Chapter 13
Log Likelihood Ratio Tests

The sensitivity of the chi-square test (Chap. 11) and the odds ratio test (Chap. 12) for testing cross-tabs is limited, and not entirely accurate if the values in one or more cells is smaller than 5. The log likelihood ratio test is an adequate alternative with generally better sensitivity, and, so, it must be absolutely recommended.

Example 1
A group of citizens is taking a pharmaceutical company to court for misrepresenting the danger of fatal rhabdomyolysis due to statin treatment.

	Patients with rhabdomyolysis	Patients without
Company	1 (a)	309,999 (b)
Citizens	4 (c)	300,289 (d)

p_{co} = proportion given by the pharmaceutical company = $a/(a+b) = 1/310{,}000$
p_{ci} = proportion given by the citizens = $c/(c+d) = 4/300{,}293$

We make use of the z-test (Chap. 10) for testing log likelihood ratios.

As it can be shown that −2 log likelihood ratio equals z^2, we can test the significance of difference between the two proportions.

$$\begin{aligned}\text{Log likelihood ratio} &= 4\ \log\frac{1/310{,}000}{4/300{,}293} + 300289\ \log\frac{1-1/310{,}000}{1-4/300{,}293}\\ &= -2.641199\\ -2\ \text{log likelihood ratio} &= -2\times -2.641199\\ &= 5.2824\ (p<0.05,\ \text{because } z>2).\\ &= z^2\end{aligned}$$

A z-value larger than 2 means a significant difference in your data (Chap. 10). Here the z-value equals $\sqrt{5.2824} = 2.29834$. The “p-calculator for z-values” in Google tells you that the exact p-value = 0.0215, much smaller than 0.05.

We should note here that both the odds ratio test and chi-square test produced a non-significant result here ($p > 0.05$). Indeed, the log likelihood ratio test is much

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more sensitive than the other tests for the same kind of data, which might once in a while be a blessing for desperate investigators.

Example 2
Two group of 15 patients at risk for arrhythmias were assessed for the development of torsade de points after calcium channel blockers treatment.

	Patients with torsade de points	Patients without
Calcium channel blocker 1	5	10
Calcium channel blocker 2	9	6

The proportion of patients with event from calcium channel blocker 1 is 5/15, from blocker 2 it is 9/15.

$$\text{Log likelihood ratio} = 9 \log\frac{5/15}{9/15} + 6 \log\frac{1-5/15}{1-9/15}$$

$$= -2.25$$

$$-2 \text{ log likelihood ratio} = 4.50$$

$$= z^2$$

$$\text{z-value} = \sqrt{4.50} = 2.1213$$

$$\text{p-value} < 0.05, \text{ because } z > 2.$$

Both odds ratio test and chi-square test were again non-significant ($p>0.05$).

Example 3
Two groups of patients with stage IV New York Heart Association heart failure were assessed for clinical admission while on two beta-blockers.

	Patients with clinical admission	Patients without
Beta blocker 1	77	62
Beta blocker 2	103	46

The proportion of patients with event while on beta blocker 1 is 77/139, while on beta blocker 2 it is 103/149.

$$\text{Log likelihood ratio} = 103 \log\frac{77/139}{103/149} + 46 \log\frac{1-77/139}{1-103/149}$$

$$= -5.882$$

$$-2 \text{ log likelihood ratio} = 11.766$$

$$= z^2$$

$$\text{z-value} = \sqrt{11.766} = 3.43016$$

$$\text{p-value} < 0.002, \text{ because } z > 3.090$$

(see the t-table on page 21).

Both the odds ratio test and chi-square test were also significant. However, at lower levels of significance, both p-values $0.01<p<0.05$.