Chapter 10
Z-Test for Cross-Tabs

Two groups of patients are assessed for being sleepy through the day. We wish to estimate whether group 1 is more sleepy than group 2. The underneath cross-tab gives the data.

	Sleepiness	No sleepiness
Treatment 1 (group 1)	5 (a)	10 (b)
Treatment 2 (group 2)	9 (c)	6 (d)

$$z = \frac{\text{difference between proportions of sleepers per group (d)}}{\text{pooled standard error difference}}$$

$$z = \frac{d}{\text{pooled SE}} = \frac{(9/15 - 5/15)}{\sqrt{(SE_1^2 + SE_2^2)}}$$

$$SE_1 \text{ (or } SEM_1) = \sqrt{\frac{p_1(1-p_1)}{n_1}} \text{ where } p_1 = 5/15 \text{ etc.....,}$$

$z = 1.45$, not statistically significant from zero, because for a $p < 0.05$ a z-value of at least 1.96 is required. This means that no significant difference between the two groups is observed. The p-value of the z-test can be obtained by using the bottom row of the t-table from page 21.

Note:

For the z-test a normal distribution approach can be used. The t-distributions are usually a bit wider than the normal distributions, and therefore, adjustment for study size using degrees of freedom (left column of the t-table) is required. With a large study size the t-distribution is equal to the normal distribution, and the t-values are equal to the z-values. They are given in the bottom row of the t-table.

T.J. Cleophas and A.H. Zwinderman, *Statistical Analysis of Clinical Data on a Pocket Calculator: Statistics on a Pocket Calculator*, DOI 10.1007/978-94-007-1211-9_10,

Note:

A single group z-test is also possible. For example in ten patients we have four responders. We question whether four responders is significantly more than zero responders.

$$
\begin{aligned}
z &= \text{proportion} / (\text{its SE}) \\
SE &= \sqrt{[(4/10 \times (1 - 4/10)) / n]} \\
&= \sqrt{(0.24/10)} \\
z &= 0.4 / \sqrt{(0.24/10)} \\
z &= 0.4 / 0.1549 \\
z &= 2.582
\end{aligned}
$$

According to the bottom row of the t-table from page 21 the p-value is < 0.01. The proportion of 0.4 is, thus, significantly larger than a proportion of 0.0.