Operational Amplifiers and Circuits

6

6.1 Operational Amplifier Properties

Operational amplifiers are general purpose, dual input, single output devices that exhibit high input impedances, low output impedances and generate large gains within its frequency bandwidth.

Important characteristics of an operational amplifier are shown in Fig. 6.1. The input currents, I_1 and I_2, of this amplifier are very close to 0 A because of the high impedance at negative and positive input terminals. A milli-volt range voltage difference between the input terminals, $(V_{IN1} - V_{IN2})$, can saturate the output of the operational amplifier to either its positive or negative power supply voltage value depending on the polarity of $(V_{IN1} - V_{IN2})$. However, we can approximate this potential difference to be 0 V for the purposes of simple circuit analysis.
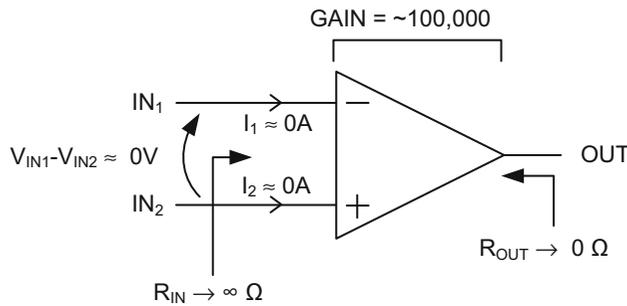

Fig. 6.1 Characteristics of an operational amplifier

6.2 Voltage Amplifier Circuits for Sensors

Inverting Voltage Sensor Amplifier

The following operational amplifier circuit in Fig. 6.2 produces voltage amplification, which results an inverted output, if one of the sensor terminals needs to be grounded. Figure 6.3 shows the current and voltage assignments of the circuit in Fig. 6.2 prior to circuit analysis.
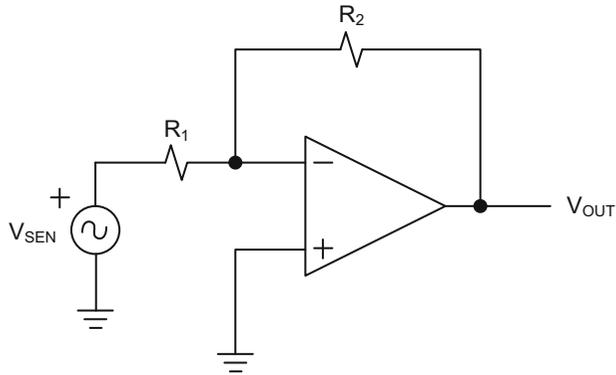
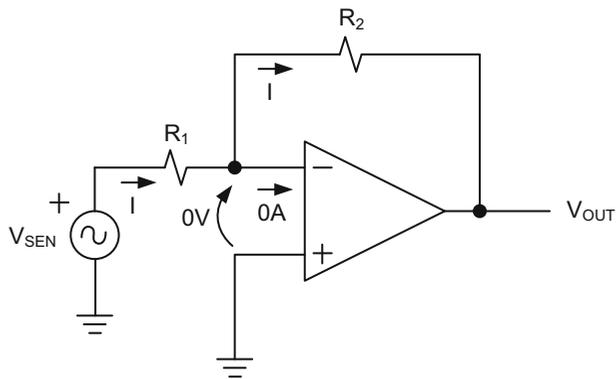**Fig. 6.2** Inverting voltage sensor amplification circuit if one sensor terminal needs to be grounded

Fig. 6.3 Current and voltage assignments of the inverting sensor amplifier in Fig. 6.2

$$V_{SEN} = R_1 I \tag{6.1}$$

$$V_{SEN} = (R_1 + R_2) I + V_{OUT} \tag{6.2}$$

Substituting $I = \dfrac{V_{SEN}}{R_1}$ into Eq. 6.2 yields:

$$V_{SEN} = (R_1 + R_2) \frac{V_{SEN}}{R_1} + V_{OUT}$$

$$V_{SEN} = V_{SEN} + \frac{R_2}{R_1} V_{SEN} + V_{OUT}$$

$$V_{OUT} = -\frac{R_2}{R_1} V_{SEN} \tag{6.3}$$

Non-inverting Voltage Sensor Amplifier

The operational amplifier circuit in Fig. 6.4 produces voltage amplification with a non-inverted output if one sensor terminal needs to be grounded. Figure 6.5 shows the current and voltage assignments of the circuit in Fig. 6.4 prior to circuit analysis.
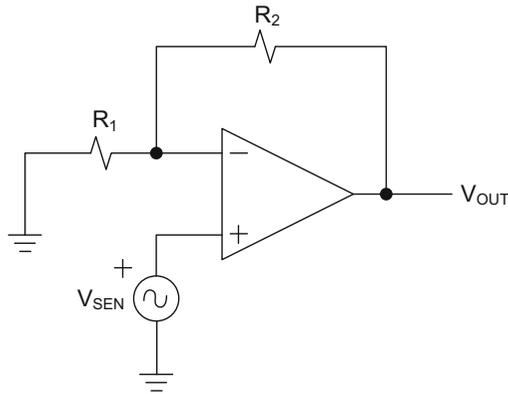

Fig. 6.4 Non-inverting voltage sensor amplification circuit if one of the sensor terminals needs to be grounded
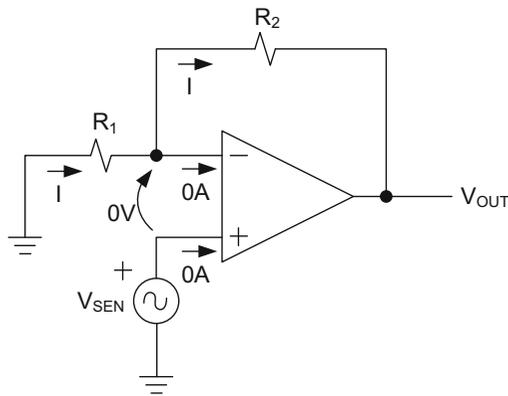

Fig. 6.5 Current and voltage assignments of the inverting sensor amplifier in Fig. 6.4

