14 Lateral Load Issues for Buildings

Abstract

Buildings are complex physical systems. Structural Engineers deal with this complexity by creating idealized structural models that define the key structural components, the overall makeup of the building structure, and the loadings that it needs to withstand. The information provided by the idealized model allows one to apply analysis and design methods directly to the model and then extrapolate the results to the actual building.

We begin the chapter with a description of the various types of building systems and the associated structural components. In general, a building consists of plane frame structures which are interconnected by floor systems. We describe approaches for establishing the *lateral loads* due to wind and earthquake excitation. These loads are evaluated at each floor level and then distributed to the individual plane frames using the concepts of center of mass and center of twist. At this point, one has the appropriate lateral loading to analyze the plane frames. The topic of loading on building frames is discussed further in the next chapter where we also consider gravity loads acting on the floor systems.

14.1 Types of Multistory Building Systems

The majority of the activities in Structural Engineering are concerned with the design of Structural Systems for buildings. Approximately 95 % of the building inventory consists of buildings having less than ten stories. Buildings of this type are classified as low-rise buildings. Figure 14.1 illustrates the typical makeup of a low-rise building. The primary structural components are beams, columns, and floor plates. Members are usually arranged in an orthogonal pattern to form a three-dimensional framework. Plate-type elements span between the beams to form the flooring system. We visualize the three-dimensional (3D) framework to be composed of plane frames which are connected by floor plates. This interpretation allows us to analyze the individual plane frames rather than the complete 3D structure.

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Fig. 14.1 Typical makeup of a structural system for a low-rise building

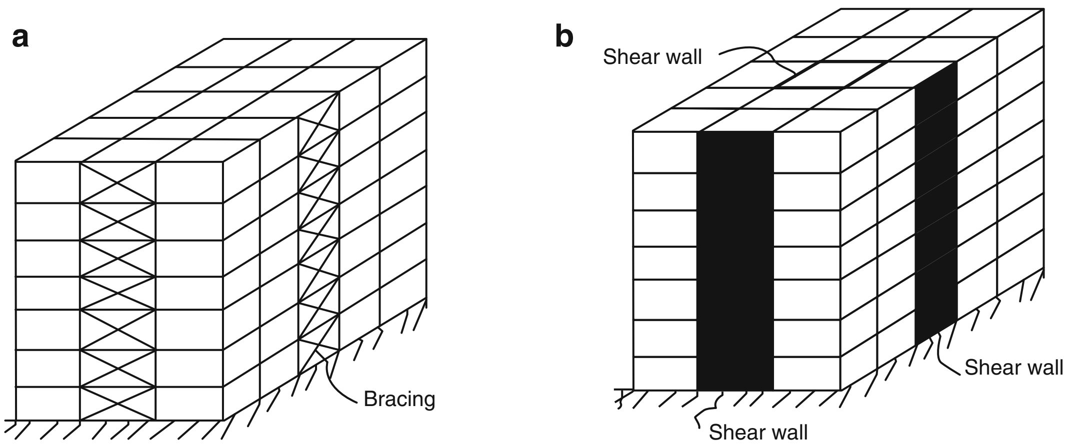


Fig. 14.2 (**a**) Steel braced frame with bracing. (**b**) Concrete frame with shear walls

Frames are designated according to how the beams and columns are interconnected at their end points. When the members are rigidly connected so that no relative rotation can occur, end moments develop under loading and the frame is said to be "rigid." Rigid frames may employ either Steel or Concrete construction.

The opposite case is when the beams are pinned to the columns. No end moments are developed and the frame behaves similar to a truss. Some form of bracing is needed since a rectangular pinned frame is unstable under lateral load. These structures are called "braced" frames. Figure 14.2a shows a typical braced frame structure. The bracing consists of sets of diagonal elements placed within certain bays and extending over the height of the structure. This system is designed to carry all the lateral loading. Note that at least two orthogonal bracing systems are needed to ensure stability under an arbitrary lateral loading. Braced frames are constructed using steel components.

Depending on the magnitude of the lateral loading, lateral stiffness systems may also be incorporated in rigid frames to carry a fraction of the lateral loading. For concrete rigid frames, the stiffening is achieved by incorporating shear walls located either within or on the exterior of the building and extending over the entire height. Figure 14.2b illustrates this scheme. These walls function as cantilever beams and provide additional lateral restraint. For steel rigid frames, the stiffening system may be either a concrete shear wall or a diagonal steel member scheme.

14.2 Treatment of Lateral Loading

Lateral loading may be due to either wind or earthquake acting on the building. These actions may occur in an arbitrary direction. For rectangular buildings, such as shown in Fig. 14.3, the directions are usually taken normal to the faces. One determines the component of the resultant force for each direction. Figure 14.4 illustrates this approach.

