in the meta-language L_{n+1} of L_n, but not in L_n itself. So, the question whether

$$\text{heterological}_n \text{ is autological}_n$$

does not make sense. What does make sense is the question whether

$$\text{heterological}_n \text{ is autological}_{n+1}. \qquad (*)$$

The answer to this question is no, since $(*)$ is equivalent to

$$\text{heterological}_n \text{ is a heterological}_n \text{ word}$$

which is meaningless.

We summarize this section in the schema below.

	proper names, definite descriptions	predicates	sentences
antinomy of	Berry (see this section)	Grelling (see this section)	the liar (see Subsection 2.10.1)
way out	distinction of object-language and meta-language: specifiable$_0$ specifiable$_1$ etc.	distinction of object-language and meta-language: true$_0$ of true$_1$ of etc.	distinction of object-language and meta-language: true$_0$ true$_1$ etc.
other way-outs			Namely-rider (see Subsection 2.10.1)

Another antinomy on the level of predicates is Russell's paradox (see Section 3.1). For further reading the reader is referred to Quine [39] and Kneale [27], Chapter XI.

Exercise 7.2. *Richard paradox, 1905* We may take the English alphabet as consisting of the blank space (to separate words), the 26 Latin letters, and the comma. By an 'expression' in the English language we may understand simply any finite sequence of these 28 symbols not beginning with a blank space. The expressions in the English language can then be enumerated by a simple device: first enumerate in alphabetical order all expressions of length 1, next all finitely many expressions of length 2, and so on.

Some English expressions, such as the expression 'the function which assigns to each natural number its square', define a number-theoretic function of one variable, i.e., a function $f : \mathbb{N} \to \mathbb{N}$. By striking out from the specified enumeration of all the expressions in the English language those which do not define a number-theoretic function, we obtain an enumeration, say $E_0, E_1, E_2, \ldots$, of those which do; say the functions defined are respectively $f_0, f_1, f_2, \ldots$. Now consider the function f defined by $f(n) = f_n(n) + 1$. This function f can be defined by an expression in the English language and hence should occur in the enumeration $f_0, f_1, f_2, \ldots$. (1)
On the other hand,

$f \neq f_1$ since $f(1) = f_1(1) + 1$,
$f \neq f_2$ since $f(2) = f_2(2) + 1$,
$f \neq f_3$ since $f(3) = f_3(3) + 1$,
and so on. Therefore, for all $i \in \mathbb{N}, f \neq f_i$. (2)

(1) and (2) are contradictory. Discover the flaw in the argument above.

7.7 The Theory of Direct Reference

According to the theory of *direct reference*, brought out by Keith Donnellan, Saul Kripke, Hilary Putnam and others, proper names ('Aristotle', 'Thales') and nouns standing for natural kinds ('gold', 'water', 'tiger') have no intension (Sinn) in the traditional sense, but only have reference; and this reference is established by a *causal chain* rather than by an associated description. For example, the reference to the person called 'Aristotle' is determined by a causal chain as follows. The person in question is given a name in a 'baptism' with the referent present. Next this name is handed on from speaker to speaker. It is in this way that we use the name 'Aristotle' referring to the person in question. We do not have to have any description of Aristotle; the information 'Aristotle was a philosopher' may be completely new to the one who is using the name 'Aristotle'.

There are at least two problems in the traditional theory of meaning:

1. In the traditional view, a proper name, like 'Jane', is identified with a description, such as 'the woman John is married to'. Now suppose that John is a bachelor. Then it would follow that Jane does not exist. This example makes clear that a

person can be referred to by his or her name even if the description of the person in question does not apply to that person.
2. According to the traditional theory, a tiger, for instance, is identified with an object which has certain properties, among which the property of having sharp teeth. Consequently, the statement 'tigers have sharp teeth' is analytic; this seems to be counter-intuitive.

In the traditional theory, the conjunction of properties which a tiger is supposed to have is called the intension of the word 'tiger' and is supposed to be the *essence* of tiger. In the traditional theory as well, intension determines extension. Similarly, in the traditional view, the proper name 'Aristotle' is identified with a description such as 'the most well-known man who studied under Plato'. As a consequence, the proposition 'Aristotle studied under Plato' would be an analytic truth. This is again against our intuition.

Typical of the theory of direct reference is the position, held by Kripke, Donnellan and others, that proper names and nouns standing for natural kinds refer independently of identifying descriptions.

In his paper [12] Donnellan distinguished between two kinds of use for definite descriptions – the *attributive*, and the *referential*. In order to make this distinction clear, Donnellan considered the use of the definite description 'Smith's Murderer' in the following two cases.

> Suppose first that we come upon poor Smith foully murdered. From the brutal manner of the killing and the fact that Smith was the most lovable person in the world, we might exclaim 'Smith's murderer is insane'. I will assume, to make it a simpler case, that in a quite ordinary sense we do not know who murdered Smith .... This, I shall say, is an *attributive use* of the definite description.

So, in the case of the attributive use, the speaker wants to say something about whoever or whatever fits the description even if he does not know who or what that is. On the other hand,

> Suppose that Jones has been charged with Smith's murder and has been placed on trial. Imagine that there is a discussion of Jones' odd behavior at his trial. We might sum up our impression of his behavior by saying 'Smith's murderer is insane'. If someone asks to whom we are referring by using this description, the answer here is 'Jones'. This, I shall say, is a *referential use* of the definite description. [K.S. Donnellan, [12], pp. 285-286.]

So, if the description 'Smith's murderer' is used referentially, the speaker is referring to Jones, even in the case that Jones turns out to be innocent. Note that in this case the description refers to Jones although it does not apply to Jones. To give another example, suppose someone asks me at a party who Mr. X is. I answer 'the man at the door with a glass of sherry in his hand'. Now suppose that the person referred to actually has a glass of white wine in his hand. Again the description may refer successfully without applying to the object referred to. These examples make clear that descriptions, when used referentially, do not always apply to the object they refer to. When using a description referentially, we have a definite object in mind whether or not it does fit the description.

It is typical of the theory of direct reference that proper names, like 'Jane', refer to some definite object, even when the description we supply, like 'the woman John is married to', does not apply to that object. This description may help us fix the reference, but it should not be taken to be the meaning of the name. And a similar view is held for nouns standing for natural kinds, like 'gold', 'water' and 'tiger'. The meaning of the word 'tiger' is its reference; identifying descriptions such as 'a tawny-coloured animal with sharp teeth, …' only help us to fix the reference of this term.

Summarizing, according to the theory of direct reference, the meaning of a proper name or a natural kind term is its reference; the descriptions given in connection with these terms only help the hearer to pick out what the speaker has in mind.

In his paper [28] Kripke in addition holds the view that a proper name, like 'Aristotle', is a *rigid designator*, i.e., it designates the very same object in all possible worlds in which this object exists. Thus, in the sentence 'Aristotle might have been a carpenter', the proper name 'Aristotle' refers to the same individual referred to in the sentence 'Aristotle was the philosopher who was a pupil of Plato and taught Alexander'. The definite description 'the most well-known man who studied under Plato', though it designates Aristotle in the actual world, may designate other individuals in other possible worlds; for it is possible that Aristotle did not study under Plato. Contrary to the traditional theory of meaning, according to the theory of direct reference, the statement 'Aristotle studied under Plato' is not necessarily true (and hence not analytic).

Now, if a and b are rigid designators and $a = b$ is true (in this world), then $\Box(a = b)$ is true, i.e., $a = b$ is true in all possible worlds accessible from this one (see Exercise 7.3). So it follows from the thesis that proper names are rigid designators that all true identity statements of the form $a = b$, where a and b are proper names, are necessarily true. In particular, it follows that 'Hesperus is Phosphorus (the Morning Star is the Evening Star)' and 'Tully is Cicero', if true (in this world) are necessarily true. On the other hand, we do not know a priori that Hesperus (the Morning Star) is Phosphorus (the Evening Star); this was discovered by empirical observation. Therefore Kripke claims in his paper [29] that sentences like 'Hesperus is Phosphorus' and 'Tully is Cicero' if true (in this world) are *necessarily true and* at the same time are *a posteriori*.

Kripke extends his insights about proper names to nouns standing for natural kinds, such as 'gold', 'water' and 'tiger'. These nouns are rigid designators too, i.e., they refer to the same substance in all possible worlds in which this substance exists. Let us consider some interesting consequences of this point of view.

'Gold' being a rigid designator, the sentence 'gold is the element with atomic number 79', if true (in this world), will be true in all worlds (accessible from this one) and hence be necessarily true. Similarly, 'water' being a rigid designator, the sentence 'water has the chemical structure H_2O', if true (in this world), will be true in any world (accessible from this one) and hence be necessarily true. So both propositions, if true (in this world), are *necessarily true and* at the same time *a posteriori*.

In Exercise 7.4 some examples are given of sentences which are *contingent*, i.e., not necessarily true, *and* at the same time *a priori*. Kripke defines a sentence A to be *analytic* if it is both necessary and a priori. Consequently, sentences like 'Hesperus is Phosphorus', 'Tully is Cicero', 'gold is the element with atomic number 79' and 'water is H_2O' are **not** analytic, since they are a posteriori, although necessarily true, if true (in this world).

Exercise 7.3. Suppose that 'a' and 'b' are rigid designators. Prove: if $a = b$ is true (in this world), then $\Box(a = b)$ is true. More precisely, for any Kripke model M and for any world w in M: if $M, w \models a = b$, then $M, w \models \Box(a = b)$.

Exercise 7.4. Regarding 'one meter' as a rigid designator, make clear that 'stick S is one meter long', where S is the standard meter in Paris, is a contingent and a priori truth. (See S.A. Kripke, [28] pp. 54-57.) Similarly, for 'water boils at hundred degrees Celsius', regarding '100 degree Celsius' as a rigid designator, and for 'I am here now'.

7.8 Analytic - Synthetic

In his *Critique of Pure Reason* (1781) Immanuel Kant [26] makes a distinction between analytic and synthetic judgments. Kant calls a judgment *analytic* if its predicate is contained (though covertly) in the subject, in other words, the predicate adds nothing to the conception of the subject. Kant gives 'All bodies are extended (Alle Körper sind ausgedehnt)' as an example of an analytic judgment; I need not go beyond the conception of *body* in order to find extension connected with it. If a judgment is not analytic, Kant calls it *synthetic*. So, a synthetic judgment adds to our conception of the subject a predicate which was not contained in it, and which no analysis could ever have discovered therein. Kant mentions 'All bodies are heavy (Alle Körper sind schwer)' as an example of a synthetic judgment.

Also in his *Critique of Pure Reason* Kant [26] makes a distinction between *a priori* knowledge and *a posteriori* knowledge. A priori knowledge is knowledge existing altogether independent of experience, while a posteriori knowledge is empirical knowledge, which has its sources in experience.

