1 Introduction to Structural Engineering

Abstract

A structure is an assemblage of components which are connected in such a way that the structure can withstand the action of loads that are applied to it. These loads may be due to gravity, wind, ground shaking, impact, temperature, or other environmental sources. Examples of structures employed in civil infrastructure are buildings, bridges, dams, tunnels, storage tanks, and transmission line towers. Non-civil applications include aerospace structures such as airplane fuselages, missiles; naval structures such as ships, offshore platforms; and automotive structures such as cars and trucks. Structural engineering is the discipline which is concerned with identifying the loads that a structure may experience over its expected life, determining a suitable arrangement of structural members, selecting the material and dimensions of the members, defining the assembly process, and lastly monitoring the structure as it is being assembled and possibly also over its life.

In this chapter, we describe first the various types of structures. Each structure is categorized according to its particular function and the configuration of its components. We then discuss the critical issues that a structural engineer needs to address when designing or assessing the adequacy of a structure. The most important issue is preventing failure, especially a sudden catastrophic failure. We describe various failure modes: initial instability, material failure, and buckling of individual structural components. In order to carry out a structural design, one needs to specify the loading which is also a critical concern. Fortunately, the technical literature contains considerable information about loadings. We present here an overview of the nature of the different loads and establish their relative importance for the most common civil structures. Conventional structural design philosophy and the different approaches for implementing this design strategy are described next. Lastly, we briefly discuss some basic analytical methods of structural engineering and describe how they are applied to analyze structures.

J.J. Connor, S. Faraji, *Fundamentals of Structural Engineering*,
DOI 10.1007/978-3-319-24331-3_1

1.1 Types of Structures and Structural Components

Structures are everywhere in the built environment. Buildings, bridges, tunnels, storage tanks, and transmission lines are examples of a "structure." Structures differ in their *makeup*, i.e., the type and configuration of the components, and also in their *function*. Our approach to describing a structure is based on identifying a set of attributes which relate to these properties.

1.1.1 Structural Components

The components are the basic building blocks of a structure. We refer to them as structural elements. Elements are classified into two categories according to their geometry [1]:

1. *Line Elements*—The geometry is essentially one-dimensional, i.e., one dimension is large with respect to the other two dimensions. Examples are cables, beams, columns, and arches. Another term for a line element is member.
2. *Surface Elements*—One dimension is small in comparison to the other two dimensions. The elements are plate-like. Examples are flat plates, curved plates, and shells such as spherical, cylindrical, and hyperbolic paraboloids.

1.1.2 Types of Structures

A structure is classified according to its function and the type of elements used to make up the structure. Typical structures and their corresponding functions are listed in Table 1.1 and illustrated in Fig. 1.1. A classification according to makeup is listed in Table 1.2 and illustrated in Fig. 1.2.

1.2 Critical Concerns of Structural Engineering

Of primary concern to a structural engineer is ensuring that the structure *will not collapse* when subjected to its design loading. This requires firstly that the engineer properly identify the extreme loading that the structure may experience over its design life and secondly, ensure that the forces generated internally within the structure due to external loading *satisfy the conditions for force equilibrium*. In general, a structure will deform, i.e., change its shape, when loaded. It may also

Table 1.1 Structures classified by function

Structural type	Function
Building	Provide shelter above ground
Bridge	Provide means of traversing above ground over a site
Tunnel	Provide means of traversing underground through a site
Tower	Support transmission lines and broadcasting devices
Retaining walls	Retain earth or other material
Containments	Provide means of storage of materials, also enclose dangerous devices such as nuclear reactors
Platforms	Provide a platform for storage of materials and machinery either onshore or offshore

Building

Tunnel

Fig. 1.1 Examples of typical structures classified by function

move as a rigid body if not properly restrained. Certain structures such as airplanes and automobiles are designed to move. However, civil structures are generally limited to small motion due to deformation, and *rigid* body motion is prohibited. Identifying the design loads is discussed later in this chapter. We focus here on the force equilibrium requirement for civil structures.

1.2.1 Reactions

Civil structures are connected to the ground at certain points called supports. When the external loading is applied to the structure, the supports develop forces which oppose the tendency of the

Offshore platforms

Bridge

Fig. 1.1 (continued)

structure to move. These forces are called reactions [2]. The nature and number of reactions depends on the type of support. Figure 1.3 shows the most common types of idealized structural supports for any planar structure. A roller support allows motion in the longitudinal direction but not in the transverse direction. A hinge prevents motion in both the longitudinal and transverse directions but allows rotation about the pin connection. Lastly, the clamped (fixed) support restrains rotation as well

Table 1.2 Structures classified by makeup

Structural type	Composition
Frame	• Composed of members rigidly or semirigidly connected in rectangular or triangular patterns
	• May be contained in a single plane (plane frame, plane grid), or in a 3D configuration (space frame)
Truss	• A type of framed structure where the members are connected together at their ends with frictionless pins (plane or space truss)
Girder/beam	• Composed of straight members connected sequentially (end to end)
	• An additional descriptor related to the type of member cross section is used
	Examples are plate girders, box girders, and tub girders
Arch	• Curved beams (usually in one plane)
Cable	• Composed of cables and possibly other types of elements such as girders
	Examples are cable-stayed bridges and tensioned grids
Shell	• Composed of surface elements and possibly also line elements such as beams
	The elements may be flat (plate structures) or curved (spherical or cylindrical roof structures)

as translation with two reaction forces and one moment. Three-dimensional supports are similar in nature. There is an increase from 2 to 3 and from 3 to 6 in the number of reactions for the 3D hinge and a clamped support.

1.2.2 Initial Stability

If either the number or nature of the reactions is insufficient to satisfy the equilibrium conditions, the structure is said to be initially unstable. Figure 1.4a illustrates this case. The structure consists of a triangular arrangement of members that are pinned at their ends. This combination of members forms a rigid body. However, the arrangement is supported on two roller supports which offer no resistance to horizontal motion, and consequently the structure is initially unstable. This situation can be corrected by changing one of the roller supports to a hinge support, as shown in Fig. 1.4b. In general, a rigid body is initially stable when translational and rotational motions are prevented in three mutually orthogonal directions.

Even when the structure is adequately supported, it still may be initially unstable if the members are not properly connected together to provide sufficient internal forces to resist the applied external forces. Consider the four member pin-connected planar structure shown in Fig. 1.5a. The horizontal force, P, cannot be transmitted to the support since the force in member 1-2 is vertical and therefore cannot have a horizontal component. Adding a diagonal member, either 1-3 or 2-4, would make the structure stable.

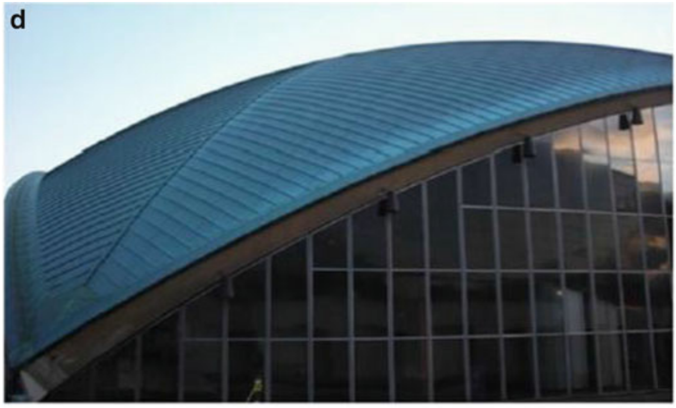
In summary, initial instability can occur either due to a *lack of appropriate supports* or to *an inadequate arrangement of members*. The test for initial instability is whether there are sufficient reactions and internal member forces to equilibrate the applied external loads. Assuming the structure is initially stable, there still may be a problem if certain structural components fail under the action of the extreme loading and cause the structure to *lose* its ability to carry load. In what follows, we discuss various failure scenarios for structures which are loaded.

Frame

Bridge girder

Space Truss

Fig. 1.2 Structures classified by makeups



Shell

Arch bridge

Cable-girder system (suspension bridge)

Fig. 1.2 (continued)

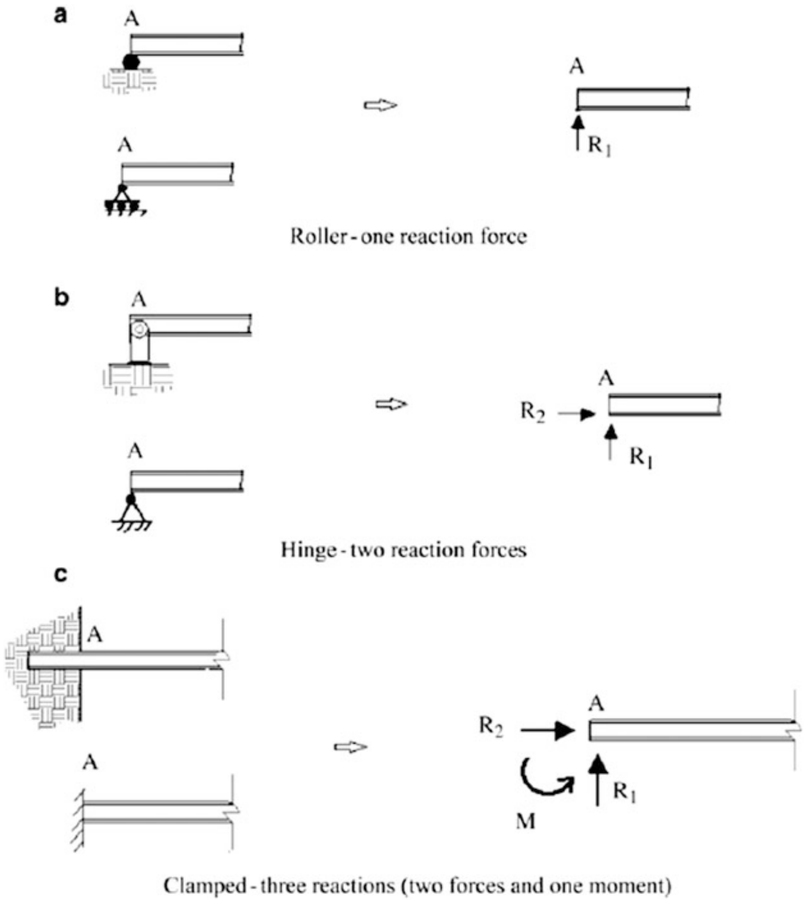


Fig. 1.3 Typical supports for planar structures

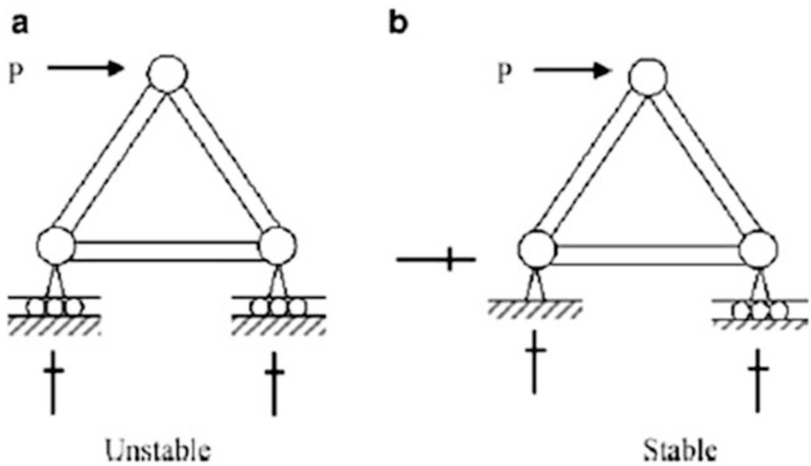


Fig. 1.4 Examples of unstable and stable support conditions—planar structure

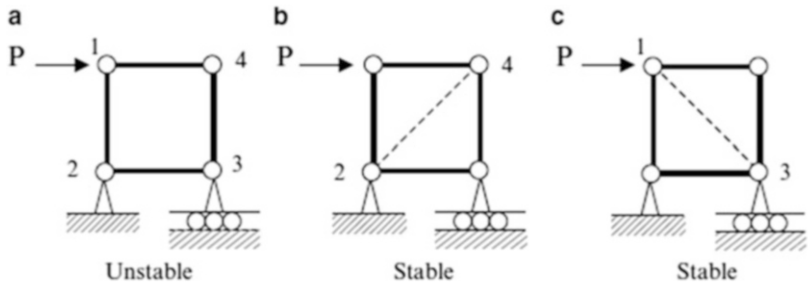


Fig. 1.5 Stabilizing an initially unstable planar structure

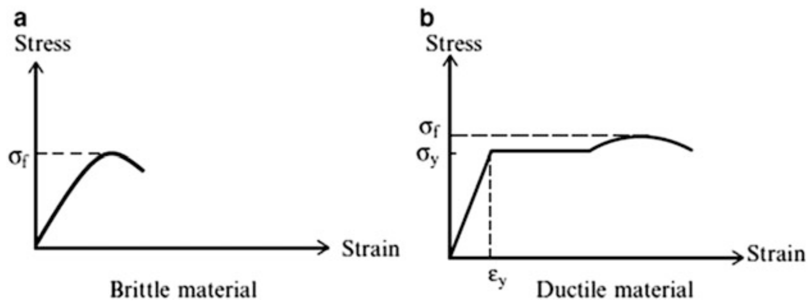


Fig. 1.6 Stress–strain behavior of brittle and ductile materials

1.2.3 Loss of Stability Due to Material Failure

In the first scenario, the level of stress in a component reaches the ultimate stress for the material, causing a material failure, which, in turn, triggers a failure of the component. This type of failure depends on the stress–strain relationship for the material. Figure 1.6 illustrates the tensile stress–extensional strain response of tension specimens fabricated from two different types of materials [3, 4]. The behavior of the first material is essentially linear up to a peak stress, σ_f, at which point the material fractures and loses its ability to carry any load. This behavior is referred to as *brittle behavior* and obviously is not desirable from a structural behavior perspective.

