# Chapter 38
# Chi-square Tests

## 1  General Purpose

Z-tests (Chap. 36) are OK for comparing the effects of two treatment modalities on numbers of responders to treatment, however, pretty laborious. Phi tests (Chap. 37) provide levels of association (or interaction) between a treatment modality and the number of responders, but no p-values. Often, we wish to know whether our result or the difference between two results are significantly different from zero. Chi-square tests make use of chi-square distributions (squared normal distributions), and produce p-values for the purpose, and these p-values are pretty much similar to those of the z-tests, but they can be obtained more easily.

## 2  Schematic Overview of Type of Data File

| Predictor (binary) | Outcome (binary) |
|---|---|
| . | . |
| . | . |
| . | . |
| . | . |
| . | . |
| . | . |
| . | . |
| . | . |
| . | . |

## 3   Primary Scientific Question

In a $2 \times 2$ interaction cross-tab of binary data, like a study of two treatment modalities and the chance of responding or not in the outcome, is there a significant difference between the two treatment modalities and the outcome.

## 4   Data Example I

The underneath table shows two groups assessed for suffering from sleepiness through the day after being treated with two different anti-sleepiness medications. We wish to know whether there is a significant difference between the proportions of subjects being sleepy.

|         | Sleepiness | no sleepiness |             |
|---------|------------|---------------|-------------|
| Group 1 | 5(a)       | 10(b)         | 15(a + b)   |
| Group 2 | 9(c)       | 6(d)          | 15(c + d)   |
|         | 14(a + c)  | 16(b + d)     | 30(a + b + c + d) |

The chi-square pocket calculator method is used for testing these data.

$$\chi^2 = \frac{(ad - bc)^2(a + b + c + d)}{(a + b)\,(c + d)\,(b + d)\,(a + c)} = \frac{(30 - 90)^2(30)}{15 \times 15 \times 16 \times 14} = \frac{3600 \times 30}{15 \times 15 \times 16 \times 14}$$

$$= \frac{108.000}{50.400} = 2.143$$

The chi-square value equals 2.143. The underneath chi-square table can tell you, whether or not the difference between the groups is statistically significant.

Chi-squared distribution

| df | Two-tailed P-value | | | |
|---|---|---|---|---|
|    | 0.10 | 0.05 | 0.01 | 0.001 |
| 1 | 2.706 | 3.841 | 6.635 | 10.827 |
| 2 | 4.605 | 5.991 | 9.210 | 13.815 |
| 3 | 6.251 | 7.851 | 11.345 | 16.266 |
| 4 | 7.779 | 9.488 | 13.277 | 18.466 |
| 5 | 9.236 | 11.070 | 15.086 | 20.515 |
| 6 | 10.645 | 12.592 | 16.812 | 22.457 |
| 7 | 12.017 | 14.067 | 18.475 | 24.321 |
| 8 | 13.362 | 15.507 | 20.090 | 26.124 |
| 9 | 14.684 | 16.919 | 21.666 | 27.877 |
| 10 | 15.987 | 18.307 | 23.209 | 29.588 |
| 11 | 17.275 | 19.675 | 24.725 | 31.264 |
| 12 | 18.549 | 21.026 | 26.217 | 32.909 |
| 13 | 19.812 | 22.362 | 27.688 | 34.527 |
| 14 | 21.064 | 23.685 | 29.141 | 36.124 |
| 15 | 22.307 | 24.996 | 30.578 | 37.698 |
| 16 | 23.542 | 26.296 | 32.000 | 39.252 |
| 17 | 24.769 | 27.587 | 33.409 | 40.791 |
| 18 | 25.989 | 28.869 | 34.805 | 42.312 |
| 19 | 27.204 | 30.144 | 36.191 | 43.819 |
| 20 | 28.412 | 31.410 | 37.566 | 45.314 |
| 21 | 29.615 | 32.671 | 38.932 | 46.796 |
| 22 | 30.813 | 33.924 | 40.289 | 48.268 |
| 23 | 32.007 | 35.172 | 41.638 | 49.728 |
| 24 | 33.196 | 36.415 | 42.980 | 51.179 |
| 25 | 34.382 | 37.652 | 44.314 | 52.619 |
| 26 | 35.536 | 38.885 | 45.642 | 54.051 |
| 27 | 36.741 | 40.113 | 46.963 | 55.475 |
| 28 | 37.916 | 41.337 | 48.278 | 56.892 |
| 29 | 39.087 | 42.557 | 49.588 | 58.301 |
| 30 | 40.256 | 43.773 | 50.892 | 59.702 |
| 40 | 51.805 | 55.758 | 63.691 | 73.403 |
| 50 | 63.167 | 67.505 | 76.154 | 86.660 |
| 60 | 74.397 | 79.082 | 88.379 | 99.608 |
| 70 | 85.527 | 90.531 | 100.43 | 112.32 |
| 80 | 96.578 | 101.88 | 112.33 | 124.84 |
| 90 | 107.57 | 113.15 | 124.12 | 137.21 |
| 100 | 118.50 | 124.34 | 135.81 | 149.45 |