Sometimes one speaks of *logically necessary* truths instead of analytic truths and of *logically contingent* truths instead of synthetic truths, to be distinguished from physically necessary truths (truths which physically could not be otherwise, true in all physically possible worlds). The distinction between necessary and contingent truth is a *metaphysical* one. In her book [21], p. 170, S. Haack stresses that this distinction 'should be distinguished from the *epistemological* distinction between *a priori* and *a posteriori* truths'. Although these – the metaphysical and the epistemological – are certainly different distinctions, it is controversial whether they coincide in extension, that is, whether all and only necessary truths are *a priori* and all and only contingent truths are *a posteriori*.

In his *Critique of Pure Reason* Kant stresses that mathematical judgments are both a priori and synthetic. 'Proper mathematical propositions are always judgments *a priori*, and not empirical, because they carry along with them the conception of necessity, which cannot be given by experience.' Why are mathematical judgments synthetic? Kant considers the proposition $7+5=12$ as an example. 'The conception of twelve is by no means obtained by merely cogitating the union of seven and five; and we may analyse our conception of such a possible sum as long as we will, still we shall never discover in it the notion of twelve.' We must go beyond this conception of $7+5$ and have recourse to an intuition which corresponds to counting using our fingers: first take seven fingers, next five fingers extra, and then by starting to count right from the beginning we arrive at the number twelve.

7:	1	1	1	1	1	1	1					
5:								1	1	1	1	1
7 + 5:	1	1	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10	11	12

'Arithmetical propositions are therefore always synthetic, of which we may become more clearly convinced by trying large numbers'. Geometrical propositions are also synthetic. As an example Kant gives 'A straight line between two points is the shortest', and explains 'For my conception of *straight* contains no notion of *quantity*, but is merely *qualitative*. The conception of the *shortest* is therefore wholly an addition, and by no analysis can it be extracted from our conception of a straight line'.

In more modern terminology, following roughly a 'Fregean' account of analyticity, one would define a proposition A to be *analytic* iff either

(i) A is an instance of a logically valid formula; e.g., 'No unmarried man is married' has the logical form $\neg\exists x[\neg P(x) \wedge P(x)]$, which is a valid formula, or

(ii) A is reducible to an instance of a logically valid formula by substitution of synonyms for synonyms; e.g., 'No bachelor is married'.

In his *Two dogmas of empiricism* W.V. Quine [40] is sceptical of the analytic/synthetic distinction. Quine argues as follows. In order to define the notion of analyticity we used the notion of synonymy in clause (ii) above. However, if one tries to explain this latter notion, one has to take recourse to other notions which directly or indirectly will have to be explained in terms of analyticity.

7.9 Logicism

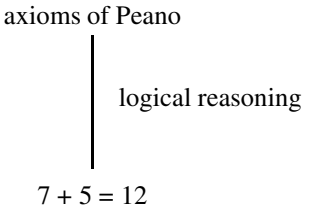
Logicism dates from about 1900, its most important representatives being G. Frege [17] in his *Grundgesetze der Arithmetik* I, II (1893, 1903) and B. Russell [41] in his *Principia Mathematica* (1903), together with A.N. Whitehead. The program of the logicists was to reduce mathematics to logic. What do they mean by this? In his Grundgesetze der Arithmetik Frege defines the natural numbers in terms of sets as follows: 1 := the class of all sets having one element, 2 := the class of all sets having two elements, and so on. Next Frege shows that all kinds of properties of natural

numbers can be logically deduced from a *naive comprehension principle*: if $A(x)$ is a property of an object x, then there exists a set $\{ x \mid A(x)\}$ which contains precisely all objects x which have property A (see Section 3.1).

Logicism tried to introduce mathematical notions by means of explicit definitions; mathematical truths would then be logical consequences of these definitions. Mathematical propositions would then be reducible to logical propositions and hence mathematical truths would be analytical, contrary to what Kant said.

The greatest achievement of Logicism is that it succeeded in reducing great parts of mathematics to one single (formal) system, namely, set theory. The logicists believed that by doing this they reduced all of mathematics to logic without making use of any non-logical assumptions, hence showing that mathematical truths are analytic. However, they mistakenly held the naive comprehension principle for a logical axiom instead of a mathematical or set theoretical principle. So, what they actually did was reduce mathematics to logic **plus** set theory. And the axioms of set theory have a non-logical status! The axioms of set theory are – in Kant's terminology – synthetic, and surely not analytic. In his later years Frege came to realize that the axioms of set theory (see Chapter 3) are not a part of logic and gave up Logicism, which he had founded himself. The interested reader is referred to K. Gödel [19], *Russell's mathematical logic*.

Another way to see that a mathematical truth like $7+5=12$ is synthetic is to realize that $7+5=12$ is not a logically valid formula; it is true under the intended interpretation, but not true under all possible interpretations: if one interprets the + symbol as negation, the formula 5 + 7 = 12 yields a false proposition. $7+5=12$ can be logically deduced from the axioms of Peano for (formal) number theory (see Chapter 5), but it cannot be proved by the axioms and rules of formal logic alone.



Again, Peano's axioms are true under the intended interpretation, but are not (logically) valid and hence they do not belong to logic.

7.10 Logical Positivism

It is an old problem to draw the line between scientifically meaningful and meaningless statements. Consider the following quotation, taken from Hume's *Enquiry Concerning Human Understanding*:

> When we run over libraries, persuaded of these principles, what havoc must we make? If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, *Does it contain any abstract reasoning concerning quantity of number*? No. *Does it contain*

> *any experimental reasoning concerning matter of fact and existence*? No. Commit it then to the flames: for it can contain nothing but sophistry and illusion [David Hume, 1711-1776].

As we learn from A.J. Ayer [2], the quotation above is a good formulation of the positivist's position. In the 1930's the adjective *logical* was added, resulting in the term *Logical Positivism*, which underscored the successes of modern logic and the expectation that the new logical discoveries would be very fruitful for philosophy.

This logical positivism was typical of the *Vienna Circle*, a group of philosophers (among them Moritz Schlick, Rudolf Carnap and Otto Neurath), scientists and mathematicians (among them Karl Menger and Kurt Gödel). According to A.J. Ayer [2], Einstein, Russell and Wittgenstein had a clear kinship to the Vienna Circle and had a great influence upon it.

In order to draw a sharp distinction between scientifically meaningful statements and scientifically meaningless statements the *verification principle* was formulated: only those statements are scientifically meaningful which can be verified in principle; in other words, the meaning of a proposition is its method of verification. However, a proposition like 'all ravens are black', which has as logical form $\forall x[R(x) \rightarrow B(x)]$, cannot be verified due to the universal quantifier, $\forall$; at the same time we consider this proposition to be (scientifically) meaningful.

However, the proposition 'all ravens are black' can be conclusively falsified, since its negation 'not all ravens are black', being of the form $\neg\forall x[R(x) \rightarrow B(x)]$, is logically equivalent to 'some raven is not black', which has the logical form $\exists x[R(x) \wedge \neg B(x)]$, and hence can be verified. For this reason the *falsification principle* was formulated: only those statements are scientifically meaningful which can be falsified in principle. This principle seems to be more in conformity with scientific practice: hypotheses are set up and rejected as soon as experimental results force us to do so.

However, Otto Neurath himself soon realized that a slightly more complex proposition, like 'all men are mortal', which has the logical form $\forall x\exists y[P(x,y)]$ (for every person there is a moment of time such that ...), can neither be verified (due to the universal quantifier $\forall x$) nor falsified (due to the existential quantifier $\exists y$), since its negation 'not all men are mortal', being of the form $\neg\forall x\exists y[P(x,y)]$, is equivalent to 'some men are immortal', which has the logical form $\exists x\forall y[\neg P(x,y)]$, and hence – due to the universal quantifier $\forall y$ – cannot be verified. Falsification of $\forall x\exists y[P(x,y)]$ is equivalent to verification of $\neg\forall x\exists y[P(x,y)]$, i.e., verification of $\exists x\forall y[\neg P(x,y)]$, which is not possible in principle due to the universal quantifier $\forall y$. At the same time we want to consider a statement like 'all men are mortal' as (scientifically) meaningful. Therefore, we have to give up not only the verification principle, but also the falsification principle. This was already realized by Otto Neurath during his stay (1938-39) in the Netherlands (oral communication by Johan J. de Iongh).

Instead of the verification or falsification principle, a weaker criterion was formulated, called the *confirmation principle*: a statement is scientifically meaningful if and only if it is to some degree possible to confirm or disconfirm it. One way to confirm (increase the degree of credibility of) universal generalizations like 'all ravens

are black' is to find things that are both ravens and black, and one way to disconfirm this proposition is to find things that are ravens but not black. The problem with this confirmation principle is that 'all ravens are black', $\forall x[R(x) \rightarrow B(x)]$, is logically equivalent to 'all non-black things are non-ravens', $\forall x[\neg B(x) \rightarrow \neg R(x)]$, and according to the confirmation principle, the latter proposition is confirmed by observations of non-black non-ravens; thus observations of brown shoes, white chalk, etc., would confirm the proposition 'all ravens are black'. Various attempts have been made to give the verification principle, in this weaker form, a precise expression, but the results have not been altogether satisfactory. For instance, a solution might be found by replacing the material implication $\rightarrow$ in $\forall x[R(x) \rightarrow B(x)]$ by the counterfactual implication $\Box\!\!\rightarrow$ (see Section 6.9), for $\forall x[A(x) \Box\!\!\rightarrow B(x)]$ is not logically equivalent to $\forall x[\neg B(x) \Box\!\!\rightarrow \neg A(x)]$.

7.11 Presuppositions

Let us start this subsection with a quotation from Frege, *Ueber Sinn und Bedeutung*.

> If anything is asserted there is always an obvious presupposition that the simple or compound proper names used have reference. If therefore one asserts 'Kepler died in misery', there is a presupposition that the name 'Kepler' designates something; but it does not follow that the sense of the sentence 'Kepler died in misery' contains the thought that the name 'Kepler' designates something. If this were the case, the negation would have to run not 'Kepler did not die in misery' but 'Kepler did not die in misery, or the name 'Kepler' has no reference'. That the name 'Kepler' designates something is just as much a presupposition for the assertion 'Kepler died in misery' as for the contrary assertion. [G. Frege, 1892, in P. Geach and M. Black [18]].

Thus, according to Frege, the sentences

(1) Kepler died in misery, and

(2) Kepler did not die in misery

both presuppose that the name 'Kepler' has a reference. This presupposition is not part of the meaning of (1) or (2) respectively, since in that case the negation of (1) would not be (2), but

(*) Kepler did not die in misery, or the name 'Kepler' has no reference.

If the presupposition is not satisfied, the speech act of asserting in (1) and (2) cannot be performed successfully.

As we have already seen in Section 7.5 on definite descriptions, Russell places the presupposition(s) of a sentence into an existentially quantified conjunction and by doing so he makes the presupposition part of the meaning of the sentence. For example, the sentence

(1a) The king of France is bald

presupposes that

(1b) There is a king of France,

but Russell translates (1a) by the expression $\exists x[x$ is king of France $\wedge\ x$ is bald $\wedge\ x$ is the only king of France], hence making the presupposition part of what is asserted.