The second response is completely different. The initial behavior is linear up to a certain stress value defined as the yield stress, σ_y. For further straining, the stress remains essentially constant. Eventually, the material stiffens and ultimately fails at a level of strain which is considerably greater than the yield strain, ε_y. This behavior is typical for *ductile* materials such as the steels used in civil structures. In practice, the maximum allowable strain is limited to a multiple of the yield strain. This factor is called the ductility ratio (μ) and is on the order of 5. Ductile behavior is obviously more desirable since a member fabricated out of a ductile material does not lose its load capacity when yielding occurs. However, it cannot carry additional loading after yielding since the resistance remains constant.

From a design perspective, the structural engineer must *avoid* brittle behavior since it can result in *sudden* catastrophic failure. Ductile behavior and the associated inelastic deformation are acceptable provided that the ductility demand is within the design limit. Limit state design is a paradigm for dimensioning structural components that assumes the component is at its limit deformation state and calculates the force capacity based on the yield stress [5]. This topic is dealt with in Chap. 16.

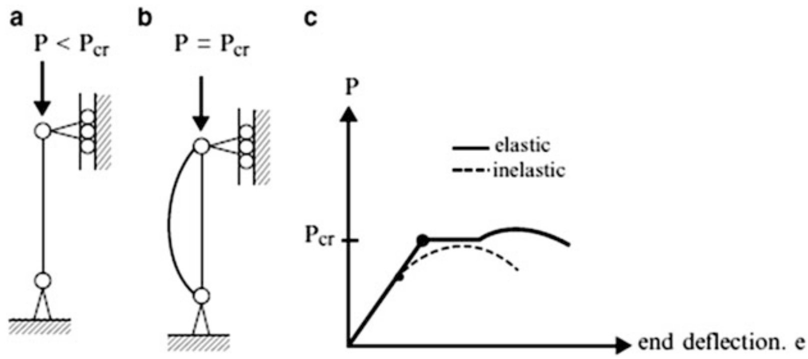


Fig. 1.7 Behavior of a flexible member

1.2.4 Buckling Failure Mode

Another possible failure scenario for a structural component is buckling. Buckling is a phenomenon associated with long slender members subjected to compressive loading [3, 4]. We illustrate this behavior using the member shown in Fig. 1.7a. As the axial loading is increased, the member remains straight until a critical load value is reached. At this point, the member adopts a deflected configuration (Fig. 1.7b) with the load remaining constant. The member force remains essentially constant as the end deflection, e, is increased (Fig. 1.7c). This load deflection behavior is similar to inelastic action in the sense that the member experiences a large deflection with essentially no increase in load. For flexible members, the critical load for buckling (P_{cr}) is generally less than the axial compressive strength based on yielding, *therefore buckling usually controls the design*.

1.2.5 Priorities for Stability

Finally, summarizing and prioritizing the different concerns discussed in the previous sections, the highest priority is ensuring the structure is initially stable. If not stable, the structure will fail under an infinitesimal load. The second priority is avoiding buckling of the members. Buckling can result in large deformation and significant loss in load capacity for a member, which could cause the structure to lose its ability to support the applied loading. The third priority is limiting inelastic deformation of members under the extreme design loading. Although there is no loss in load capacity, the member cannot provide any additional load capacity, and therefore the deformation will increase significantly when the external loading is increased. We discuss this topic further in Sect. 1.4 where we present design philosophies.

1.3 Types of Loads

As described above, structures must be proportioned so that they will not fail or deform excessively under the loads they may be subjected to over their expected life. Therefore, it is critical that the nature and magnitude of the loads they may experience be accurately defined. Usually, there are a number of different loads, and the question as to which loads may occur simultaneously needs to be addressed when specifying the design loading. In general, the structural engineer works with codes, which specify design loadings for various types of structures. General building codes such as the "International Building Code" [6] specify the requirements of governmental agencies for minimum design loads for structures and minimum standards for construction. Professional technical societies such as the American Society of Civil Engineers (ASCE) [7], the American Concrete Institute (ACI) [8], the American Institute of Steel Construction (AISC) [9], and the British Standards Institute (BSI) [10] publish detailed technical standards that are also used to establish design loads and structural performance requirements. In what follows, we present an overview of the nature of the different loads and provide a sense of their relative importance for the most common civil structures.

1.3.1 Source of Loads

Loads are caused by various actions: the interaction of the structure with the natural environment; carrying out the function they are expected to perform; construction of the structure; and terrorist activities.

1.3.1.1 Interaction with the Environment

Interaction with the natural environment generates the following types of loads:

- Gravity—gravitational force associated with mass
- Snow—gravity-type loading
- Wind—steady flow, gusts
- Earthquake—ground shaking resulting from a seismic event
- Water—scour, hydrostatic pressure, wave impact
- Ice—scour, impact
- Earth pressure—soil–structure interaction for foundations and underground structures
- Thermal—seasonal temperature variations

The relative importance of these sources depends on the nature of the structure and the geographical location of the site. For example, building design is generally governed by gravity, snow, wind, and possibly earthquake loads. Low-rise buildings in arctic regions tend to be governed by snow loading. Underground basement structures and tunnels are designed for earth pressure, hydrostatic pressure, and possibly earthquake loads. Gravity is the dominant source of load for bridge structures. Wave and ice action control the design of offshore platforms in coastal arctic waters such as the coasts of Alaska and Newfoundland. Structures located in California need to be designed for high seismic load. Structures located in Florida need to be designed for high wind load due to hurricanes. Thermal loads occur when structural elements are exposed to temperature change and are not allowed to expand or contract.

1.3.1.2 Function

Function-related loads are structure specific. For bridges, vehicular traffic consisting of cars, trucks, and trains generates gravity-type load, in addition to the self-weight load. Office buildings are intended to provide shelter for people and office equipment. A uniformly distributed gravity floor load is specified according to the nature of the occupancy of the building. Legal offices and libraries tend to have a larger design floor loading since they normally have more storage files than a normal office. Containment structures usually store materials such as liquids and granular solids. The associated loading is a distributed internal pressure which may vary over the height of the structure.

1.3.1.3 Construction

Construction loading depends on the process followed to assemble the structure. Detailed force analyses at various stages of the construction are required for complex structures such as segmented long-span bridges for which the erection loading dominates the design. The structural engineer is responsible for approving the construction loads when separate firms carry out engineering and construction. A present trend is for a single organization to carry out both the engineering design and construction (the design-build paradigm where engineering companies and construction companies form a joint venture for the specific project). In this case, a team consisting of structural engineers and construction engineers jointly carries out the design. An example of this type of partnering is the construction of the Millau Viaduct in southwestern France, shown in Fig. 1.8. The spans were constructed by cantilevering segments out from existing piers, a technically challenging operation that required constant monitoring. The bridge piers are the highest in the world: the central pier is 280 m high.

1.3.1.4 Terrorist Loads

Terrorist loads are a new problem for structural engineers, driven primarily by the need to protect essential facilities from terrorist groups. Design criteria are continuously evolving, and tend to be directed more at providing multilevel defense barriers to prevent incidents, rather than to design for a specific incident. Clearly, there are certain incidents that a structure cannot be designed to safely handle, such as the plane impacts that destroyed the World Trade Center Towers. Examining progressive collapse mechanisms is now required for significant buildings and is the responsibility of the structural engineer.

Fig. 1.8 Millau Viaduct

1.3.2 Properties of Loadings

The previous discussion was focused on the source of loadings, i.e., environmental, functional, construction, and terrorist activity. Loadings are also characterized by attributes, which relate to properties of the loads. Table 1.3 lists the most relevant attributes and their possible values.

Duration relates to the time period over which the loading is applied. Long-term loads, such as self-weight are referred to as dead loads. Loads whose magnitude or location changes are called temporary loads. Examples of temporary loads are the weight of vehicles crossing a bridge, stored items in buildings, wind and seismic loads, and construction loads.

Most loads are represented as being applied over a finite area. For example, a line of trucks is represented with an equivalent uniformly distributed load. However, there are cases where the loaded area is small, and it is more convenient to treat the load as being concentrated at a particular point. A member partially supported by cables such as a cable-stayed girder is an example of concentrated loading.

Temporal distribution refers to the rate of change of the magnitude of the temporary loading with time. An impulsive load is characterized by a rapid increase over a very short duration and then a drop off. Figure 1.9 illustrates this case. Examples are forces due to collisions, dropped masses, brittle fracture material failures, and slamming action due to waves breaking on a structure. Cyclic loading alternates in direction (+ and −) and the period may change for successive cycles. The limiting case of cyclic loading is harmonic excitation where the amplitude and period are constant. Seismic excitation is cyclic. Rotating machinery such as printing presses, electric generators, and turbines produce harmonic excitation on their supports when they are not properly balanced. Quasi-static loading is characterized by a relatively slow build up of magnitude, reaching essentially a steady state. Because they are applied slowly, there is no appreciable dynamic amplification and the structure responds as if the load was a *static* load. Steady winds are treated as quasi-static; wind gusts are impulsive. Wind may also produce a periodic loading resulting from vortex shedding. We discuss this phenomenon later in this section.

The design life of a structure is that time period over which the structure is expected to function without any loss in operational capacity. Civil structures have long design lives vs. other structures such as motorcars, airplanes, and computers. A typical building structure can last several centuries. Bridges are exposed to more severe environmental actions, and tend to last a shorter period, say 50–75 years. The current design philosophy is to extend the useful life of bridges to at least 100 years.

Table 1.3 Loading attributes

Attribute	Value
Duration	Temporary or permanent
Spatial distribution	Concentrated or distributed
Temporal distribution	Impulsive; cyclic; quasi-static
Degree of certainty	Return period; probability of occurrence

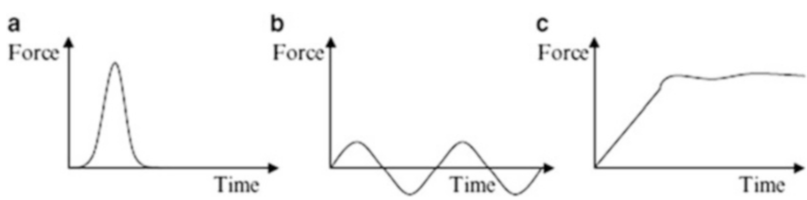


Fig. 1.9 Temporal variation of loading. (**a**) Impulsive. (**b**) Cyclic. (**c**) Quasi-static

The Millau viaduct shown in Fig. 1.8 is intended to function at its full design capacity for at least 125 years.

Given that the natural environment varies continuously, the structural engineer is faced with a difficult problem: the most critical natural event, such as a windstorm or an earthquake that is likely to occur during the design life of the structure located at a particular site needs to be identified. To handle this problem, natural events are modeled as stochastic processes. The data for a particular event, say wind velocity at location x, is arranged according to return period which can be interpreted as the average time interval between occurrences of the event. One speaks of the 10-year wind, the 50-year wind, the 100-year wind, etc. Government agencies have compiled this data, which is incorporated in design codes. Given the design life and the value of return period chosen for the structure, the probability of the structure experiencing the chosen event is estimated as the ratio of the design life to the return period. For example, a building with a 50-year design life has a 50 % chance of experiencing the 100-year event during its lifetime. Typical design return periods are ≈50 years for wind loads and between 500 and 2500 years for severe seismic loads.

Specifying a loading having a higher return period reduces the probability of occurrence of that load intensity over the design life. Another strategy for establishing design loads associated with uncertain natural events is to increase the load magnitude according to the importance of the structure. Importance is related to the nature of occupancy of the structure. In ASCE Standards 7-05 [7], four occupancy categories are defined using the *potential hazard to human life in the event of a failure* as a basis. They are listed in Table 1.4 for reference.

The factor used to increase the loading is called the importance factor, and denoted by I. Table 1.5 lists the values of I recommended by ASCE 7-05 [7] for each category and type of loading.

For example, one increases the earthquake loading by 50 % for an essential structure (category 4).