Therefore,

$$0 = R_1 I + V_{SEN} \tag{6.4}$$

$$0 = (R_1 + R_2) I + V_{OUT} \tag{6.5}$$

Substituting $I = -\dfrac{V_{SEN}}{R_1}$ into Eq. 6.5 yields:

$$0 = -(R_1 + R_2)\frac{V_{SEN}}{R_1} + V_{OUT}$$

$$V_{OUT} = \frac{R_1 + R_2}{R_1}V_{SEN} \tag{6.6}$$

Isolation Circuit with Unity Gain

Figure 6.6 represents the isolation circuit if the sensor requires very large input impedance before being interfaced to another circuitry. This circuit practically eliminates any output loading (isolation) on the sensor. One terminal of the sensor has to be grounded in order to use the isolation circuit.
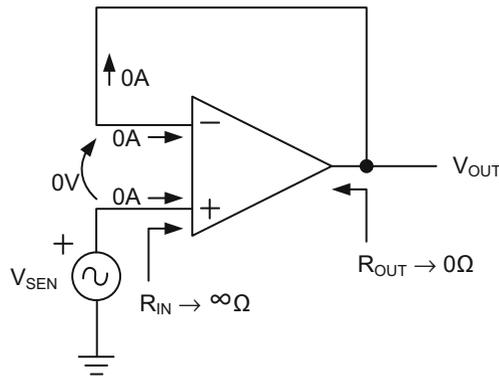

Fig. 6.6 Isolation circuit with unity gain

The only equation available for this circuit is:

$$V_{SEN} = 0 + V_{OUT} \tag{6.7}$$

$$V_{OUT} = V_{SEN} \tag{6.8}$$

Voltage Sensor Amplifier with Signal and Reference Inputs

If the sensor output needs to be compared against a reference voltage, a differential amplifier circuit in Fig. 6.7 is used. This circuit produces a differential amplification of the input signals, ($V_{SEN} - V_{REF}$), provided that one of the sensor (and the reference signal) terminals is connected to ground. Figure 6.8 shows the current and voltage assignments of the circuit in Fig. 6.7 prior to circuit analysis.
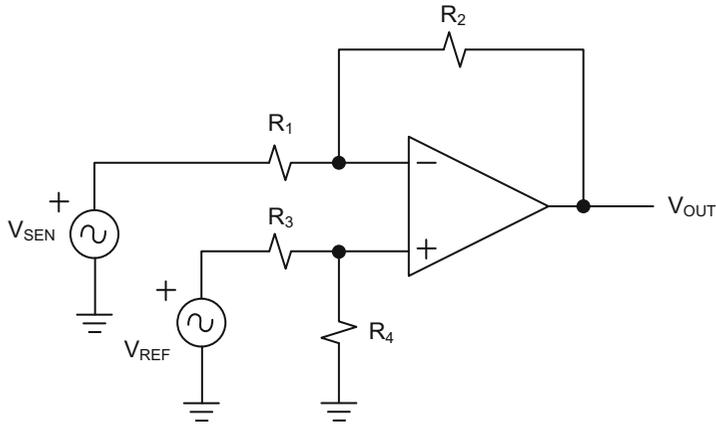
Fig. 6.7 Voltage sensor amplification with a reference voltage if one sensor terminal is grounded
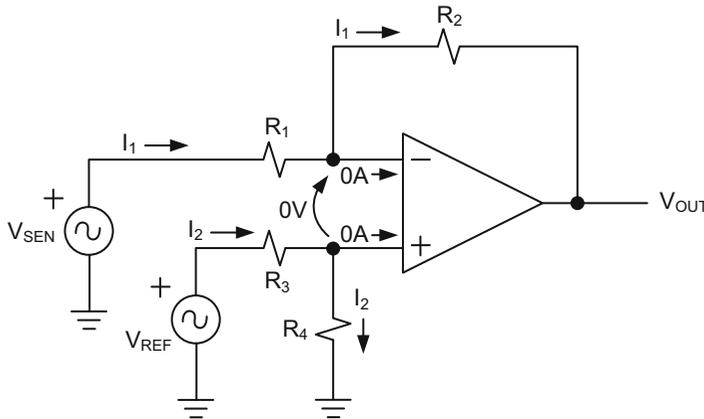

Fig. 6.8 Current and voltage assignments of the sensor amplifier in Fig. 6.7

$$V_{SEN} = (R_1 + R_2)I_1 + V_{OUT} \tag{6.9}$$

$$V_{REF} = (R_3 + R_4) I_2 \tag{6.10}$$

$$R_4I_2 = R_2I_1 + V_{OUT} \tag{6.11}$$

From Eq. 6.11,

$$I_2 = \frac{R_2}{R_4}I_1 + \frac{V_{OUT}}{R_4} \tag{6.12}$$

Substituting Eq. 6.12 into Eq. 6.10 yields:

$$V_{REF} = (R_3 + R_4)\frac{R_2}{R_4}I_1 + (R_3 + R_4)\frac{V_{OUT}}{R_4} \tag{6.13}$$

or

$$I_1 = \frac{R_4 V_{REF} - (R_3 + R_4)V_{OUT}}{R_2(R_3 + R_4)} \tag{6.14}$$

Finally, substituting Eq. 6.14 into Eq. 6.9 yields:

$$V_{SEN} = (R_1 + R_2)\frac{R_4 V_{REF} - (R_3 + R_4)V_{OUT}}{R_2(R_3 + R_4)} + V_{OUT} \tag{6.15}$$

Reorganizing the terms in this equation produces:

$$V_{SEN} = \frac{R_4}{R_2}\frac{(R_1 + R_2)}{(R_3 + R_4)}V_{REF} - \frac{R_1}{R_2}V_{OUT}$$

Thus,

$$V_{OUT} = -\frac{R_2}{R_1}V_{SEN} + \frac{R_4}{R_1}\frac{(R_1 + R_2)}{(R_3 + R_4)}V_{REF} \tag{6.16}$$

If $R_1 = R_3$ and $R_2 = R_4$, then Eq. 6.16 simplifies as:

$$V_{OUT} = -\frac{R_2}{R_1}V_{SEN} + \frac{R_2}{R_1}\frac{(R_1 + R_2)}{(R_1 + R_2)}V_{REF}$$

$$V_{OUT} = -\frac{R_2}{R_1}(V_{SEN} - V_{REF}) \tag{6.17}$$

In Eq. 6.17, the amplification is adjusted by the ratio of $\frac{R_2}{R_1}$.