As we have seen above, sentences containing proper names and sentences containing definite descriptions in subject position carry or induce presuppositions. R. van der Sandt gives in his book [42], on which this section is based, the following examples in which the (a)-sentences presuppose the corresponding (b)-sentences.

Quantifiers (all, a few, at least one, ...)
(2a) All John's children are asleep.
(2b) John has children.

Aspectual verbs (begin, stop)
(3a) Charles has stopped smoking.
(3b) Charles used to smoke.

Presuppositional adverbs (only, even, also, ...)
(4a) Only John voted for Harry.
(4b) John voted for Harry.

Contrastive stress
(5a) The *butcher* killed the goose.
(5b) Someone killed the goose.

Factive verbs (realize, regret, discover, ...)
(6a) Tom regrets that the goose has been killed.
(6b) The goose has been killed.

Cleft constructions
(7a) It was John who caught the thief.
(7b) Someone caught the thief.

It is widely accepted that presuppositions are characterized by the following three tests.

1. The *negation test*: presuppositions are preserved when the original sentence is embedded under negation. The negation of each (a)-sentence above still presupposes the corresponding (b)-sentence. For instance, the negation of (3a), 'Charles has not stopped smoking', still presupposes (3b), 'Charles used to smoke'.
2. The *modality test*: presuppositions are preserved when the original sentence is embedded under a possibility operator. For instance, 'It is possible that Charles has stopped smoking' still presupposes (3b), 'Charles used to smoke'.
3. The *antecedent test*: presuppositions are preserved when the original sentence is taken as the antecedent of a conditional statement. For instance, 'If Charles stopped smoking, his wife would be happy' still presupposes (3b) 'Charles used to smoke'.

(8) Charles managed to leave the country
entails
(9) Charles left the country.
But (8) does not presuppose (9), since (9) is not preserved in the application of the tests mentioned above.
(8i) Charles did not manage to leave the country,

(8ii) Perhaps Charles managed to leave the country, and
(8iii) If Charles has managed to leave the country, then he will never come back,
do not suggest that (9) is true.

The verb 'manage' is called *implicative*, i.e., each sentence containing this verb entails the complement of that verb.

Next, consider sentence
(10) Charles was glad that he had left the country.
Applying the negation, modality and antecedent test to (10) teaches us that (9) is a presupposition of (10). These examples show (or at least suggest) that the tests mentioned above eliminate entailments, but preserve presuppositions. ([42], 2.3.)

When a presupposition is induced by a positive or negative *polarity element*, the application of the negation test is problematic.

Negative polarity elements, such as *at all, ever, anymore, matter that, mind that*. In order to form a grammatically correct expression, these elements are accompanied by a negation. When a presupposition is induced by a negative polarity element, the negation test cannot be applied, since the original sentence has no grammatical non-negated counterpart. The following examples are from van der Sandt [42].
(11a) Dick does not mind that his theory is wrong
presupposes
(11b) Dick's theory is wrong.
And
(12a) It does not matter that John was fired
presupposes
(12b) John was fired.
The presupposition-inducing elements *mind that* and *matter that* are negative polarity elements; so the non-negative versions of (11a) and (12a) are not grammatical. Thus the negation test cannot be applied in these cases. However, the modality test and the antecedent test can be applied successfully to (11a) and (12a). For this reason (11b) and (12b) are considered to be presuppositions of (11a) and (12a) respectively.

Positive polarity elements, such as *still, plenty of, perhaps, a lot, certainly, swarm with, be delighted*. When a presupposition is induced by a positive polarity element, the negation test may yield the wrong results since the negated sentence can evoke some kind of echo-effect. By this we mean that the negated sentence may suggest that the speaker rejects the original sentence, because he does not accept its presupposition. Consequently, when the original sentence is embedded under negation, the presupposition may get lost. Consider the following examples, again from [42].
(13a) John still believes in David's theory
presupposes
(13b) John believed in David's theory until recently.
And
(14a) Dick is delighted that his book is published
presupposes
(14b) Dick's book is published.

The presupposition inducing elements *still* and *be delighted* are positive polarity elements. Now, consider the negations of (13a) and (14a) respectively:

John does not still believe in David's theory; (he never believed in it).

Dick is not delighted that his book is published; (his book is not published).

On their most natural reading, the negations of (13a) and (14a) evoke the echo-effect that the speaker rejects the original sentence, not accepting its presupposition. So, application of the negation test would yield a negative result. On the other hand, if we apply the modality test and the antecedent test to (13a) and (14a) the truth of (13b) and (14b) is preserved. For this reason (13b) and (14b) are considered to be presuppositions of (13a) and (14a) respectively.

The negation, modality and antecedent test enable us to determine the elementary presuppositions of simple sentences. These elementary presuppositions are induced directly by certain lexical elements or syntactic constructions. The *projection problem* is the problem of finding out whether there is an algorithm – and if so, which one – which determines the presuppositions of a complex sentence on the basis of the presuppositions of its components.

In this connection it is important to realize that *presupposing is* **not** a binary relation between sentences and propositions, but *a ternary relation between sentences, propositions and contexts*. Whether a sentence presupposes a certain proposition or not may depend on the context! This can be illustrated by the following two examples, again taken from [42].

(15a) John will regret that there is a bouncer at the party

presupposes

(15b) There will be a bouncer at the party.

In the context that Charles is a competent bouncer and that problems are expected to arise, in sentence

(16) If Charles comes to the party, John will not regret that there is a bouncer at the party

presupposition (15b) is lost. In this context (16) does not presuppose (15b). However, in the context in which John gives a party and Charles, a notorious brawler, is one of the potential guests, the elementary presupposition (15b) is preserved. In such a context (16) does presuppose (15b).

(17a) Peter drinks too

presupposes

(17b) Someone other than Peter drinks.

Without any specification of a context, more precisely, in an empty context

(18) If Peter drinks too, the bottle is empty

preserves the presupposition (17b). However, in the context 'If John drinks, he drinks at least half a bottle', (18) does not presuppose (17b).

As is clear from these examples, in order to determine the presuppositions of a sentence one has to take into account not only its elementary presuppositions and its mode of composition, but also the relevant contextual information.

Exercise 7.5. The counterpart of the antecedent test for identifying elementary presuppositions would be the 'succedent test'. However, make clear that an elementary

presupposition that is induced by the succedent need not be a presupposition of the entire sentence. Hint: consider the sentence 'If the king of France exists, then the king of France is bald'.

Exercise 7.6. Inspired by the negation test, 'A presupposes B' has been defined as: $A \models B$ and $\neg A \models B$. Make clear that this definition does not make sense. Hint: use the fact that $\models A \vee \neg A$.

7.12 Wittgenstein on meaning

> My *whole* task consists in explaining the nature of proposition. (Wittgenstein)

Wittgenstein (1889-1951) is probably unique in the history of philosophy having earned the name of being the author of two completely different, original and highly influential systems of philosophy. His early philosophy, represented in the *Tractatus* (1922) [55], and his later philosophy, represented mainly in the *Philosophical Investigations* [56] (1952), are regarded as two major classics in the philosophy of language. The present survey is chiefly concerned with Wittgenstein's later philosophy of language. However, reference to his earlier philosophy of language will be necessary since his later philosophy, though different from the earlier one, can never be viewed as separate from it. There is an underlying continuity between the two systems that unites them. This is his critique of language or the problem of meaning, a theme with which Wittgenstein was preoccupied in all his philosophical writings. Wittgenstein's remark quoted above 'My *whole* task consists in explaining the nature of proposition' (*Notebooks* 1914-1916 [54], p. 39) characterizes this central concern and also indicates the theme that dominates the different phases of his philosophical thought. To explain the nature of proposition is to explain the nature and function of language. That is why Wittgenstein was interested in questions like: what makes it possible to say something?; how can words in combination signify something?; how can the sense of an expression be communicated to others?. But the difference lies in his approach to these questions. The questions are the same, but the answers to these questions are different in the two phases of Wittgenstein's philosophy of language. The later Wittgenstein was not convinced of the answers he offered to these questions in the Tractatus period. As a result, there was a new set of answers offering a new understanding of the nature and function of language.

To put the difference in perspective, in the earlier period a proposition says something because it is a picture or model of reality. The reason for its being a picture of reality is that it has an isomorphic relation to reality. Further, it is held that the sense of a sentence is determined by its truth conditions. But when we come to the later period, this perspective is changed. A sentence is no longer thought to be the picture of reality nor its meaning to be determined by its truth conditions. A sentence, on the other hand, is compared with a tool to be used to perform various functions including that of describing reality. Its meaning is determined by the rule or the convention associated with it.

One of the most important thrusts of the earlier approach was to show that ultimately a proposition consists only of names and that every name stands for an object. Elementary propositions are thus placed at the very bottom of the system of the Tractatus and all other compound (molecular) propositions are truth-functionally related to elementary propositions. For an elementary proposition to be true, its pictorial form must be isomorphic to the state of affairs it represents. In other words, its structure must mirror the actual structure of the state of affairs. Failure to do so makes the proposition false (though the proposition will be meaningful because it represents a possible state of affairs). According to what has been called the *picture theory of meaning*, the sense of an elementary sentence is determined by its being true or false with respect to the reality of which it is a picture or model. The same holds true of molecular propositions since they are truth-functionally connected with elementary propositions. The truth or falsity of a molecular proposition is dependent on the truth or falsity of the elementary propositions. Similarly, the meaning of a molecular sentence is determined by the specific truth conditions which are related to the elementary propositions. The basic presupposition underlying all these theoretical moves is that the main function of language is to depict reality.

In Wittgenstein's later philosophy we are confronted with a different story regarding how linguistic expressions get their meaning. The aim of the analysis was not to arrive at elementary propositions conceived as the essence of language. The entire depth-grammar approach to language was abandoned. For the later Wittgenstein, language is ordinary language. The question of reducing it to something more basic, such as elementary propositions, does not arise. Language has to be described and understood the way it is found. Its function is not just to describe reality or to picture facts. It has varied functions to perform – it has multiple uses. As Wittgenstein said:

> It is wrong to say that in philosophy we consider an ideal language as opposed to our ordinary one. For this makes it appear as though we thought we could improve an ordinary language. But ordinary language is all right. [*The Blue and Brown Books* [57], p. 28]

The expression, 'ordinary language is all right' needs clarification. Wittgenstein does not mean that ordinary language is free of problems. It is all right in so far as it is used in the way it ought to be used. But very often it is found that language has not been used correctly. These are cases of misuse which give rise to a number of problems, including philosophical problems. Wittgenstein argued that many of the traditional disputes in philosophy arose precisely because of the misuse of language. Such disputes, therefore, do not have any real basis. What is required is a correct diagnosis which will reveal the pseudo nature of the philosophical problems arising due to the misuse of language. The need to reform ordinary language is ruled out since ordinary language is very rich in its content and also because it contains all the nuances regarding a particular concept. Later, Wittgenstein's entire approach to language took a radical turn, giving utmost importance to the notion of use. Language is to be understood not in terms of any predesigned fixed model but in the way it is used, i.e., the way it is used to perform various functions. This has far reaching consequences for semantics. The meaning of an expression is determined by its use and not by its truth conditions.