1.3.3 Gravity Live Loads

Gravitational loads are the dominant loads for bridges and low-rise buildings located in areas, where the seismic activity is moderate. They act in the downward vertical direction and are generally a combination of fixed (dead) and temporary (live) loads. The dead load is due to the weight of the construction materials and permanently fixed equipment incorporated into the structure. As

Table 1.4 Occupancy categories

Category	Description
I	Structures that represent a low hazard to human life in the event of a failure
II	All structures outside of categories I, III, and IV
III	Structures that represent a substantial hazard to human life in the event of failure
IV	Essential structures. Failure not allowed

Table 1.5 Values of I

	Wind			
Category	Non-horizontal	Horizontal	Snow	Earthquake
I	0.87	0.77	0.80	1.00
II	1.00	1.00	1.00	1.00
III	1.15	1.15	1.10	1.25
IV	1.15	1.15	1.20	1.50

Table 1.6 Uniformly distributed live loads (ASCE 7-05)

Occupancy	Magnitude lbs/ft^2 (kN/m^2)
Computer equipment	150 (7.18)
Dormitories	80 (3.83)
File room	125 (6.00)
Court rooms	50–100 (2.4–4.79)
Scientific laboratories	100 (4.79)
Public rooms	100 (4.79)
Rest rooms	60 (2.87)
Laundries	150 (7.18)
Foundries	600 (28.73)
Ice manufacturing	300 (14.36)
Transformer rooms	200 (9.58)
Storage, hay, or grain	300 (14.36)

mentioned earlier, temporary live loads depend on the function of the structure. Typical values of live loads for buildings are listed in Table 1.6. A reasonable estimate of live load for office/residential facilities is ≈100 lbs/ft^2 (4.8 kN/m^2). Industrial facilities have higher live loadings, ranging up to 600 lbs/ft^2 for foundries.

Live loading for bridges is specified in terms of standard truck loads. In the USA, bridge loads are defined by the American Association of State Highway and Transportation Officials (AASHTO) [11]. They consist of a combination of the Design truck or tandem, and Design lane load.

The design truck loading has a total weight of 72 kip (323 kN), with a variable axle spacing is shown in Fig. 1.10. The design tandem shall consist of a pair of 25 kip (112 kN) axles spaced 4 ft (1.2 m) apart. The transverse spacing of wheels shall be taken as 6 ft (1.83 m). The design lane load shall consist of a load of 0.64 kip/ft^2 (30.64 kN/m^2) uniformly distributed in the longitudinal direction and uniformly distributed over a 10 ft (3 m) width in the transverse direction.

1.3.4 Wind Loading

1.3.4.1 Wind Pressure Distribution

The effect of wind acting on a building is represented by a pressure loading distributed over the exterior surface. This pressure loading depends on the geometry of the structure and the geographic location of the site. Figure 1.11 illustrates the flow past a low-rise, single story, flat roof structure. The sharp corners such as at point A causes flow separation, resulting in eddies forming and turbulence zones on the flat roof, side faces, and leeward face. The sense of the pressure is positive (inward) on the incident face and negative (outward) in the turbulence zones.

In general, the magnitude of the pressure varies over the faces, and depends on both the shape of the structure and the design wind velocity at the site. The influence of shape is illustrated by Fig. 1.12, which shows the effect of roof angle on the pressure distribution. When $\theta > 45°$, there is a transition from negative to positive pressure on face AB of the inclined roof. This shift is due to the flow separation point moving from A to B for steeply inclined roofs.

1.3.4.2 Wind Velocity

The effect of the site is characterized firstly by the topography at the site, and secondly by the regional wind environment. Exposure categories are defined to describe the local topography and to establish the level of exposure to wind. ASCE 7-05 adopts the following definitions of exposure categories.

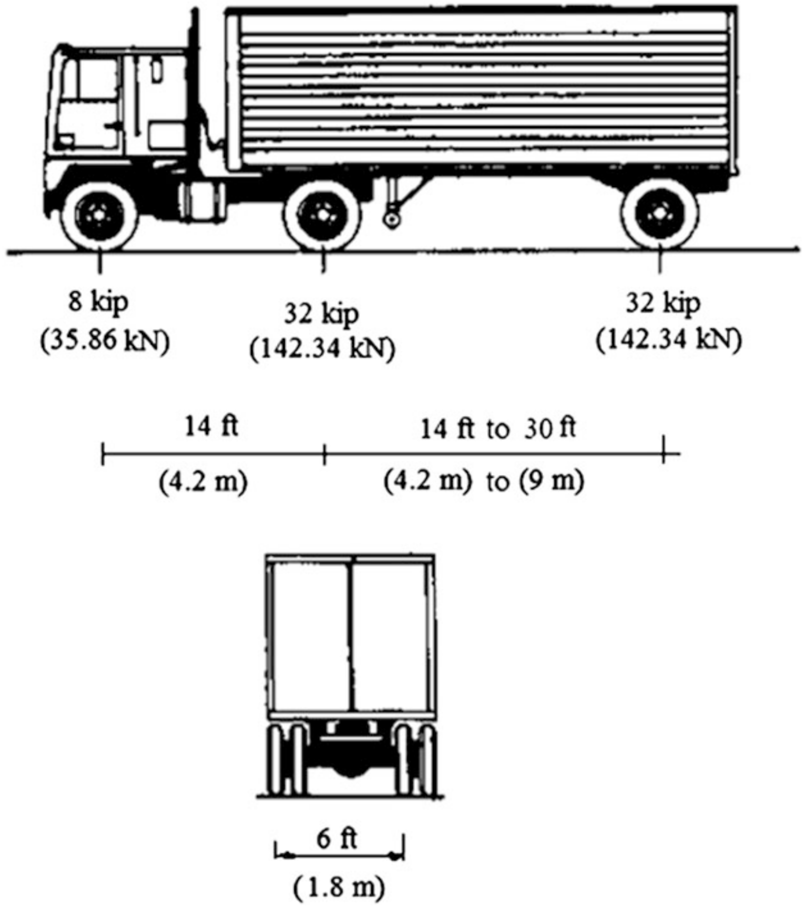


Fig. 1.10 Characteristics of the AASHTO design truck

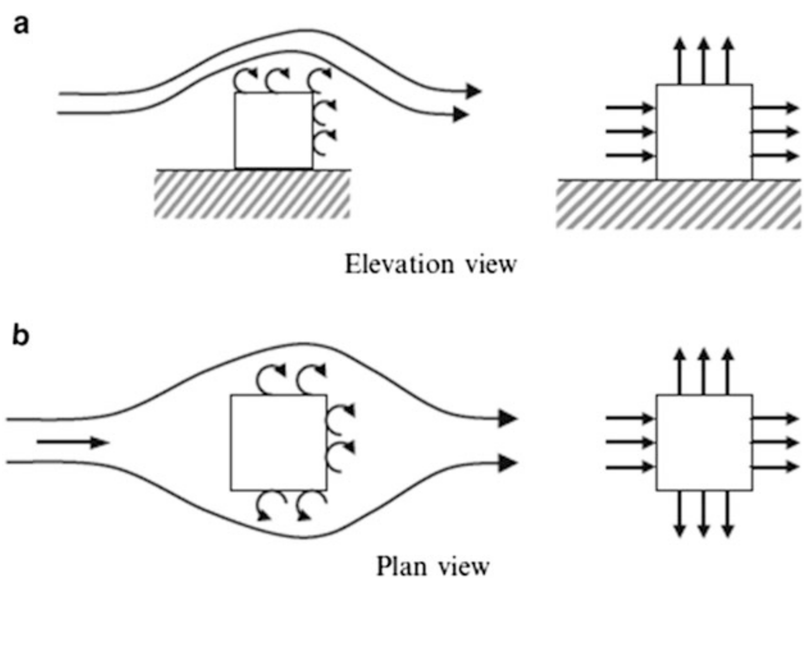
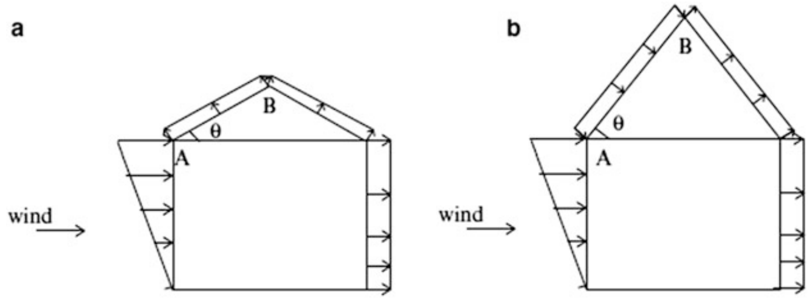


Fig. 1.11 Flow lines and pressure distributions due to wind

Fig. 1.12 Wind pressure profiles for a gable roof. (**a**) $\theta < 45°$. (**b**) $\theta > 45°$



Category B: Site located within an urban or suburban area having numerous closely spaced obstructions similar in size to a single family dwelling, and extending at least 2600 ft from the site.
Category D: Site located in a flat unobstructed area or on a water surface outside hurricane prone regions, and extending at least 5000 ft from the site.
Category C: All cases where exposure categories B and D do not apply.

Regional wind environments are represented by maps containing wind speed data for a specified return period and exposure category. Figure 1.13 shows US data for the 50-year wind speed observed at 10 m elevation corresponding to Exposure C. The higher wind speeds along the East and Gulf Coasts reflects the occurrence of hurricanes in these regions. Typical 50-year wind speeds are on the order of 100 miles per hour (45 m/s).

Given a site, one can establish the 50-year wind speed at 10 m elevation using Fig. 1.13. In general, the wind velocity increases with distance from the ground. A typical approximation is a power law:

$$V(z) = \overline{V}\left(\frac{z}{\overline{z}}\right)^{1/\alpha} \tag{1.1}$$

where z is the elevation above the ground, $\overline{V}$ is the velocity measured at elevation $\overline{z}$, and $\alpha \approx 7$. For US data, one takes $\overline{z} = 10\,\text{m}$ and $\overline{V}$ given by Fig. 1.13.

1.3.4.3 Pressure Profiles

The next step is to establish the vertical pressure distribution associated with this velocity distribution, and then modify it to account for the shape of the building. Pressure and velocity are related by Bernoulli's Equation, which is a statement of conservation of energy. Specialized for steady irrotational inviscid flow of a weightless fluid, the Law states that [12]

$$E = \text{Energy per unit volume} = p + \frac{1}{2}\rho V^2 \tag{1.2}$$

is constant along a streamline. Here, p is the pressure energy, ρ is the mass density, and $1/2\rho V^2$ is the kinetic energy per unit volume. Assuming the pressure is zero in the free stream flow regime away from the structure, and taking point (1) in the free stream and point (2) at the structure, one obtains

$$p_2 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) \tag{1.3}$$

The free stream velocity, V_1, is defined by (1.1). Considering the flow to be stopped by the structure, ($V_2 \approx 0$), it follows that the maximum pressure energy associated with the free stream velocity is estimated as

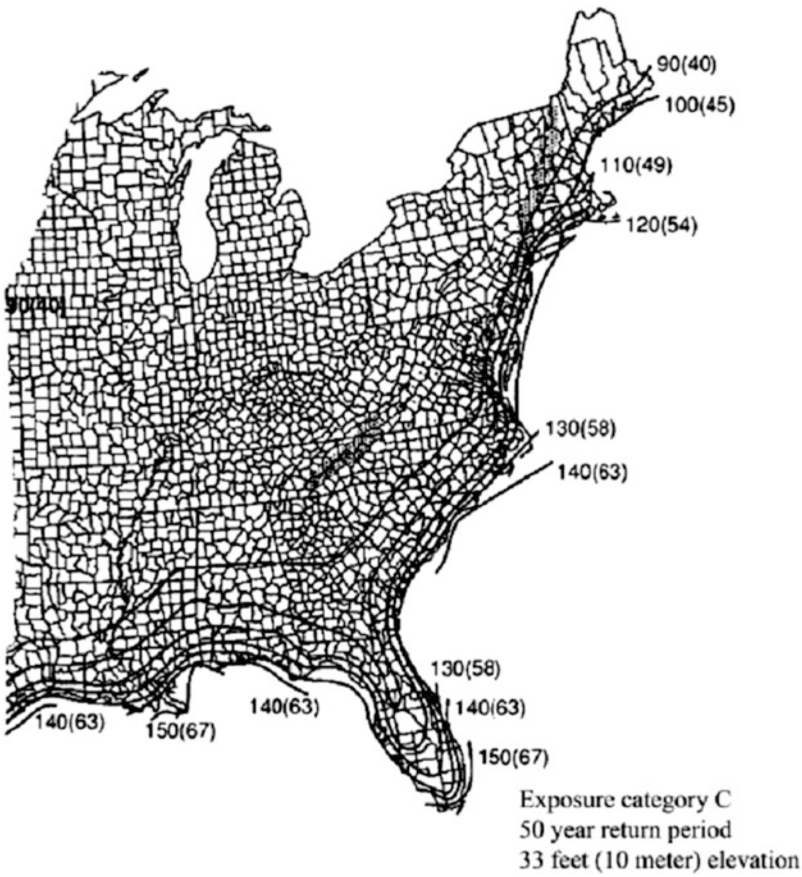


Fig. 1.13 Basic wind speed miles per hour (meter per second) for the East coast of the USA

$$(p_2)_{\max} = \frac{1}{2}\rho V_1^2 = p_{\text{stag}} \tag{1.4}$$

This pressure is called the stagnation pressure and is generally expressed in terms of the reference velocity, $\overline{V}$, at $z = 10$ m and a function $k(z)$ which defines the vertical distribution.

$$p_{\text{stag}} = \frac{1}{2}\rho \overline{V}^2 k(z) \tag{1.5}$$

ASCE 7-05 tabulates values of $k(z)$ vs. z.