Summation Voltage Sensor Amplifier

It is possible to add two sensor inputs according to Fig. 6.9 provided that both sensors have one of their terminals grounded. Figure 6.10 shows the current and voltage assignments of the circuit in Fig. 6.9 prior to circuit analysis.
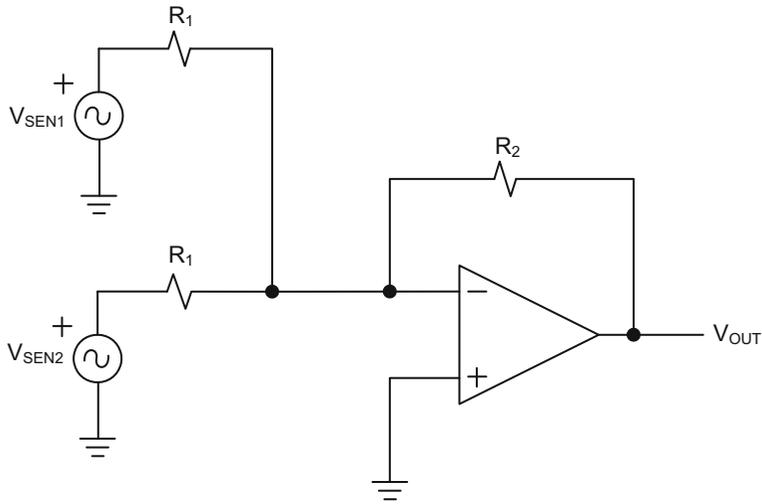
Fig. 6.9 Summation voltage sensor amplifier if one of the sensor terminals is grounded
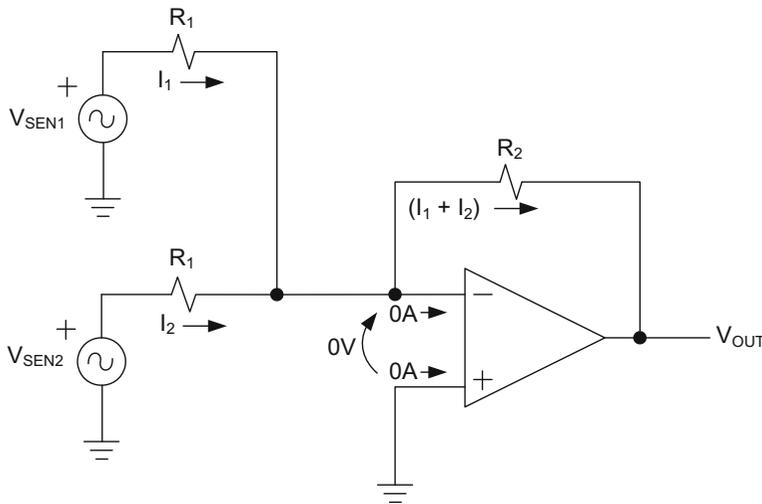

Fig. 6.10 Current and voltage assignments of the summation sensor amplifier in Fig. 6.9

$$V_{SEN1} = R_1 I_1 + R_2(I_1 + I_2) + V_{OUT} \tag{6.18}$$

$$V_{SEN2} = R_1 I_2 + R_2(I_1 + I_2) + V_{OUT} \tag{6.19}$$

$$R_2(I_1 + I_2) + V_{OUT} = 0 \tag{6.20}$$

Eliminating I_1 and I_2 in Eqs. 6.18, 6.19 and 6.20 results in:

$$V_{OUT} = -\frac{R_2}{R_1}(V_{SEN1} + V_{SEN2}) \qquad (6.21)$$

The output voltage in Eq. 6.21 forms the inverted sum of the sensor voltages, V_{SEN1} and V_{SEN2}, with a amplification factor of $\frac{R_2}{R_1}$.

Voltage Sensor Amplifier with Ungrounded Sensor Terminals

If neither of the sensor terminals can be grounded, then the circuit in Fig. 6.11 is used to obtain an amplified but an inverted sensor signal at the output. This schematic is similar to the differential amplifier schematic in Fig. 6.7; however, the sensor in this circuit is not grounded. The resistors, R_3 and R_4, are also replaced with the resistors, R_1 and R_2, respectively.
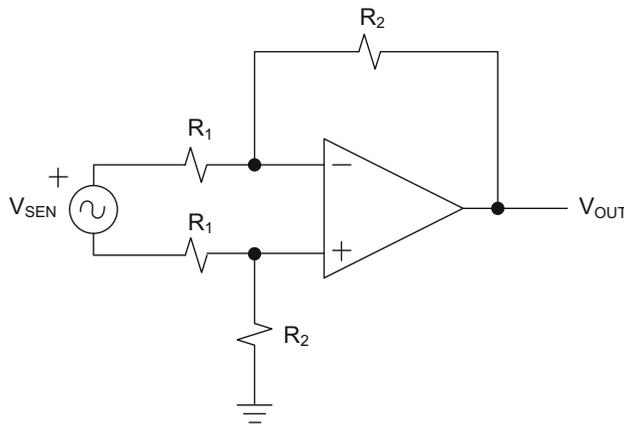

