Chapter 28
Value Theory (Axiology)

Erik Carlson

Abstract This chapter deals with an area of study sometimes called "formal value theory" or "formal axiology". Roughly characterized, this area investigates the structural and logical properties of value properties and value relations, such as goodness, badness, and betterness. There is a long-standing controversy about whether goodness and badness can, in principle, be measured on a cardinal scale, in a way similar to the measurement of well-understood quantitative concepts like length. Sect. 28.1 investigates this issue, mainly by comparing the properties of the relations "longer than" and "better than". In Sect. 28.2, some attempts to define goodness and badness in terms of the betterness relation are discussed, and a novel suggestion is made. Sect. 28.3, finally, contains an attempt to define the recently much discussed value relation "on a par with" in terms of the more familiar betterness relation.

This chapter deals with an area of study sometimes called "formal value theory" or "formal axiology". Roughly characterized, this area investigates the structural and logical properties of value properties and value relations, such as goodness, badness, and betterness. In contrast, "substantial value theory" or "substantial axiology" seeks to determine what is good and bad, and what is better than what. A third branch of value theory, usually called "meta-ethics", although "meta-axiology" would perhaps be a more appropriate term, discusses the ontological status of value properties, and the semantics of value terms and value judgements. Most philosophers would agree that the demarcations between the three areas are not sharp. Some of the issues to be discussed in this chapter arguably straddle the distinction between formal and substantial axiology, in particular.

The main focus of most axiological investigations is *intrinsic* or *final* value; i.e., the value a thing has "in itself", or "for its own sake". There is not space here for

E. Carlson (✉)
Uppsala University, Uppsala, Sweden
e-mail: Erik.Carlson@filosofi.uu.se

S. O. Hansson, V. F. Hendricks (eds.), *Introduction to Formal Philosophy*, Springer Undergraduate Texts in Philosophy, https://doi.org/10.1007/978-3-319-77434-3_28

trying to provide precise definitions of these concepts. [A helpful discussion and overview of the literature on intrinsic value can be found in the "Introduction" to Rønnow-Rasmussen and Zimmerman [29].]

28.1 "Longer than" and "Better than"

A useful way of introducing several central problems of formal axiology is to ask to what extent value (i.e., goodness and badness) is similar to a familiar and well-understood quantitative concept like length. Many philosophers have assumed that value can, at least in principle, be measured additively, analogously to the measurement of length. [An early example is Bentham [4]] Others have denied this. Often, however, this discussion is conducted without much recognition of what it takes for the additivity assumption to be true. Let us therefore state the conditions necessary for standard extensive or additive measurement, and then ask whether value can reasonably be thought to satisfy these conditions. Primarily, this will amount to comparing the properties of the relations "longer than" and "better than".

Letting $\succsim$ denote "at least as long as", we can define "longer than" , denoted $\succ$, and "equally long as", denoted $\sim$, as follows: $a \succ b$ iff (if and only if) $a \succsim b \wedge \neg(b \succsim a)$; $a \sim b$ iff $a \succsim b \wedge b \succsim a$. If $X = \{a, b, c \ \ldots\}$ is a set of items that have length, the relational structure $(X, \succsim)$ has the following properties:

Completeness. For any a and b in X, $a \succsim b \vee b \succsim a$.
Transitivity. If $a \succsim b \wedge b \succsim c$, then $a \succsim c$.
Concatenation. Any a and b in X can be put together, or "concatenated", into an item, denoted $a \circ b$, that also has length.
Monotonicity. $a \succsim b$ iff $a \circ c \succsim b \circ c$ iff $c \circ a \succsim c \circ b$.
Weak associativity. $a \circ (b \circ c) \sim (a \circ b) \circ c$.
Archimedeanness. For any a and b in X, there is a positive integer n, such that $na \succ b$, i.e., a concatenation of n copies of a is longer than b.

The last condition involves a certain amount of idealization, since there may not actually exist a sufficient number of copies of a.

These properties together imply that $(X, \succsim, \circ)$ is a "closed extensive structure". [This is a slight simplification. Actually, a somewhat more complicated Archimedean condition is needed. See [21], 73.] This, in turn, means that there is a function f with real numbers as values, such that (i) $f(a) \geq f(b)$ iff $a \succsim b$, and (ii) $f(a \circ b) = f(a) + f(b)$. Further, another function g satisfies properties (i) and (ii) iff g is a "similarity transformation" of f; i.e., iff there is a real number $x > 0$, such that, for all a in X, $g(a) = xf(a)$. This amounts to measurement on a ratio scale. Thus, if $f(a) = 4$ and $f(b) = 2$, it follows that a is twice as long as b.