The actual pressure distribution is influenced by the geometric shape which tends to change both the magnitude and sense of the pressure. Figures 1.11 and 1.12 illustrate this effect for flat and gable roof structures. Design codes handle this aspect by introducing "shape" factors for different regions of the structural surface. They also include a gust factor for "dynamic" loading, and an importance factor for the structure. The final expression for the design pressure has the following general form:

$$P_{\text{design}} = IGC_{\text{p}}(z)p_{\text{stag}} \tag{1.6}$$

where $C_{\text{p}}(z)$ is the pressure coefficient that accounts for the shape, G is the gust factor, and I is the importance factor corresponding to the occupancy category. Values for these parameters are code dependent. The determination of the design pressure can be labor intensive if one wants to account fully for the spatial distribution of design pressure. A reasonable estimate can be obtained using the simplified procedure illustrated in the following example which is appropriate for low-rise buildings.

Example 1.1 Wind Pressure Distribution on a Low-Rise Gable Roof Structure

Given: The structure shown in Fig. E1.1a. There are four surface areas included in the sketch. Zone (1) is the windward face, zone (2) is the leeward face, and zones (3) and (4) are on the gable roof.

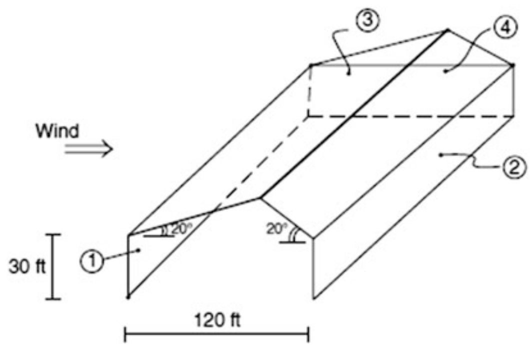


Fig. E1.1a

Determine: The wind pressure distribution on the interior zone away from the ends. Assume

$$\overline{V} = 100\,\text{mph and exposure } C$$

Solution: Applying (1.5) leads to

$$p_{\text{stag}} = (0.00256)\left(10^4\right)k(z) = 25.6k(z)\left(\text{lb/ft}^2\right)$$

Values of $k(z)$ and the corresponding p_{stag} are listed below

z (ft)	k (z)	p_{stag} (lb/ft^2)
15	0.85	21.8
20	0.90	23.0
25	0.94	24.1
30	0.98	25.1

We assume the structure is Category III and use $I = 1.15$. For low-rise buildings with $h < 60$ ft, the factors G and C_{p} are combined and specified as constant for each zone. Using data from ASCE 7-05, the values are

Zone	GC_{p}	IGC_{p}
1	0.53	0.609
2	−0.43	−0.495
3	−0.69	−0.794
4	−0.48	−0.552

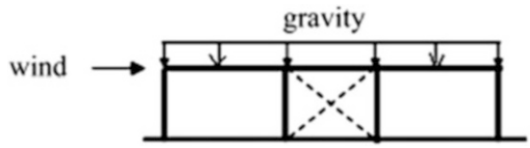


Fig. 1.14 Lateral bracing system

Lastly, we compute the design pressure using (1.6). The ASCE 7-05 code assumes that p_{design} varies on the windward force (zone 1), but specifies constant distributions for the other zones. The details are listed below.

Zone 1	$p_{\text{design}} = 0.609\ (25.6)\ k(z) = 15.59\ k(z)$
Zone 2	$p_{\text{design}} = -0.495\ (25.6)\ k(30) = -12.42$ psf
Zone 3	$p_{\text{design}} = -0.791\ (25.6)\ k(30) = -19.92$ psf
Zone 4	$p_{\text{design}} = -0.552\ (25.6)\ k(30) = -13.85$ psf

Pressure distributions generated with (1.6) define the quasi-static wind load, which acts predominately in the horizontal (lateral) direction. For low-rise buildings, gravity loads are the dominant loads and generally control the structural dimensioning process for vertical members. Since the wind loads are horizontal, whereas the gravity loads are vertical, lateral structural bracing systems such as shown in Fig. 1.14 need to be incorporated in certain types of structures such as a braced frames. This topic is addressed further in Chaps. 11, 14, and 15.

1.3.4.4 Vortex Shedding Pressure

The action of a steady wind on a structure is represented by quasi-static forces. However, a steady wind also creates periodic forces due to the shedding of vortices from the turbulence zones at the leeward face [12]. Consider the rectangular cross-section plan view shown in Fig. 1.15. As the incident flow velocity increases, eddies are created at the upper and lower surfaces and exit downstream. This shedding pattern develops a cyclic mode, shedding alternately between the upper and lower surfaces, which result in an antisymmetric pressure distribution. The net effect is a periodic force, F_t, acting in the transverse direction with frequency, f_s. An estimate for the shedding frequency is

$$f_s(\text{cycles per second}) \approx \frac{0.2V}{D} \tag{1.7}$$

where D is a representative dimension in the transverse direction, and V is the free stream velocity. Vortex shedding is a major concern for tall buildings and slender long-span horizontal structures since these structures are flexible and consequently more susceptible to transverse periodic excitation with a frequency close to f_s. Low-rise buildings are stiffer and relatively insensitive to vortex shedding-induced transverse motion.

1.3.5 Snow Loading

Design snow loads for a structure are based on ground snow load data for the region, where the structure is located. Snow loads act on the roof zones of structures. For a flat roof, defined as a roof with a slope angle less than 5°, the snow load is represented as a uniform downward pressure, p_f. The

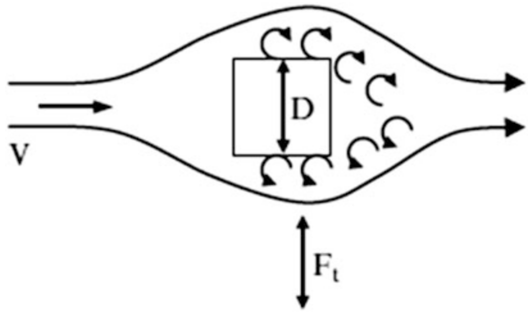


Fig. 1.15 Vortex shedding patterns—plan view

magnitude of p_f depends on the exposure category and regional environment at the site, as well as the importance of the structure. We express p_f as

$$p_f = Cp_g \tag{1.8}$$

where p_g is the ground snow pressure given by Fig. 1.16a and C is a factor that incorporates the exposure and importance parameters. A typical value of C is ≈1. The ground snow pressure varies from 0 in the southeastern zone of the USA up to ≈100 psf in northern New England.

A sloped roof is defined as a roof with a slope angle greater than 5°. The snow load on a sloped roof is expressed in terms of the *horizontal projected area* rather than the actual surface area. Figure 1.16b illustrates this definition.

The sloped roof pressure depends on the slope angle as well as the other parameters mentioned earlier.

$$p_s = C_S p_f \tag{1.9}$$

where C_s is a slope coefficient. In general, $C_s \leq 1$. For $\theta \lesssim 30°$, one usually assumes $C_s \approx 1$ and takes $p_s \approx p_f$.

When the roof has projections as illustrated in Fig. 1.17, a nonuniform snow loading can result due to the drifting on both the windward and leeward faces produced by wind. Drifts are modeled as triangular surcharge loadings. The details are code dependent.

1.3.6 Earthquake Loading

The structural engineer's task is to design structures such that they can resist the ground shaking associated with an earthquake without collapsing. Since an earthquake may occur anytime during the design life, the first task is to identify the magnitude of peak ground acceleration (ρga) that has a specified probability of occurrence during the design life. A common value is 2 % probability of occurrence in 50 years, which corresponds to a return period of 2500 years. Earthquake ground motion is site specific in that it depends on the location and soil conditions for the site. Sites near known faults and sites on soft soils such as soft clay experience more intense ground motion. Factors such as the importance of the building, the geographic location of the site, and the type of soil must be taken into account when specifying the design magnitude for ρga.

In order to understand how buildings respond to ground motion, one needs to examine the dynamic response. Consider the three-story frame shown in Fig. 1.18a. We approximate it with the simple beam/mass system defined in Fig. 1.18b. This approximation, known as a single degree-of-freedom

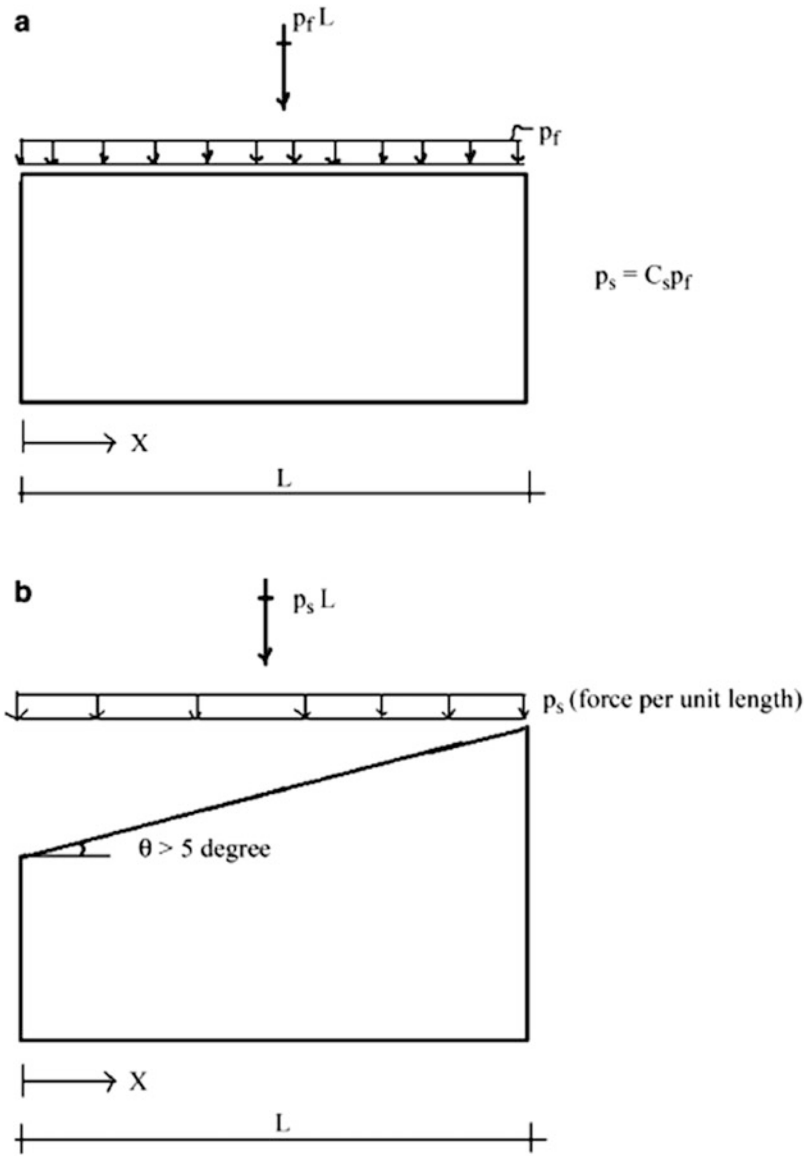
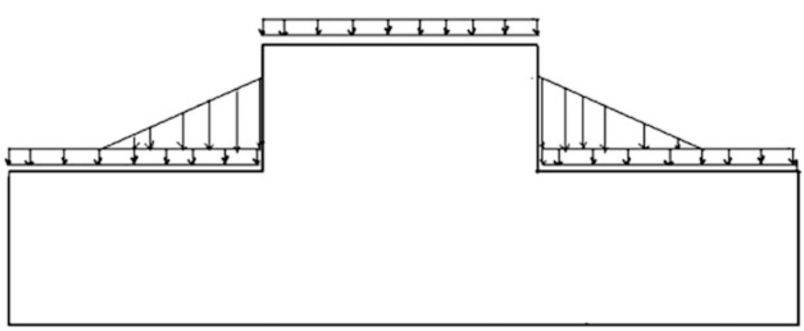


Fig. 1.16 Snow loadings on sloped and flat roofs. (**a**) Flat roof. (**b**) Sloped roof

Fig. 1.17 Snow drift profiles



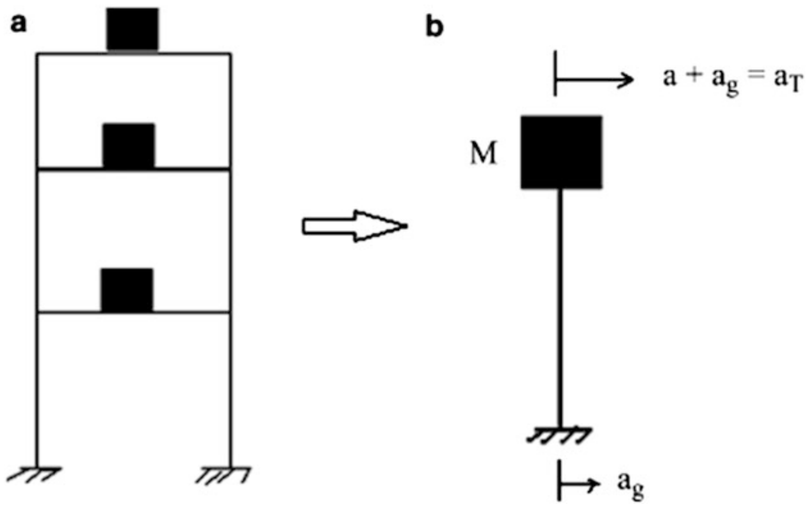


Fig. 1.18 A typical three-story frame and the corresponding one degree-of-freedom model

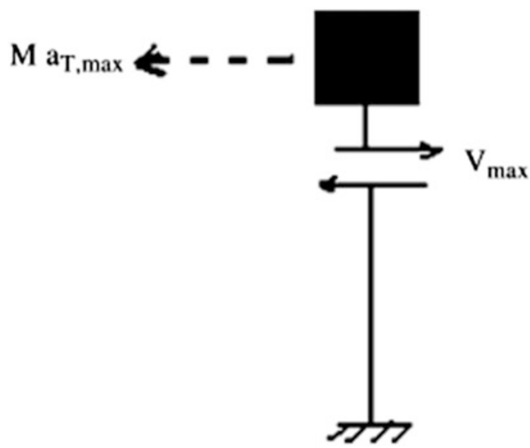


Fig. 1.19 Peak lateral inertia force

model, provides useful information concerning the influence of certain structural properties on the response.