Fig. 6.11 Voltage amplification of a sensor if no sensor terminal is grounded

Figure 6.12 shows the current and voltage assignments of the circuit in Fig. 6.11 prior to circuit analysis. In this circuit, the voltage drop at the input of the operational amplifier is assumed 0 V. Similarly, the currents going to the positive and negative input terminals of the operational amplifier are assumed to be very close to 0 A.
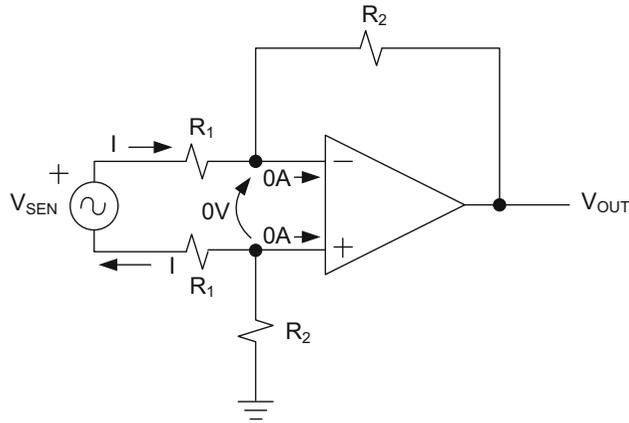
Fig. 6.12 Current and voltage assignments of the sensor amplifier in Fig. 6.11

Therefore, from Fig. 6.12 we have:

$$I(R_1 + R_2) - V_{SEN} + I(R_1 + R_2) + V_{OUT} = 0$$

or

$$V_{SEN} - V_{OUT} = 2I(R_1 + R_2) \tag{6.22}$$

We also have:

$$V_{SEN} = I(R_1 + R_1) = 2I\,R_1$$

$$I = \frac{V_{SEN}}{2R_1} \tag{6.23}$$

Substituting Eq. 6.23 into Eq. 6.22 yields:

$$V_{SEN} - V_{OUT} = 2(R_1 + R_2)\frac{V_{SEN}}{2R_1}$$

or

$$V_{OUT} = -\frac{R_2}{R_1}V_{SEN} \tag{6.24}$$

Equation 6.24 shows the sensor voltage at the input of Fig. 6.11 is inverted and amplified with a ratio of $\frac{R_2}{R_1}$.

6.3 Trans-resistance Amplifier Circuits for Sensors

Trans-resistance amplifiers are for sensors that generate current instead of voltage. These sensors are usually opto-electronic devices such photodiodes, photo-detectors and solar cells. These specialty amplifiers are designed to convert microampere-range currents into voltages.

Inverting Trans-resistance Sensor Amplifier

The operational amplifier circuit in Fig. 6.13 produces inverted sensor amplification if one sensor terminal is grounded. Figure 6.14 shows the current and voltage assignments of the circuit in Fig. 6.13 prior to circuit analysis.
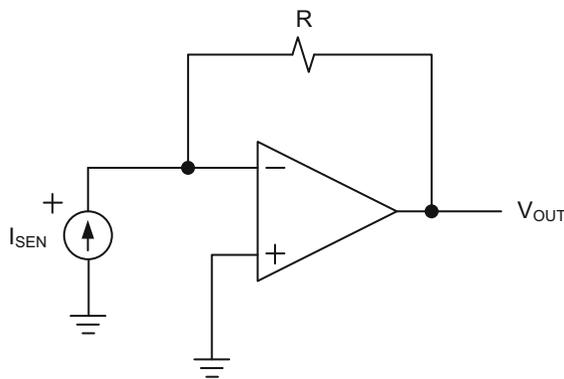

Fig. 6.13 Inverting trans-resistance sensor amplifier if one sensor terminal is grounded
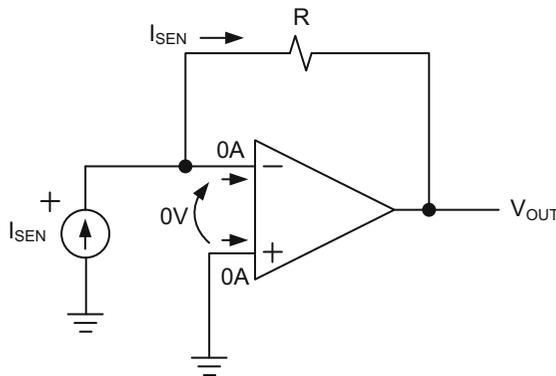

Fig. 6.14 Current and voltage assignments of the trans-resistance amplifier in Fig. 6.13

The Kirchoff's voltage law for this circuit becomes:

$$0 = R \, I_{SEN} + V_{OUT} \tag{6.25}$$

Therefore,

$$V_{OUT} = -R \ I_{SEN} \tag{6.26}$$

Here, the input sensor current, I_{SEN}, is inverted and amplified by the magnitude of R at the output.

Non-inverting Trans-resistance Sensor Amplifier

The following operational amplifier circuit in Fig. 6.15 produces non-inverted sensor amplification if one sensor terminal is grounded. This circuit basically combines the inverting trans-resistance amplifier in Fig. 6.13 and the inverting voltage amplifier in Fig. 6.2 with unity gain.
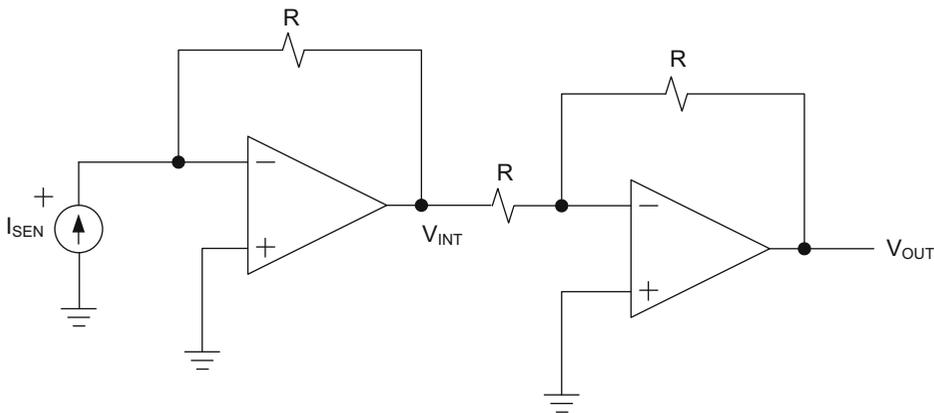

Fig. 6.15 Non-inverting trans-resistance sensor amplifier if one sensor terminal is grounded

