Chapter 29
Fuzzy Modeling for Imprecise and Incomplete Data

1 General Purpose

Fuzzy modeling is a methodology that works with partial truths: it can answer questions to which the answers are "yes" and "no" at different times or partly "yes" and "no" at the same time. It can be used to match any type of data, particularly incomplete and imprecise data, and it is able to improve precision of such data. It can be applied with any type of statistical distribution and it is, particularly, suitable for uncommon and unexpected non linear relationships. This chapter assesses the use of fuzzy modeling of clinical data.

2 Schematic Overview of Type of Data File

Outcome
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3 Primary Scientific Question

Is fuzzy modeling able to provide improved precision of imprecise clinical data.

4 Fuzzy Terms

4.1 Universal Space

Defined range of input values, defined range of output values.

4.2 Fuzzy Memberships

The universal spaces are divided into equally sized parts called membership functions

4.3 Linguistic Membership Names

Each fuzzy membership is given a name, otherwise called linguistic term.

4.4 Triangular Fuzzy Sets

A common way of drawing the membership function with on the x-axis the input values, on the y-axis the membership grade for each input value.

4.5 Fuzzy Plots

Graphs summarizing the fuzzy memberships of (for example) the input values.

4.6 Linguistic Rules

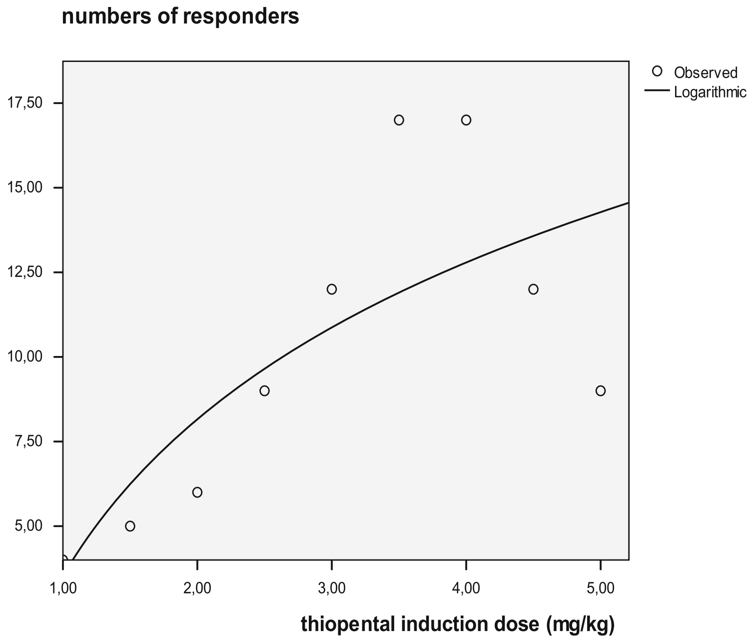
The relationships between the fuzzy memberships of the input data and those of the output data (the method of calculation is shown in the underneath examples).

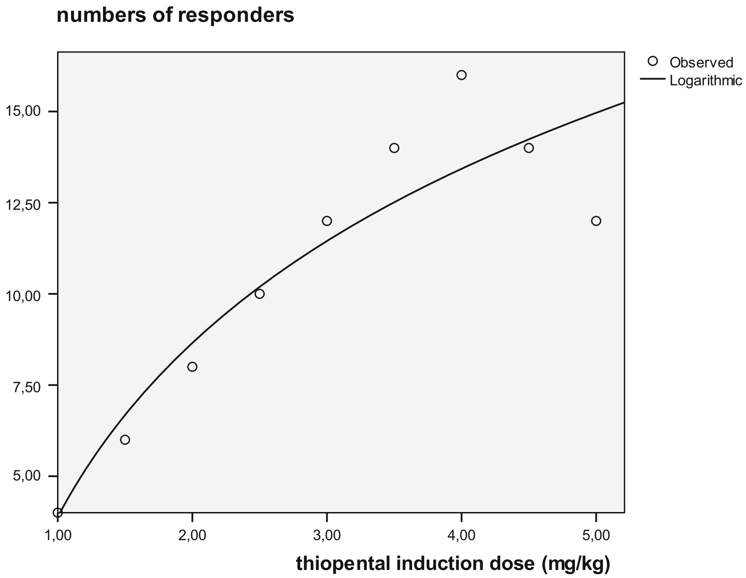
5 Data Example 1

Underneath are the quantal pharmacodynamic effects of different induction dosages of thiopental on numbers of responding subjects.