It may be of interest to mention the historical event that profoundly influenced Wittgenstein in this new direction of thinking. On March 10, 1928 in Vienna, Wittgenstein along with Herbert Feigl and Friedrich Waismann attended a lecture given by L.E.J. Brouwer on 'Mathematics, Science, and Language'. (See Hacker, *Insight and Illusion* [22], p. 120.) Feigl noted that this lecture made a tremendous impact on Wittgenstein and he was found to be visibly disturbed after the lecture was over. The theme of Brouwer's lecture was closer to the Kantian tradition of epistemology. Following Kant, Brouwer defends the constructive function of the human mind which provides the structure for organizing the data of experience. Both mathematics and language are examples of these constructive activities of the mind. Accordingly, as the argument goes, mathematical truths are not something to be discovered; they are to be invented. There are no independent, eternal truths that mathematics discloses. The *Tractatus* was not conceived in this constructivistic line of thinking. On the contrary, the essential thrust of the *Tractatus* was always realistic. The greatest evidence of this was found in the conception of logic and mathematics by the early Wittgenstein. Logic discloses the necessary structure which is inherent in all possible states of affairs and similarly mathematics, conceived as a set of tautologies, discloses the necessities inherent in the structure of reality. Finally, Wittgenstein comes to the determination of sense. The sense of all sentences must be determinate so that they can correspond with the objects of the world. Further, for the determination of sense, a well formulated language is required in which they can be completely articulated. The constructivistic theme of Brouwer's lecture, as Feigl reported, appealed to Wittgenstein so much that he thought of coming back to philosophy with a new approach and a new orientation to language. The result was the post-*Tractatus* writings of Wittgenstein upholding the constructivistic and conventionalistic view of language and meaning. Meaning of an expression does not correspond to any independent structure of reality. It is, on the other hand, determined by rules of use or conventions that people devise and adopt. With these introductory remarks explaining the transition of Wittgenstein from his early phase to his later phase, we now directly come to Wittgenstein's later view on meaning. In this attempt we shall be mainly concerned with the explication of the view that meaning is to be understood as use. This *'meaning as use'* view assumes a long chain of involved and interconnected arguments. In our presentation we shall follow an order which at the end will establish why meaning is to be understood as use.

Wittgenstein's revolt against essentialism was probably the first step towards his new approach to language and meaning. *Essentialism* is a view which says that there are common, uniform and essential properties. Acceptance of such essential properties becomes necessary, otherwise there cannot be any proper understanding of any thing. Accordingly, the search for common essences was a dominant theme in Western philosophy. Belief in essences was epitomized in Plato's doctrine of ideas. A similar tendency was found in Russell's attempt to discover the ultimate constituent of matter. The *Tractatus* is not an exception to this. Essentialism is ingrained in the very texture of the philosophical thinking of the *Tractatus*. One of the major goals of the *Tractatus* is to determine the limits of language which implies drawing the boundary that will separate sense from nonsense or what can be said from what can-

not be said. In this task of separating the two, logic plays an important role. The task of logic is to make the distinction between sense and nonsense in such a way that the distinction becomes universal, necessary and a priori. In his search for universal essences Wittgenstein further talked about the general form of propositions and the essence of language. Wittgenstein tried to establish the essence of language by going back to the core from which all the propositions of language except those core propositions follow. These are elementary propositions and all other propositions are truth functions of these propositions. As stated earlier, elementary propositions consist of names, each of which designates a simple object in the world. Further, the configuration of names in a proposition depicts a state of affairs. This is, in brief, Wittgenstein's idea of the essence of all languages and also the essence of the relation between language and the world.

Later, Wittgenstein rejected this search for essences or 'craving for generality' as he called it. For him this whole search is illusory. But why do we look for such essences? Wittgenstein analysed it and offered some reasons for this craving for generality. In the following we shall present some of the reasons which will help us to understand why the search for essences is illusory.

There is a peculiar tendency in us that we always look for something common. We bring the particulars under a general term, for example, all houses are brought under the general term 'house', all tables are brought under the general term 'table' and so on. We believe that these general terms are meant to express the common properties which reside in the relevant particulars. But the falsity of this entire move is evident if we analyse what is meant by a common property as expressed by a general term. Wittgenstein (*The Blue and Brown Books* [57], pp. 17 to 20) explained this with reference to the analogy of games. In accordance with our belief in essentialism, we think that there must be something common to all games and the general term 'game' is meant to express this common property. But Wittgenstein says that the analysis of the term 'game' shows that it does not stand for any common property. The reason is that different games form a family in the sense that members 'exhibit family likeness', for example, 'some of them have the same nose, others the same eyebrows and others again the same way of walking'. What is important here is to note that these similarities or likenesses overlap, but by no means do they convey a general property. In other words, the overlapping likenesses cannot be mistaken for a general property.

Second, as Wittgenstein pointed out, there is a tendency to think that a man who has learnt a general term, say 'leaf', has come to possess a general picture of a leaf. This general picture is over and above the particular pictures of leaves. To have a general picture of leaf is to have a 'visual image' which 'contains what is common to all leaves'. Thus the meaning of a word is associated with an image which is correlated with the word. This is how essentialism arises and it gives rise to a questionable notion of meaning.

The next important source of our craving for generality is our preoccupation with the method of science. Philosophers are always influenced by the method of natural science which seeks to discover essences beneath the multiplicities. Natural phenomena are thus explained with reference to some universal laws. In this way the

method of natural science reduces multiplicities to some general patterns or regularities. Influenced by this method, philosophers have made similar attempts and offered generalisations in the same way as it is done in the sciences. Metaphysics is the prime example of this or as Wittgenstein put it 'this tendency is the real source of metaphysics' (*ibid*).

Finally, as Wittgenstein holds, this craving for generality also has its source in our 'contemptuous attitude towards the particular case' (*ibid*). We are averse to studying particular cases.

It has already been mentioned that for Wittgenstein the whole search for essences is illusory since there is nothing to be called a common property as has been shown in connection with the analysis of game. This calls for a drastic change in our philosophical method. To put it in concrete terms, Wittgenstein's suggestion is that our aim should be to study individual cases and while studying them we should take note of both their similarities and differences. A study of this sort will make us realise that there is no property common to all these cases. Instead of commonness, we will find that there are overlapping similarities on the basis of which we use general terms. As mentioned earlier, Wittgenstein compared this situation with a human family. The members of a family may resemble each other, but it is not the case that one particular characteristic is necessarily shared by all. What in traditional philosophy is taken as a common property expressing essence is in reality what Wittgenstein calls family resemblance (*Philosophical Investigations* [56], sec. 67). The same holds true of a game. The term 'game' covers the multitude of cases where all of them are found to have family resemblances. To put it in Wittgenstein's words:

> ... we see a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities, sometimes similarities of detail (*ibid*, sec. 66).

This is how Wittgenstein moved from essentialism to the notion of family resemblance which provided the basis of his theory of language and meaning understood as use.

In the light of the above considerations, Wittgenstein made an analysis of language. Wittgenstein argued that we often make a common mistake by trying to discover certain fixed essences in language. According to St. Augustine's theory of language, the primary function of language is to use names to refer to entities or objects. Wittgenstein had a basic objection to any view which tries to define language in essentialistic terms. His objection is that language cannot be defined as having one single homogeneous nature. Language has a complex nature with an enormously complex variety of uses. Accordingly, to suggest that there is a certain use of language which is more fundamental than the other uses is wrong. There is no fundamental use of language. On the contrary, any attempt to do so will be a falsification of what language is.

That language has a variety of uses was explained by Wittgenstein with the help of a number of interesting and revealing analogies. To cite some of them: words in a language have been compared with tools in a tool box (*Philosophical Investigations* [56], Section 11). In a tool box there are different kinds of instruments, e.g., ham-

mers, pliers, screwdrivers, etc. All of them have to perform different functions. In a similar way 'functions of words are as diverse as the functions of these objects'. Again, language has been compared with the inside of a cabin of a locomotive (*ibid*, Section 12). If you look into its cabin 'we see handles all looking more or less alike'. But each handle has its own function to perform. These analogies show why there cannot be any fundamental use of language. Language has a multiplicity of uses. A view which holds that all words are names for objects is wrong from the point of view of the actual functioning of language. To restrict language only to one such function will be like saying that the only role of money is to buy objects. But this is not true since money has many other uses (*ibid*, Section 120).

Wittgenstein further holds that many of the traditional disputes in philosophy arise because of this mistaken view that all words function like a proper name. As a result, philosophers have always been preoccupied with the task of showing what is designated by such abstract terms as 'mind', 'time', 'proposition', etc. That is why they ask questions like: 'What is time?', 'What is mind?', 'What is meaning?', 'What is length?', etc. These questions demand that corresponding to the terms expressed in these questions there must be objects existing outside. But at the same time we feel that there is nothing which we can point out as objects corresponding to these terms. Yet, there is pressure to say that 'we ought to point to something' (*The Blue and Brown Books* [57], p. 1). This produces in us a 'mental cramp' which becomes 'the great source of philosophical bewilderment' (*ibid*).

Wittgensteinian remedy to this bewilderment is that we must give up this fallacious view that words must name something. The very question: 'What is meaning?' or 'What is time?' is wrong. Instead, we should ask: 'What is an explanation of meaning?', 'How do we measure length?', 'How are numerical expressions used?'. Alternatively, to put it in a general form, meaning is to be seen as use.

The next crucially important concept that is intimately related to Wittgenstein's concept of meaning as use is the notion of a language game. A language game is a theoretical construct developed by Wittgenstein which offers a justification for the irreducible complexities that language exhibits. Language is not a monolithic system. On the contrary, it consists of different types of language games. These are as many language games as there are uses. St. Augustine's account of language, in spite of its limited application, is also considered to be a description of one kind of language game. It is a simple language game in which human beings communicate only by means of names where every name refers to some object. In Wittgenstein's famous example (*Philosophical Investigations* [56], sec. 2, 3) it is the language game played by the builder and the assistant in which the builder uses such words as 'slab' or 'block' to get the assistant to bring them out for him. This is a simple language game in which words are used as names for objects and thereby the purpose of communication is fulfilled, namely, giving the order and obeying it. There are also more complicated language games and Wittgenstein cites many instances of such games (*ibid*, sec. 23).

With the notion of language game, Wittgenstein introduced the notion of form of life. A language game is associated with a definite form of life. It is something which is embedded in the game itself. There is a controversy as to the exact meaning

of form of life and its implications. Without going into these controversies, it is possible to offer a simple idea of what a form of life may mean. A form of life expresses the typical life situation within which a particular language game is played. It is the set of activities that define the form of life. Finally, coming back to use, to say that the meaning of an expression is to be understood in terms of its use is really to assert that it is the use of an expression in a particular language game that determines its meaning. In Wittgenstein's metaphor, a language game is the 'original home' of meaning and use.

