Chapter 36
Z-Test for Cross-Tabs

1 General Purpose

If, as shown in the previous chapter, multiple similar a selective samples of binary data have a normal frequency distribution, then t-tests, like the ones for continuous data (Chap. 7), should be OK for their analysis. Consequently, if we compare two binary samples, for example the outcomes of two treatment modalities, then a two sample t-test should be OK for statistical testing. The two sample t-test for binary outcomes is called the two sample z-test. This chapter shows how it works.

2 Schematic Overview of Type of Data File

Outcome (binary)	predictor (binary)
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T.J. Cleophas, A.H. Zwinderman, *Clinical Data Analysis on a Pocket Calculator*,
DOI 10.1007/978-3-319-27104-0_36

3 Primary Scientific Question

Is the two sample z-test adequate for comparing the differences in numbers of responders to two different treatment modalities.

4 Data Example, Two Group Z-Test

Two groups of patients are assessed for being sleepy through the day. We wish to estimate whether group 1 is more sleepy than group 2. The underneath cross-tab gives the data.

	Sleepiness	no sleepiness
Treatment 1 (group 1)	5 (a)	10 (b)
Treatment 2 (group 2)	9 (c)	6 (d)

$$z = \frac{\text{difference between proportions of sleepers per group (d)}}{\text{pooled standard error difference}}$$

$$z = \frac{d}{\text{pooled SE}} = \frac{(9/15 - 5/15)}{\sqrt{(SE_1^2 + SE_2^2)}}$$

$$SE_1 \text{ (or } SEM_1 \text{)} = \sqrt{\frac{p_1 (1 - p_1)}{n_1}} \text{ where } p_1 = 5/15 \text{ etc........,}$$

$z = 1.45$, not statistically significant from zero, because for a two- tail p – value <0.05 a z-value of at least 1.96 is required. This means that no significant difference between the two groups is observed. The p-value of the z-test can be obtained by using the bottom row of the underneath t-table.

df	One-Tail = .4	.25	.1	.05	.025	.01	.005	.0025	.001	.0005
	Two-Tail = .8	.5	.2	.1	.05	.02	.01	.005	.002	.001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The t-table has a left-end column giving degrees of freedom (≈ sample sizes), and two top rows with p-values (areas under the curve = p – values), one-tail meaning that only one end of the curve, two-tail meaning that both ends are assessed simultaneously. The t-table is, furthermore, full of t-values, that, with ∞ degrees of freedom, are equal to z-values. The z-values and t-values are to be understood as mean results of studies, but not expressed in mmol/l, kilograms, or proportions of responders, but in so-called SEM-units (Standard error of the mean units), that are obtained by dividing your mean result by its own standard error. For continuous outcome data, with many degrees of freedom (large samples) the curve will be a little bit narrower, and more in agreement with nature. For binary outcome data, nature has determined that the curves will always be as narrow as can be, according to the row at the bottom.

5 Single Group Z-Test

A single group z-test is also possible (see also Chap. 35). For example, in 10 patients we have 4 responders. We question whether 4 responders is significantly more than 0 responders.

$$
\begin{aligned}
z &= \text{proportion}/(\text{its SE}) \\
SE &= \sqrt{[(4/10 \times (1 - 4/10))/n]} \\
&= \sqrt{(0.24/10)} \\
z &= 0.4/\sqrt{(0.24/10)} \\
z &= 0.4/0.1549 \\
z &= 2.582
\end{aligned}
$$

According to the bottom row of the t-table the p-value is <0.01. A proportion of 0.4 is, thus, significantly larger than a proportion of 0.0.

6 Conclusion

As multiple similar a selective samples of binary data have a normal frequency distribution, the t-tests like the ones for continuous data (Chap. 7) are OK for their analysis. Consequently, for comparing two binary samples, for example the outcomes of two treatment modalities, two sample t-test is OK for statistical testing. The two sample t-test for binary outcomes is called the two sample z-test. This chapter shows how it works.

7 Note

More background, theoretical and mathematical information is given in the Statistics applied to clinical studies 5th edition, Springer Heidelberg Germany, Chap. 3, 2012, from the same authors.