Chapter 11
Power Equations

1 General Purpose

Power can be described as statistical conclusive force. It can be defined as the chance of finding a difference where there is one. Other chances are the chance of finding no difference where there is one (type II error) and the chance of finding a difference where there is none (type I) error (Chap. 3). A study result is often expressed in the form of the mean result and its standard deviation (SD) or standard error (SE). With the mean result getting larger and the standard error getting smaller, the study will obtain increasing power. This chapter is to show how to compute from a study's mean and standard error its statistical power.

2 Schematic Overview of Type of Data File

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T.J. Cleophas, A.H. Zwinderman, *Clinical Data Analysis on a Pocket Calculator*,
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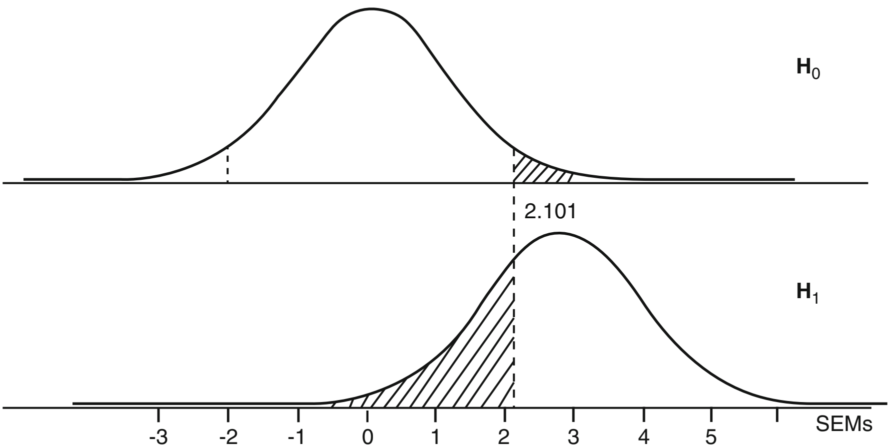
3 Primary Scientific Question

What is the power of a study with its mean study result and its standard error given.

4 Power Assessment

Important hypotheses are hypothesis 0 (H0, no difference from a 0 effect), and hypothesis 1 (H1, real difference from a 0 effect). For the purpose of power assessment, we will, particularly, emphasize hypothesis 1. The underneath figure shows graphs of H0 and H1.



H1 is a graph based on a 2-group trial with a sample size of 20. Mean ± SEMs are on the x-axis (commonly called z-axis here). H0 is the same graph with mean 0. H1 is also the summary of the means many trials similar to ours. H0 is also the summary of the means many trials similar to ours, but with an overall mean effect of 0. If the hypothesis 0 is true, then the mean of our study is part of H0, if the hypothesis 1 is true, then the mean of our study is part of H1. So, the mean of our study may be part of H0, or of H1. We can't prove anything, but we can calculate the chance of either of these possibilities.

A mean result of 2.9 is far distant from 0. Suppose, it belongs to H0. Only 5 % of the H0 trials > 2.1 SEMs distant from 0. The chance, that it belongs to H0 is, thus, < 5 %. Reject this small possibility. Now, suppose the result belongs to H1.

Up to 30 % of the H1 trials are < 2.1 SEMs distant from 0. These 30 % cannot reject null hypothesis of no effect. The trials right from 2.1 SEMs (corresponding with 70 % of the area under the curve (AUC)) can do so.

We can conclude from all of the above considerations: if H0 is true, we will have < 5 % chance to find it; if H1 is true, we will have 70 % chance to find it. And so, we will decide to reject the null hypothesis of no effect at $p < 0.05$, and to do so with a power of 70 %.

5 Data Example

A blood pressure study shows a mean decrease in blood pressure of 10.8 mm Hg with a standard error of 3.0 mm Hg. Results from study samples are often given in grams, liters, Euros, mm Hg etc. For the calculation of power we have to first standardize our study result, which means that the mean result has to be divided by its own standard error (SE or SEM):

Mean ± SE =
mean /SE ± SE/SE =
t-value ± 1.

All of the t-values, as found in the t-table, can be looked at as the standardized mean results of all kinds of studies. In our blood pressure study the t-value = 10.8 / 3.0 = 3.6. The unit of the t-value is not mm Hg, but rather SE-units or SEM-units. The question is: what power will the study have, if we assume a type I error (alpha) = 5 % and a sample size of n = 20.

The question is: what will the power of this study be, if we assume a type I error (alpha) of 5 %, and a sample size of n = 20.

A. 90 % < power < 95 %,
B. power > 80 %,
C. power < 75 %,
D. power > 75 %.

n = 20 indicates 20–2 = 18 degrees of freedom in the case of 2 groups of 10 patients.

We will use the following power equation (prob = probability, z = value on the z-line (the x-axis of the t-distribution)

$$\text{Power} = 1 - \text{prob}\left(z < t - t^1\right)$$

t	= the t-value of your results,
t^1	= the t –value, that matches a p-value of 0.05 = 2.1;
t	= 3.6; $t^1 = 2.1$; $t\text{-}t^1 = 1.5$;
prob $(z < t - t^1)$	= beta = type II error = between 0.05 and 0.1
1-beta = power	= between 0.9 and 0.95 = between 90 and 95 %.

So, there is a very good power here. Explanation of the above calculation is given in the next few lines.

6 T-Table

The t-table has a left-end column giving degrees of freedom (≈ sample sizes), and two top rows with p-values (areas under the curve = p-values), one-tail meaning that only one end of the curve, two-tail meaning that both ends are assessed simultaneously. The t-table is, furthermore, full of t-values, that, with ∞ degrees of freedom, are equal to z-values (Chap. 36). The t-values are to be understood as mean results of studies, but not expressed in mmol/l, kilograms, but in so-called SEM-units (Standard error of the mean units), that are obtained by dividing your mean result by its own standard error. With many degrees of freedom (large samples) the curve will be a little bit narrower, and more in agreement with nature. The current chapter shows how the t-table can also be applied for computing statistical power.

With a t-value of 3.6 as shown in the previous section, and 18 degrees of freedom, the term $(t - t^1)$ equals 1.5. This value is between 1.330 and 1.734. Look right up at the upper top row for finding beta (type II error = the chance of finding no difference where there is one). We have two top rows here, one for one-tail testing one for two-tail testing. Power is always tested one-tail. We are between 0.1 and 0.05 (10 and 5 %). This is an adequate estimate of the type II error. The power, thus, equals (100 % − beta) = between 90 and 95 % in our example.

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

7 Conclusion

Power can be defined as the chance of finding an effect (or a difference from zero), where there is one. It is equal to 1 minus the type II error ($=1 - \beta$). A study result is often expressed in the form of the mean result and its standard deviation (SD) or standard error (SE or SEM). With the mean result getting larger and the standard error getting smaller, the study will obtain increasing power. This chapter shows, how to compute a study's statistical power from its mean and standard error.

8 Note

More background, theoretical and mathematical information of power assessments is given in Statistics applied to clinical studies 5th edition, Chap. 6, Springer Heidelberg Germany, 2012, from the same authors.