Repeating Eq. 6.26 for the first stage yields:

$$V_{INT} = -R \ I_{SEN}$$

For the second stage, we have:

$$V_{OUT} = -\frac{R}{R} V_{INT} = -V_{INT} \tag{6.27}$$

Substituting V_{INT} in Eq. 6.27 yields:

$$V_{OUT} = -(-R \ I_{SEN}) = R \ I_{SEN} \tag{6.28}$$

6.4   Analog Voltage Comparator

Voltage comparators are used to compare two analog inputs to produce an output signal either equal to the positive voltage supply of the operational amplifier or to 0 V (the negative voltage supply can be 0 V or a negative value depending on the operational amplifier datasheet). Figure 6.16 shows the circuit diagram of a comparator. Here, voltage levels at the N and P inputs are compared with each other. If the voltage at P becomes larger than N, V_{OUT} transitions to V_{CC}, and the LED turns on. Otherwise, V_{OUT} stays at 0 V, and the LED turns off.
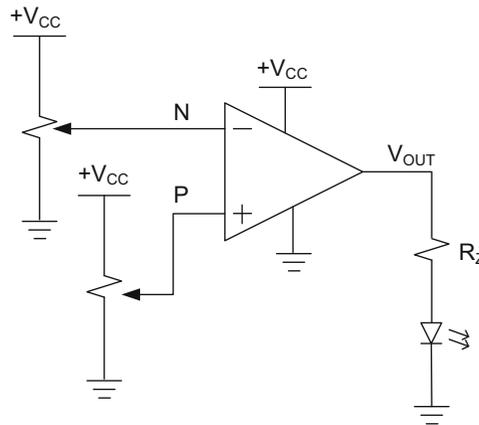

Fig. 6.16 Basic comparator circuit

A good application of a voltage comparator is a voltage monitoring circuit. Suppose we want to monitor the changes in the 2.5 V CPU power supply voltage constantly, and generate a reset signal if the supply voltage drops below 2 V.

A circuit shown in Fig. 6.17 can be used as a voltage monitor. In this circuit, a Zener diode with a value of 1.25 V is connected in series with R_Z, which provides the biasing current for the diode. The voltage at the negative terminal of the operational amplifier must also be set at 1.25 V by $R_1 = R_2$. Thus,

$$V_S = \frac{R_2}{(R_1 + R_2)} V_{CC} = \frac{V_{CC}}{2} \tag{6.29}$$

Now, if V_{CC} drops below 2.5 V, such as 2 V, V_S proportionally changes:

$$V_S = \frac{V_{CC}}{2} = \frac{2}{2} = 1V$$

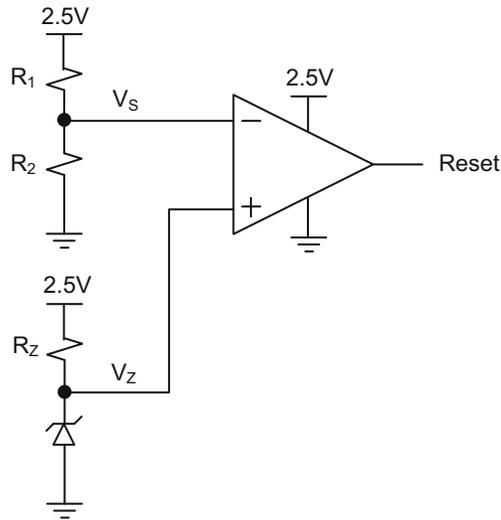Since this voltage is less than $V_Z = 1.25$ V, Reset output in Fig. 6.17 switches to 2.5 V and resets the CPU.

Fig. 6.17 A voltage monitor circuit

6.5 Schmitt Trigger

Most TTL and CMOS logic gates require fast high and low logic transitions at their inputs. If the edges of the input signal are too slow to rise or fall, excessive current drainage, even uncontrolled oscillations at the output of a logic gate become possible. Schmitt triggers are used to remove slow transitions or noisy edges from signals and transform them into forms that will meet the input rise and fall time specifications of a logic gate.

A normal Schmitt trigger produces a low-to-high logic transition at a logic level much higher than the threshold voltage of a gate. Similarly, a high-to-low transition in the Schmitt trigger occurs at a logic level much lower than the threshold voltage of a gate. This creates hysteresis at the output of the Schmitt trigger despite noise-free and sharp logic transitions.

The basic Schmitt trigger circuit is shown in Fig. 6.18. In this figure, the output of the device transitions from one supply voltage ($+V_{CC}$ or $-V_{CC}$) to the next as soon as the input voltage exceeds a set voltage limit. Schmitt trigger operates under the same principle of the voltage comparator. However, it compares the input voltage against two set values instead of one, and its feedback loop ensures that a portion of the output value is always taken into account when the comparison is performed at the input.
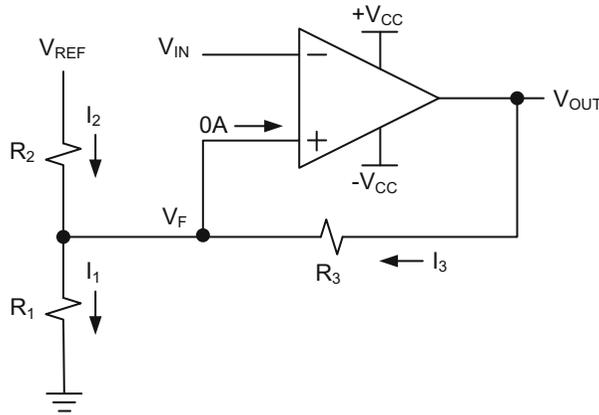
Fig. 6.18 Basic Schmitt trigger

We can adjust and set the voltage values in a Schmitt trigger by adjusting the values of R_1, R_2 and R_3.