The resultant force is distributed to the individual floors, and then each floor load is distributed to the nodes on the floor. This process leads to a set of nodal forces acting on the individual frames. We express the force acting at floor j of frame i as

$$P_x\Big|\text{frame } i\,\text{floor}\, j = f_{ij} R_x \qquad (14.1)$$

How one establishes f_{ij} is discussed in Sect. 14.3.

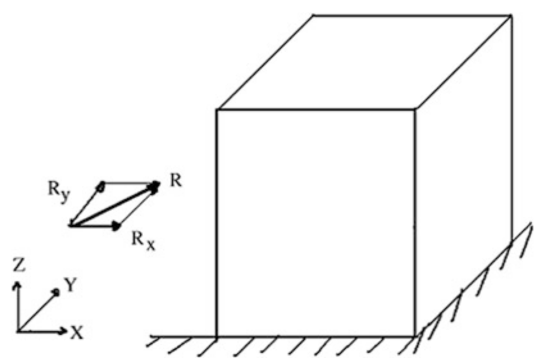


Fig. 14.3 Rectangular building

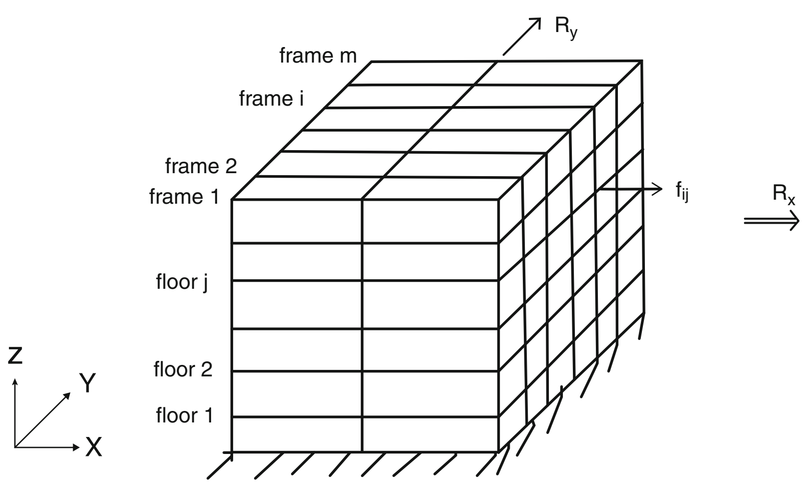


Fig. 14.4 Resultant force components

14.2.1 Wind Loading

We suppose the wind acts in the x direction, as shown in Fig. 14.5a. The normal pressure varies in the vertical direction according to a power law (e.g., $p \sim z^{1/7}$). We approximate the distribution with a set of step functions centered at the floor levels and generate the resultant force for each floor by integrating over the tributary area associated with the floor. This process is illustrated in Fig. 14.5b, c. The individual floor forces are given by

$$P_i = p(z_i)\left[\frac{z_{i+1} - z_i}{2} + \frac{z_i - z_{i-1}}{2}\right]B = p(z_i)\left[\frac{z_{i+1} - z_{i-1}}{2}\right]B \qquad i = 1, 2, \ldots, n \tag{14.2}$$

