Chapter 13
Confidence Intervals

1 General Purpose

The 95% confidence interval of a study represents an interval covering 95% of the means of many studies similar to that of our study. It tells you something about what you can expect from future data: if you repeat the study, you will be 95% sure that your mean outcome will be within the 95% confidence interval. The chapter shows how it can be calculated.

2 Schematic Overview of Type of Data File

Outcome
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3 Primary Scientific Question

Can the 95% confidence intervals be used as an alternative to statistical significance testing? What are the advantages?

4 Data Example, Continuous Outcome Data

The 95% confidence interval of a study represents an interval covering 95% of the means of many studies similar to that of our study. It tells you something about what you can expect from future data: if you repeat the study, you will be 95% sure that the outcome will be within the 95% confidence interval. The 95% confidence of a study is found by the equation

95% confidence interval = mean ± 2 x standard error (SE)

The SE is equal to the standard deviation (SD) / √n, where n = the sample size of your study. The SD can be calculated from the procedure reviewed in the Chap. 2.

With an SD of 1.407885953 and a sample size of n = 8,

your SE = 1.407885953 / √ 8
= 0.4977,

with a mean value of your study of 53.375,
with a 95% confidence interval = 53.375 ± 2 × 0.4977
= between 52.3796 and 54.3704.

The mean study results are often reported together with 95% confidence intervals. They are also the basis for equivalence studies and noninferiority studies, which will be reviewed in the Chaps. 14 and 15. Also for study results expressed in the form of numbers of events, proportion of deaths, odds ratios of events, etc., 95 % confidence intervals can be readily calculated.

We should add that the equation

95% confidence interval = mean ± 2 × standard error (SE),

is a pretty rough approximation, and that a more precise estimate would be the equation

95% confidence interval = mean ± t^1 × standard error (SE),

where t^1 = the critical t-value corresponding to a two-sided p-value of 0.05.

5 T-Table and 95% Confidence Intervals

The t-table has a left-end column giving degrees of freedom (≈ sample sizes), and two top rows with p-values (areas under the curve = p-values), one-tail meaning that only one end of the curve, two-tail meaning that both ends are assessed simultaneously. The t-table is, furthermore, full of t-values, that, with ∞ degrees of freedom, are equal to z-values (Chap. 36). The t-values are to be understood as mean results of studies, but not expressed in mmol/l, kilograms, but in so-called SEM-units (Standard error of the mean units), that are obtained by dividing your mean result by its own standard error. With many degrees of freedom (large samples) the curve will be a little bit narrower, and more in agreement with nature.

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

In the fifth column of t-values of the above t-table all of the t^1-values are given. For example, with a sample of 120 the t^1-value equals 1.980, with a sample size of close to 8 the t^1-value rise to 2.306.

6 Data Example, Binary Outcome Data

What is the standard error (SE) of a study with events in 10% of the patients, and a sample size of 100 (n). Ten % events means a proportion of events of 0.1. The standard deviation (SD) of this proportion is defined by the equation

$\sqrt{[\text{proportion} \times (1 - \text{proportion})]} = \sqrt{(0.1 \times 0.9)} = \sqrt{0.09} = 0.3,$

the standard error $= \text{standard deviation}/\sqrt{n},$
$= 0.3/10 = 0.03,$

the 95 % confidence interval is given by
proportion given $\pm 1.960 \times 0.03 = 0.1 \pm 1.960 \times 0.03,$
$= 0.1 \pm 0.06,$
$=$ between 0.04 and 0.16.

7 Conclusion

The 95% confidence interval of a study represents an interval covering 95% of the means of many studies similar to that of our study. It tells you something about what you can expect from future data: if you repeat the study, you will be 95% sure that your mean outcome will be within the 95% confidence interval. The 95% confidence interval can be used as an alternative to statistical significance testing. The advantages are that the intervals picture expected mean results of future data, and that they can be applied for studying therapeutic equivalence and noninferiority (Chaps. 14 and 15).

8 Note

More background, theoretical and mathematical information of confidence intervals are given in the Chaps. 14 and 15 of this volume.