Now, let X be a set of value bearers, and let $\succsim$, $\succ$, and $\sim$ denote "at least as good as", "better than", and "equally good as", respectively. Which of the above conditions can be expected to hold? It appears that they are all more or less controversial. Let us briefly consider each condition in turn.

Completeness Many substantial axiologies are pluralistic, recognizing value bearers of different kinds. Suppose, for example, that friendship and pleasure both have value. It then seems somewhat implausible that $a \succsim b \vee b \succsim a$ holds for every instance a of friendship and every instance b of pleasure. Such appeals to intuition, against completeness, are sometimes buttressed by the "small improvement argument". Let a be an instance of friendship and let b be an instance of pleasure, such that we are disinclined to claim either that $a \succ b$, or that $b \succ a$. Does it follow that $a \sim b$? If so, anything better than b must be better than a (given that $\sim$ is an equivalence relation). Consider an instance of pleasure b^+, which is just like b, only slightly more intense. Although $b^+ \succ b$, we will probably not judge that $b^+ \succ a$. Hence, the small improvement argument concludes, $\neg(a \sim b)$, implying that $\succsim$ is not complete.

An objection to the small improvement argument is that our unwillingness to judge that $b^+ \succ a$ only proves that we, in the first comparison, did not (rationally) judge that $a{\sim}b$. It does not prove that we judged that $\neg(a{\sim}b)$. To refrain from making a judgement of equality is not to make a judgement of nonequality. The small improvement argument presupposes, however, a judgement of the latter kind [27].

Transitivity The transitivity of $\succsim$ has been questioned by a number of philosophers. An interesting type of alleged counterexample is due to Stuart Rachels [26] and Larry Temkin [31]. Their examples can be seen as applications of the following general assumptions:

(1) For any painful experience, no matter what its intensity and duration, it would be better to have that experience than one that was only slightly less intense but twice as long.
(2) There is a continuum of painful experiences ranging in intensity from extreme forms of torture to the mild discomfort of, say, a hangnail.
(3) A mild discomfort for the duration of one's life would be preferable to two years of excruciating torture, no matter the length of one's life.

Rachels and Temkin argue from assumptions 1 to 3 to the conclusion that $\succ$ is not transitive. (If $\succ$ is not transitive, $\succsim$ cannot be transitive, either. For suppose that $\succsim$ is transitive, while $\succ$ is not. There is then a case such that $a \succ b \wedge b \succ c \wedge a \sim c$. This implies that $c \succsim a \wedge a \succsim b \wedge \neg(c \succsim b)$, contradicting the assumption that $\succsim$ is transitive.)

Other philosophers claim that we can know *a priori* that $\succ$ is always transitive, since transitivity is part of the meaning of comparatives like "better than". Thus, John Broome finds it "self-evident" that any comparative relation is necessarily transitive. Since this is a conceptual truth, "not much argument is available to support it directly" ([6], 51). A defense of the transitivity assumption must then consist mainly in responses to apparent counterexamples. Broome notes that many such examples involve large numbers, and argues that our intuitions about large numbers are unreliable. We may, for example, be unable to grasp what it would be like to have a hangnail for thousands of years. It is far from clear, though, that intuitively plausible counterexamples to transitivity must involve large numbers (see, e.g., [23]).

If Rachels and Temkin are right, $\succ$ is sometimes not only nontransitive, but *cyclical*. That is, there are value bearers $a, b, c, \ldots, y, z$, such that $a \succ b$, $b \succ c, \ldots, y \succ z$, and $z \succ a$. Let us call a structure of this kind a "betterness cycle". Whether or not there are betterness cycles, certain structural restrictions apply to any such case. For example, no betterness cycle can contain both good and bad options. The following propositions are surely universal truths:

(4) No option is both good and bad (all things considered).
(5) If a is good and $b \succ a$, then b is good.
(6) If a is bad and $a \succ b$, then b is bad.