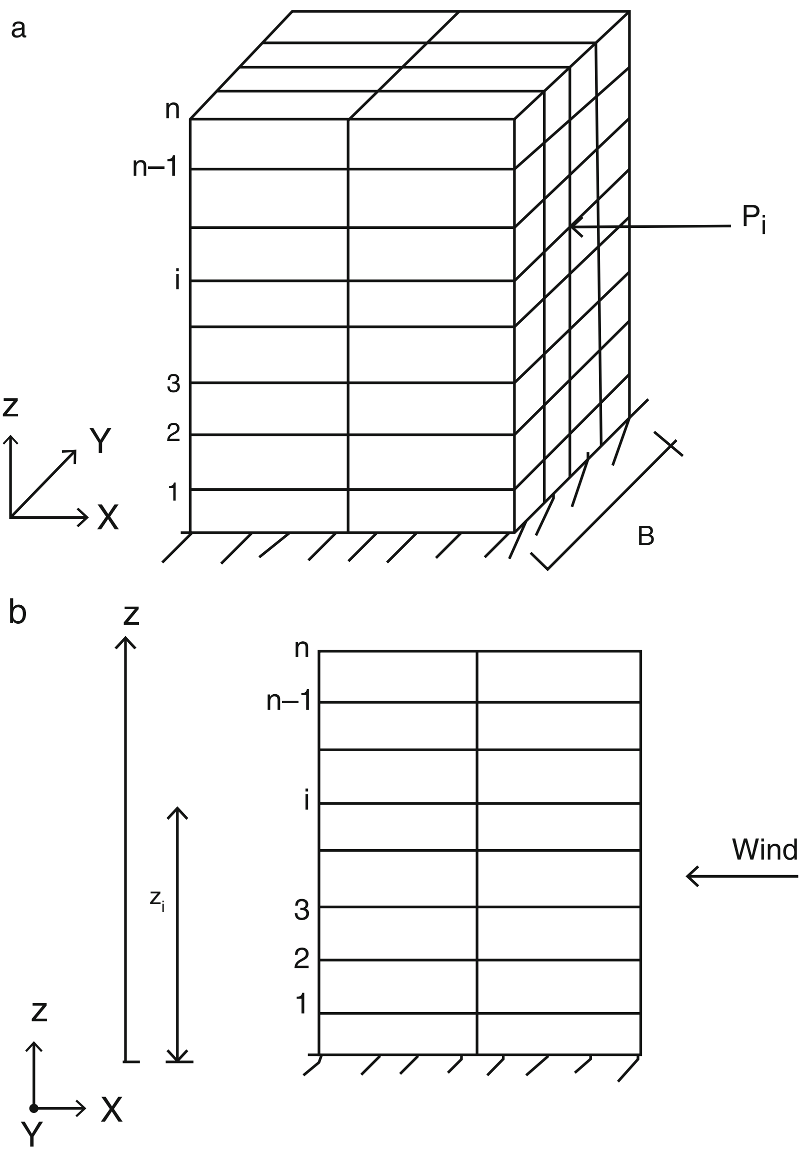


Fig. 14.5 Lateral floor forces due to wind pressure. (**a**) Wind in X direction. (**b**) Wind on Y–Z face. (**c**) Floor loads due to wind load on Y–Z face

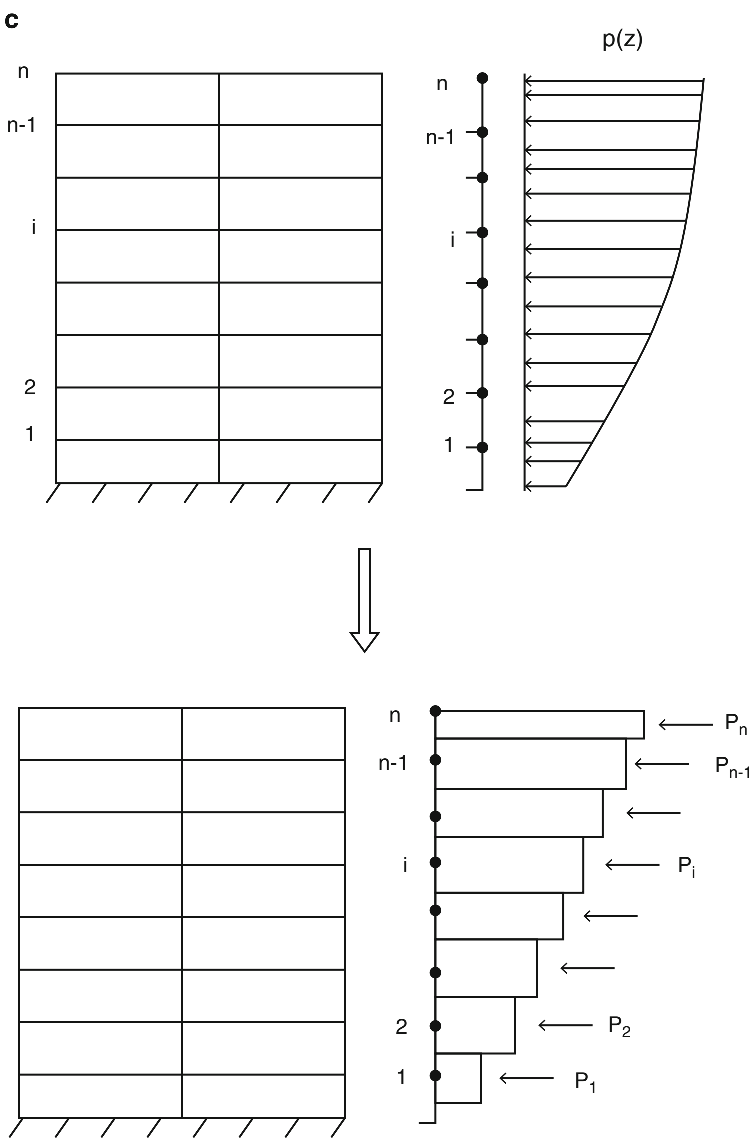


Fig. 14.5 (continued)

This computation is repeated for wind acting in the *Y* direction. It remains to distribute the loads acting at the floor levels of the facade to nodes of the individual plane frames. The final result is a set of lateral nodal loads for each plane frame.