The ground acceleration is defined as a_g. This motion causes the mass to vibrate. We define $a_{T,max}$ as the peak total acceleration of the mass (Fig. 1.19). If the frame is very stiff, $a_{T,max}$ is essentially equal to $a_{g,max}$, the peak ground acceleration. When the frame is very flexible, $a_{T,max}$ *is small in comparison to* $a_{g,max}$. It follows that the *stiffness* of the structure has a *significant influence* on the peak total acceleration response. The peak acceleration also depends on the geographic location and the soil conditions at the site. Data concerning earthquake accelerations is published by the US Geological Survey on their Web site [13]. This site contains an extensive set of earthquake ground motion records for the USA and other major seismically active regions throughout the world.

The motion of the mass generates an inertia force which is resisted by the lateral shear force in the system. The maximum value of the lateral shear force is denoted as V_{max}.

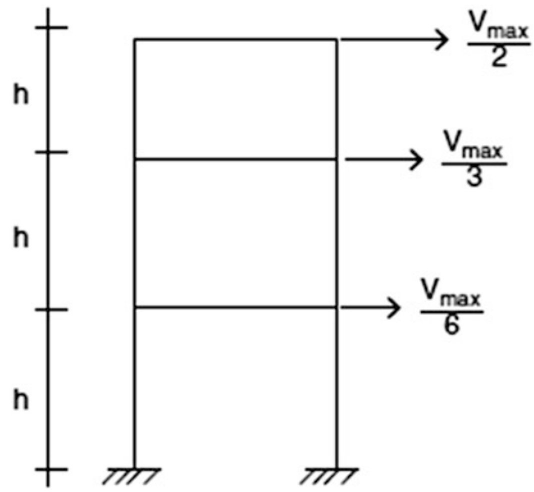


Fig. 1.20 Seismic lateral load profile

$$V_{\max} = Ma_{\mathrm{T,max}} = W\left(\frac{a_{\mathrm{T,max}}}{g}\right) \tag{1.10}$$

Given the structural weight, W, and the peak total acceleration, one can estimate the peak total lateral load that the structure will experience due to seismic excitation. This load is assumed to be distributed linearly throughout the height of the structure, as indicated in Fig. 1.20, and used to generate an initial structural design. The final design is checked with a more refined dynamic analysis. Seismic design is an advanced topic within the field of structural engineering. We discuss this topic in more detail in Chap. 14.

1.4 Structural Design Philosophy

Conventional structural design philosophy is based on satisfying two requirements, namely safety and serviceability [7]. Safety relates to extreme loadings, which have a very low probability of occurrence, on the order of 2 %, during a structure's life, and is concerned with the collapse of the structure, major damage to the structure and its contents, and loss of life. The most important priority is ensuring sufficient structural integrity so that sudden collapse is avoided. Serviceability pertains to medium to large loadings, which are likely to occur during the structure's lifetime. For service loadings, the structure should remain operational. It should suffer minimal damage, and furthermore, the motion experienced by the structure should not exceed specified comfort levels for humans and motion-sensitive equipment mounted on the structure. Typical occurrence probabilities for service loads range from 10 to 50 %.

Safety concerns are satisfied by requiring the resistance, i.e., the strength of the individual structural elements to be greater than the demand associated with the extreme loading. Once the structure is dimensioned, the stiffness properties are derived and used to check the various serviceability constraints such as elastic behavior. Iteration is usually necessary for convergence to an acceptable structural design. This approach is referred to as strength-based design since the elements are dimensioned initially according to strength requirements.

Applying a strength-based approach for preliminary design is appropriate when strength is the dominant design requirement. In the past, most structural design problems have fallen in this category. However, the following developments have occurred recently that limit the effectiveness of the strength-based approach. Firstly, the trend toward more flexible structures such as tall buildings and long-span horizontal structures has resulted in more structural motion under service loading, thus shifting the emphasis toward serviceability. Secondly, some new types of facilities such as micro device manufacturing centers and hospital operating centers have more severe design constraints on motion than the typical civil structure. For example, the environment for micro device manufacturing must be essentially motion free. Thirdly, recent advances in material science and engineering have resulted in significant increases in the strength of traditional civil engineering materials. However, the material stiffness has not increased at the same rate. The lag in material stiffness vs. material strength has led to a problem with satisfying the requirements on the various motion parameters. Indeed, for very high strength materials, the motion requirements control the design. Fourthly, experience with recent earthquakes has shown that the cost of repairing structural damage due to inelastic deformation is considerably greater than anticipated. This finding has resulted in a trend toward decreasing the reliance on inelastic deformation to dissipate energy and shifting to other type of energy dissipating and energy absorption mechanisms.

Performance-based design [14] is an alternate design paradigm that addresses these issues. The approach takes as its primary objective the satisfaction of motion-related design requirements such as restrictions on displacement and acceleration and seeks the optimal deployment of material stiffness and motion control devices to achieve these design targets as well as satisfy the constraints on strength. Limit state design can be interpreted as a form of performance-based design, where the structure is allowed to experience a specific amount of inelastic deformation under the extreme loading.

1.5 Basic Analytical Tools of Structural Analysis

Engineering a structure involves not only dimensioning the structure but also evaluating whether the structure's response under the construction and design loadings satisfy the specified design criteria. Response evaluation is commonly referred to as structural analysis and is carried out with certain analytical methods developed in the field of Engineering Mechanics and adopted for structural systems. In this section, we review these methods and illustrate their application to some simple structures. Most of this material is covered in textbooks dealing with Statics and Mechanics of Materials [2–4] and Structural Analysis [15–17]. Heyman's text [18] contains an excellent description of the "underlying science of Structural Engineering."

1.5.1 Concept of Equilibrium: Concurrent Force System

We begin with a discussion of static equilibrium conditions for solid bodies. This topic is relevant to structural engineering since structures are solid bodies subjected to loads, and we need to ensure that a structure remain at rest, i.e., that it is in a state of equilibrium.

The simplest case is a body subjected to a set of concurrent forces. By definition, the lines of action of the forces comprising a concurrent force system intersect at a common point. Figure 1.21 illustrates this case. For static equilibrium, the resultant of the force system must be a null vector.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0} \tag{1.11}$$

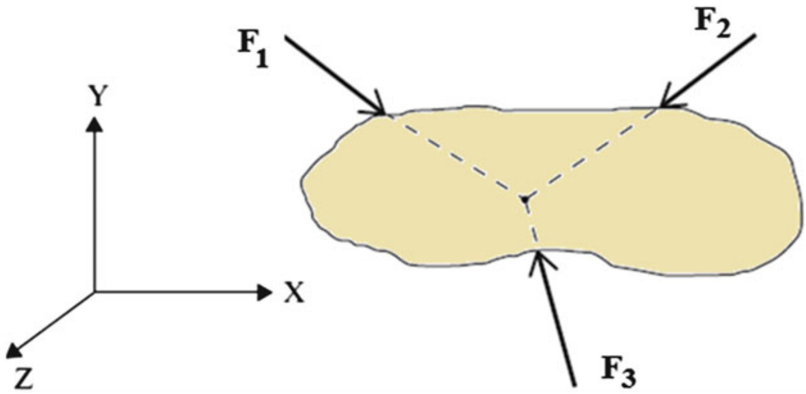


Fig. 1.21 Concurrent force system

We convert the vector equilibrium over to a set of algebraic equations by resolving the force vectors into their components with respect to an arbitrary set of orthogonal directions (X, Y, Z). This operation leads to

$$\begin{aligned} \sum_{i=1}^{3} F_{i,x} &= F_{1,x} + F_{2,x} + F_{3,x} = 0 \\ \sum_{i=1}^{3} F_{i,Y} &= F_{1,Y} + F_{2,Y} + F_{3,Y} = 0 \\ \sum_{i=1}^{3} F_{i,z} &= F_{1,z} + F_{2,z} + F_{3,z} = 0 \end{aligned} \tag{1.12}$$

We find it more convenient to work with (1.12) rather than (1.11).

When all the force vectors are in one plane, say the $X - Y$ plane, the force system is called a planar force system and (1.12) reduces to two equations. Most of the force systems that we deal with will be planar systems.

1.5.2 Concept of Equilibrium: Nonconcurrent Force System

The next level of complexity is a body subjected to a nonconcurrent planar force system. Referring to Fig. 1.22, the forces tend to rotate the body as well as translate it. Static equilibrium requires the resultant force vector to vanish and, in addition, the resultant moment vector about an arbitrary point to vanish.

$$\begin{aligned} \mathbf{R} = \mathbf{F_1} + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0} \\ \mathbf{M_o} = \mathbf{0} \end{aligned} \tag{1.13}$$

Resolving the force and moment vectors into their X, Y, Z components leads to six scalar equations, three for force and three for moment. When the force system is planar, say in the $X - Y$ plane, (1.13) reduce to three scalar equations

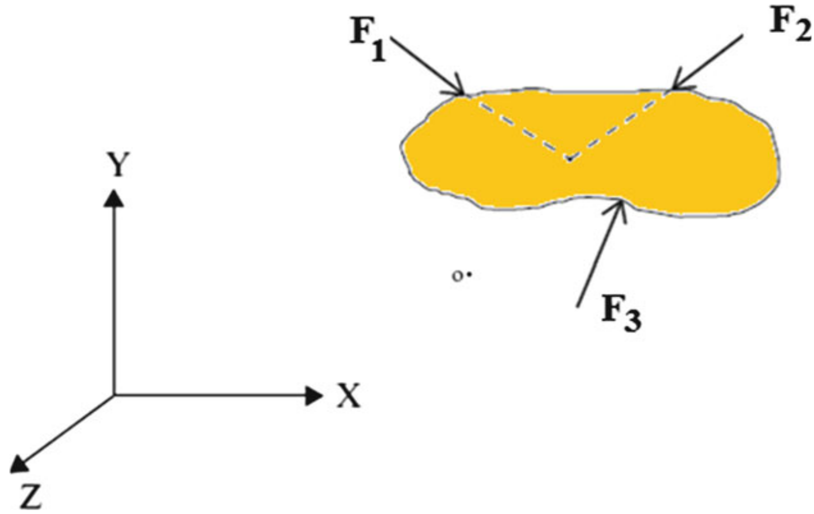


Fig. 1.22 Nonconcurrent force system

$$
\begin{gathered}
\sum_{i=1}^{3} F_{i,x} = 0 \\
\sum_{i=1}^{3} F_{i,Y} = 0 \\
\sum M_o = 0
\end{gathered}
\tag{1.14}
$$

where o is an arbitrary point in the $x - y$ plane. Note that now for a planar system there are three equilibrium conditions vs. two for a concurrent system. Note also that since there are three equilibrium equations, one needs to apply three restraints to prevent planar rigid body motion.

Example 1.2 Equilibrium Equations

Given: The rigid body and force system shown in Fig. E1.2a. Forces A_x, A_Y, and B_Y are unknown.