Let a be an arbitrary option in a betterness cycle. If a is good, iterated applications of (5) entail that every option in the ordering is good. If a is bad, iterated applications of (6) entail that every option is bad. It thus follows from (4) to (6) that no betterness cycle contains both good and bad options. This is of some significance, since certain putative examples of betterness cycles contain intuitively good as well as intuitively bad options (see, e.g., [25]).

Concatenation Whether the concatenation condition is satisfied may depend on what kinds of entities are bearers of value. If the value bearers are taken to be propositional entities, concatenation is naturally identified with conjunction. This immediately leads to a problem, however, since conjunction is idempotent; i.e., $a \wedge a = a$. Given reflexivity of $\sim$, this means that $a \circ a \sim a$. If monotonicity and weak associativity hold, the assumption that $a \circ a \sim a$, for all a, implies that all value bearers are equally good.

A possible solution to this problem is to define $a \circ a$ as the conjunction of a with a numerically different but qualitatively identical propositional entity. For example, if a is the state of affairs that Alf is happy to degree 10, $a \circ a$ could be identified with the conjunction $a \wedge a^*$, where a^* is the state that Alf's counterpart in some other possible world is happy to degree 10. ($a^* \circ a^*$ then has to be identified with the conjunction of a^* and a third state a^{**}.)

If other kinds of entities, for example material objects, are among the value bearers, concatenation might be defined in terms of mereological fusion, rather than conjunction. Since also mereological fusion is usually understood as idempotent, the problem of how to understand self-concatenation remains. It should be noted, though, that self-concatenation must be defined by means of identical "copies" also in the context of length or mass measurement (See [21], 3f.)

Monotonicity The monotonicity condition is closely connected to G. E. Moore's famous principle of "organic unities". Moore claimed that, in some cases, "the intrinsic value of a whole is neither identical with nor proportional to the sum of the values of its parts" ([22], 184). One of Moore's examples of an organic unity is the state of being conscious of a beautiful object. Moore took such a state to be of great intrinsic value, containing as parts (in some sense) the object and the state of being conscious. But neither of these parts has, according to Moore, much intrinsic value considered in isolation.

However, Moore's way of formulating the principle of organic unities is unfortunate, since it is meaningful to add the values of two items only if value is measurable on an additive ratio scale. Measurability on such a scale implies, in turn, that the value of a whole *is* proportional to the sum of the values of its parts (see [12]). On a literal interpretation, therefore, Moore's claim does not make much sense. Arguably, what Moore really meant to assert was simply that the monotonicity condition does not hold. It is, indeed, easy to think of putative counterexamples to monotonicity. To borrow a case from Roderick Chisholm ([16], 306) suppose that a and b are two identical beautiful paintings, and that c is a beautiful piece of music. Suppose also that the value of contemplating a, b or c is the same. It nevertheless seems that the whole consisting in the contemplation of a and c is better than the whole consisting in the contemplation of a and b. Some philosophers have suggested, however, that a restricted version of the monotonicity assumption suffices for the purposes of value measurement [18, 32].

Weak Associativity If concatenation is identified with conjunction or mereological fusion, the weak associativity condition probably holds. As regards a concatenation operation involving physical interaction between objects, on the other hand, weak associativity may be questionable. As Fred Roberts notes, "combining a with b first and then bringing in c might create a different object from that obtained when b and c are combined first. To give an example, if a is a flame, b is some cloth, and c is a fire retardant, then combining a and b first and then combining with c is quite different from combining b and c first and then combining with a." ([28], 125) Clearly, this difference could be evaluatively relevant.

Archimedeanness The Archimedean condition has been denied by many philosophers. To cite just two examples, Franz Brentano judged it "quite possible for there to be a class of goods which could be increased *ad indefinitum* but without exceeding a given finite good" [5], while W. D. Ross believed that, although virtue and pleasure are both good, "*no* amount of pleasure is equal to any amount of virtue, [. . .] in fact virtue belongs to a higher order of value, beginning at a point higher on the scale of value than pleasure ever reaches [. . .]". ([30], 150)

Gustaf Arrhenius has argued that the existence of such "superior goods" has an implausible implication. Assuming a and b to be good, Arrhenius defines a as "weakly superior" to b iff there is a positive integer m, such that $ma \succ nb$, for every positive integer n. Now let $a_1, \ldots, a_k$ be a finite sequence of items, such that $a_1 \succ a_2 \succ \ldots \succ a_{k-1} \succ a_k$, and a_1 is weakly superior to a_k. Arrhenius shows that any such sequence must contain a pair a_i, a_{i+1}, such that a_i is weakly superior to a_{i+1}. However, he believes that for most or all types of goods, the sequence $a_1, \ldots, a_k$ can be chosen so that the difference in value between adjacent items is only marginal. Hence, the assumption that a_1 is weakly superior to a_k implies that a_i

is only marginally better than, although weakly superior to a_{i+1}. This, Arrhenius contends, is implausible ([1], 301).