The underlying strategy for this approach is based on analyzing plane frames vs. a three-dimensional system. This approach works when the structural geometry is composed of parallel plane frames which produce an orthogonal pattern of columns and beams. If the structural geometry is irregular, one has to analyze the full 3D structural system. In this case, one subdivides the façade area into panels centered on the structural nodes contained in the façade area and generates the force for structural node *j* with

$$P_j = p(z_j)A_j \tag{14.3}$$

where A_j is the tributary area for structural node *j*.

14.2.2 Earthquake Loading

Seismic loading is generated by an earthquake passing through the site. An earthquake is the result of slippage between adjacent tectonic plates which releases energy in the form of pressure waves that produce both horizontal and vertical ground motion. For civil structures, in seismically active regions, the horizontal motion produces the most critical lateral loading since the design of civil structures is usually controlled by vertical gravity loading. Data on earthquake ground motion is continuously collected and distributed by the US Geological Survey National Earthquake Information Center [1]. Figure 14.6 contains a typical plot of ground acceleration vs. time for the 1994 Northridge California earthquake. The information of interest is the peak ground acceleration, denoted as pga, with respect to g, the acceleration due to gravity. In this case, the pga is equal to $0.6g$. We point out that seismic loading is cyclic, of varying amplitude, and of short duration, on the order of 20–30 s for a typical earthquake.

Seismic loading is discussed in Sect. 1.3.6. We briefly review the important features of seismic loading here and then describe how one uses this information to generate the lateral loading for a building. The lateral forces produced by the horizontal ground motion require the incorporation of lateral bracing systems. Structures located in high seismic activity regions, such as Japan, Greece, and the Western parts of the USA, are required to meet more extreme performance standards, and the design is usually carried out by firms that specialize in seismic design.

Figure 14.7 illustrates how a typical low-rise rigid frame building responds to horizontal ground motion. The floor slabs act like rigid plates and displace horizontally with respect to the ground due to bending of the columns. Since there are no external loads applied to the floors, *the deformation has to be due to the inertia forces associated with the floor masses*. The magnitude of these forces depends on the floor masses and the floor accelerations.

The lateral displacement profile is assumed to be a linear function of Z as indicated in Fig. 14.8.

$$u_i(z) = u_{\text{ground}} + \frac{Z_i}{H} u(H)$$

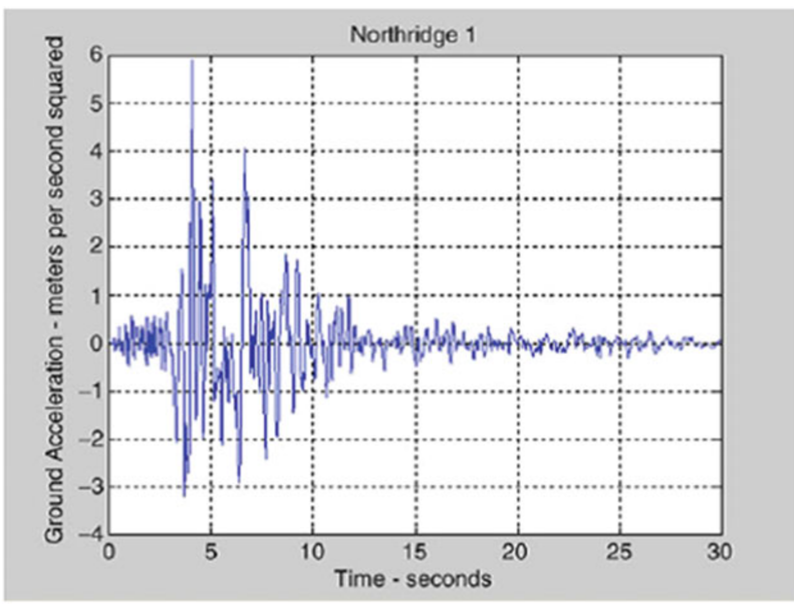


Fig. 14.6 Ground acceleration time history—Northbridge (1994) California Earthquake