If I_1, I_2 and I_3 are the currents passing through R_1, R_2 and R_3 in Fig. 6.18, then we can write the following:

$$I_1 = I_2 + I_3 \tag{6.30}$$

However,

$$I_1 = \frac{V_F}{R_1} \tag{6.31}$$

$$I_2 = \frac{(V_{REF} - V_F)}{R_2} \tag{6.32}$$

$$I_3 = \frac{(V_{OUT} - V_F)}{R_3} \tag{6.33}$$

Substituting Eqs. 6.31, 6.32 and Eq. 6.33 into Eq. 6.30 yields:

$$\frac{V_F}{R_1} = \frac{(V_{REF} - V_F)}{R_2} + \frac{(V_{OUT} - V_F)}{R_3}$$

$$V_F \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{REF}}{R_2} + \frac{V_{OUT}}{R_3}$$

If we define $\dfrac{1}{R_{123}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$, then

$$\frac{V_F}{R_{123}} = \frac{V_{REF}}{R_2} + \frac{V_{OUT}}{R_3}$$

$$V_F = \frac{R_{123}}{R_2} V_{REF} + \frac{R_{123}}{R_3} V_{OUT} \qquad (6.34)$$

If $V_{OUT} = +V_{CC}$ then V_F becomes:

$$V_{FPOS} = \frac{R_{123}}{R_2} V_{REF} + \frac{R_{123}}{R_3} V_{CC} \qquad (6.35)$$

Similarly,
If $V_{OUT} = -V_{CC}$ then V_F becomes:

$$V_{FNEG} = \frac{R_{123}}{R_2} V_{REF} - \frac{R_{123}}{R_3} V_{CC} \qquad (6.36)$$

Here, V_{FPOS} and V_{FNEG} are the two feedback voltages at the output terminal when $V_{OUT} = +V_{CC}$ and $V_{OUT} = -V_{CC}$, respectively.

In this example, let us set the voltage level for the low-to-high logic transition to 3.3 V, and for the high-to-low logic transition to 0 V for an input voltage that swings between +5 V and −5 V. Therefore, we choose:

$$R_1 = R_2 = R_3 = 3K\Omega$$

$$V_{REF} = 5V, \ + V_{CC} = 5V \ \text{ and } \ -V_{CC} = -5V$$

Then:

$$R_{123} = 1K\Omega$$

$$V_{FPOS} = \left(\frac{R_{123}}{R_2}\right) V_{REF} + \left(\frac{R_{123}}{R_3}\right) V_{CC} = \left(\frac{1}{3}\right)5 + \left(\frac{1}{3}\right)5 = \frac{10}{3} = 3.3V$$

$$V_{FNEG} = \left(\frac{R_{123}}{R_2}\right) V_{REF} - \left(\frac{R_{123}}{R_3}\right) V_{CC} = \left(\frac{1}{3}\right)5 - \left(\frac{1}{3}\right)5 = 0V$$

The operation and the output waveform generation of the Schmitt trigger are explained in Fig. 6.19 according to these numerical values. If V_{IN} is less than the first set value, $V_{FPOS} = 3.3$ V, then V_{OUT} transitions to $+V_{CC} = 5$ V. For input voltages greater than 3.3 V, V_{OUT} transitions to $-V_{CC} = -5$ V, and V_F becomes $V_{FNEG} = 0$ V. Here, V_{OUT} stays at −5 V until V_{IN} drops below the second set value, 0 V. As soon as V_{IN} is less than 0 V,
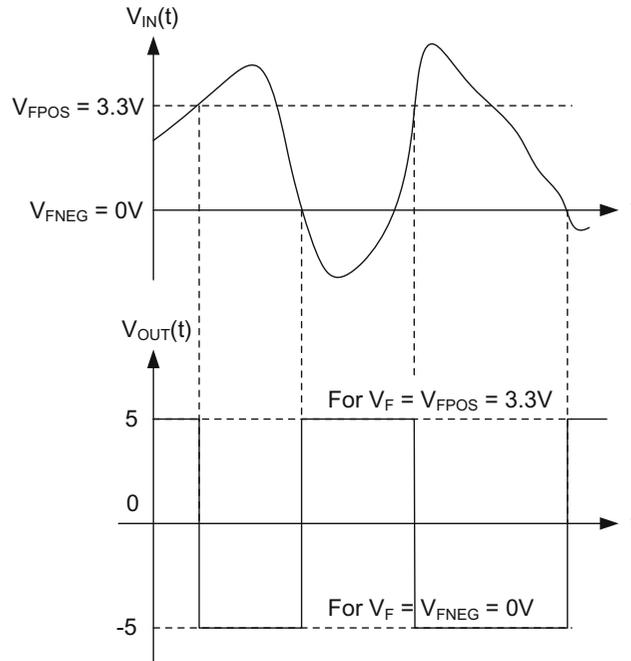
Fig. 6.19 Schmitt trigger input and output waveforms

V_{OUT} transitions back to $+V_{CC} = 5$ V, and V_F becomes $V_{FPOS} = 3.3$ V. The next output change to -5 V does not take place until V_{IN} reaches above 3.3 V.

6.6 Square Waveform Generator

Operational amplifiers are excellent candidates to generate square waveforms with varying duty cycles. The circuit in Fig. 6.20 generates a square waveform at V_{OUT} based on an RC feedback circuit.

Suppose the operational amplifier has dual power supplies at $+V_{CC}$ and $-V_{CC}$. Assume that the initial value of $V_{OUT} = +V_{CC}$ and $V_{SIG} = -V_{CC}$ since the output changes between $+V_{CC}$ and $-V_{CC}$. When $V_{OUT} = +V_{CC}$, V_{REF} becomes:

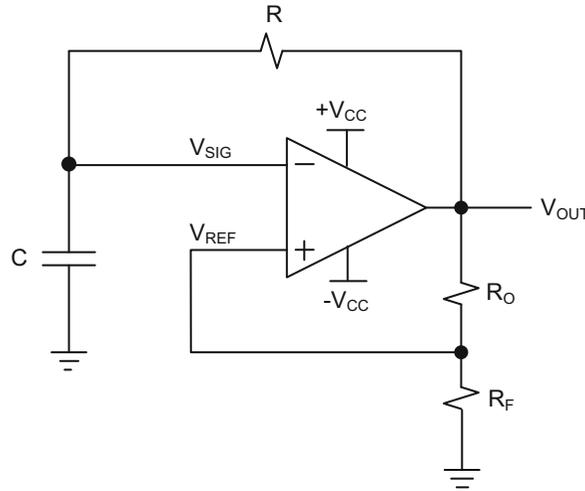$$V_{REF} = V_{OUT} \frac{R_F}{R_F + R_O} = V_{CC} \frac{R_F}{R_F + R_O} \tag{6.37}$$