The defender of superior goods can retort that since Arrhenius does not explain what a "marginal" value difference is, he provides no ground for denying that weak superiority is compatible with a merely marginal difference. As Arrhenius acknowledges, there need not be a pair a_j, a_{j+1} in the above sequence, such that a_j is *strongly* superior to a_{j+1}, in the sense that $a_j \succ na_{j+1}$, for all positive integers n. It might thus be claimed that strong, but not weak, superiority is incompatible with a merely marginal difference ([1], 301; [2], 138). Another response to Arrhenius' argument would be to deny the claim that any value difference can be spanned in a finite number of steps, such that each step involves only a marginal difference. If we share Ross' view that any amount of virtue is better than any amount of pleasure, why should we believe that a finite number of marginal worsenings could bridge the value gap between an instance of virtue and an instance of pleasure? Indeed, it could be argued that the claim of superiority essentially involves the denial of this contention. If so, Arrhenius' argument begs the question.

The most problematic of the conditions we have discussed are perhaps completeness, Archimedeanness, and monotonicity. It can be shown, however, that if value is represented by other mathematical entities than real numbers, measurement on a kind of generalized ratio scale is possible even if the former two conditions do not hold [7, 8, 10]. On the other hand, the truth of some version of the monotonicity condition appears essential for any form of extensive measurement, and, in fact, for measurement on any scale stronger than an ordinal scale.

28.2 Defining "Good" and "Bad" in Terms of "Better than"

On the face of it, there is another important difference between value, on the one hand, and quantities like length, on the other. There are *bad* things, i.e., things with *negative* value, but there are no things with negative length (speculative physics aside). Moreover, the "zero point", dividing the good from the bad things, appears to be absolute, rather than dependent on the scale of measurement. (This is in contrast to, e.g., temperature. Some temperatures are positive if measured on the Fahrenheit scale, but negative if measured on the Celsius scale.) Many philosophers have attempted to define goodness and badness in terms of betterness. Such definitions, if possible, would arguably be desirable for reasons of theoretical simplicity. In a very influential paper, Roderick Chisholm and Ernest Sosa [17] proposed the following definitions, with "I", "N", "G", and "B" standing for, respectively, "intrinsically indifferent", "intrinsically neutral", "intrinsically good", and "intrinsically bad":

D1. $a \sim b$ iff $\neg(a \succ b) \wedge \neg(b \succ a)$.
D2. Ia iff $\neg(a \succ \neg a) \wedge \neg(\neg a \succ a)$.
D3. Na iff $(\exists b)$ (I$b \wedge a \sim b$).
D4. Ga iff $(\exists b)$ (I$b \wedge a \succ b$).

D5. Ba iff $(\exists b)$ (I$b \wedge b \succ a$).

Chisholm's and Sosa's theory assumes equivalents to the following axioms:

A1. If $a \succ b$, then $\neg(b \succ a)$.
A2. If $a \succ c$, then $a \succ b \vee b \succ c$.
A3. If I$a \wedge$ Ib, then $a \sim b$.
A4. If Ga or B$\neg a$, then $a \succ \neg a$.

An important feature of Chisholm's and Sosa's theory is that, unlike many previous theories, it does not assume that a state of affairs is good if it is better than its negation, and bad if it is worse than its negation. The state that there are happy egrets is, they assume, intrinsically good, while the state that there are no happy egrets is intrinsically neutral. (The latter state is not intrinsically bad, since the mere absence of happiness does not "rate any possible universe a minus".) Analogously, the state that there are unhappy egrets is intrinsically bad, whereas the state that there are no unhappy egrets is neutral. (The latter state is not intrinsically good, since the mere absence of unhappiness does not "rate any possible universe a plus".)