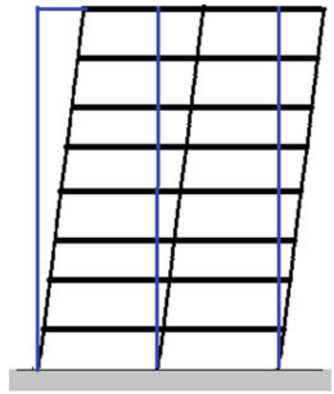


Fig. 14.7 Seismic response of low-rise frames with respect to ground

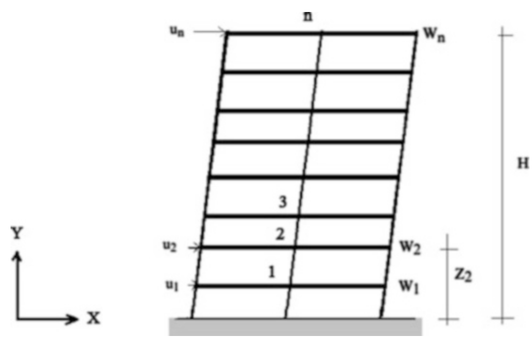


Fig. 14.8 Lateral displacement profile with respect to ground

where $u(H)$ is the relative displacement of the top floor with respect to the ground, and Z_i is the vertical coordinate of floor i. This assumption leads to the following expression for the total acceleration of a typical floor:

$$a\big|_{\text{floor } i} = a_g(t) + \frac{Z_i}{H}\frac{\mathrm{d}^2u(H)}{\mathrm{d}t^2} \tag{14.4}$$

where $a_g(t)$ is the ground acceleration time history. Applying Newton's law, the force required to accelerate floor i, assuming it moves as a rigid body and the lateral displacement profile is linear, is

$$P\big|_{\text{floor } i} = \frac{W_i}{g}\left\{a_g(t) + \frac{Z_i}{H}\frac{\mathrm{d}^2u(H)}{\mathrm{d}t^2}\right\} \tag{14.5}$$

This force is provided by the shear forces in the columns adjacent to the floor.

Given an earthquake ground motion time history, one applies this floor loading to a structure and determines the structural response. The solution for the acceleration at the top floor is expressed as [2]:

$$\frac{\mathrm{d}^2u(H)}{\mathrm{d}t^2} = -\Gamma\big(a_g(t) + \theta(t)\big) \tag{14.6}$$

where $\theta(t)$ depends on the earthquake ground motion and the structural period; Γ is a dimensionless parameter that depends on the distribution of floor masses,

$$\Gamma = \frac{\sum_{i=1}^{N} m_i \frac{Z_i}{H}}{\sum_{i=1}^{N} m_i \left(\frac{Z_i}{H}\right)^2} = \frac{H \sum_{i=1}^{N} W_i Z_i}{\sum_{i=1}^{N} W_i (Z_i)^2} \tag{14.7}$$

Substituting for the top floor acceleration, the inertia force expands to

$$P\Big|_{\text{floor}\, i} = \frac{W_i}{g} a_{\text{g}}(t) \left\{ 1 - \Gamma \frac{Z_i}{H} \right\} + \frac{W_i}{g} \left\{ -\Gamma \frac{Z_i}{H} \theta(t) \right\} \tag{14.8}$$

The peak values of $a_{\text{g}}(t)$ and $\theta(t)$ do not generally occur at the same time. Also the magnitude of Γ is of order one and the maximum value of $\theta(t)$ is usually larger than $a_{\text{g}_{\max}}$. Therefore, the *peak force* at floor i is approximated as:

$$P\Big|_{\text{floor}\, i} \approx \frac{Z_i W_i}{H} \left\{ \Gamma \frac{S_{\text{a}}}{g} \right\} \tag{14.9}$$

where S_{a} is defined as the maximum absolute value of $\theta(t)$. In the seismic literature, S_{a} is called the spectral acceleration. It is *the maximum acceleration that an equivalent single degree of freedom system experiences when subjected to the earthquake*. Summing up the floor forces leads to the resultant force which is also equal to the maximum shear force at the base.

$$V\Big|_{\text{base}} = \sum P\Big|_{\text{floor}\ i} \approx \frac{\left(\sum_{i=1}^{N} W_i Z_i\right)^2}{\sum_{i=1}^{N} W_i (Z_i)^2} \frac{S_{\text{a}}}{g} \tag{14.10}$$

Finally, we express the force for floor i in terms of $V|_{\text{base}}$.

$$P\Big|_{\text{floor}\, i} = \left(\frac{W_i Z_i}{\sum W_i Z_i} \right) V\Big|_{\text{base}} \tag{14.11}$$

The spectral acceleration measure, S_{a}, depends on the ground motion time history $a_{\text{g}}(t)$ and the period of the structure, T.