Input values induction dosage of thiopental (mg/kg)	output values numbers of responders (n)	fuzzy-modeled output numbers of responders (n)
1	4	4
1.	5	5
2	6	8
2.5	9	10
3	12	12
3.5	17	14
4	17	16
4.5	12	14
5	9	12

The effects of different induction dosages of thiopental on numbers of responding subjects are in the above table, left two columns. The right column gives the fuzzy-modeled output. The figures below show that the un-modeled curve (upper curve) fits the data less well than does the modeled (lower curve).





We fuzzy-model the input and output relationships (figures below).

First of all, we create linguistic rules for the input and output data.

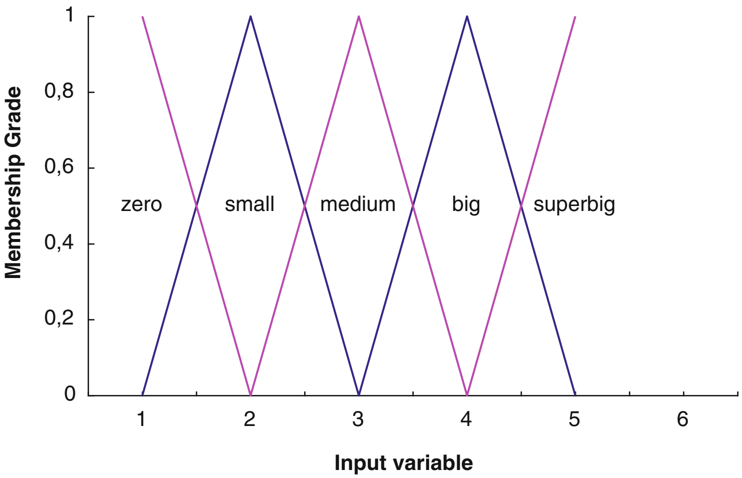
For that purpose we divide the universal space of the input variable into fuzzy memberships with linguistic membership names:

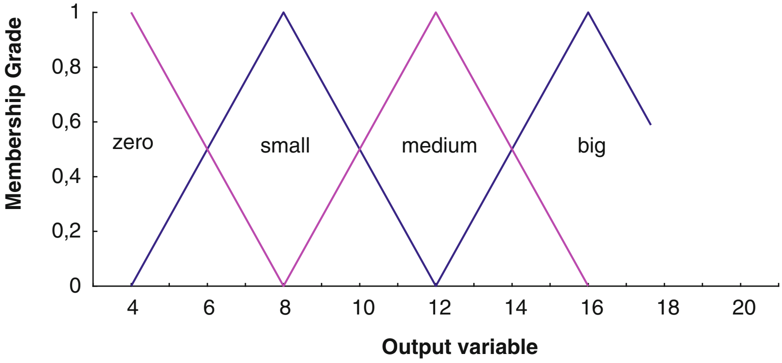
input-*zero, -small, -medium, -big, -superbig*.

Then we do the same for the output variable:

output-*zero, -small, -medium, -big*.

Subsequently, we create linguistic rules.





The above figure shows that input-*zero* consists of the values 1 and 1.5.

The value 1 (100 % membership) has 4 as outcome value (100 % membership of output-*zero*).
The value 1.5 (50 % membership) has 5 as outcome value (75 % membership of output-*zero,* 25 % of output-*small*).

The input-*zero* produces 100%x100% + 50%x75% = 137.5 % membership to output-*zero*, and 50% × 25% = 12.5 % membership to output-*small*, and so, output-*zero* is the most important output contributor here, and we forget about the small contribution of output-*small.*

Input-*small* is more complex*,* it consists of the values 1.5, and 2.0, and 2.5.

The value 1.5 (50 % membership) has 5 as outcome value (75 % membership of output-*zero,* 25 % membership of output-*small*).
The value 2.0 (100 % membership) has 6 as outcome value (50 % membership of outcome-*zero*, and 50 % membership of output-*small).*
The value 2.5 (50 % membership) has 9 as outcome value (75 % membership of output-*small* and 25 % of output-*medium).*

The input-*small* produces 50 % × 75 % + 100 % × 50 % = 87.5 % membership to output-*zero*, 50 % × 25 % + 100 % × 50 % + 50 % × 75 % = 100 % membership to output-*small*, and 50 % × 25 % = 12.5 % membership to output-*medium*. And so, the output-*small* is the most important contributor here, and we forget about the other two.

For the other input memberships similar linguistic rules are determined:

Input-*medium* → output-*medium*
Input-*big* → output-*big*
Input-*superbig* → output-*medium*

We are, particularly interested in the modeling capacity of fuzzy logic in order to improve the precision of pharmacodynamic modeling.

The modeled output value of input value 1 is found as follows.