In the above we have tried to show at a very rudimentary level the grounds that Wittgenstein offered to establish his new approach to meaning. However, his conception of meaning has various other dimensions and implications all of which together give rise to a philosophy of language which makes a new breakthrough in our understanding of language and meaning. It is also important to note in this connection that many of the implications of his theory of meaning are not restricted to the semantics of natural language. Wittgenstein extended his approach to some other frontiers of philosophical inquiry, such as the foundations of mathematics, philosophical psychology, philosophy of social sciences, etc. It is not possible in this brief survey to go into the various aspects of Wittgenstein's theory of meaning nor it is possible to show the application of this theory to other conceptual territory.

7.13 Syntax - Semantics - Pragnatics

Syntax is the study of sentences. It specifies the grammatical rules according to which well-formed expressions are built from basic expressions or from the letters of a given alphabet. The *syntax* of a language is concerned only with the *form* of the expressions, while the *semantics* is concerned with their *meaning*. So, the rules according to which the well-formed expressions of a language are formed and the rules belonging to a formal proof system, such as $\frac{A \quad A \to B}{B}$ and $\frac{A}{A \vee B}$, belong to the syntax of the language in question. These rules can be manipulated mechanically; a machine can be instructed to apply the rule Modus Ponens and to write down a B once it sees both A and $A \to B$, while the machine does not know the meanings of A, B and $\to$. The notions of (formal) proof and deduction, as well as the notions of (formal) provability and deducibility, clearly belong to the syntax: they are only concerned with the form of the formulas involved.

Semantics is the study of propositions. By a proposition we mean the cognitive meaning (Sinn, see Section 7.2) of a sentence. Semantics is the study of truth conditions for the sentences of certain languages in isolation from the context in which those sentences are uttered. Truth tables belong to the semantics, because they say how the truth value (meaning) of a composite proposition is related to the truth values (meanings) of the components from which it is built. The notions of validity and valid consequence also belong to the semantics: they are concerned with the meaning of the formulas in question.

Now consider the proposition expressed by 'Nixon is the president of the USA'. There is (was) a possible world in which this proposition is true, and there are possible worlds in which this proposition is false. Therefore, R.C. Stalnaker gave in his paper *Pragmatics* [50] the following mathematical definition of a proposition.

A *proposition* is a function from the set W of possible worlds to the set $\{0,1\}$ of truth values. So a proposition assigns to each possible world a truth value. Stalnaker explains that under this definition, propositions have all the properties that have traditionally been ascribed to them:

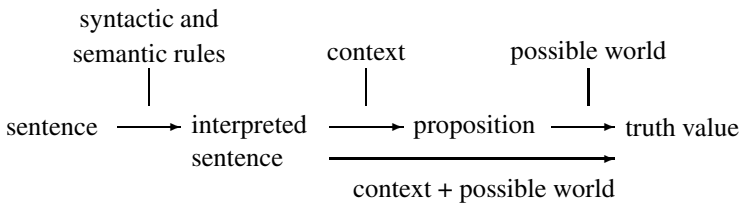
1. A proposition is independent of the particular language and the linguistic formulation in which it is expressed. 'John writes a book', 'Johan schreibt ein Buch' and 'a book is written by John' all express the same proposition.
2. A proposition is independent of the speech act in which it figures. The same proposition 'I will come' may figure in the speech act of promising, in the speech act of asserting and in many others (see Section 7.4 on speech acts).

Pragmatics is the study of speech acts and the contexts in which they are performed. Typical examples of problems to be solved within pragmatics are:

- To find necessary and sufficient conditions for the successful performance of a speech act, such as promising and many others.
- To characterize the features of the context which help determine which proposition is expressed by a given sentence. For instance, the sentence 'I am here now' expresses different propositions in different contexts.
- To determine how the presuppositions of a given sentence depend on the context (see Section 7.11).

The syntactical and semantical rules for a language enable us to interpret a sentence like 'I am here now', although we do not have to know what the indexical expressions 'I', 'here' and 'now' stand for. This interpreted sentence will result in different propositions depending on the context: 'Harry is in Amsterdam on November 12, 1989', 'Mary is in New York on December 5, 1989', and so on. Therefore, the following mathematical definition (again from R.C. Stalnaker) of an interpreted sentence seems appropriate:

An *interpreted sentence* is a function from the set C of contexts to the set P of propositions. So an interpreted sentence assigns to each context a proposition. As we have seen above, the set P of all propositions is the set $\{0,1\}^W$ of all functions from the set W of possible worlds to the set $\{0,1\}$ of truth values. Therefore, an interpreted sentence is a function from C to $\{0,1\}^W$, where C is the set of contexts and W is the set of possible worlds. The overall picture is the following:



Stalnaker explains further that pragmatics-semantics could be treated as the study of the way in which truth values are dependent on context and a possible world in which case propositions are not explicitly taken into consideration on the road from sentences to truth values. However, propositions are of some independent interest: they are the objects of illocutionary acts, such as asserting, promising, questioning, etc., and of propositional attitudes, such as believing, knowing, hoping, wishing, and so on. This justifies the extra step on the road from sentences to truth values.

7.14 Conversational Implicature

P. Grice in the 1967 William James Lectures (published in 1989 in [20]) works out a theory in *pragmatics* which he calls the theory of *conversational implicature*. Generally speaking, in conversation we usually obey or try to obey rules something like the following:

Quantity:	Be informative
Quality:	Tell the truth
Relation:	Be relevant
Mode:	Avoid obscurity, prolixity, etc.

If the fact that *A* has been said, plus the assumption that the speaker is observing the above rules, plus other reasonable assumptions about the speaker's purposes and intentions in the context, logically entails that *B*, then we can say *A conversationally implicates B*.

It is possible for *A* to conversationally implicate many things which are in no way part of the *meaning* of *A*. For example, if X says 'I'm out of gas' and Y says 'there's a gas station around the corner', Y's remark conversationally implicates that the station in question is open. (Since the information that the station is there would be *irrelevant* to X's predicament otherwise.) If X says 'Your hat is either upstairs in the back bedroom or down in the hall closet', this remark conversationally implicates 'I don't know which', since if X did know which, this remark would not be the most *informative* one he could provide.

Grice shows how philosophers have sometimes mistaken conversational implicatures for elements of meaning. For instance, Strawson sometimes claims not-knowing-which must be part of the *meaning* of 'or' (and therefore the traditional treatment of disjunction in logic is misleading or false). Grice claims this is mistaking the conversational implicature cited above for an aspect of meaning.

Sometimes it is possible to *cancel* a conversational implicature by adding something to one's remark. For example, in the gas station case, 'I'm not sure whether it's open' and in the hat case, 'I know, but I'm not saying which' (one might say this if locating the hat was part of some sort of parlor game). The possibility of cancellation shows that the conversational implicatures definitely are not part of the *meaning* of the utterance.

7.15 Conditionals

In the examples below (from Adams, [1]) the conditional in (1) is in the *indicative* mood, while the conditional in (2) is a *subjunctive* one.

(1) If Oswald did not kill Kennedy, someone else did.

(2) If Oswald had not killed Kennedy, someone else would have.

(1) is true: someone killed Kennedy; but (2) is probably false. Therefore, different analyses are needed for indicative and for subjunctive conditionals.

A *counterfactual* conditional is an expression of the form 'if A were the case, then B would be the case', where A is supposed to be false. Not all subjunctive conditionals are counterfactual. Consider the argument: 'The murderer used an ice pick. But if the butler had done it, he wouldn't have used an ice pick. So the murderer must have been someone else'. If this subjunctive conditional were a counterfactual, then the speaker would be presupposing that the conclusion of his argument is true. (This example is from R.C. Stalnaker, [51].) Counterfactuals are discussed in Section 6.9. In this section we will restrict our attention from now on to indicative conditionals.

In Chapter 2 we have considered the so-called *paradoxes of material implication*: the following two inferences for material implication '$\rightarrow$' are valid, whereas the corresponding English versions seem invalid.

$$\frac{\neg A}{A \rightarrow B} \qquad \frac{\text{There is no oil in my coffee}}{\text{If there is oil in my coffee, then I like it}}$$

$$\frac{B}{A \rightarrow B} \qquad \frac{\text{I'll ski tomorrow}}{\text{If I break my leg today, then I'll ski tomorrow}}$$

(The latter example is from R. Jeffrey [25], p.74.)

So, the truth-functional reading of 'if ..., then ...', in which $A \rightarrow B$ is equivalent to $\neg A \vee B$, seems to conflict with judgments we ordinarily make. The paradoxical character of these inferences disappears if one realizes that

1. the material implication $A \rightarrow B$ has the same truth-table as $\neg A \vee B$,
2. speaking the truth is only one of the conversation rules one is expected to obey in daily discourse; one is also expected to be as relevant and informative as possible.

Now, if one has at one's disposal the information $\neg A$ (or B, respectively) and at the same time provides the information $A \rightarrow B$, i.e., $\neg A \vee B$, then one is speaking the truth, but a truth calculated to mislead, since the premiss $\neg A$ (or B, respectively) is so much simpler and more informative than the conclusion $A \rightarrow B$. If one knows the premiss $\neg A$ (or B, respectively), the conversation rules force us to assert this premiss instead of $A \rightarrow B$. Quoting R. Jeffrey [25], pp. 77-78:

> Thus defenders of the truth-functional reading of everyday conditionals point out that the disjunction '$\neg A \vee B$' shares with the conditional 'if A, then B' the feature that normally it is not to be asserted by someone who is in a position to deny 'A' or to assert 'B. '... Normally, then, conditionals will be asserted only by speakers who think the antecedent false or the consequent true, but do not know which. Such speakers will think they know of some connection between the components, by virtue of which they are sure (enough for the purposes at hand) that the first is false or the second is true. [R. Jeffrey, [25], pp. 77-78]

Summarizing in a slogan:

indicative conditional = material implication + conversation rules.

So H.P. Grice uses principles of conversation to explain facts about the use of conditionals that seem to conflict with the truth-functional analysis of the ordinary indicative conditional. In [50] R.C. Stalnaker follows another strategy, rejecting the material conditional analysis and in [49] Brian Skyrms also claims that the indicative conditional cannot be construed as the material implication '$\rightarrow$' plus conversational implicature. The dispute between advocates of the truth-functional account of conditionals and the advocates of other, more complex but seemingly more adequate accounts is as old as logic itself. The truth-functional account is first known to have been proposed by Philo of Megara ca. 300 B.C. in opposition to the view of his teacher Diodorus Cronus. We know of this through the writings of Sextus Empiricus some 500 years later, the earlier documents having been lost. According to Sextus,

> Philo says that a sound conditional is one that does not begin with a truth and end with a falsehood. ... But Diodorus says it is one that neither could nor can begin with a truth and end with a falsehood. [W. & M. Kneale [27], p. 128]

There can be no doubt that what Sextus refers to is precisely the truth-functional connective that we have symbolized by the '$\rightarrow$', for he says elsewhere,

> So according to him there are three ways in which a conditional may be true, and one in which it may be false. For a conditional is true when it begins with a truth and ends with a truth, like 'If it is day, it is light'; and true also when it begins with a falsehood and ends with a falsehood, like 'If the earth flies, the earth has wings'; and similarly a conditional which begins with a falsehood and ends with a truth is itself true, like 'If the earth flies, the earth exists'. A conditional is false only when it begins with a truth and ends with a falsehood, like 'If it is day, it is night'. [W. & M. Kneale [27], p. 130]

So Sextus reports Philo as attributing truth values to conditionals just as in our table for $\rightarrow$, except for the order in which he lists the cases. Diodorus probably had in mind what later was called 'strict implication'; see Section 6.8. For relevant implication see Section 6.10.

