Chapter 11
Chi-Square Tests for Cross-Tabs

First Example Cross-Tab

The underneath table shows two separate groups with patients assessed for suffering from sleepiness through the day. We wish to know whether there is a significant difference between the proportions of subjects being sleepy.

	Sleepiness	No sleepiness	
Group 1	5 (a)	10 (b)	15 (a+b)
Group 2	9 (c)	6 (d)	15 (c+d)
	14 (a+c)	16 (b+d)	30 (a+b+c+d)

The chi-square pocket calculator method is used for testing these data.

$$\chi^2 = \frac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(b+d)(a+c)} = \frac{(30-90)^2(30)}{15\times15\times16\times14} = \frac{3{,}600\times30}{15\times15\times16\times14}$$

$$= \frac{108.000}{50.400} = 2.143$$

The chi-square value equals 2.143. The chi-square table can tell us whether or not the difference between the groups is significant. See next page for the procedure to be followed.

Chi-Square Table (χ^2-Table)

The underneath chi-square table gives columns and rows: the upper row gives the p-values. The first column gives the degrees of freedom which is here largely in agreement with the numbers of cells in a cross-tab. The simplest cross-tab has 4 cells, which means $2\times2=4$ cells. The table has been constructed such that we have here $(2-1)\times(2-1)=1$ degree of freedom. Look at the row with 1 degree of freedom: a chi-square value of 2.143 is left from 2.706. Now look from here right up at the

T.J. Cleophas and A.H. Zwinderman, *Statistical Analysis of Clinical Data on a Pocket Calculator: Statistics on a Pocket Calculator*, DOI 10.1007/978-94-007-1211-9_11,
© Springer Science+Business Media B.V. 2011

Chi-squared distribution

	Two-tailed *P*-value			
df	0.10	0.05	0.01	0.001
1	2.706	3.841	6.635	10.827
2	4.605	5.991	9.210	13.815
3	6.251	7.815	11.345	16.266
4	7.779	9.488	13.277	18.466
5	9.236	11.070	15.086	20.515
6	10.645	12.592	16.812	22.457
7	12.017	14.067	18.475	24.321
8	13.362	15.507	20.090	26.124
9	14.684	16.919	21.666	27.877
10	15.987	18.307	23.209	29.588
11	17.275	19.675	24.725	31.264
12	18.549	21.026	26.217	32.909
13	19.812	22.362	27.688	34.527
14	21.064	23.685	29.141	36.124
15	22.307	24.996	30.578	37.698
16	23.542	26.296	32.000	39.252
17	24.769	27.587	33.409	40.791
18	25.989	28.869	34.805	42.312
19	27.204	30.144	36.191	43.819
20	28.412	31.410	37.566	45.314
21	29.615	32.671	38.932	46.796
22	30.813	33.924	40.289	48.268
23	32.007	35.172	41.638	49.728
24	33.196	36.415	42.980	51.179
25	34.382	37.652	44.314	52.619
26	35.563	38.885	45.642	54.051
27	36.741	40.113	46.963	55.475
28	37.916	41.337	48.278	56.892
29	39.087	42.557	49.588	58.301
30	40.256	43.773	50.892	59.702
40	51.805	55.758	63.691	73.403
50	63.167	67.505	76.154	86.660
60	74.397	79.082	88.379	99.608
70	85.527	90.531	100.43	112.32
80	96.578	101.88	112.33	124.84
90	107.57	113.15	124.12	137.21
100	118.50	124.34	135.81	149.45

upper row. The corresponding p-value is larger than 0.1 (10%). There is, thus, no significant difference in sleepiness between the two groups. The small difference observed is due to the play of chance.

Second Example Cross-Tab

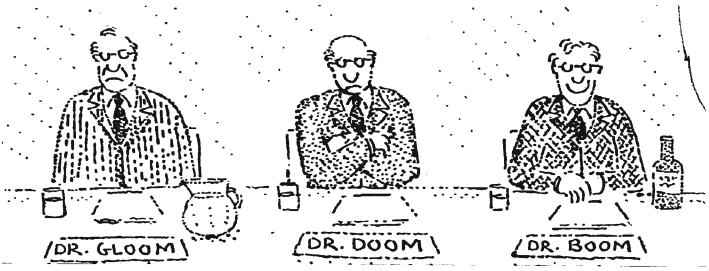
Two partnerships of internists have the intention to associate. However, in one of the two a considerable number of internists has suffered from a burn-out.

	Burn out	No burn out	
Partnership 1	3 (a)	7 (b)	10 (a+b)
Partnership 2	0 (c)	10 (d)	10 (c+d)
	3 (a+c)	17 (b+d)	20 (a+b+c+d)

$$\chi^2 = \frac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(b+d)(a+c)} = \frac{(30-0)^2(20)}{10\times10\times17\times3} = \frac{900\times20}{.....} = 3.529$$

According to the chi-square table of the previous page a p-value is found of <0.10.

This means that no significant difference is found, but a p-value between 0.05 and 0.10 is looked upon as a trend to significance. The difference may be due to some avoidable or unavoidable cause. We should add here that values in a cell lower than 5 is considered slightly inappropriate according to some, and another test like the log likelihood ratio test (Chap. 13) is more safe.



Example for Practicing 1

Example	2 × 2 table	Events	No events	
	Group 1	15 (a)	20 (b)	35 (a+b)
	Group 2	15 (c)	5 (d)	20 (c+d)
		30 (a+c)	25 (b+d)	55 (a+b+c+d)

Pocket calculator

$$\frac{(ad-bc)^2\,(a+b+c+d)}{(a+b)(c+d)(b+d)(a+c)} = \qquad p = ...$$

Example for Practicing 2

Another example	2×2 table	Events	No events	
	Group 1	16 (a)	26 (b)	42 (a+b)
	Group 2	5 (c)	30 (d)	35 (c+d)
		21 (a+c)	56 (b+d)	77 (a+b+c+d)

Pocket calculator

$$\frac{(ad-bc)^2\,(a+b+c+d)}{(a+b)(c+d)(b+d)(a+c)} = \qquad p = ...$$