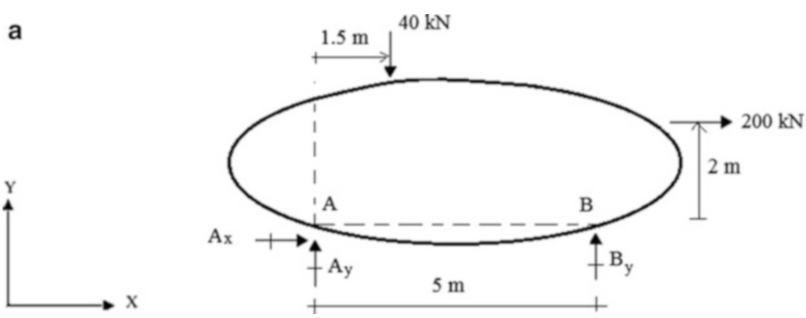


Fig. E1.2a

Determine: The forces A_x, A_Y, and B_Y

Solution: We sum moments about A, and solve for B_Y

$$\sum M_{\mathrm{A}} = -40(1.5) - 200(2) + B_Y(5) = 0$$
$$B_Y = +92 \Rightarrow B_Y = 92\,\mathrm{kN} \uparrow$$

Next, summing forces in the X and Y directions leads to (Fig. E1.2b)

$$\sum F_x \rightarrow^{+} = A_x + 200 = 0 \Rightarrow A_x = -200 \Rightarrow A_x = 200\,\mathrm{kN} \leftarrow$$
$$\sum F_y \uparrow^{+} = A_Y + 92 - 40 = 0 \Rightarrow A_Y = -52 \Rightarrow A_Y = 52\,\mathrm{kN} \downarrow$$

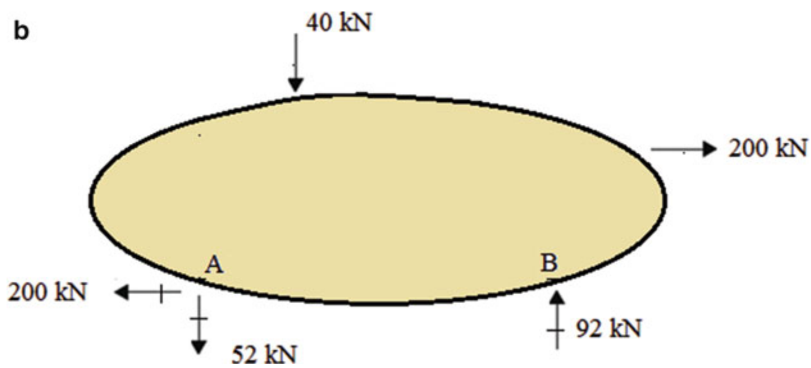


Fig. E1.2b

1.5.3 Idealized Structure: Free Body Diagrams

Generating an idealization of an actual structure is the key step in applying the equilibrium equations. Given a structure acted upon by external loads and constrained against motion by supports, one idealizes the structure by identifying the external loads and supports, and replacing the supports with their corresponding unknown reaction forces. This process is called constructing the free body diagram (FBD). Figure 1.23a, b illustrates the details involved.

One applies the equilibrium equations to the FBD. Note that this diagram has four unknown reaction forces. Since there are only three equilibrium equations, one cannot determine all the reaction forces using only the equilibrium conditions. In this case, we say that the structure is *statically indeterminate*.

Constructing an FBD is *an essential step* in applying the equilibrium equations. The process is particularly useful when the structure is actually a collection of interconnected structural components such as a framed structure. One first generates an FBD for the entire structure and then works with separate FBDs for the individual members. We illustrate this approach throughout the text.

1.5.4 Internal Forces

Consider the body shown in Fig. 1.24a. Suppose we pass a cutting plane as indicated and separate the two segments. We represent the action of body "n" on body "m" by a force $\vec{F}$ and moment $\vec{M}$. From Newton's third law, the action of body m on body n is opposite in sense.

Once the reaction forces are known, we can determine $\vec{F}$ and $\vec{M}$ by applying the equilibrium conditions to either segment. These force quantities are called "*internal forces*" in contrast to the

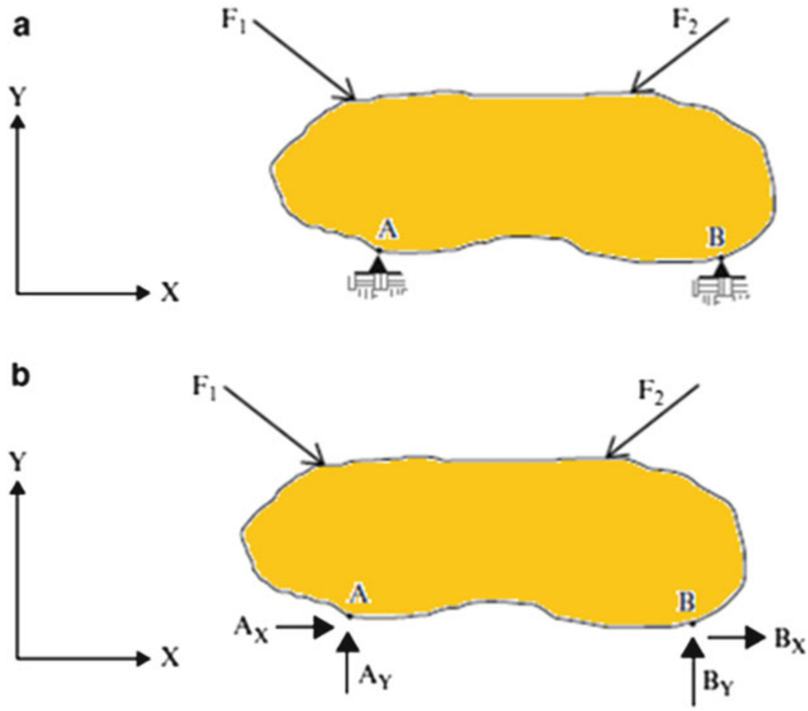


Fig. 1.23 Constructing the free body diagram (FBD)

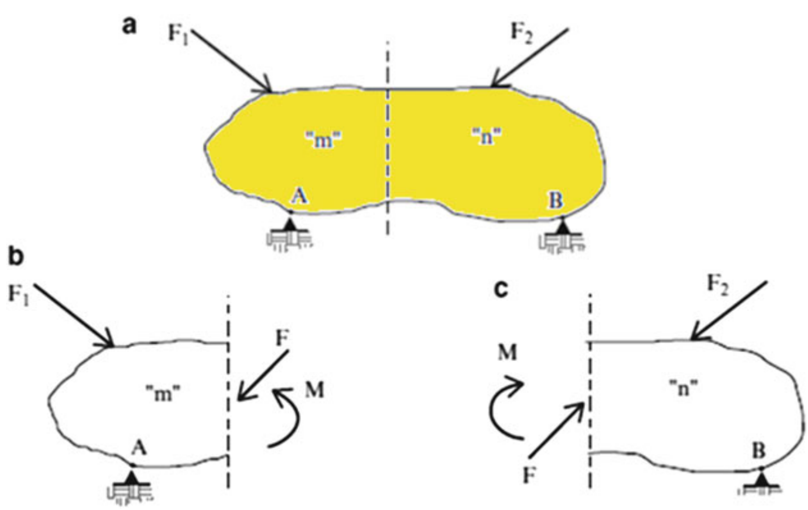


Fig. 1.24 Definition of internal forces

reactions which are "*external forces*." Note that the magnitude of the internal forces varies with the location of the cutting plane. The following example illustrates the process of computing internal forces.

Example 1.3

Given: The body and loading shown in Fig. E1.3a.

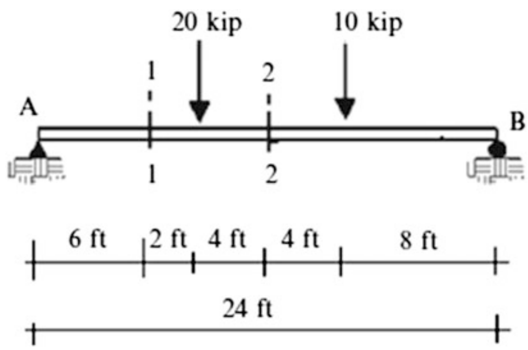


Fig. E1.3a

Determine: The internal forces at Sects. 1-1 and 2-2

Solution: First, we determine the reactions at A and B by applying the equilibrium conditions to entire body AB (Fig. E1.3b).

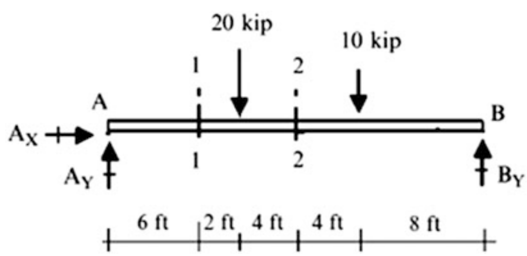


Fig. E1.3b

The static equilibrium equations are

$$\sum F_x = 0 \quad A_x = 0$$

$$\sum F_Y = 0 \quad A_Y + B_Y = 30$$

$$\sum \mathrm{M}_{\text{about A}} = 0 \quad 8(20) + 16(10) - 24B_Y = 0$$

We solve for B_Y, $B_Y = \frac{20}{3} + \frac{2}{3}(10) = 13.3\,\text{kip} \uparrow$
and then A_Y $A_Y = 16.7\,\text{kip} \uparrow$

Next, we work with the FBDs shown below. We replace the internal force vector with its normal and tangential components, F and V (Figs. E1.3c and E1.3d).

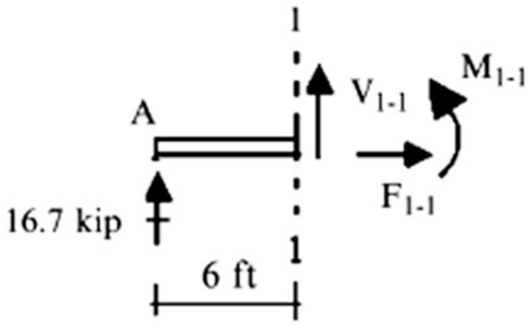


Fig. E1.3c Left segment-cutting plane 1-1

Applying the equilibrium conditions to the above segment leads to

$$\sum F_x \rightarrow^{+} F_{1-1} = 0$$
$$\sum F_y \uparrow^{+} = 0 \quad V_{1-1} + 16.7 = 0 \Rightarrow V_{1-1} = 16.7\,\text{kip} \downarrow$$
$$\sum M_{\text{about}\,1-1} = 0 \quad M_{1-1} - 16.7(6) = 0 \Rightarrow M_{1-1} = 100.2\,\text{kip ft counterclockwise}$$

Applying the equilibrium conditions to the segment shown below leads to

$$\sum F_x \rightarrow^{+} F_{2-2} = 0$$
$$\sum F_y \uparrow^{+} = 0 \quad V_{2-2} - 20 + 16.7 = 0 \Rightarrow V_{2-2} = 3.3\,\text{kip} \downarrow$$
$$\sum M_{\text{about}\,2-2} = 0 \quad M_{2-2} - 12(16.7) + 4(20) = 0 \Rightarrow M_{2-2} = 120.4 \text{ kip ft counterclockwise}$$

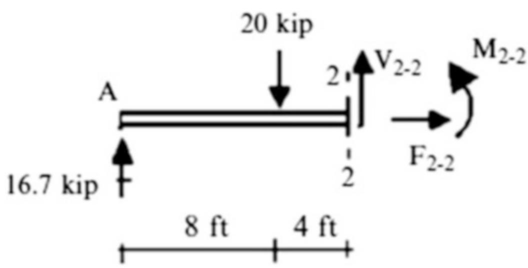


Fig. E1.3d Left segment-cutting plane 2–2

Note that the sense of V_{1-1} and V_{2-2} are opposite to the directions we chose initially.

1.5.5 Deformations and Displacements

When a body is subjected to external loads, internal forces are developed in order to maintain equilibrium between the internal segments. These forces produce stresses which in turn produce strains that cause the body to change its shape and displace from its unloaded position.

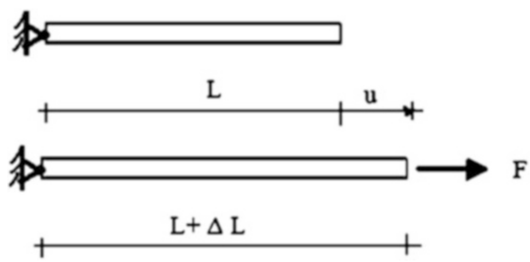


Fig. 1.25 Unreformed and deformed states

Consider the member shown in Fig. 1.25. We apply an axial force which generates the axial stress, σ, equal to

$$\sigma = \frac{F}{A} \tag{1.15}$$

where A is the cross-sectional area. The resulting strain depends on E, the modulus of elasticity for the material [3, 4].

$$\varepsilon = \frac{\sigma}{E} \tag{1.16}$$

By definition, the extensional strain is the relative change in length.

$$\varepsilon = \frac{\Delta L}{L} \tag{1.17}$$

Then,

$$\Delta L = L\varepsilon = \left(\frac{L}{AE}\right)F \tag{1.18}$$

We refer to the movement due to strain as the displacement and denote it by u. It follows that ΔL is equal to u. Finally, we write (1.18) as

$$u = \left(\frac{L}{AE}\right)F \tag{1.19}$$

Strains are generally referred to as deformations since they relate to a change in shape. This example illustrates that displacements are a consequence of deformations which are due to forces. Note that deformations are dimensionless quantities whereas displacements have geometric units such as either length (translation) or angle (rotation). The coefficient of F in (1.19) has units of displacement/force. We interpret this coefficient as a measure of the flexibility of the member. Here, we are defining flexibility as displacement per unit force. The inverse of flexibility is called stiffness. Stiffness relates the force required to introduce a unit displacement. Inverting (1.19) leads to

$$F = \left(\frac{AE}{L}\right)u \tag{1.20}$$

It follows that the stiffness of an axial loaded member is equal to $\frac{AE}{L}$.

Stiffness and flexibility are important concepts in structural engineering. We use them to reason qualitatively about the change in behavior of a structure when we introduce modifications to the geometry and structural members. Obviously, to reduce displacements, one makes the structure stiffer. How this is achieved is one of the themes of this text.