7.16 Leibniz

We will here pay attention to only a few aspects of G. Leibniz (1646-1716). For more information the reader is referred to W. & M. Kneale, *The Development of Logic* [27] and to B. Mates, *Elementary Logic*, [37] , Chapter 12. What follows in this section is based on these works.

One of Leibniz' ideals was to develop a *lingua philosophica* or *characteristica universalis*, an artificial language that in its structure would mirror the structure of thought and that would not be affected with ambiguity and vagueness like ordinary language. His idea was that in such a language the linguistic expressions would be pictures, at it were, of the thoughts they represent, such that signs of complex thoughts are always built up in a unique way out of the signs for their composing parts. Leibniz believed that such a language would greatly facilitate thinking

and communication and that it would permit the development of mechanical rules for deciding all questions of consistency or consequence. The language, when it is perfected, should be such that 'men of good will desiring to settle a controversy on any subject whatsoever will take their pens in their hands and say *Calculemus* (let us calculate)'. If we restrict ourselves to propositional logic, Leibniz' ideal has been realized: classical propositional logic is decidable (see Chapter 2). However, A. Church and A. Turing proved in 1936 that (classical) predicate logic is undecidable, i.e., there is no mechanical method to test logical consequence (in predicate logic), let alone philosophical truth.

Leibniz also developed a theory of identity, basing it on *Leibniz' Law*: *eadem sunt quorum unum potest substitui alteri salva veritate* – those things are the same if one may be substituted for the other with preservation of truth. Leibniz' Law is also called the substitutivity of identity and it is frequently formulated as follows.

$$a = b \to (\ldots a \ldots \rightleftarrows \ldots b \ldots),$$

where $\ldots a \ldots$ is a context containing occurrences of the name a, and $\ldots b \ldots$ is the same context in which one or more occurrences of a have been replaced by b; if $a = b$, then what holds for a holds for b and vice versa.

A consequence of Leibniz' law is that from 'it is necessary that $9 > 7$' and from 'the number of planets (in this world) = 9' it follows that 'it is necessary that the number of planets (in this world) > 7'. This result has been seen as problematic, in particular if one talks about 'the number of planets' instead of 'the number of planets in this world'. The definite description 'the number of planets' assigns different numbers to different possible worlds, but the phrase 'the number of planets in this world' is a rigid designator, referring to the number 9. So, the alleged problem is caused by a sloppy use of language and can be remedied by a careful and precise use of language. See Section 6.11.1.

Leibniz made a distinction between *truths of reason* and *truths of fact*. The truths of reason are those which could not possibly be false, i.e., – in modern terminology – which are *necessarily true*. Examples of such truths are: $2+2=4$, living creatures cannot survive fire, and so on. Truths of fact are called contingent truths nowadays; for example, unicorns do not exist, Amsterdam is the capital of the Netherlands, and so on. Leibniz spoke of the truths of reason as *true in all possible worlds*. He imagined that there are many *possible worlds* and that our actual world is one of them. '$2+2=4$' is true not only in this world, but also in any other world. 'Amsterdam is the capital of the Netherlands' is true in this world, but we can think of another world in which this proposition is false. In 1963, S. Kripke extended the notion of possible world with an *accessibility relation* between possible worlds, which enabled him to give adequate semantics for the different modal logics (see Chapter 6). The idea is that some worlds are accessible from the given world, and some are not. For instance, one could postulate (and one usually does) that worlds with different mathematical laws are not accessible from the present world.

7.17 De Dicto - De Re

If one wants to translate the sentence

It is possible that a Republican will win

into a logical formula, it becomes evident that this sentence is ambiguous. Using '$\Diamond$' for 'it is possible that', the predicate symbol R for 'being a Republican' and the symbol W for 'will win', there are two different translations of the sentence in question:

(1) $\exists x[R(x) \wedge \Diamond W(x)]$, and
(2) $\Diamond \exists x[R(x) \wedge W(x)]$.

(1) says, literally, that there is some particular individual (in the actual world) who is a Republican (in the actual world) and who may possibly win (in some imaginary world).
(2) says, literally, that it is possible that some Republican or other will win; more precisely, there is an imaginary world in which a person exists who is a Republican (in that world) en who wins (in that world)

(1) is called the *de re* or referential reading of the sentence above. Typical of the *de re* reading is that the possibility operator $\Diamond$ occurs within the scope of the (existential) quantifier. (2) is called the *de dicto* or non-referential reading of the sentence above. Typical of the *de dicto* reading is that the (existential) quantifier occurs within the scope of the possibility operator $\Diamond$.

The example above demonstrates that sentences containing modalities such as 'possibly', 'necessarily', 'John believes that …', etc., in combination with existential or universal quantifiers may give rise to ambiguities. Speaking in terms of possible worlds (see Chapter 6) and interpreting '$\Diamond A$ (A is possible)' as 'there is some world accessible from the given world in which A holds', (1) says that in the given world there is a person who is a Republican and who will win in some world accessible from the given one, while (2) says that there is a world accessible from the given one in which there is a person who in that world is Republican and will win.

The proposition 'John finds a unicorn' can be properly translated as $\exists x[U(x) \wedge F(j,x)]$ where $U(a)$ stands for 'a is a unicorn', j stands for 'John' and $F(a,b)$ stands for 'a finds b'. But $\exists x[U(x) \wedge S(j,x)]$, where $S(a,b)$ stands for 'a seeks b' would be an improper translation of 'John seeks a unicorn', because the use of the existential quantifier commits us to an ontology in which unicorns do exist. In his '*The Proper Treatment of Quantification in Ordinary English*' R. Montague [38] develops a 'categorial' language in which 'John seeks a unicorn' can be properly translated.

7.18 Grammars

In the sixties Noam Chomsky developed the notion of grammar which turned out to be important not only for linguistics but also for computer science, for instance, in building parsers and compilers.

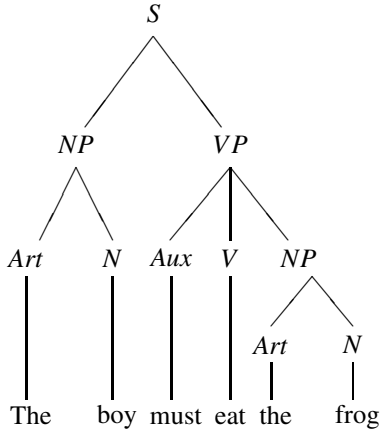
One of the main reasons for Chomsky to introduce the notion of grammar was to explain the linguistic competence of people; that they are able to produce new sentences they have never read or heard before. In order to do so, Chomsky assumed that everybody is equipped with certain grammatical rules which can be applied again and again to produce more and more linguistic expressions.

It is well-known that a sentence (S) can be built from a noun phrase (NP) and a verb phrase (VP). Chomsky's basic idea was to represent this fact as a *production rule* $S \to NP + VP$. This rule should be read as follows: whenever symbol S occurs, it is allowed to rewrite S as the string consisting of the symbols NP and VP. Similar rewrite or production rules, also called *phrase structure rules*, exist for NP and VP. For instance, $NP \to Art\ +\ N$, expressing that a noun phrase (NP) can be built from an article (Art) and a noun (N); and $VP \to Aux + V + NP$, expressing that a verb phrase (VP) may consist of an auxiliary verb (Aux), a main verb (V) and a noun phrase (NP).

In order to produce English sentences, we also need rewrite or production rules of the form $Aux \to$ can, $Aux \to$ may, $Aux \to$ will, $Aux \to$ must, usually represented by $Aux \to$ (can, may, will, must) for short. And we also need rules such as $V \to$ (read, hit, eat), $Art \to$ (a, the) and $N \to$ (boy, man, frog).

The expressions 'can', 'read', 'a', 'boy', etc., are called *terminals* because there are no production rules starting with these expressions. The symbols S, NP, VP, Art, N, Aux, V, on the other hand, are called *non-terminals* for obvious reasons.

Starting with the symbol S, we can, by repeated application of the production rules above, generate terminal strings to which no production rules can be applied. For instance, starting with S, the production rules mentioned above can generate the English sentence 'the boy must eat the frog'. The *derivation tree* or *phrase marker* for this sentence looks as follows.



In this way the production rules above can generate a small fragment of English. Together they form what Chomsky called a *grammar*. In his view, the production rules in a grammar represent the linguistic competence of a speaker and are part of everyone's innate mental equipment.

Definition 7.1 (Grammar). A (type 0) *grammar* G is a quadruple $\langle V_N, V_T, P, S\rangle$, where
1) V_N is a finite set; the elements of V_N are called *non-terminals*.
2) V_T is a finite set such that V_N and V_T have no elements in common; the elements of V_T are called *terminals*.
3) P is a finite set of expressions of the form $\alpha \to \beta$, where α and β are strings of finite length composed of symbols of V_N and/or V_T (i.e., $\alpha, \beta \in (V_N \cup V_T)^*$) and the length of α is at least 1 (i.e., $|\alpha| \geq 1$); the elements of P are called *productions*.
4) $S \in V_N$; S is called the sentence- or *start-symbol*.

Example 7.1. Let $G_1 = \langle V_N, V_T, P, S\rangle$ where
$V_N = \{S, NP, VP, Art, N, Aux, V\}$,
$V_T = \{$can, may, will, must, read, hit, eat, a, the, boy, man, frog$\}$, and
P consists of the following productions:

$S \to NP$ - VP	$Aux \to$ (can, may, will, must)
$NP \to Art$ - N	$V \to$ (read, hit, eat)
$VP \to Aux$ - V - NP	$Art \to$ (a, the)
	$N \to$ (boy, man, frog).

Example 7.2. Let $G_2 = \langle \{S\}, \{0,1\}, \{S \to 0S1, S \to 01\}, S\rangle$.
Here, S is the only non-terminal, 0 and 1 are terminals and there are two productions, $S \to 0S1$ and $S \to 01$.

By putting certain restrictions on the productions in P one obtains grammars of type 1, 2 and 3, respectively.

Definition 7.2 (Derivable from). Let $G = \langle V_N, V_T, P, S\rangle$ be a grammar and let α and β be finite strings composed of elements of V_N and/or V_T ($\alpha, \beta \in (V_N \cup V_T)^*$). $\alpha \overset{*}{\underset{G}{\Rightarrow}} \beta$ (β is *derivable from* α in grammar G) := β can be obtained from α by application of one or more productions in P. By convention, $\alpha \overset{*}{\underset{G}{\Rightarrow}} \alpha$ for each string α.