Value 1 is 100 % member of input-*zero*, meaning that according to the above linguistic rules it is also associated with a 100 % membership of output-*zero* corresponding with a value of 4.

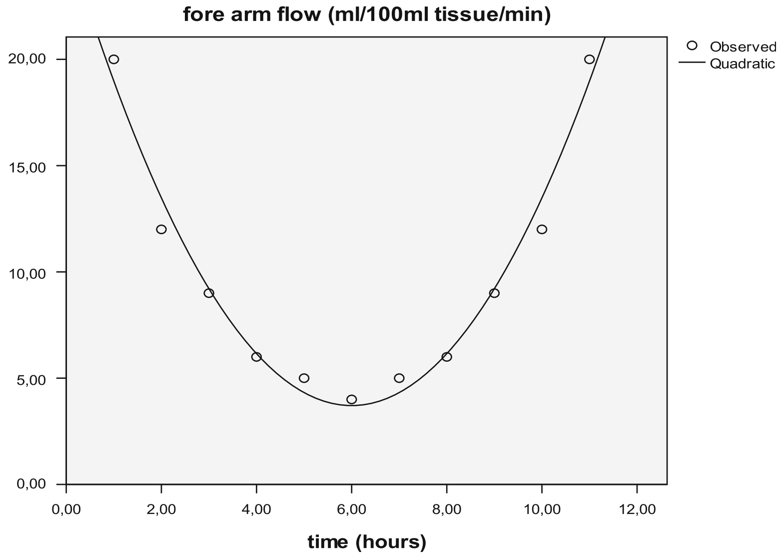
Value 1.5 is 50 % member of input-*zero* and 50 % input-*small*. This means it is 50 % associated with the output-*zero* and –*small* corresponding with values of 50 % $\times$ (4 + 8) = 6.

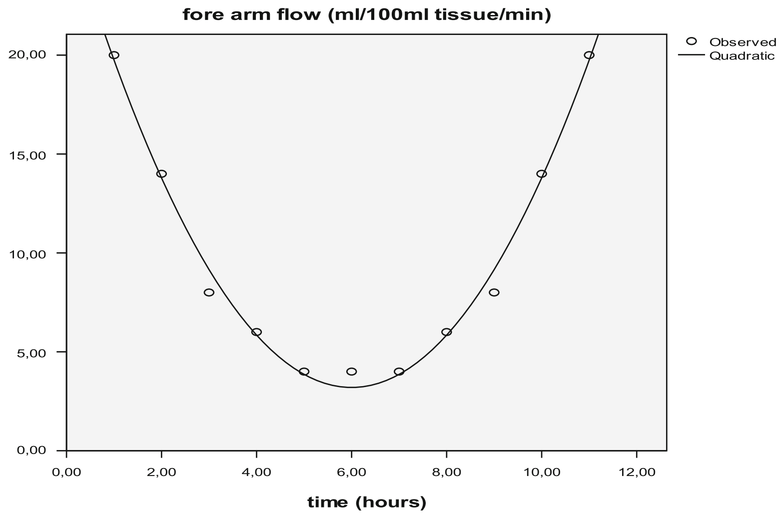
For all of the input values modeled output values can be found in this way. The table of the current section, right column shows the results.

6 Data Example 2

In the underneath example the fuzzy-modeled output has been given. Try and fuzzy-model the data for yourself. Time-response effect of single oral dose of 120 mg propranolol on peripheral arterial flow.

Input values Hours after oral Administration of 120 mg propranolol	output values peripheral arterial flow (ml/100 ml tissue/min)	fuzzy-modeled output peripheral arterial flow (ml/100 ml tissue/min)
1	20	20
2	12	14
3	9	8
4	6	6
5	5	4
6	4	4
7	5	4
8	6	6
9	9	8
10	12	14
11	20	20





Pharmacodynamic relationship between the time after oral administration of 120 mg of propranolol (x-axis, hours) and absolute change in fore arm flow (y-axis, ml/100 ml tissue/min) are in the above graphs. The un-modeled curve (upper curve) fits the data slightly less well than does the modeled (lower curve).

7 Conclusion

Fuzzy modeling is a methodology that works with partial truths: it can answer questions to which the answers are "yes" and "no" at different times or partly "yes" and "no" at the same time. This chapter assesses the use of fuzzy modeling of clinical data. The examples given show that the fuzzy models better fit the data than do the un-modeled data. The figures were drawn with SPSS module regression (curve estimation).

8 Note

More background, theoretical and mathematical information of fuzzy modeling is given in Statistics applied to clinical studies 5th edition, Chap. 59, Springer Heidelberg Germany, 2012, from the same authors.