1.5.6 Structural Behavior: Structural Analysis

When a structure is subjected to an external loading, it responds by developing internal forces which lead to internal stresses. The stresses generate strains, resulting in displacements from the initial unloaded position. Figure 1.26 illustrates the displacement process for a beam-type member subjected to a transverse loading. This process continues until the internal stresses reach a level at which the external loading is equilibrated by the internal forces. The final displacement profile corresponds to this equilibrium state.

Structural analysis is concerned with quantifying the response of structures subjected to external loading. The scope includes determining the magnitude of the reactions, internal forces, and displacements. The analysis is generally carried out in the order shown in Fig. 1.26.

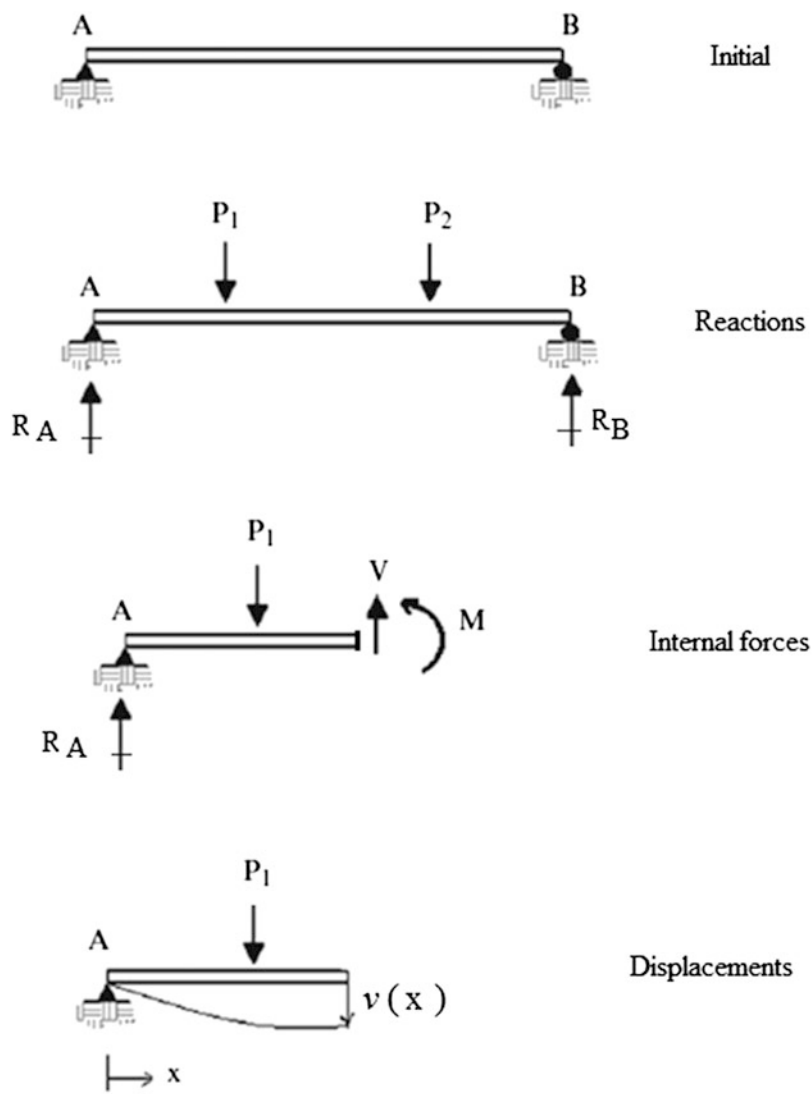


Fig. 1.26 Simple beam response

1.5.6.1 Study Forces

In the study of forces, we apply the equilibrium equations to various FBDs. We work initially with the FBD for the structure treated as a single body and determine the reactions. Once the reactions are known, we select various cutting planes and determine the corresponding internal forces. This phase involves some heuristic knowledge as to "the best" choice of cutting planes.

1.5.6.2 Study Displacements

Displacements are the geometric measures that define the position of the structure under the applied external loading. Displacements are a consequence of internal stresses and are usually expressed in terms of the internal forces. The form of the "force-displacement" relations depends on the type of structural member, e.g., a truss member and a beam. We discuss this topic in more detail in Chaps. 2 and 3. In what follows, we illustrate these computations for some fairly simple structures.

Example 1.4

Given: The structure defined in Fig. E1.4a. Member AB is a rigid member. It is connected to a hinge support at A, and supported at B by a cable, BC.

Determine: The reactions, cable tension, and vertical displacement at B. Assume $E_C = 200$ Gpa, $A_C = 600\ \text{mm}^2$, $h = 4$ m, $L = 10$ m, and $P = 80$ kN

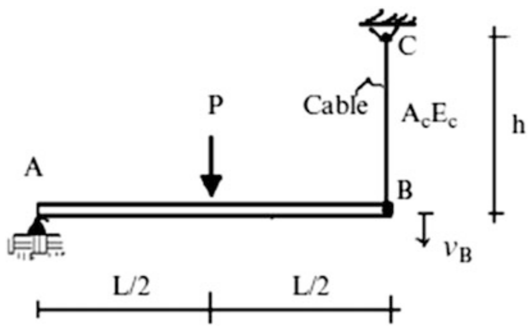


Fig. E1.4a

Solution: We start with the FBD of the entire structure shown in Fig. E1.4b. We note that the cable force is tension. Requiring the sum of the moments of the forces with respect to point A to vanish leads to the T_C

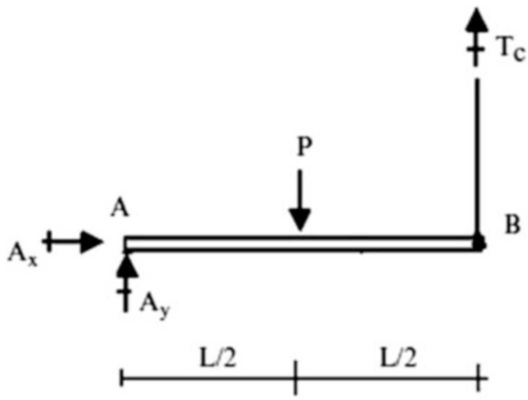


Fig. E1.4b

$$\sum M_{\text{about A}} = \frac{L}{2}P - LT_{\mathrm{C}} = 0$$
$$\Downarrow$$
$$T_{\mathrm{C}} = \frac{P}{2}$$

Next, we determine the reactions at A using force summations.

$$\sum F_Y = 0 \quad A_y + T_{\mathrm{C}} - P = 0 \Rightarrow A_y = \frac{P}{2}$$
$$\sum F_x = 0 \quad A_x = 0$$

The vertical displacement of B is equal to the extension of the cable. Noting (1.19), the expression for v_{B} is

$$v_{\mathrm{B}} = \left(\frac{h}{A_{\mathrm{c}}E_{\mathrm{c}}}\right)T_{\mathrm{C}} = \frac{h}{A_{\mathrm{c}}E_{\mathrm{c}}}\left(\frac{P}{2}\right) = \frac{4{,}000}{(600)(200)}\left(\frac{80}{2}\right) = 1.33\,\mathrm{mm}\downarrow$$

In what follows, we illustrate the application of the general analysis procedure to the idealized structure defined in Fig. 1.27. Member ABCD is considered to be rigid. It is supported by a hinge at A and springs at C and D. The force, P, is constant. Replacing the hinge support and springs with their corresponding forces results in the FBD shown in Fig. 1.27b. There are four unknown forces; A_X, A_Y, F_{c}, and F_{d}. Setting the resultant force equal to zero leads to

$$\begin{aligned} A_X &= 0 \\ A_Y + F_{\mathrm{c}} + F_{\mathrm{d}} &= P \end{aligned} \tag{1.21}$$

Next, we require that the moment vanish at A.

$$\frac{l}{4}P = \frac{l}{2}F_{\mathrm{c}} + lF_{\mathrm{d}} \tag{1.22}$$

Since there are more force unknowns than force equilibrium equations, the structure is statically indeterminate.

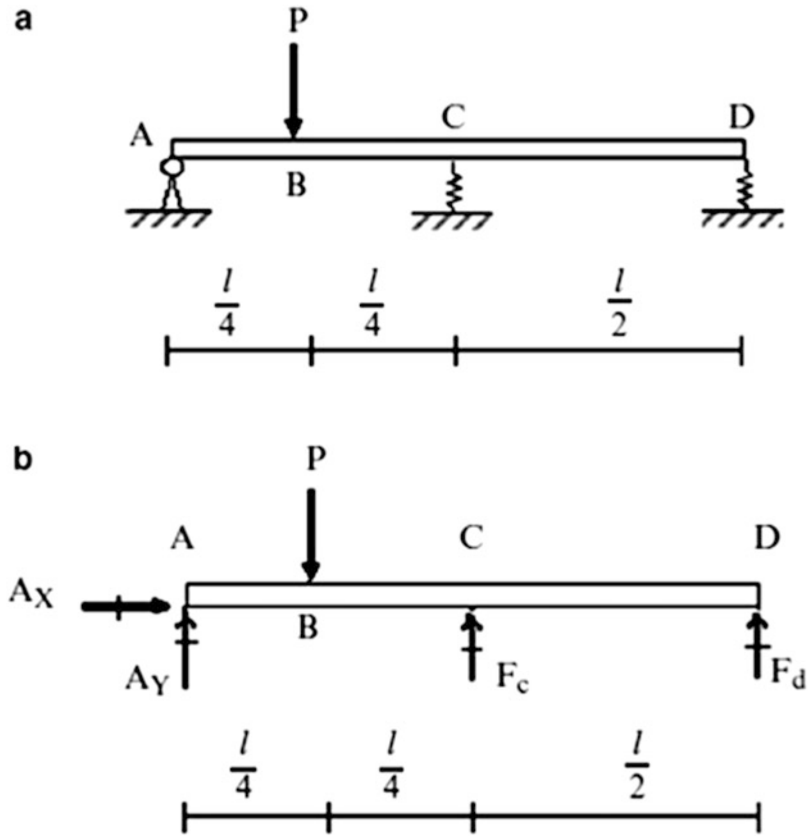


Fig. 1.27 Rigid member on springs

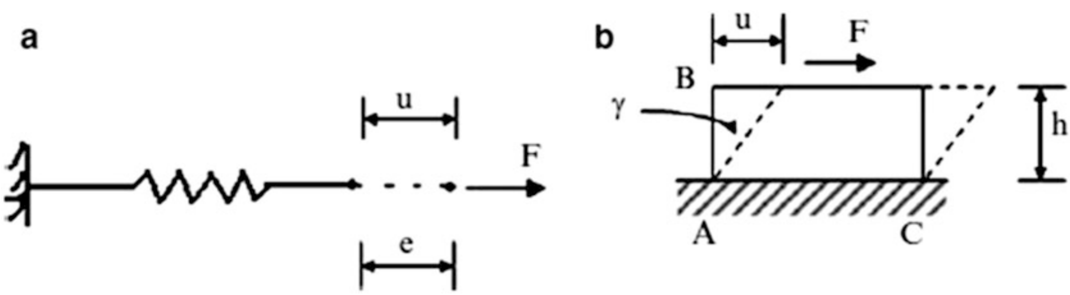


Fig. 1.28 Deformation modes. (**a**) Extension. (**b**) Shear

We generate additional equations by examining how the structure deforms. Deformation is a consequence of applying a force to a material. Deformation is associated with a change in shape. Figure 1.28 illustrates various deformation modes: the first is extension of a spring; the second is shear. A rigid body is an idealized case: the deformations are considered to be negligible.

An important phase in the analysis of a deformable body is the study of deformations. One first identifies the displacement variables that define the "deformed" position and then, using geometric analysis, establishes the expressions relating the deformations of the deformable structural elements with the displacements. We illustrate this process for the structure defined in Fig. 1.29.

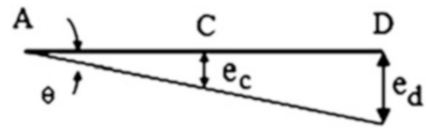


Fig. 1.29 Deformation–displacement relations

Member ABCD is assumed to be rigid and therefore remains straight when the load is applied. Deformation occurs in the springs at C and D, causing ABCD to rotate about the hinge at A. There is only one independent displacement variable. We take it to be the rotation angle θ shown in Fig. 1.29. With this choice of sense, the springs compress. When θ is small, the spring deformations can be approximated as linear functions of θ. This approximation is valid for most cases.

$$\begin{aligned} e_c &= \left(\frac{l}{2}\right)\theta \\ e_d &= l\theta \end{aligned} \tag{1.23}$$

The last step in the analysis involves relating the deformations and the corresponding internal forces. For this example structure, the internal forces are the spring forces, F_c and F_d. In general, the relationship between the force and deformation of a component is a function of the makeup of the component (i.e., the material used and the geometry of the component). Here, we assume the behavior is linear and write

$$\begin{aligned} F_c &= k_c e_c \\ F_d &= k_d e_d \end{aligned} \tag{1.24}$$

where k_c and k_d are the spring stiffness factors. Note that the units of k are force/length since e has units of length.

At this point, we have completed the formulation phase. There are seven equations, (1.21)–(1.23), relating the seven variables consisting of the four forces, one displacement, and two deformations. Therefore, the problem is solvable. How one proceeds through the solution phase depends on what variables one wants to determine first.

