Chapter 46
Log Likelihood Ratio Tests for the Best Precision

1 General Purpose

The sensitivity of the chi-square test (Chap. 38), and the odds ratio test (Chap. 44) for testing cross-tabs is limited, and, not entirely, accurate, if the values in one or more cells is smaller than 5. The log likelihood ratio test may be an adequate alternative with generally better sensitivity, and, so, it must be absolutely considered. This chapter assesses how it works.

2 Schematic Overview of Type of Data File

Outcome
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T.J. Cleophas, A.H. Zwinderman, *Clinical Data Analysis on a Pocket Calculator*,
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3 Primary Scientific Question

Is the log likelihood ratio test adequate and more sensitive for testing 2×2 contingency table (cross-tabs, 2×2 interaction tables) compared to standard methods.

4 Example 1

A group of citizens is taking a pharmaceutical company to court for misrepresenting the danger of fatal rhabdomyolysis due to statin treatment.

	Patients with rhabdomyolysis	patients without
company	1 (a)	309999 (b)
citizens	4 (c)	300289 (d)

p_{co} = proportion given by the pharmaceutical company = $a/(a + b) = 1/310000$
p_{ci} = proportion given by the citizens = $c/(c + d) = 4/300293$

We make use of the z-test (Chap. 36) for testing log likelihood ratios.

As it can be shown that -2 log likelihood ratio equals z^2, we can test the significance of difference between the two proportions.

$$
\begin{aligned}
\text{Log likelihood ratio} &= 4 \log \frac{1/310000}{4/300293} + 300289 \ \log \frac{1-1/310000}{1-\ 4/300293} \\
&= -2.641199 \\
-2 \text{ log likelihood ratio} &= -2 \times -2.641199 \\
&= 5.2824 \ (p < 0.05, \text{ because } z > 2). \\
&= z^{2}
\end{aligned}
$$

A z – value larger than 2 (actually 1.960, see bottom row of underneath t-table for all z-values) means a significant difference in your data. Here the z-value equals $\sqrt{5.2824} = 2.29834$. The "p-calculator for z-values" in Google tells you that a more precise p – value = 0.0215, anyway much smaller than 0.05.

We should note here that both the odds ratio test and chi-square test produced a non-significant result of these data ($p > 0.05$). Indeed, the log likelihood ratio test is much more sensitive than the other tests for the same data, which might once in a while be a blessing for desperate investigators.

df	One-Tail = .4	.25	.1	.05	.025	.01	.005	.0025	.001	.0005
	Two-Tail = .8	.5	.2	.1	.05	.02	.01	.005	.002	.001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The above t-table gives in the left-end column degrees of freedom (df), (≈ sample sizes), two top rows with p-values (areas under the curve), and furthermore plenty t-values, that, with ∞ degrees of freedom, have become equal to z-values. We should emphasize, that, instead of the above t-table, the one-degree-of-freedom-row of the chi-square table can be used, because this row produces z^2 – values.

The underneath chi-square table has an upper row with areas under the curve, a left-end column with degrees of freedom, and a whole lot of chi-square values. The one-degree-of-freedom-row of the chi-square table has values equal to the squared values of the bottom row of the t-table.

Chi-squared distribution

	Two-tailed *P*-value			
df	0.10	0.05	0.01	0.001
1	2.706	3.841	6.635	10.827
2	4.605	5.991	9.210	13.815
3	6.251	7.851	11.345	16.266
4	7.779	9.488	13.277	18.466
5	9.236	11.070	15.086	20.515
6	10.645	12.592	16.812	22.457
7	12.017	14.067	18.475	24.321
8	13.362	15.507	20.090	26.124
9	14.684	16.919	21.666	27.877
10	15.987	18.307	23.209	29.588
11	17.275	19.675	24.725	31.264
12	18.549	21.026	26.217	32.909
13	19.812	22.362	27.688	34.527
14	21.064	23.685	29.141	36.124
15	22.307	24.996	30.578	37.698
16	23.542	26.296	32.000	39.252
17	24.769	27.587	33.409	40.791
18	25.989	28.869	34.805	42.312
19	27.204	30.144	36.191	43.819
20	28.412	31.410	37.566	45.314
21	29.615	32.671	38.932	46.796
22	30.813	33.924	40.289	48.268
23	32.007	35.172	41.638	49.728
24	33.196	36.415	42.980	51.179
25	34.382	37.652	44.314	52.619
26	35.536	38.885	45.642	54.051
27	36.741	40.113	46.963	55.475
28	37.916	41.337	48.278	56.892
29	39.087	42.557	49.588	58.301
30	40.256	43.773	50.892	59.702
40	51.805	55.758	63.691	73.403
50	63.167	67.505	76.154	86.660
60	74.397	79.082	88.379	99.608
70	85.527	90.531	100.43	112.32
80	96.578	101.88	112.33	124.84
90	107.57	113.15	124.12	137.21
100	118.50	124.34	135.81	149.45

5 Example 2

Two group of 15 patients at risk for arrhythmias were assessed for the development of torsade de points after calcium channel blockers treatment

	Patients with torsade de points	patients without
Calcium channel blocker 1	5	10
Calcium channel blocker 2	9	6

The proportion of patients with event from calcium channel blocker 1 is 5/15, from blocker 2 it is 9/15.

Log likelihood ratio $= 9 \log \frac{5/15}{9/15} + 6 \log \frac{1-5/15}{1-9/15}$
$= -2.25$
-2 log likelihood ratio $= 4.50$
$= z^2$
z – value $= \surd\, 4.50 = 2.1213$
p – value <0.05, because $z > 2$.

The traditional chi-square test of these data was non-significant ($p > 0.05$), (Chap. 38). You can check for yourself, that, with the odds ratio test (Chap. 44), this will be equally so.

6 Example 3

Two groups of patients with stage IV New York Heart Association heart failure were assessed for clinical admission while on two beta-blockers.

	Patients with clinical admission	patients without
Beta blocker 1	77	62
Beta blocker 2	103	46

The proportion of patients with event while on beta blocker 1 is 77/139, while on beta blocker 2 it is 103/149.

Log likelihood ratio $= 103 \log \frac{77/139}{103/149} + 46 \log \frac{1-77/139}{1-103/149}$
$= -5.882$
-2 log likelihood ratio $= 11.766$
$= z^{\,2}$
z – value $= \surd\, 11.766 = 3.43016$
p – value <0.002, because $z > 3.090$ (see the above t-table).

Both the odds ratio test and chi-square test were also significant. However, at lower levels of significance, both p-values $0.01 < p < 0.05$.

7 Conclusion

The sensitivity of the traditional tests for testing cross-tabs is limited, and, not entirely, accurate, with cells smaller than 5. The log likelihood ratio test is an adequate alternative with generally better sensitivity. It is lovely to use it, if your traditional test can not reject the null hypothesis of your study with a p-value a bit larger than 0.05. Your chance is big, that the current test will produce a p-value just under 0.05.

8 Note

More background, theoretical and mathematical information of log likelihood ratio tests is given in Statistics applied to clinical studies 5th edition, Chap. 4, Springer Heidelberg Germany, 2012, from the same authors.