Example 7.3. $S \overset{*}{\underset{G_1}{\Rightarrow}} Art$ - N - Aux - eat - NP and $VP \overset{*}{\underset{G_1}{\Rightarrow}} Aux$ - eat - the - frog.

$S \overset{*}{\underset{G_2}{\Rightarrow}} 00S11$.

Definition 7.3 (Language generated by a Grammar). Let $G = \langle V_N, V_T, P, S\rangle$ be a grammar. $L(G) := \{w \in V_T^* \mid S \overset{*}{\underset{G}{\Rightarrow}} w\}$, i.e., $L(G)$, called *the language generated by* G, is by definition the set of all strings (or words) w of terminals such that $S \overset{*}{\underset{G}{\Rightarrow}} w$.

So, a string w is in $L(G)$ iff 1) w consists solely of terminals and 2) w is derivable from S in G.

Example 7.4. The reader easily verifies that the sentences 'the boy must eat the frog' and 'a man may hit the boy' both are in $L(G_1)$, where G_1 is the grammar from Example 7.1. $L(G_2) = \{0^n 1^n \mid n \geq 1\}$, i.e., the language generated by G_2 (see Example 7.2) consists of all finite sequences composed of n 0's followed by the same number n of 1's ($n \geq 1$).

Consider the following three sentences.

1. Locusta is an alleged poisoner.
2. Locusta is a Roman poisoner.
3. Locusta is a skillful poisoner.

These three sentences have similar surface structures, but still intuitively we feel they are quite different in meaning. (An alleged poisoner is not another kind of poisoner along with Roman and Carthaginian or skillful and clumsy; a Roman poisoner is one who is both Roman and a poisoner, but a skillful strangler may be a clumsy poisoner.)

In order to explain this, Chomsky distinguishes the *deep structure* of a sentence, determined by the production rules of a grammar and the *surface structure* of a sentence which results by applying to the derivation tree of the sentence certain transformation rules. Corresponding with their quite different meanings, the sentences 1, 2 and 3 above have radically different deep structures. Transformation rules transform these different deep structures into similar surface structures. See Exercise 7.7.

Postulating that the deep structure determines the meaning of a sentence, Chomsky explains in this way that the sentences 1, 2 and 3 above have quite different meanings although they have a similar surface structure.

The sentences 'a man may hit the boy' and 'the boy may be hit by a man', on the other hand, have the same meaning, although they are syntactically different. This can also be explained in terms of deep and surface structures. These sentences have the same deep structure and hence the same meaning. And the surface structure of one of these sentences is obtained by applying certain transformation rules to its deep structure. See Exercise 7.8.

So, in Chomsky's view, the syntax of a language consists of two components:
1) a *base component*, containing the production (or phrase structure) rules. These rules generate the *deep structure* of each sentence.
2) a *transformational component*, containing the transformation rules which map derivation trees into (other) derivation trees. The transformation rules take as input a deep structure and generate as output a *surface structure*.

The deep structure determines the meaning of a sentence; the surface structure determines its sound.

In the case of the sentences 'a man will hit the boy', in the active mood, and 'the boy will be hit by a man', in the passive mood, two surface structures are derived from one deep structure. And in the case of 'Locusta is an alleged/Roman/skillful poisoner', similar surface structures are derived from several different deep structures.

One can show that the languages generated by a (type 0) grammar are precisely the languages recognized by a Turing machine; see, for instance, de Swart [53].

Exercise 7.7. Let G be a grammar with the following phrase structure rules (productions):

$S \to S$ - [and] - S
$S \to NP$ - VP
$NP \to (Art)$ - (Adj) - N
$VP \to V$ - [that] - S
$VP \to (Adv)$ - V - N
$VP \to Copula$ - NP
$VP \to Copula$ - Adj
$Art \to$ (a, an, the)
$Adj \to$ (roman, alleged, skilful)
$N \to$ (someone, Locusta, poisoner)
$V \to$ (allege, poison)
$Copula \to$ be
$Adv \to$ skilfully

Generate the phrase markers (derivation trees) in G for the deep structures of the following sentences:

1. Locusta is an alleged poisoner.
2. Locusta is a Roman poisoner.
3. Locusta is a skilful poisoner.

Various transformations give rise to the same surface structure.

Exercise 7.8. Let G be the same grammar as in Exercise 7.7 and consider the following transformation rule: N_1 - V - $N_2 \ \to \ N_2$ - [be] - V - [ed] - [by] - N_1.
Check that this transformation rule, applied to the phrase marker (derivation tree) in G for the deep structure of 'someone is poisoned by Locusta', which is the same as the one of 'Locusta poisons someone', yields the phrase marker for the surface structure of 'someone is poisoned by Locusta'.

Exercise 7.9. Construct a grammar which generates precisely all formulas of propositional logic built from the atomic formulas Q, Q', Q'', ... by means of the connectives $\wedge$, $\vee$ and $\neg$.

Exercise 7.10. Let $G = \langle \{S\},\ \{0,1\},\ \{S \to 0S1, S \to 01\},\ S \rangle$. Show that $L(G) = \{0^n 1^n \mid n \geq 1\}$, where 0^n stands for 0 repeated n times and similarly for 1^n.

Exercise 7.11. Give a grammar generating the set of all finite strings w of 0's and 1's such that w does not contain two consecutive 1's.

Exercise 7.12. Give a grammar generating the set of all finite strings w of a's, b's and c's such that w consists of equal numbers of a's, b's and c's.

Exercise 7.13. Let $\{a,b\}^*$ be the set of all finite strings of a's and b's, including the empty string e of length 0. Let $L = \{w \in \{a,b\}^* \mid w$ contains an even number of b's$\}$. Check that $L - \{e\}$ is generated by the grammar $G = \langle \{S,B\},\ \{a,b\},\ P,\ S \rangle$ with $P = \{S \to aS,\ S \to a,\ S \to bB,\ B \to aB,\ B \to bS,\ B \to b\}$.

Exercise 7.14. With $\{a,b\}^*$ as in Exercise 7.13, let $L = \{w \in \{a,b\}^* \mid w$ does not contain three consecutive b's$\}$. Verify that $L - \{e\}$ is generated by the grammar $G = \langle \{S,B,C,D\},\ \{a,b\},\ P,\ S \rangle$ with P consisting of the following productions:

$$
\begin{array}{llll}
S \to aS, & B \to aS, & C \to aS, & D \to aD, \\
S \to a, & B \to a, & C \to a, & D \to bD. \\
S \to bB, & B \to bC, & C \to bD, & \\
S \to b, & B \to b. &&
\end{array}
$$

7.19 Solutions

Solution 7.1.

(a) true	(b) false; The Iliad is an epic poem.
(c) true	(d) true
(e) false; $7+5=12$.	(f) true
(g) true	(h) false; 'Saul' is another name of Paul.
(i) true	(j) false; '$2+2=4$' is synthetic.
(k) true	(l) true

Solution 7.2. The difficulty lies in our assumption that one can determine mechanically whether or not an alleged definition of a function is indeed such a definition. Since the function f, defined by $f(n) := f_n(n)+1$, is not definable in the English language, our only recourse is to conclude: There is no algorithm that enables one to decide whether an alleged definition of a number-theoretic function is indeed such a definition. In other words, there is no algorithm that enables one to decide mechanically for any expression in the English language whether it defines a number-theoretic function or not.

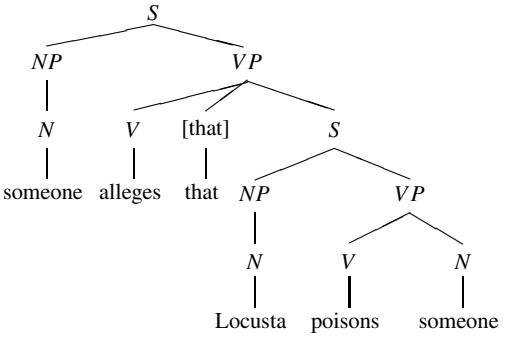
Solution 7.3. Suppose that a and b are rigid designators. If '$a=b$' is true, so that 'a' and 'b' designate the same object in the actual world, then, since both names, being rigid designators, designate the same object in all possible worlds, '$a=b$' is true in all possible worlds, that is to say, it is necessarily true that $a=b$.

Solution 7.4. Since stick S is the standard meter in Paris, stick S is by definition one meter long. Therefore, the epistemological status of the statement 'stick S is one meter long' is that this statement is an *a priori* truth. Conceiving 'one meter' as a rigid designator, indicating the same length in all possible circumstances (worlds), the metaphysical status of 'stick S is one meter long' will be that of a *contingent* statement, since the length of stick S can vary with the temperature, humidity, etc.

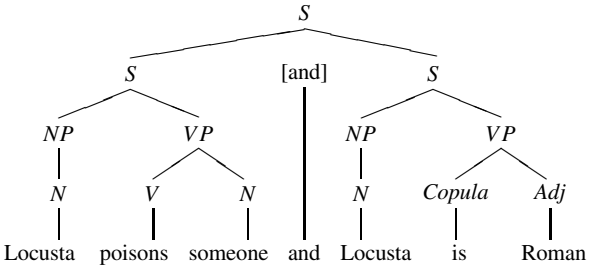
Solution 7.5. 'The king of France is bald' induces the presupposition that there is a king of France. This presupposition is not induced by the sentence 'If the king of France exists, then the king of France is bald'.

Solution 7.6. Suppose we define 'A presupposes B' as: $A \models B$ and $\neg A \models B$. Then $A \vee \neg A \models B$. But $\models A \vee \neg A$. Therefore, $\models B$. So, 'A presupposes B' would mean that $\models B$. This is counter-intuitive.

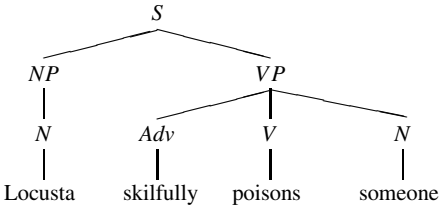
Solution 7.7. 1.



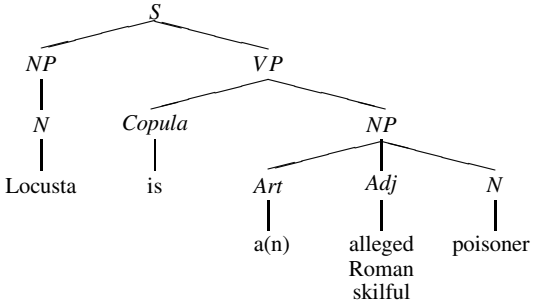
2.



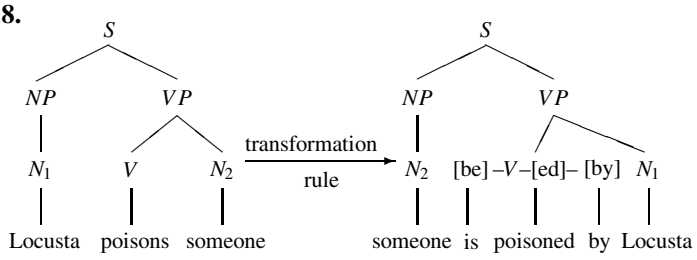
3.



