Chapter 43
Logarithmic Transformations, a Great Help to Statistical Analyses

1 General Purpose

Non-linear relationships in clinical research are often linear after logarithmic transformations. Also, logarithmic transformation normalizes skewed frequency distributions and is used for the analysis of likelihood ratios. Basic knowledge of logarithms is, therefore, convenient for a better understanding of many statistical methods. Almost always natural logarithm (ln), otherwise called Naperian logarithm, is used, i.e., logarithm to the base e. Log is logarithm to the base 10, ln is logarithm to the base e (2.718281828). This chapter is for showing that knowledge of logarithms is helpful for understanding many statistical methods.

2 Schematic Overview of Type of Data File

Outcome binary	predictor binary
.	.
.	.
.	.
.	.
.	.
.	.
.	.
.	.
.	.

T.J. Cleophas, A.H. Zwinderman, *Clinical Data Analysis on a Pocket Calculator*,
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3 Primary Scientific Question

Is logarithmic transformation relevant for statistical modeling.

4 Theory and Basic Steps

log 10 = 10 log 10 = 1
log 100 = 10 log 100 = 2
log 1 = 10 log 1 = 0
antilog 1 = 10
antilog 2 = 100
antilog 0 = 1

Casio fx-825 scientific, Scientific Calculator, Texas TI-30XA, Sigma, Commodore
Press: 100. . ..log. . ..2
Press: 2. . ..2ndf. . ..log. . .100

Electronic Calculator, Kenko KK-82MS-5
Press: 100. . .. = ...log. . .. = ...2
Press: 2. . .. = ...shift. . .log. . ..100

ln e = e log e = 1
ln e^2 = e log e^2 = 2
ln 1 = e log 1 = 0
antiln 1 = 2.718. . .
antiln 2 = 7.389. . .
antiln 0 = 1

Casio fx-825 scientific, Scientific Calculator, Texas TI-30XA, Sigma
Press: 7.389. . ..ln. . ..2
Press: 2. . ..2ndf. . ..ln. . .7389

Electronic Calculator, Kenko KK-82MS-5
Press: 7.389. . .. = ...ln. . .. = ...2
Press: 2. . .. = ...shift. . .ln. . ..7.389

5 Example, Markov Model

In patients with diabetes mellitus (* = sign of multiplication):

After 1 year 10 % has beta-cell failure, and 90 % has not.
2 90 * 90 = 81 % has not.
3 90 * 90 * 90 = 73 % has not.

When will 50 % have beta-cell failure?

$0.9^x = 0.5$

$x \log 0.9 = \log 0.5$

$x = \log 0.5/\log 0.9 = 6.5788$ years.

6 Example, Odds Ratios

	events	no events	
	numbers of patients		
group 1	15(a)	20(b)	35(a + b)
group 2	15(c)	5(d)	20(c + d)
	30(a + c)	25(b + d)	55(a + b + c + d)

The odds of an event = the number of patients in a group with an event divided by the number without. In group 1 the odds of an event equals = a/b.

The odds ratio (OR) of group 1 compared to group 2

$= (a/b)/(c/d)$

$= (15/20)/(15/5)$

$= 0.25$

lnOR $= \ln 0.25 = -1.386$ (ln = natural logarithm)

The standard error (SE) of the above term

$= \sqrt{(1/a + 1/b + 1/c + 1/d)}$

$= \sqrt{(1/15 + 1/20 + 1/15 + 1/5)}$

$= \sqrt{0.38333}$

$= 0.619$

The odds ratio can be tested using the z-test.

The test-statistic = z-value

= (ln odds ratio)/(SE ln odds ratio)

$= -1.386/0.619$

$= -2.239$

The underneath t-table, bottom row, shows, that, if this value is smaller than −1.96 or larger than +1.96, then the odds ratio is significantly different from 1 with a two-tail p-value < 0.05. There is, thus, a significant difference in numbers of events between the two groups.

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The left-end column of the above t-table gives degrees of freedom (≈ sample sizes), two top rows with p-values (areas under the curve), and, furthermore, the t-table is full of t-values, that, with ∞ degrees of freedom, become equal to z-values.

7 Conclusion

We conclude that basic knowledge of logarithms is convenient for a better understanding of many statistical methods. Odds ratio tests (Chap. 44), log likelihood ratio tests (Chap. 46), Markov modeling (Chap. 55), and many regression models use logarithmic transformations.

8 Note

More background, theoretical and mathematical information of logarithmic transformations is given in the Chaps. 28, 30, 44, 45, 46, 47, 48, 49, and 55.