Chapter 7
Power Equations

Power can be defined as statistical conclusive force. A study result is often expressed in the form of the mean result and its standard deviation (SD) or standard error (SE). With the mean result getting larger and the standard error getting smaller, the study obtains increasing power.

What is the power of the underneath study?

A blood pressure study shows a mean decrease in blood pressure of 10.8 mm Hg with a standard error of 3.0 mm Hg. Results from study samples are often given in grams, liters, Euros, mm Hg etc. For the calculation of power we have to standardize our study result, which means that the mean result has to be divided by its own standard error:

$$
\begin{aligned}
&\text{Mean} \pm \text{SE} \\
&= \text{mean} / \text{SE} \pm \text{SE} / \text{SE} \\
&= \text{t-value} \pm 1.
\end{aligned}
$$

The t-values are found in the t-table, can be looked upon as standardized results of all kinds of studies.

T.J. Cleophas and A.H. Zwinderman, *Statistical Analysis of Clinical Data on a Pocket Calculator: Statistics on a Pocket Calculator*, DOI 10.1007/978-94-007-1211-9_7,

In our blood pressure study the t-value = 10.8/3.0 = 3.6. The unit of the t-value is not mm Hg, but rather SE-units. The question is: what power does the study have, if we assume a type I error (alpha) = 5% and a sample size of n = 20.

The question is: what is the power of this study if we assume a type I error (alpha) of 5%, and will have a sample size of n = 20.

A. 90% < power < 95%,
B. power > 80%,
C. power < 75%,
D. power > 75%.

n = 20 indicates 20–2 = 18 degrees of freedom in the case of two groups of ten patients each.

We will use the following power equation (prob = probability, z = value on the z-line (the x-axis of the t-distribution)

$$\text{Power} = 1 - \text{prob}\ (z < t - t^1)$$

$$t = \text{the t-value of your results,}$$

$$t^1 = \text{the t-value, that matches a p-value of } 0.05 = 2.1;$$

$$t = 3.6;\ t^1 = 2.1;\ t - t^1 = 1.5;$$

$$\text{prob}\ (z < t - t^1) = \text{beta} = \text{type II error} = 0.05 - 0.1$$

$$\text{1-beta} = \text{power} = 0.9 - 0.95 = \text{between } 90\% \text{ and } 95\%.$$

So, there is a very good power here. See below for explanation of the calculation.

Explanation of the above calculation.

The t-table on the next page is a more detailed version of the t-table of page 21, and is adequate for power calculations. The degrees of freedom are in the left column and correlate with the sample size of a study. With large samples the frequency distribution of the data will be a little bit narrower, and that is corrected in the table. The t-values are to be looked upon as mean results of studies, but not expressed in mmol/l, kilograms, but in so-called SE-units (Standard error units), that are obtained by dividing your mean result by its own standard error. With a t-value of 3.6 and 18 degrees of freedom $t - t^1$ equals 1.5. This value is between 1.330 and 1.734. Look right up at the upper row for finding beta (type II error = the chance of finding no difference where there is one). We are between 0.1 and 0.05 (10% and 5%). This is an adequate estimate of the type II error. The power equals 100% − beta = between 90% and 95% in our example.

t-Table

v	*Q* = **0.4** 2Q = **0.8**	**0.25** **0.5**	**0.1** **0.2**	**0.05** **0.1**	**0.025** **0.05**	**0.01** **0.02**	**0.005** **0.01**	**0.001** **0.002**
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.326

(continued)

t-Table (continued)

3	0.277	0.765	1.638	2.353	3.182	4.547	5.841	10.213
4	0.171	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.261	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.269	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.269	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.685	1.319	1.714	2.069	2.600	2.807	3.485
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.684	1,316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.654	1,315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.684	1,314	1.701	2.052	2.473	2.771	3.421
28	0.256	0.683	1,313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.254	0.677	1.289	1.658	1.950	2.358	2.617	3.160
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090

The upper row shows p-values = Areas under the curve (AUCs) of t-distributions. The second row gives two-sided p-values, it means that left and right end of the AUCs of the Gaussian-like curves are added up. The left column gives the adjustment for the sample size. If it gets larger, then the corresponding Gaussian-like curves will get a bit narrower. In this manner the estimates become more precise and more in agreement with reality. The t-table is empirical, and has been constructed in the 1930s of the past century with the help of simulation models and practical examples. It is till now the basis of modern statistics, and all modern software makes extensively use of it