Starting with (1.23), we observe that the reaction A_v can be determined once the spring forces are known. Therefore, we hold this equation in reserve, and focus on the remaining equations. We can combine (1.23) and (1.24) by substituting for the deformations. The resulting equations together with (1.24) are

$$F_c = \left(k_c \frac{l}{2}\right)\theta, \quad F_d = (k_d l)\theta \tag{1.25a}$$

$$\frac{l}{4}P = \frac{l}{2}F_c + lF_d \tag{1.25b}$$

The most convenient strategy is to substitute for F_c, F_d in the second equation. Then,

$$\frac{l}{4}P = l\left(\frac{l}{4}k_c + k_d\right)l\theta$$

and

$$\theta = \frac{P}{(4k_{\mathrm{d}} + k_{\mathrm{c}})l} \tag{1.26}$$

Finally, the spring forces corresponding to this value of θ are

$$\begin{aligned} F_{\mathrm{c}} &= \frac{k_{\mathrm{c}}}{2(4k_{\mathrm{d}} + k_{\mathrm{c}})} P \\ F_{\mathrm{d}} &= \frac{k_{\mathrm{d}}}{(4k_{\mathrm{d}} + k_{\mathrm{c}})} P \end{aligned} \tag{1.27}$$

An alternate strategy is to solve first for one of the spring forces. Suppose we take F_{c} as the primary force variable. Using (1.25b), we solve for F_{c}.

$$F_{\mathrm{c}} = \frac{1}{2}P - 2F_{\mathrm{d}} \tag{1.28}$$

Another equation relating F_{c} and F_{d} is obtained by eliminating θ in (1.25a). The steps are

$$\theta = \frac{1}{k_{\mathrm{c}}(l/2)} F_{\mathrm{c}} \tag{1.29}$$

and

$$F_{\mathrm{d}} = \frac{2k_{\mathrm{d}}}{k_{\mathrm{c}}} F_{\mathrm{c}} \tag{1.30}$$

Equation (1.30) represents a constraint on the spring forces. The deformations of the springs are not arbitrary; they must satisfy (1.23), which can be written as:

$$e_{\mathrm{d}} = 2e_{\mathrm{c}} \tag{1.31}$$

Finally, substituting for F_{d} in (1.28) and solving for F_{c} leads to

$$F_{\mathrm{c}} = \left(\frac{1/2}{1 + 4k_{\mathrm{d}}/k_{\mathrm{c}}} \right) P \tag{1.32}$$

The rotation angle is determined with (1.29) and F_{c} with (1.30).

We refer to the first solution procedure as the *displacement or stiffness method*. It is relatively simple to execute since it involves only substitution. Most of the structural analysis computer programs are based on this method. The second procedure is called the *force or flexibility method*. Some manipulation of the equations is required when the structure is statically indeterminate and consequently the method is somewhat more difficult to apply in this case. However, the Force Method is more convenient to apply than the displacement method when the structure is statically determinate, since the forces can be determined using only the equilibrium equations. The approach in part I of the text is based on the Force Method. Later in part II, we discuss the Force and Displacement methods in more detail.

1.5.7 The Importance of Displacements

Displacements are important for two reasons. Firstly, the serviceability requirement for structures is usually specified as a limit on the magnitude of certain displacements. Secondly, for indeterminate structures, one cannot determine the internal forces using only the equations of static equilibrium.

One needs to consider the displacements and internal forces simultaneously. This topic is addressed in part II of the text. The following example illustrates one of the strategies employed for a statically indeterminate beam.

Example 1.5: A Statically Indeterminate Beam

Given: The beam shown in Fig. E1.5a.

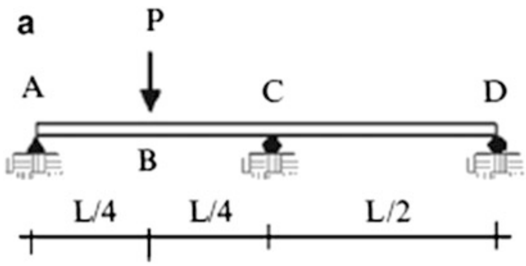


Fig. E1.5a

Determine: The reactions.

Solution: First, we construct the FBD for the beam (Figs. E1.5b and E1.5c).

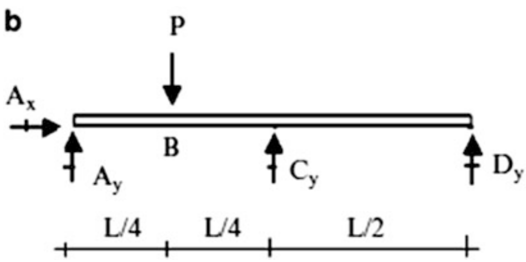


Fig. E1.5b

Considering summation of forces in the X and Y directions and summation of moments about A, we obtain the following three equations.

$$\sum F_x \rightarrow^+ = 0 \quad \Rightarrow \quad A_x = 0$$
$$\sum \mathrm{F}_y \uparrow^+ = 0 \quad \Rightarrow \quad A_y + C_y + D_y = P$$
$$\sum M_{\text{about A}} = 0 \quad \Rightarrow \quad \frac{L}{4}P = \frac{L}{2}C_y + LD_y$$

We have only two equations for the three vertical reactions, A_y, C_y, and D_y. Therefore, we *cannot* determine their magnitude using only the force equilibrium equations.

The Force (or Flexibility) method for this problem is based on releasing one of the roller supports, say support C, replacing it with an unknown force, C_y, and allowing point C to move vertically under the action of the applied loads. First, we take $C_y = 0$ and apply P. Point C moves an amount $\Delta_c\big|_p$ shown in the figure below. Then, we take $P = 0$, $C_y = 1$, and determine $\Delta_c\big|_v$ the corresponding movement at C due to a unit upward force at C. Assuming the support at C is unyielding, the net movement must be zero. Therefore, we increase the force C_y until this condition is satisfied. Once C_y is known, we can find the remaining forces using the equations of static equilibrium. In order to carry out this solution procedure, one needs to have a method for computing displacements of beams. These methods are described in Chap. 3.

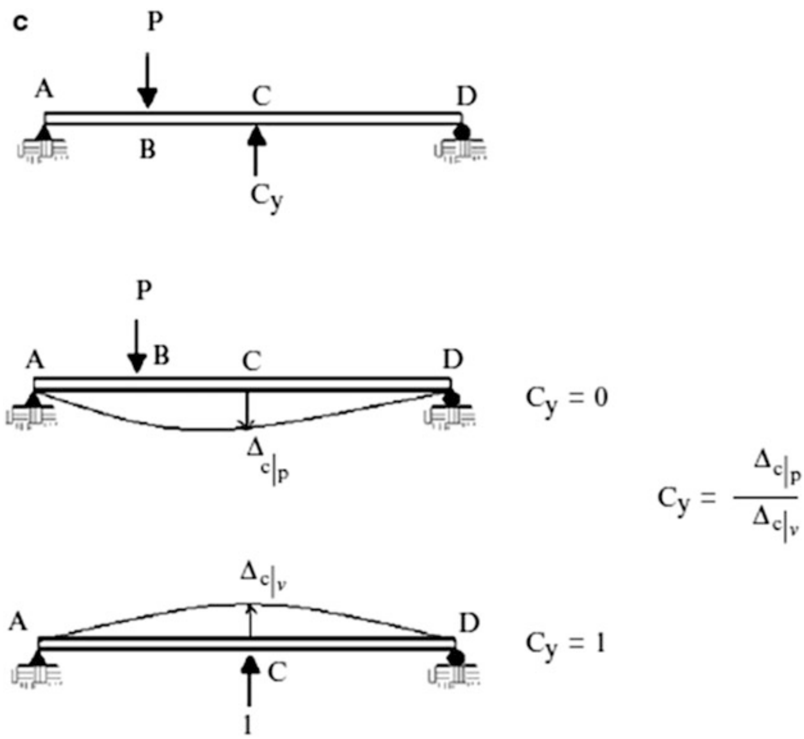


Fig. E1.5c

1.6 Summary

1.6.1 Objectives of the Chapter

- Provide an overview of the set of issues that a structural engineer needs to address as a practicing professional engineer.
- Introduce the basic analytical methods of structural analysis and describe how they are applied to determine the response of a structure.

1.6.2 Key Issues and Concepts Introduced

- A structure is an assemblage of structural components which are arranged in such a way that the structure can withstand the action of the loads that are applied to it. Structures are classified according to their makeup such as trusses, frames, and their functions such as bridges, office buildings.

- The primary concern of a structural engineer is to ensure that the structure *will not collapse* during its expected lifetime. This requires firstly that the engineer properly identify the extreme loading that the structure is likely to experience over its design life, and secondly, that the structure is dimensioned so that it has adequate capacity to resist the extreme loading.
- Structures are restrained against rigid body motion by supports. When the structure is loaded, reaction forces are developed by the supports. A minimum of three nonconcurrent reaction forces are necessary to prevent rigid body motion for a planar structure.
- Initial instability occurs when the reactions are insufficient or the members are not properly arranged to resist applied external forces. In this case, the structure will fail under an infinitesimal load. This condition can be corrected by modifying the supports or including additional members.
- Loss of stability under loading can occur when a primary structural member loses its capacity to carry load due to either elastic buckling or failure of the material. There are two modes of material failure: "brittle" and "ductile." Brittle failure occurs suddenly with a complete loss in load capacity. One should avoid this mechanism. Ductile failure is evidenced by substantial inelastic deformation and loss in stiffness. The limit state design procedure allows for a limited amount of inelastic deformation.
- Loads applied to civil structures are categorized according to direction. Vertical loads are due to gravitational forces and are defined in terms of the weight of objects. Lateral loads are produced by natural events such as wind and earthquake. The relative importance of these loads depends on the nature of the structure and the geographical location of the site.
- Loads are also generated during the construction of the structure. The design loading for certain types of structures such as segmented concrete girders is controlled by the construction process. Most structural failures occur during the construction process.
- Loads are also classified according to the time period over which the loads are applied. Long-term loads, such as self-weight, are called "dead "loads. Loads whose magnitude or location changes are called "live" loads. Typical live loads are produced by vehicles crossing bridges, and people occupying buildings.
- Extreme loads such as wind and earthquakes are defined in terms of their return period, which is interpreted as the average time interval between occurrences of the event. One speaks of the 50-year wind, the 50-year earthquake, etc. The magnitude of the load increases with increasing return period.
- The design life of a structure is the time period over which the structure is expected to function without any loss in functionality. Bridges are designed to last at least 100 years. Industrial buildings are expected to have design lives usually greater than 100 years. The probability that a structure will experience an extreme event over its design life is approximately equal to the ratio of the design life to the return period.
- The effect of wind acting on a building is represented by a pressure loading distributed over the external surfaces. The magnitude and spatial variation of the pressure depends on the shape of the building and the local wind environment. Positive pressure is generated on windward vertical forces and steeply inclined roofs. Turbulence zones occur on flat roof and leeward faces and result in negative pressure.
- Design codes specify procedures for computing the spatial distribution of wind pressure given the expected extreme wind velocity at the geographic location. The wind velocity tends to be larger in coastal regions. A typical wind velocity for coastal regions of the USA is 100 miles per hour. The corresponding wind pressure is approximately 20 psf.
- Snow loading is represented as a uniform download pressure acting on the roof zones of a structure. Design snow pressure is based on ground snow data for the region where the structure

is located. Snow is an important loading for the northern part of the USA, Canada, and Eastern Europe.

- Earthquakes produce sudden intense short-term ground motion which causes structures to vibrate. The lateral floor loading is due to the inertia forces associated with the acceleration generated by the ground shaking and is generally expressed as $(W_f/g)a_{max}$, where W_f is the floor weight, and a_{max} is the peak value of floor acceleration. Seismic engineering is specialized technical area which is beyond the scope of this textbook. However, the reader should be knowledgeable of the general seismic design strategy.
- Conventional structural design philosophy is based on satisfying two requirements: safety and serviceability. Safety relates to extreme loading and is concerned with preventing collapse and loss of life. Safety is achieved by providing more resistance than is required for the extreme loading. Serviceability relates to loading which occurs during the structure's lifetime. One needs to ensure that the structure remains operational and has no damage.
- Motion-Based Design, also called performance-based design, is an alternate design methodology that takes as its primary objective the satisfaction of motion-related design requirements such as displacement and acceleration. The goal here is to provide sufficient stiffness and energy dissipation mechanisms to limit the motion under extreme loading.
- Structural analysis is concerned with quantifying the response of a structure subjected to external loading. This effort involves determining the reactions, internal forces, and displacement profiles. One generally carries out the analysis in two steps: study of forces and study of displacements. In the study of forces, one applies the force equilibrium equations to isolated segments of the structure called FBDs. Selecting appropriate FBDs is a skill acquired through practice. In the study of displacements, one first uses a geometric-based approach to express the deformation measures in terms of displacement measures. The displacement measures are then related to the internal forces by introducing certain material properties such as the elastic modulus. These relations allow one to determine the displacements, given the internal forces. We apply this approach throughout part II of the text. It provides the basis for the analysis of statically indeterminate structures.

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