Fig. 6.20 Square wave generator circuit

When V_{SIG} departs from $-V_{CC}$ and approaches towards $+V_{CC}$ with a time constant of RC, it produces the following expression:

$$V_{SIG} = -V_{CC} + 2V_{CC}\left[1 - \mathbf{exp}\left(-\frac{t}{RC}\right)\right] \tag{6.38}$$

However, as soon as V_{SIG} climbs above $V_{REF} = \dfrac{R_F}{R_F + R_O} V_{CC}$, V_{OUT} changes its polarity and it becomes $-V_{CC}$. Now, the accumulated charge on the capacitor starts discharging through R and the operational amplifier's output impedance. As a result, V_{SIG} starts decreasing from $\dfrac{R_F}{R_F + R_O} V_{CC}$ towards $-V_{CC}$ with the same time constant, RC. Thus,

$$V_{SIG} = V_{CC}\left(1 + \frac{R_F}{R_F + R_O}\right)\mathbf{exp}\left(-\frac{t}{RC}\right) - V_{CC} \tag{6.39}$$

This discharge cycle continues until V_{SIG} reaches a new V_{REF} value, $-\dfrac{R_F}{R_F + R_O} V_{CC}$, at which the output changes its polarity again to $+V_{CC}$. With $V_{OUT} = +V_{CC}$, V_{REF} transitions back to $\dfrac{R_F}{R_F + R_O} V_{CC}$. Now, V_{SIG} starts increasing from $-\dfrac{R_F}{R_F + R_O} V_{CC}$ towards $+V_{CC}$, but stops at $\dfrac{R_F}{R_F + R_O} V_{CC}$ because the output transitions to $-V_{CC}$ as mentioned previously. The waveforms at V_{SIG} and V_{OUT} are shown in Fig. 6.21.
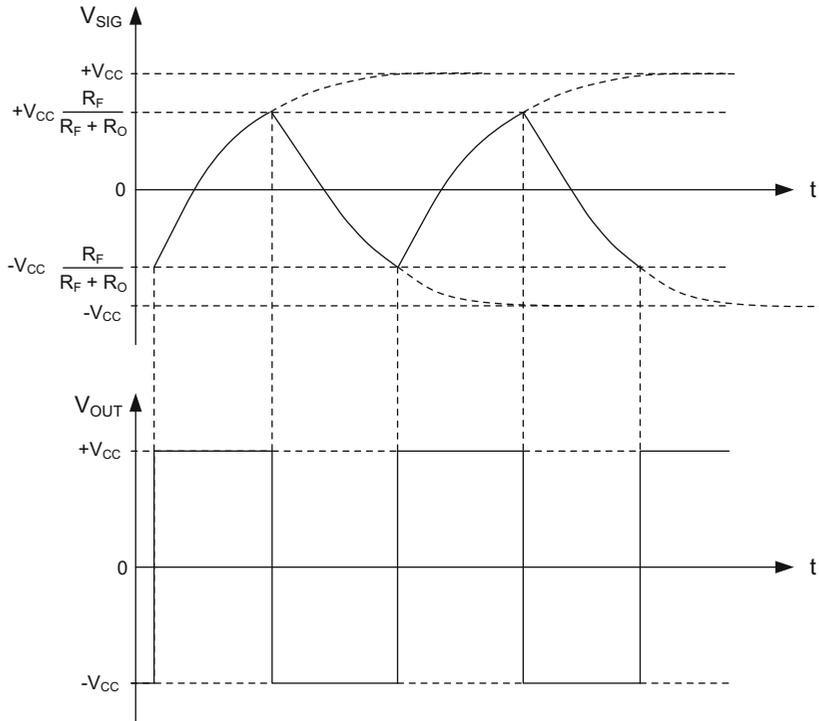
Fig. 6.21 V_{SIG} and V_{OUT} waveforms of the square wave generator in Fig. 6.20

Review Questions

1. Determine the amplification, $V_{OUT}(t)/V_S(t)$, in the circuit below.

   The operational amplifier is non-ideal, and there is a voltage drop between the negative and positive input terminals, Δv, as shown in the figure. The amplifier has also a limited gain, A, between its differential input, Δv, and its output, $V_{OUT}(t)$.
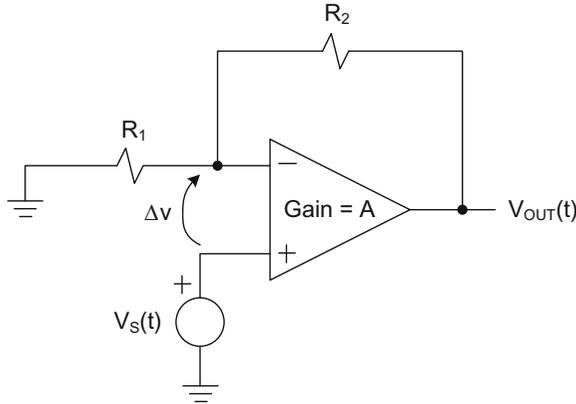
2. Assuming the operational amplifier is ideal in the circuit below, find the output voltage, $V_{OUT}(t)$, in terms of $V_{IN}(t)$ and $V_{REF}(t)$.
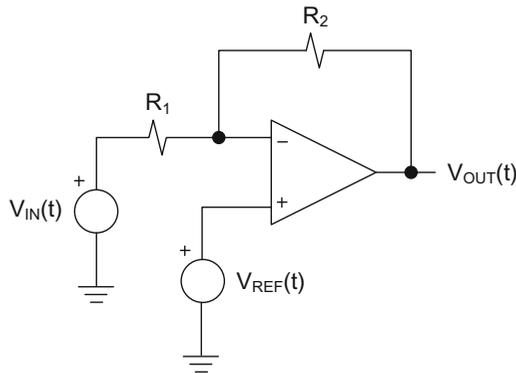