In summary, the chi-square table has an upper row with areas under the curve (p-values), a left-end column with degrees of freedom, and a whole lot of chi-square values.

The chi-square table has columns and rows. The upper row are the p-values. The left-end column are the degrees of freedom (df), which are, here, largely in agreement with the numbers of cells in a cross-tab. The simplest cross-tab has 4 cells, which means $2 \times 2 = 4$ cells. The table has been constructed such, that we have here $(2–1) \times (2–1) = 1$ degree of freedom. Look at the row with 1 degree of freedom: a chi-square value of 2.143 is left from 2.706. Now look from here right up at the upper row. The corresponding p-value is larger than 0.1 (10 %). There is, thus, no significant difference in sleepiness between the two groups. The small difference observed is due to the play of chance.

## 5   Data Example II

Two partnerships of internists have the intention to associate. However, in one of the two a considerable number of internists has suffered from a burn-out.

|               | burn out | no burn out |             |
|---------------|----------|-------------|-------------|
| partnership 1 | 3(a)     | 7(b)        | 10(a + b)   |
| partnership 2 | 0(c)     | 10(d)       | 10(c + d)   |
|               | 3(a + c) | 17(b + d)   | 20(a + b + c + d) |

$$\chi^2 = \frac{(ad - bc)^2 (a + b + c + d)}{(a + b)\ (c + d)\ (b + d)\ (a + c)} = \frac{(30 - 0)\ 2(20)}{10 \times 10 \times 17 \times 3} = \frac{900 \times 20}{\ldots\ldots} = 3.529$$

According to the chi-square table of the previous page a p-value is found of <0.10. This means that no significant difference is found, but a p-value between 0.05 and 0.10 is looked upon as a trend to significance. The difference may be due to some avoidable or unavoidable cause. We should add, here, that values in a cell lower than 5 is considered slightly inappropriate according to some, and another test like the log likelihood ratio test (Chap. 46) is more safe.

## 6   Example for Practicing 1

| Example 2 × 2 table | events   | no events |             |
|---------------------|----------|-----------|-------------|
| group 1             | 15(a)    | 20(b)     | 35(a + b)   |
| group 2             | 15(c)    | 5(d)      | 20(c + d)   |
|                     | 30(a + c)| 25(b + d) | 55(a + b + c + d) |

Pocket calculator

$$\frac{(ad - bc)^2 (a + b + c + d)}{(a + b)(c + d)(b + d)(a + c)} \quad = \quad p=\ldots$$

## 7 Example for Practicing 2

| Another example 2 × 2 table | events | no events | |
|---|---|---|---|
| group 1 | 16(a) | 26(b) | 42(a + b) |
| group 2 | 5(c) | 30(d) | 35(c + d) |
| | 21(a + c) | 56(b + d) | 77(a + b + c + d) |

Pocket calculator

$$\frac{(ad - bc)^2 (a + b + c + d)}{(a + b)(c + d)(b + d)(a + c)} \quad = \quad p=\ldots$$

## 8 Conclusion

Phi tests (previous chapter) provide levels of association (or interaction) between a treatment modality and the number of responders, but no p-values. Often, we wish to know whether our result is significantly different from a zero-result. Chi-square tests make use of the chi-square distribution (squared normal distribution), and produce p-values for the purpose. A $p$-value $< 0.05$ means that our result is better than a zero-result with over 95 % certainty, or with less than 5 % chance of a type I error (the chance of finding a difference where there is none). A pleasant thing about the pocket-calculator-chi-square-test is, that it can be readily performed during working hours by physicians, for the purpose of solving the scientific questions of their daily life (Chaps. 58, and 59). This means no need for starting a major statistical software program like SPSS, SAS, or R.

## 9 Note

More background, theoretical and mathematical information of chi-square tests is given in Statistics applied to clinical studies 5th edition, Chap. 3, Springer Heidelberg Germany, 2012, from the same authors.