D1 excludes the possibility that a and b are incomparable with respect to intrinsic value, in the sense that $\neg(a \succ b) \wedge \neg(b \succ a) \wedge \neg\,(a \sim b)$. (Cf. the discussion of completeness of $\succsim$, in Sect. 28.1) If incomparability is possible, D1 is not an appropriate definition of "is equal in intrinsic value to". Furthermore, b may be incomparable to each of a and c, although $a \succ c$. This would mean that A2 is violated.

Philip Quinn [24] has objected to Chisholm's and Sosa's logic on precisely the grounds that it illegitimately rules out the possibility of incomparability. Even if we assume a hedonistic axiology, Quinn remarks, it is far from obvious that the value of Smith's enjoying the taste of apples is comparable to the value of her enjoying the sound of Beethoven's Ninth Symphony. Quinn argues, nonetheless, that universal comparability can be shown to be true. He retains Chisholm's and Sosa's axioms A3 and A4, and proposes, in addition, the following axioms:

A5. *Transitivity.* If $a \succsim b \wedge b \succsim c$, then $a \succsim c$.
A6. $(a \succsim (a \vee b) \vee b \succsim (a \vee b)) \wedge ((a \vee b) \succsim a \vee (a \vee b) \succsim b)$.
A7. $(a \succsim (a \vee b) \vee (a \vee b) \succsim a)$ iff $(b \succsim (a \vee b) \vee (a \vee b) \succsim b)$.

Quinn shows that A5 to A7 entail that $a \succsim b$ or $b \succsim a$ holds for all a and b. In other words, universal comparability (completeness) is true.

A6 is hardly unassailable, though. To use Quinn's own example, what are the grounds for assuming that either $a =$ *Smith enjoys apples*, or $b =$ *Smith enjoys Beethoven*, is at least as good as $a \vee b$? If it is intuitively plausible to judge a and b incomparable, it appears equally plausible to judge each of these states incomparable to their disjunction.

Sven Ove Hansson [20] has suggested a more general definition of goodness and badness in terms of betterness. Hansson's proposal assumes neither universal comparability nor transitivity of betterness. However, his and, to the best of my knowledge, every other extant proposal presuppose that the value bearers are propositional entities, which can be negated. Since many value theorists believe that non-propositional entities, such as persons or material objects, can have intrinsic or final value, a definition format that does not rely on negation or indifference is desirable. Assuming that at least some value bearers can be concatenated, there is a fairly simple way to construct such a format [Carlson [13] contains further discussion of the proposal sketched below].

First, we define an item a as "universally null" iff, for all value bearers b, such that $a \circ b$ is a value bearer: $a \circ b \sim b$. Thus, a universally null item does not affect the intrinsic value of any whole of which it is a part. (In Chisholm's and Sosa's parlance, such an item rates any possible universe a zero.)

Let us assume the following four axioms:

A8. $\succsim$ is a quasi-order; i.e., reflexive and transitive.
A9. There is at least one universally null item that is a value bearer.
A10. For any universally null value bearers a and b, whose coexistence is logically possible, $a \circ b$ is a value bearer.
A11. For any complex value bearer $a \circ b$, it holds that $a \circ b \sim b \circ a$.

On the basis of these axioms, and letting "UN" abbreviate "universally null", we may propose the following definitions of "intrinsically good", "intrinsically bad", and "intrinsically neutral":

D6. Ga iff $(\exists b)$ (UN$b \wedge a \succ b$).
D7. Ba iff $(\exists b)$ (UN$b \wedge b \succ a$).
D8. Na iff $(\exists b)$ (UN$b \wedge a \sim b$).

If some value bearers are incomparable, there may be reason to assume the existence of a fourth value category, in addition to intrinsic goodness, badness, and neutrality (See [11]). This category, which may be labelled "intrinsic indeterminacy", is readily incorporated into our proposal. Defining "is incomparable in intrinsic value to", symbolized $||$, as $a \,||\, b$ iff $\neg(a \succsim b) \wedge \neg(b \succsim a)$, we may define "intrinsically indeterminate", abbreviated "IND", as follows:

D9. INDp iff $(\exists b)$ (UN$b \wedge a \,||\, b$).

The following twenty propositions can be derived from A8 to A11 and D6 to D9:

(i) If UN$a \wedge$ UNb, then $a \sim b$.
(ii) For any a, exactly one of the following is true: Ga, Ba, Na, or INDa.
(iii) If G$a \wedge b \succsim a$, then Gb.
(iv) If G$a \wedge a \,||\, b$, then G$b \vee$ INDb.
(v) If B$a \wedge a \succsim b$, then Bb.
(vi) If B$a \wedge a \,||\, b$, then B$b \vee$ INDb.
(vii) If N$a \wedge b \succ a$, then Gb.
(viii) If N$a \wedge a \succ b$, then Bb.
(ix) If N$a \wedge a \sim b$, then Nb.
(x) If N$a \wedge a \,||\, b$, then INDb.