3. Assuming the operational amplifier in the circuit below is ideal, find the output voltage, $V_{OUT}(t)$, in terms of $V_{IN}(t)$ using time-domain analysis. Plot the output voltage as a function of time if a square wave is applied at its input as shown below. The value of V in the input waveform is assumed to be smaller than the power supply voltage of the operational amplifier.
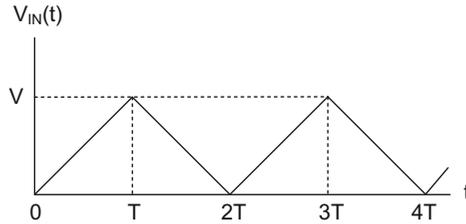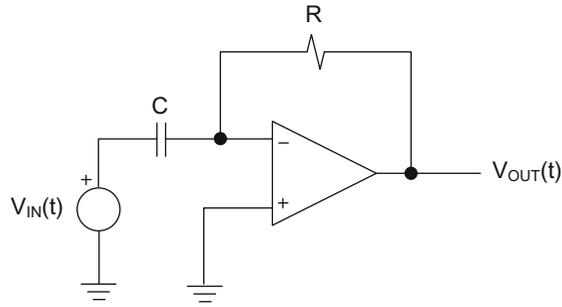
4. The following circuit shows a non-ideal operational amplifier where there is the voltage drop, Δv, between the positive and negative input terminals. Also, the amplifier exhibits a limited voltage gain, A, between its differential input and output. However, the input impedance, R_{IN}, at each input terminal is considerably high, approaching infinity. Thus, the current going into any input terminal is assumed to be 0 A.

(a) Find the overall voltage gain, $V_{OUT}(t)/V_{IN}(t)$, of the circuit.
(b) Now, assume the operational amplifier is ideal with $\Delta v = 0$ V, and A approaching to infinity. Apply a small signal sinusoidal voltage to $V_{IN}(t)$. What value does $V_{OUT}(t)/V_{IN}(t)$ approach as frequency increases? Compute the absolute value of $V_{OUT}(t)/V_{IN}(t)$.
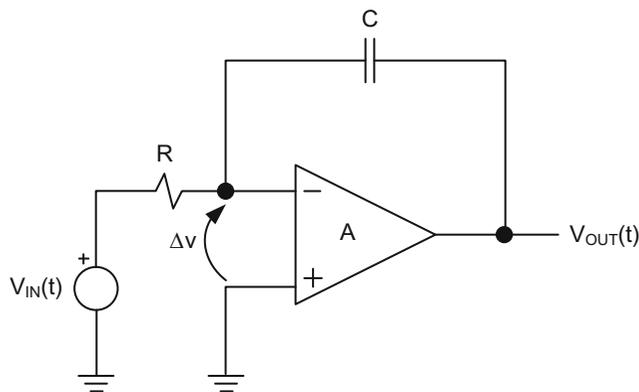
5. The following non-inverting operational amplifier circuit is designed to have a positive amplification of 10, and it is used to eliminate the DC component of the sensor signal. If V_O is the DC component and ΔV_S is the time varying true component of the sensor signal, $V_S = V_O + \Delta V_S$, find the relative values of R_1 and R_2, and compute the offset voltage, V_{OFFSET}, to eliminate V_O.
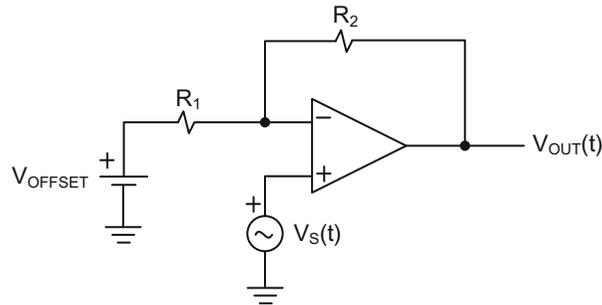
6. The following operational amplifier RC oscillator generates square waveform whose values change between $+V_{CC}$ and $-V_{CC}$. Consider $R_O = R_F = R = 10\ K\Omega$ in this circuit.

   (a) Describe the operation of the circuit. First assume V_{OUT} is at $+V_{CC}$. Explain how the switching to $-V_{CC}$ takes place at the output. Draw the waveform for $V_{OUT}(t)$, $V_{REF}(t)$ and $V_{SIG}(t)$.
   (b) Calculate the value of the capacitor, C, if the period of oscillation is 2 μs. That means that $V_{OUT}(t)$ is at $+V_{CC}$ for a period of 1 μs and at $-V_{CC}$ for a period of 1 μs.
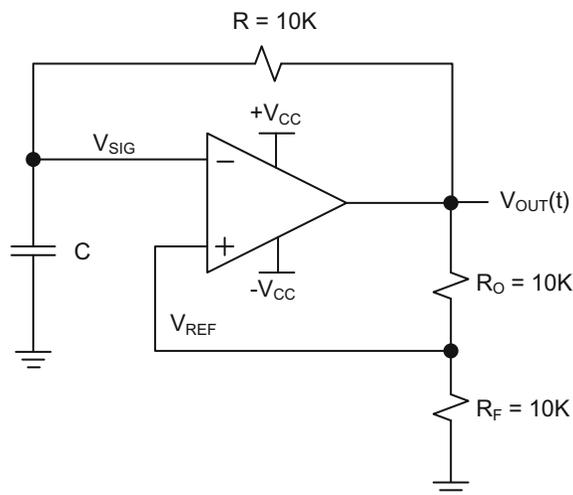
7. The following differential equation is given:

$$y(t) = A \frac{dx(t)}{dt} + Bx(t)$$

where A and B are constants, x(t) is a time varying voltage produced by a sensor, and y(t) is the analog output.

Generate y(t) using ideal operational amplifiers. Find the constants, A and B, in terms of the components used in the circuit.

8. Assuming the operational amplifier in the circuit below is non-ideal, and there is a voltage drop, Δv, between the negative and positive inputs as shown in the figure. The amplification factor between the differential input and the output is A. Derive the equation for $V_{OUT}(t)$ in terms of $V_1(t) - V_2(t)$.