

# Chapter 12

## Odds Ratios

The odds ratio test is just like the chi-square test applicable for testing cross-tabs. The advantage of the odds ratio test is that a odds ratio value can be calculated. The odds ratio value is just like the relative risk an estimate of the chance of having an event in group 1 compared to that of group 2. An odds ratio value of 1 indicates no difference between the two groups.

### Example 1

	Events	No events	
	Numbers of patients		
Group 1	15 (a)	20 (b)	35 (a+b)
Group 2	15 (c)	5 (d)	20 (c+d)
	30 (a+c)	25 (b+d)	55 (a+b+c+d)

The odds of an event = the number of patients in a group with an event divided by the number without. In group 1 the odds of an event equals = a/b.

The odds ratio (OR) of group 1 compared to group 2

$$\begin{aligned}
 &= (a / b) / (c / d) \\
 &= (15 / 20) / (15 / 5) \\
 &= 0.25
 \end{aligned}$$

$$\ln OR = \ln 0.25 = -1.386 \text{ (ln = natural logarithm)}$$

The standard error (SE) of the above term

$$\begin{aligned}
 &= \sqrt{(1 / a + 1 / b + 1 / c + 1 / d)} \\
 &= \sqrt{(1 / 15 + 1 / 20 + 1 / 15 + 1 / 5)} \\
 &= \sqrt{0.38333} \\
 &= 0.619
 \end{aligned}$$

The odds ratio can be tested using the z-test (Chap. 10).

$$\begin{aligned}
 \text{The test-statistic} &= z\text{-value} \\
 &= (\text{odds ratio}) / \text{SE} \\
 &= -1.386 / 0.619 \\
 &= -2.239
 \end{aligned}$$

If this value is smaller than  $-2$  or larger than  $+2$ , then the odds ratio is significantly different from 1 with  $p < 0.05$ . An odds ratio of 1 means that there is no difference in events between group 1 and group 2. The bottom row of the t-table (page 21) gives the z-values matching Gaussian distributions. Look at a z-value of 1.96 right up at the upper row. We will find a p-value here of 0.05. And, so, a z-value larger than 1.96 indicates a p-value of  $< 0.05$ . There is a significant difference in event between the two groups.

*Example 2*

	Events	No events	
	Number of patients		
Group 1	16 (a)	26 (b)	42 (a+b)
Group 2	5 (c)	30 (d)	35 (c+d)
	21 (a+c)	56 (b+d)	77 (a+b+c+d)

Test with OR whether there is a significant difference between group 1 and 2.

See for procedure also example 1.

$$\text{OR} = (16/26) / (5/30)$$

$$= 3.69$$

$$\ln\text{OR} = 1.3056 \text{ (ln = natural logarithm see the above example)}$$

$$\text{SE} = \sqrt{(1/16 + 1/26 + 1/5 + 1/30)}$$

$$= \sqrt{0.334333}$$

$$= 0.578$$

$$z\text{-value} = 1.3056 / 0.578$$

$$= 2.259$$

Because this value is larger than 2, a p-value of  $< 0.05$  is observed, 0.024 to be precise (numerous “p-calculator for z-values” sites in Google will help you calculate an exact p-value if required).