(xi) If IND$a \wedge b \succ a$, then G$b \vee$ INDb.
(xii) If IND$a \wedge a \succ b$, then B$b \vee$ INDb.
(xiii) If IND$a \wedge a \sim b$, then INDb.
(xiv) If G$a \wedge$ Bb, then $a \succ b$.
(xv) If G$a \wedge$ Nb, then $a \succ b$.
(xvi) If G$a \wedge$ INDb, then $a \succ b \vee a \mid\mid b$.
(xvii) If B$a \wedge$ Nb, then $b \succ a$.
(xviii) If B$a \wedge$ INDb, then $b \succ a \vee a \mid\mid b$.
(xix) If N$a \wedge$ Nb, then $a \sim b$.
(xx) If N$a \wedge$ INDb, then $a \mid\mid b$.

The proofs of these propositions are simple and will not be stated here.

Apart from being more generally applicable than earlier suggestions, D6 to D9 have the virtue of not prejudging questions in substantial axiology. Importantly, they permit organic unities of various sorts. For example, it is consistent with these definitions that a concatenation of intrinsically good items is intrinsically bad, or that a concatenation of intrinsically bad items is intrinsically good.

28.3 Comparability and Parity

It has usually been taken for granted, in line with our definition of incomparability in Sect. 28.2, that two value bearers, a and b, are comparable with respect to value iff $a \succ b$, or $b \succ a$, or $a \sim b$. If so, universal comparability just means that $\succsim$ is complete. In recent years, however, this view has been questioned. Ruth Chang has argued that there is a positive value relation, "on a par with", that is incompatible with the familiar relations. If two items are on a par, they are comparable with respect to value, although neither item is better than the other, and they are not equally good. As possible examples of parity Chang mentions the value relationships between two artists, such as Mozart and Michelangelo, or between two careers, such as one in accounting and one in skydiving, or between two Sunday enjoyments, such as an afternoon at the museum and one hiking in the woods [15].

Chang's characterization of the parity relation is sketchy and not very clear. She does not, for example, discuss the logical properties of the relation. Symmetry should surely hold. If a is on a par with b, then b is on a par with a. Further, since parity is assumed to be incompatible with value equality, and since equality is a reflexive relation, parity must be irreflexive. A symmetric and irreflexive relation cannot be transitive. Since *in*transitivity is out of the question, parity, at least as conceived by Chang, thus has to be nontransitive; i.e., neither transitive nor intransitive. (Chang has confirmed, in personal communication, that she understands the relation as symmetric, irreflexive, and nontransitive.)

Given that parity, as understood by Chang, has these logical properties, it can be defined in terms of the standard value relations. We retain our assumption that $\succsim$

is a quasi-order on the relevant set X of value bearers, and introduce the following definitions:

D10. An item $a \in X$ is an "upper semibound" of a set $S \subseteq X$ iff there is no $b \in S$, such that $b \succ a$.
D11. An item $a \in X$ is a "minimal upper semibound" of a set $S \subseteq X$ iff there is no upper semibound b of S, such that $a \succ b$.
D12. An item $a \in X$ is a "lower semibound" of a set $S \subseteq X$ iff there is no $b \in S$, such that $a \succ b$.
D13. An item $a \in X$ is a "maximal lower semibound" of a set $S \subseteq X$ iff there is no lower semibound b of S, such that $b \succ a$.

Next, we define a relation $\succcurlyeq$, which we may call "almost better than":

D14. $a \succcurlyeq b$ iff a is either (i) a minimal upper semibound of the set of $c \in X$, such that $\neg(c \succ b)$, or (ii) a maximal lower semibound of the set of $d \in X$, such that $\neg(b \succ d)$.

Let us say that a is "almost worse than" b iff $b \succcurlyeq a$. With the help of $\succcurlyeq$, we define "on a par with", denoted by $\asymp$:

D15. $a \asymp b$ iff (i) $\neg(a \succsim b) \wedge \neg(b \succsim a)$, and (ii) $(\exists c)$ $(c \succ a \wedge c \succcurlyeq b$, or $c \succ b \wedge c \succcurlyeq a$, or $a \succ c \wedge b \succcurlyeq c$, or $b \succ c \wedge a \succcurlyeq c)$.