A simple approximation for T for a low-rise building is

$$T \approx \frac{N}{10} \text{s} \tag{14.12}$$

where N is the number of stories. Values of S_{a} vs. the structural period, T, have been complied by various agencies, such as the US Geological Survey's National Earthquake Information Center [1] for a range of earthquakes, and used to construct design plots such as illustrated below in Fig. 14.9. One estimates T and determines S_{a} with this plot. The limiting values for the plot, such as S_{DS} and S_{D1}, depend on S_{MS} and S_{M1} which are defined for a particular site and seismic design code [3]. Values of S_{MS}, S_{M1}, and T_{L} are listed on the USGS Web site, usgs/gov/hazards: S_{M1} is usually taken as the spectral acceleration for 5 % damping and 1 s period (i.e., $T_1 = 1$ s): S_{MS} is the spectral acceleration for 5 % damping and 0.2 s period and T_{L} is the long transition period. The worst case scenario is for the structural period to be between $0.2T_{\text{S}}$ and T_{S}. When $T \gg T_{\text{S}}$, the seismic load is significantly less than the load corresponding to the region $0.2T_{\text{S}} < T < T_{\text{S}}$.

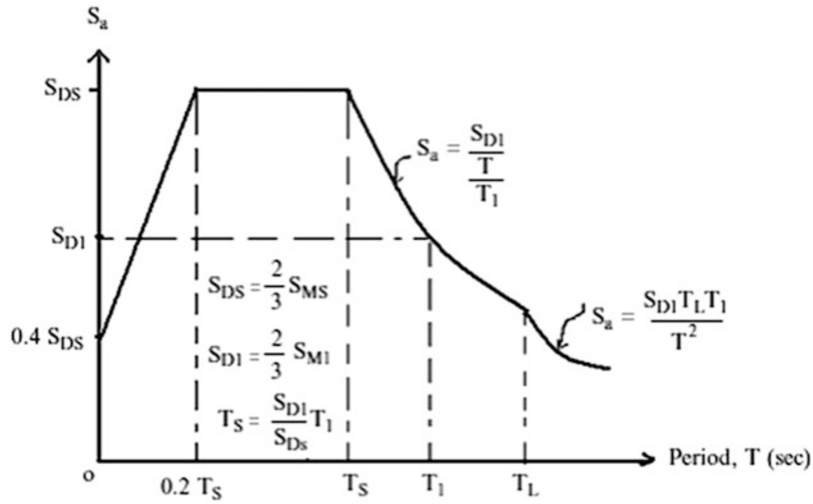


Fig. 14.9 Peak acceleration vs. structural period [3]

Example 14.1

Given: The three-story building shown in Figs. E14.1a and E14.1b. Assume the building is subjected to an earthquake in the North–South direction. Take the spectral acceleration as $S_a = 0.4$ g

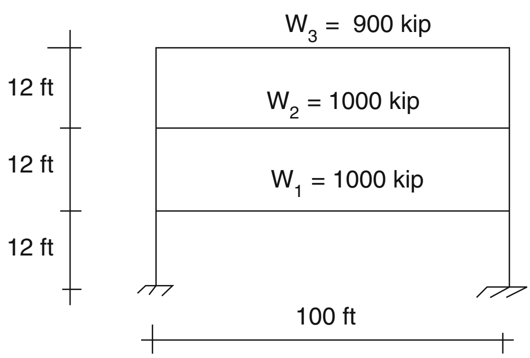


Fig. E14.1a Elevation N-S

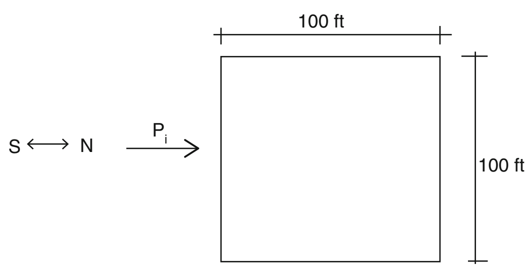


Fig. E14.1b Typical floor plan

Determine: The base shear and earthquake forces on the individual floors.

Solution: We use (14.10). The base shear is given by

$$V\big|_{\text{base}} = \frac{\left(\sum_{i=1}^{3} Z_i W\right)^2}{\sum_{i=1}^{3} W_i (Z_i)^2} \frac{S_a}{g} = \frac{\{1000(12) + 1000(24) + 900(36)\}^2}{\left\{1000(12)^2 + 1000(24)^2 + 900(36)^2\right\}} \frac{S_a}{g} = 3420(0.4) = 1368\,\text{kip}$$

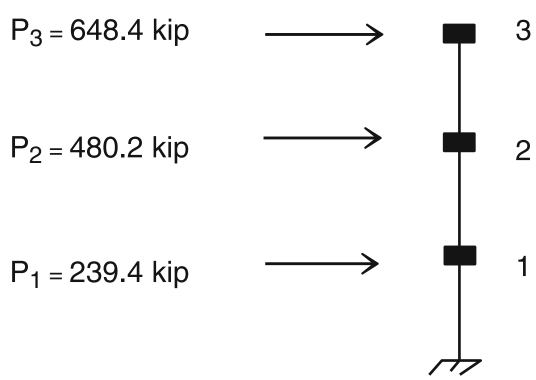
Then, applying (14.11), we obtain the individual floor loads.

