Chapter 3
t-Tests

1 Sample t-Test

As an example, the mean decrease in blood pressure after treatment is calculated with the accompanying p-value. A p-value <0.05 indicates that there is less than 5% probability that such a decrease will be observed purely by the play of chance. There is, thus, >95% chance that the decrease is the result of a real blood pressure lowering effect of the treatment. We call such a decrease statistically significant.

Patient	mm Hg decrease
1	3
2	4
3	−2
4	3
5	1
6	−2
7	4
8	3

Is this decrease statistically significant?

$$\text{Mean decrease} = 1.75 \text{ mmHg}$$
$$\text{SD} = 2.49 \text{ mmHg}$$

From the standard deviation the standard error (SE) can be calculated using the equation

$$\text{SE} = \text{SD} / \sqrt{n} \quad (n = \text{sample size})$$
$$\text{SE} = 2.49 / \sqrt{8} = 0.88$$

De t-value is the test-statistic of the t-test and is calculated as follows:

$$t = 1.75 / 0.88 = 1.9886$$

T.J. Cleophas and A.H. Zwinderman, *Statistical Analysis of Clinical Data on a Pocket Calculator: Statistics on a Pocket Calculator*, DOI 10.1007/978-94-007-1211-9_3,

Because the sample size is 8, the test has here 8−1 = 7 degrees of freedom.

The t-table on the pages 7–8 shows that with 7 degrees of freedom the p-value should equal: $0.05 < p < 0.10$. This result is close to statistically significant, and is called a trend to significance.

Paired t-Test

Two rows of observations in ten persons are given underneath:

Observation 1:

6.0, 7.1, 8.1, 7.5, 6.4, 7.9, 6.8, 6.6, 7.3, 5.6

Observation 2:

5.1, 8.0, 3.8, 4.4, 5.2, 5.4, 4.3, 6.0, 3.7, 6.2

Individual differences

0.9, −0.9, 4.3, 3.1, 1.2, 2.5, 2.5, 0.6, 3.8, −0.6

A. not significant
B. $0.05 < p < 0.10$
C. $P < 0.05$
D. $P < 0.01$

Is there a significant difference between the observation 1 and 2, and which level of significance is correct?

Mean difference	= 1.59
SD of mean difference	= 1.789
$SE = SD/\sqrt{10}$	= 0.566
$t = 1.59/0.566$	= 2.809

10–1 = 9 degrees of freedom, because we have 10 patients and 1 group of patients.

According to the t-table of page XXX the p-value equals <0.05, and we can conclude that a significant difference between the two observations is in the data: the values of row 1 are significantly higher than those of row 2. The answer C is correct.

Unpaired t-Test

Two matched groups of patients are compared with one another.

Group 1:
6.0, 7.1, 8.1, 7.5, 6.4, 7.9, 6.8, 6.6, 7.3, 5.6
Group 2:
5.1, 8.0, 3.8, 4.4, 5.2, 5.4, 4.3, 6.0, 3.7, 6.2

Mean Group 1 = 6.93 SD = 0.806 SE = SD/√10 = 0.255
Mean Group 2 = 5.21 SD = 1.299 SE = SD/√10 = 0.411

A. not significant
B. $0.05 < p < 0.10$
C. $p < 0.05$
D. $P < 0.01$

Is there a significant difference between the two groups, which level of significance is correct?

Mean	Standard deviation (SD)
6.93	0.806
5.21–	1.299
1.72	pooled SE $= \sqrt{\left(\frac{0.806^2}{10} + \frac{1.299^2}{10}\right)} = 0.483$

The t-value = (6.93−5.21)/0.483 = 3.56.

20−2 = 18 degrees of freedom, because we have 20 patients and 2 groups.

According to the t-table of page the p-value is <0.01, and we can conclude that that a very significant difference exists between the two groups. The values of group 1 are higher than those of group 2. The answer D is correct.

t-Table

df	0.1	0.05	0.01	0.002
1	6.314	12.706	63.657	318.31
2	2.920	4.303	9.925	22.326
3	2.353	3.182	5.841	10.213
4	2.132	2.776	4.604	7.173
5	2.015	2.571	4.032	5.893
6	1.943	2.447	3.707	5.208
7	1.895	2.365	3.499	4.785
8	1.860	2.306	3.355	4.501
9	1.833	2.262	3.250	4.297

(continued)

t-Table (continued)

df	0.1	0.05	0.01	0.002
10	1.812	2.228	3.169	4.144
11	1.796	2.201	3.106	4.025
12	1.782	2.179	3.055	3.930
13	1.771	2.160	3.012	3.852
14	1.761	2.145	2.977	3.787
15	1.753	2.131	2.947	3.733
16	1.746	2.120	2.921	3.686
17	1.740	2.110	2.898	3.646
18	1.734	2.101	2.878	3.610
19	1.729	2.093	2.861	3.579
20	1.725	2.086	2.845	3.552
21	1.721	2.080	2.831	3.527
22	1.717	2.074	2.819	3.505
23	1.714	2.069	2.807	3.485
24	1.711	2.064	2.797	3.467
25	1.708	2.060	2.787	3.450
26	1.706	2.056	2.779	3.435
27	1.701	2.052	2.771	3.421
28	1.701	2.048	2.763	3.408
29	1.699	2.045	2.756	3.396
30	1.697	2.042	2.750	3.385
40	1.684	2.021	2.704	3.307
60	1.671	2.000	2.660	3.232
120	1.658	1.950	2.617	3.160
∞	1.645	1.960	2.576	3.090

The rows give t-values adjusted for degrees of freedom. The numbers of degrees of freedom largely correlate with the sample size of a study. With large samples the frequency distribution of the data will be a little bit narrower, and that is corrected in the table. The t-values are to be looked upon as mean results of studies, but not expressed in mmol/l, kilograms, but in so-called SE-units (Standard error units), that are obtained by dividing your mean result by its own standard error. A t-value of 3.56 with 18 degrees of freedom indicates that we will need the row no. 18 of the table. The upper row gives the area under the curve of the Gaussian-like t-distribution. The t-value 3.56 is left from 3.610. Now look right up to the upper row: we are right from 0.01. The p-value equals <0.01