Less formally put, two items are on a par just in case neither is at least as good as the other, but there is a third item that is either better than one of them and almost better than the other, or worse than one of them and almost worse than the other.

Given the following four assumptions, of which (9) is the axiologically most significant one, it can be shown that D15 yields a *necessary* condition for parity:

(7) If a and b are on a par, then $\neg(a \succsim b) \wedge \neg(b \succsim a)$.
(8) If $(\forall c)$ $(c \succ a$ iff $c \succ b$, and $a \succ c$ iff $b \succ c)$, then a and b are not on a par.
(9) If a and b are on a par, there is an item that is either (i) better than any item that is better than exactly one of a and b (call such an item "superior" to a and b), or (ii) worse than any item that is worse than exactly one of a and b (call such an item "inferior" to a and b).
(10) Every nonempty set $S \subseteq X$ with an upper (lower) semibound has a minimal upper (maximal lower) semibound.

Assumptions (7) to (10) imply that if a and b are on a par, then $a \asymp b$. Suppose that two items a and b are on a par. Hence, by (7), they are not standardly related. By (9), there is a superior or an inferior item, relative to a and b. Suppose that there is a superior item. By (8), there is an item that is better, or an item that is worse, than exactly one of a and b. Assume, first, that the former possibility obtains; e.g., that there is an item that is better than a, but not better than b. Let $S = \{c\colon c \succ a \wedge \neg(c \succ b)\}$. Since there is a superior item, S has an upper semibound. Hence, by (10), S has a minimal upper semibound, e. Let $S^* = \{d\colon \neg(d \succ b)\}$. We shall show, by *reductio*, that e is a minimal upper semibound of S^*, implying that $e \succcurlyeq b$ and $a \asymp b$.

If e is not an upper semibound of S^*, there is an $f \in S^*$, such that $f \succ e$. Since $e \succ a$, transitivity yields that $f \succ a$. But then $f \in S$, contradicting the assumption that e is an upper semibound of S. Hence, e is an upper semibound of S^*. If e is not a *minimal* upper semibound of S^*, there is an upper semibound g of S^*, such that $e \succ g$. But, since $S \subseteq S^*$, if g is an upper semibound of S^*, it is an upper semibound of S. This contradicts the assumption that e is a minimal upper semibound of S. Hence, e is a minimal upper semibound of S^*. By D14, therefore, $e \succcurlyeq b$. Since $e \succ a$, it follows that $a \asymp b$.

Now, suppose instead that there is no item that is better than exactly one of a and b. By (8), there is then an item that is worse than exactly one of the two items. Assume, thus, that $a \succ c \wedge \neg(b \succ c)$. Since there is a superior item, there is an item better than b. Further, since by assumption, any item that is better than b is better than a, and since $a \succ c$, it holds, for all d, that $d \succ b$ implies $d \succ c$. Hence, b is an upper semibound of the set $S = \{d: \neg(d \succ c)\}$. Moreover, since $b \in S$, b is a *minimal* upper semibound of S. It follows that $b \succcurlyeq c$. Since $a \succ c$, we conclude that $a \asymp b$.

We have thereby shown that if a and b are on a par, assumptions (7), (8) and (10) imply that $a \asymp b$, given the existence of an item superior to a and b. A similar argument shows that $a \asymp b$ follows from (7), (8) and (10), if there is an inferior item. Hence, (7) to (10) imply that if a and b are on a par, then $a \asymp b$. That is, D15 states a necessary condition for parity.

Does D15 also state a *sufficient* condition? In other words, is it always the case that if $a \asymp b$, then a and b are on a par? For this to hold, the following two claims must be true:

(11) If $a \asymp b$, then a and b are comparable.
(12) If $\neg(a \succsim b) \wedge \neg(b \succsim a)$, and a and b are comparable, then they are on a par.

Chang argues explicitly for (12), and (11) appears plausible given other assumptions she makes. Hence, D15 seems to be a satisfactory definition of Chang's notion of parity. [Carlson [9] contains a discussion of the plausibility of assumptions (7), (8), (9), (11) and (12), as well as of a different version of (10).]

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