$$\sum_{i=1}^{3} W_i Z_i = \{(12(1000) + 24(1000) + 36(900)\} = 68,400$$

$$P_1 = \frac{12(1000)}{68,400} V\big|_{\text{base}} = 0.175(1368) = 239.4\,\text{kip}$$

$$P_2 = \frac{24(1000)}{68,400} V\big|_{\text{base}} = 0.351(1368) = 480.2\,\text{kip}$$

$$P_3 = \frac{36(900)}{68,400} V\big|_{\text{base}} = 0.474(1368) = 648.4\,\text{kip}$$



Example 14.2

Given: The three-story building shown in Figs. E14.2a and E14.2b. The floor weights are indicated. There is also an additional weight located on the top floor.

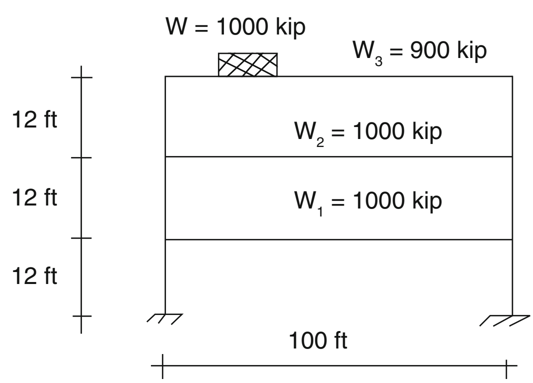


Fig. E14.2a Elevation

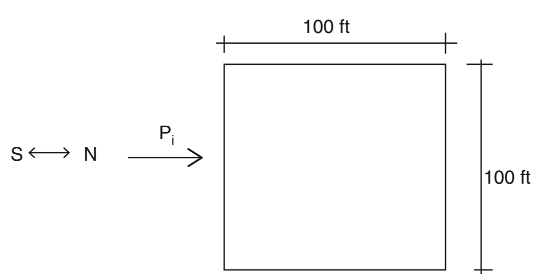


Fig. E14.2b Typical floor plan

Determine: The earthquake floor forces for a North–South earthquake of intensity $S_a = 0.4g$.

Solution: The computations are organized in the following table.

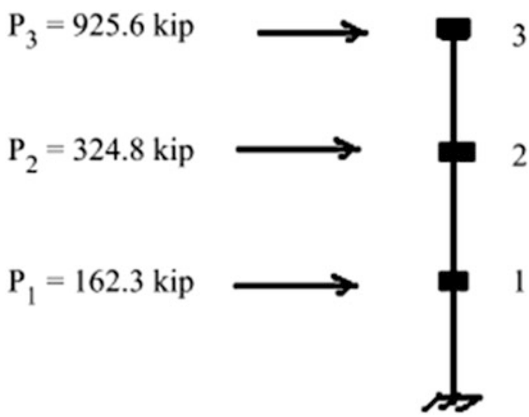
Floor	Z_i	W_i	W_iZ_i (10^3)	$W_i(Z_i)^2(10^3)$
1	12	1000	12	144
2	24	1000	24	596
3	36	900	32.4	1050
Roof	36	1000	36	1296
			104.4×10^3	3086×10^3

$$V\big|_{\text{base}} = (0.4)\frac{(104.4(1000))^2}{3086(1000)} = 1412.7\,\text{kip}$$

$$P_1 = \frac{12,000}{104.4(1000)}(1412.7) = 162.3\,\text{kip}$$

$$P_2 = \frac{24(1000)}{104.4(1000)}(1412.7) = 324.8\,\text{kip}$$

$$P_3 = \frac{(32.4 + 36)(1000)}{104.4(1000)}(1412.7) = 925.6\,\text{kip}$$



Note that the shear in the top story is increased considerably due to the additional mass on the third floor.

14.3 Building Response Under Lateral Loads

Up to this point, we have discussed how one generates the lateral loads acting at the floor levels. These loads are resisted by the frames which support the floors. In this section, we develop a methodology for distributing a floor load to the frames which support the floor.

We model the building as a set of *rigid floors* supported by columns and braces between the floors. When subjected to horizontal loading, the floor plates displace horizontally, resulting in bending of the columns and shearing deformation in the braces. The horizontal load is resisted by the shear forces developed in the columns and braces. We know from the examples studied in Chaps. 9 and 10 that stiffness attracts force. Therefore, one should expect that the distribution of floor load to the supporting elements, i.e., the columns and braces, will depend on the relative stiffness of these elements.