Phrase-marker for the surface structure of the sentences in 1, 2 and 3:



Solution 7.8.



Solution 7.9. $G = \langle V_N,\ V_T,\ P,\ S\rangle$, where $V_T = \{Q,\ ',\ \wedge,\ \vee,\ \neg,\ (,\ )\}$, $V_N = \{S,A\}$, and $P = \{S \to A,\ S \to (S \wedge S),\ S \to (S \vee S),\ S \to (\neg S),\ A \to A',\ A \to Q\}$.

Solution 7.10. Each string of the form $0^n 1^n,\ n \geq 1$, is generated by G:
$S \to 0S1 \to 00S11 \to 0^3S1^3 \to \ldots \to 0^{n-1}S1^{n-1} \to 0^n1^n$.
Furthermore it is easy to see that these are the only strings in $L(G)$.

Solution 7.11. $G = \langle \{S,A\},\ \{0,1\},\ P,\ S\rangle$, where P contains the following productions:

$$\begin{array}{ll} S \to 1 \ , & A \to 0, \\ S \to 0 \ , & A \to 0S, \\ S \to 0S\ , & S \to 1A. \end{array}$$

Solution 7.12. $G = \langle \{S,A,B,C\},\ \{a,b,c\},\ P,\ S\rangle$, where P consists of the following productions:

$$\begin{array}{llll} S \to ABC, & A \to a, & AB \to BA, & BC \to CB, \\ S \to SS, & B \to b, & AC \to CA, & CA \to AC, \\ & C \to c, & BA \to AB, & CB \to BC. \end{array}$$

Solution 7.13. We have to show that $L - \{e\} = L(G)$, i.e. that $L - \{e\}$ and $L(G)$ have the same elements.
1) So suppose $w \in L - \{e\}$, i.e., w contains an even number of b's and $w \neq e$. Then it is not hard to see that w can be generated by G, i.e., $S \overset{*}{\underset{G}{\Rightarrow}} w$, and hence $w \in L(G)$.
2) Conversely, suppose $w \in L(G)$. Then it follows from the definition of the productions in G that w contains an even number of b's and $w \neq e$, and hence that $w \in L - \{e\}$.

Solution 7.14. Similar to Solution 7.13. Note that if an expression u generated by the grammar $(S \overset{*}{\underset{G}{\Rightarrow}} u)$ contains three or more consecutive b's, it must also contain the nonterminal D, and hence $u \notin V_T^*$, so $u \notin L(G)$.

References

1. Adams, E., Subjunctive and Indicative Conditionals. *Foundations of Language* 6, 89-94, 1970.
2. Ayer, A.J., (ed.), *Logical Positivism*. Editor's Introduction. Greenwood Press, Westport, C.T., 1959, 1978.
3. Austin, J.L., *How to Do Things with Words*. Oxford University Press, 1962.
4. Austin, J.L., Performative-Constative. In J.R. Searle, (ed.), *Philosophy of Language*.
5. Bergmans, L.J.M., Gerrit Mannoury. *Encyclopédie Philosophique Universelle*, vol. III *Oeuvres*. Les Presses Universitaire de France, Paris, 1992, columns 1778-1783.
6. Brouwer, L.E.J., *Leven, Kunst en Mystiek*. J. Waltman, Jr., Delft, 1905.
7. Brouwer, L.E.J., Boekbespreking van J.I. de Haan, Rechtskundige Significa en hare Toepassing op de begrippen: "Aansprakelijk, Verantwoordelijk, Toerekeningsvatbaar". Akademisch Proefschrift, Thesis, Amsterdam: W. Versluys, 1916. *Groot-Nederland* 14, 1916, pp. 333-336.
8. Bűcher, K., *Arbeit und Rhythmus*. (3rd edition) B.G.Teubner, Leipzig, 1902.
9. Carnap, R., *Meaning and Necessity*. University of Chicago Press, 1947, 1967, 2008.

10. Church, A., *Introduction to Mathematical Logic*. Princeton Univ. Press, 1956, 1996.
11. Dantzig, D. van, On the principles of intuitionistic and affirmative mathematics. *Indigationes mathematicae*, vol. IX, 4 & 5, 1947.
12. Donnellan, K., Reference and Definite Descriptions. *Philosophical Review* 75, 281-304, 1966.
13. Dummett, M., *Frege, Philosophy of Language*. Duckworth, 1981.
14. Frege, G., *Begriffsschrift und andere Aufsätze*. I. Angelelli (ed.), Olms, Hildesheim, 1964.
15. Frege, G., *Philosophical Writings*. Translated by P. Geach and M. Black. Basil Blackwell, Oxford, 1970.
16. Frege, G., *Funktion, Begriff, Bedeutung*. Fünf Logische Studien. G. Patzig (ed.), VandenHoeck & Ruprecht, Göttingen, 1975.
17. Frege, G., *Grundgesetze der Arithmetik*. 1893, 1903, 2009.
18. Geach, P. and M. Black (eds.) *Translations from the Philosophical Writings of Gottlob Frege*. Oxford University Press, 1980.
19. Gödel, K., *Russell's mathematical logic*. in P.A. Schilpp (ed.), *The philosophy of Bertrand Russell*, Tudor, N.Y., 1944; and in P. Benacerraf and H. Putnam, *Philosophy of Mathematics, Selected readings*, New York, 1964, 1983, 1998.
20. Grice, P., *Studies in the Way of Words*. Harvard University Press, 1989
21. Haack, S., *Philosophy of Logics*. Cambridge University Press, 1978, 1999.
22. Hacker, P.M.S., *Insight and Illusion: Wittgenstein on Philosophy and Metaphysics of Experience*. Oxford University Press, 1972.
23. Heijenoort, J. van, *From Frege to Gödel*. Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought. Harvard University Press, Cambridge, 1967.
24. Heijerman, E. and Schmitz, H.W., (eds.), *Significs, Mathematics and Semiotics. The Signific Movement in the Netherlands*. Proceedings of the International Conference held at Bonn, November 19-21, 1986. Modus Publikationen, Műnster, 1990.
25. Jeffrey, R., *Formal Logic: its scope and limits*. McGraw-Hill, New York, 1967, 1981, 2006.
26. Kant, I., *The Critique of Pure Reason*. William Benton, Publisher, Encyclopaedia Brittanica, in particular pp. 14-18.
27. Kneale, W. & M., *The Development of Logic*. Clarendon Press, Oxford, 1962, 2008.
28. Kripke, S.A., *Naming and Necessity*. Basil Blackwell, Oxford, 1980.
29. Kripke, S.A., *Identity and Necessity*. In [46], pp 66-101.
30. Mannoury, G., De filosofische grondslagen der verantwoordelijkheid. *Accountancy* 15 (12), december 1917, pp. 97-104.
31. Mannoury, G., *Mathesis en Mystiek: Een signifiese studie van kommunisties standpunt*. Maatschappij voor goede en goedkope lectuur, Amsterdam, 1925.
32. Mannoury, G., Die signifischen Grundlagen der Mathematik. *Erkenntnis*, Vol. IV, 1934.
33. Mannoury, G., *Les Fondements Psycho-Linguistiques des Mathématiques*. F.G. Kroonder, Bussum/Editions du Griffon, Neuchâtel, 1947.
34. Mannoury, G., *Handboek der Analytische Signifika; deel I: Geschiedenis der Begripskritiek*. F.G. Kroonder, Bussum, 1947.
35. Mannoury, G., *Handboek der Analytische Signifika; deel II: Hoofdbegrippen en Methoden der Signifika. Ontogenese en Fylogenese van het Verstandhoudingsapparaat*. F.G. Kroonder, Bussum, 1948.
36. Mannoury, G., *Polairpsychologische Begripssynthese*. F.G. Kroonder, Bussum, 1953.
37. Mates, B., *Elementary Logic*. Oxford University Press, 1965, 1972.
38. Montague, R., *The Proper Treatment of Quantification in Ordinary English*. Reprinted in R.H. Thomason (ed.), *Formal Philosophy; Selected Papers of Richard Montague*. Yale University Press, 1974.
39. Quine, W.V., *The Ways of Paradox and other Essays*. Harvard University Press, 1976.
40. Quine, W.V., *Two Dogmas of Empiricism*. In W.V. Quine, *From a Logical Point of View*. Harvard University Press, 1953, 1999.
41. Russell, B. and A.N. Whitehead, *Principia Mathematica*. 1903, 2011.
42. Sandt, R. van de, *Context and Presupposition*. Routledge, 1988.
43. Schmitz, H.W., Mannoury and Brouwer: Aspects of their relationship and cooperation. *Methodology and Science*, 20, 1987, pp. 40-62.

44. Schmitz, H.W., *De Hollandse Signifika: Een reconstructie van de geschiedenis van 1892 tot 1926*. Translated by J. van Nieuwstadt, Van Gorcum, Assen, 1990.
45. Schmitz, H.W., Empirical Methods of Signific Analysis of Meaning: Transformation and Exhaustion of Linguistic Acts. In Heijerman, E., and Schmitz, H.W., (eds.), 1990.
46. Schwartz, S.P., *Naming, Necessity and Natural Kinds*. Introduction, pp. 13-41. Cornell University Press, 1977-1979.
47. Searle, J.R. (ed.), *The Philosophy of Language*; Introduction. Oxford Univ. Pr., 1971.
48. Searle, J.R., What is a Speech Act. In: J.R. Searle, (ed.), *Philosophy of Language*.
49. Skyrms, B., *Causal Necessity*. Yale University Press, 1980.
50. Stalnaker, R.C., *Pragmatics*. In Davidson and Harman (eds.), *Semantics of Natural Language*. Reidel, Dordrecht, 1972.
51. Harper, W.L., R.C. Stalnaker, G. Pearce (eds.), *IFS*; Conditionals, Belief, Decision, Chance and Time. Reidel, Dordrecht, 1981.
52. Strawson, P.F., (ed.), *Philosophical Logic*; The Thought. Oxford Univ. Press, 1968.
53. Swart, H.C.M. de, *Logic and Computer Science*. Verlag Peter Lang, Frankfurt, 1994.
54. Wittgenstein, L., *Notebooks 1914-1916*. 2nd edition. Edited by G.H. von Wright and G.E.M. Anscombe. Translated by G.E.M. Anscombe. Basil Blackwell, Oxford 1961.
55. Wittgenstein, L., *Tractatus Logico-Philosophicus*. Translated by D.F. Pears and B.F. McGuiness. 2nd edition. Kegan Paul, London, 1961.
56. Wittgenstein, L., *Philosophical Investigations*. 3rd edition. Translated by G.E.M. Anscombe. Basil Blackwell, Oxford, 1967.
57. Wittgenstein, L., *The Blue and Brown Books*. 2nd edition. Basil Blackwell, Oxford, 1969.
58. Wundt, W., *Völkerpsychologie: Eine Untersuchung der Entwicklungsgesetze von Sprache, Mythus und Sitte; vol. I: Die Sprache*. W. Engelmann, Leipzig, 1900.