Chapter 5
One-Sample Continuous Data (One-Sample *T*-Test, One-Sample Wilcoxon Test)

1 General Purpose

Studies where a single outcome per patient is compared to zero may be analyzed with the one-sample *t*-test (see also Chap. 4). The *t*-test is only adequate, if the data can be assumed to follow a Gaussian-like frequency distribution. For non-Gaussian-like data the one-sample Wilcoxon test will be appropriate.

2 Schematic Overview of Type of Data File

Outcome
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T.J. Cleophas, A.H. Zwinderman, *Clinical Data Analysis on a Pocket Calculator*,
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3 Primary Scientific Question

Are the one-sample *t*-test and one-sample Wilcoxon test adequate for testing whether the result of a one-sample study is significantly different from a zero result?

4 Data Example, One-Sample *T*-test

In 10 patients the mean blood pressure reduction after treatment is calculated with the accompanying p-value. A p-value < 0.05 indicates, that there is less than 5 % probability that such a decrease will be observed purely by the play of chance. There is, thus, > 95 % chance that the decrease is the result of a real blood pressure lowering effect of the treatment. We call such a decrease statistically significant.

Patient number	mm Hg decrease
1	3
2	4
3	−2
4	3
5	1
6	−3
7	4
8	3
9	2
10	1

Is this decrease statistically significant?

Mean decrease = 1.6 mm Hg
SD = 2.4129 mm Hg

From the standard deviation (SD) the standard error (SE) can be calculated using the equation

$SE = SD/\sqrt{n}$ (n = sample size)
$SE = 2.4129/\sqrt{10} = 0.7636$

De t-value (t) is the test-statistic of the *t*-test, and is calculated as follows:

$$t = 1.6 / 0.7636 = 2.095$$

Because the sample size is 10, the test has here $10–1 = 9°$ of freedom.

The t-table underneath shows that with 9° of freedom the t-value should be > 2.262 in order to obtain a two-tail result significantly different from zero at $p < 0.05$. With a t-value of 2.095 the level equals: $0.05 < p < 0.10$. This result is close to statistically significant, and is called a trend to significance.

5 T-Table

The underneath t-table has a left-end column giving degrees of freedom (≈ sample sizes), and two top rows with p-values (areas under the curve = p-values), one-tail meaning that only one end of the curve, two-tail meaning that both ends are assessed simulataneously. The t-table is, furthermore, full of t-values, that, with ∞ degrees of freedom, are equal to z-values (Chap.36). The t-values are to be understood as mean results of studies, but not expressed in mmol/l, kilograms, but in so-called SEM-units (Standard error of the mean units), that are obtained by dividing your mean result by its own standard error. With many degrees of freedom (large samples) the curve will be a little bit narrower, and more in agreement with nature.

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

6 Data Example, One-Sample Wilcoxon Test

Patient number	mm Hg	Decrease	Smaller one of two add-up sums
1	3	6.5	
2	4	9.5	
3	−2	3.5	3.5
4	3	6.5	
5	1	1.5	
6	−3	6.5	6.5
7	4	9.5	
8	3	6.5	
9	2	3.5	
10	1	1.5	

The example of the previous section will be applied once more for a Wilcoxon analysis. We will first put the differences from zero in ascending order. The patients 5 and 10 are equal (1 mm Hg different from zero), we will give them rank number 1.5 instead of 1 and 2. Patient 3 and 9 are equal (have equal distances from zero), and will be given rank number 3.5 instead of 3 and 4.

The patients 1, 4, 6, and 8 are again equal and will be given rank number 6.5 instead of 5, 6, 7, 8. When each patient has been given an appropriate rank number, all of the positive and all of the negative rank numbers will be added up, and the smaller number of the two will be used for estimating the level of statistical significance. Our add-up sum of negative outcome values is the smaller number, and adds the values of the patients 3 and 6, and equals $3.5 + 6.5 = 10$. According to the underneath Wilcoxon table with 10 number of paires the add-up value of 10 indicates that our p-value equals < 0.10. This result is very similar to the result of the above *t*-test. Again a trend to significance is observed at $0.05 < P < 0.10$.

Wilcoxon Test Table

Number of pairs	P < 0.10	P < 0.05	P < 0.01
7	3	2	0
8	5	2	0
9	8	6	2
10	10	8	3
11	13	11	5
12	17	14	7
13	20	17	10
14	25	21	13
15	30	25	16
16	35	30	19

The first column in the above Wilcoxon test table gives the numbers of patients in your file. Rank numbers of positive and negative differences from zero are separately added up. The second, third, and fourth columns give the smaller one of the two add-up sums required for statistical significance of increasing levels.

7 Conclusion

Studies where one outcome in one patient is compared with zero can be analyzed with the one-sample *t*-test. The *t*-test is adequate, if the data can be assumed to follow a Gaussian-like frequency distribution. For non-Gaussian-like data the one-sample Wilcoxon test is appropriate. The example given shows that levels of statistical significance of the two tests are very similar.

8 Note

More background, theoretical and mathematical information of testing null-hypothesis testing with t-tests and Wilcoxon tests is given in Statistics applied to clinical studies 5th edition, Chap. 1, Springer Heidelberg Germany, 2012, from the same authors.