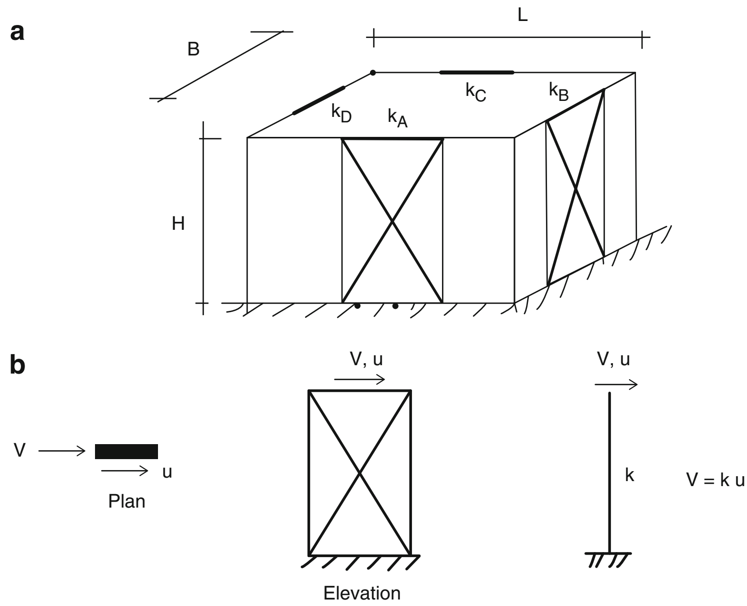


Fig. 14.10 (**a**) One-story braced frame. (**b**) Shear spring model for brace

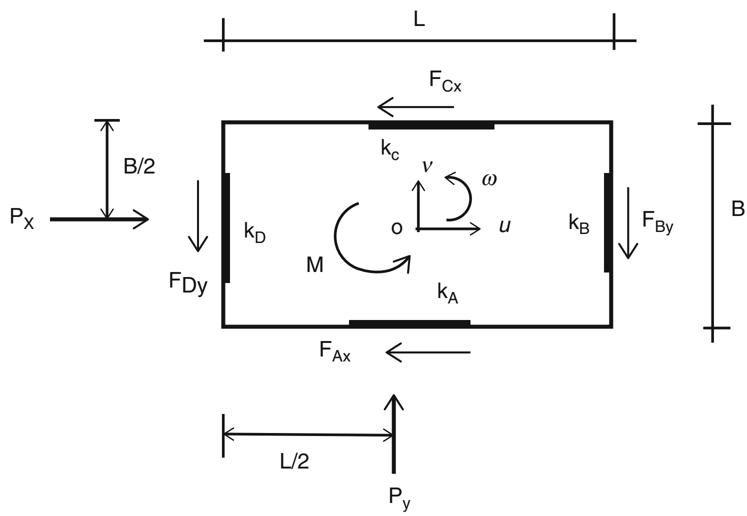


Fig. 14.11 Plan view

14.3.1 Center of Twist: One-Story Frame

We consider first the one-story braced frame structure shown in Fig. 14.10a. The braces located at the midpoints of the sides provide the resistance to horizontal load. We represent the braces as simple shear springs. Figure 14.10b illustrates this modeling strategy. Each brace provides a force which acts in the plane of the wall that contains the brace.

We locate the origin of the X–Y coordinate system at the geometric center of the floor and assume the floor is subjected to external forces, P_x, P_y, M. Under the action of these forces, the floor will experience translation (u, v) and rotation ω about the origin. These displacements produce shear forces in the springs which oppose the motion. The free body diagram for the floor is shown in Fig. 14.11.

Noting the free body diagram shown, the equilibrium equations expand to

$$\begin{aligned} + \rightarrow \sum F_x &= P_x - F_{Ax} - F_{Cx} = 0 \\ + \uparrow \sum F_y &= P_y - F_{By} - F_{Dy} = 0 \\ \sum M_0 &= M + (F_{Cx} - F_{Ax})\frac{B}{2} + \left(F_{Dy} - F_{By}\right)\frac{L}{2} = 0 \end{aligned} \tag{14.13}$$

Assuming the floor plate is rigid; the shear forces are related to the displacements by

$$\begin{aligned} F_{Ax} &= k_A\left(u + \frac{B}{2}\omega\right) \\ F_{Cx} &= k_C\left(u - \frac{B}{2}\omega\right) \\ F_{By} &= k_B\left(v + \frac{L}{2}\omega\right) \\ F_{Dy} &= k_D\left(v - \frac{L}{2}\omega\right) \end{aligned} \tag{14.14}$$

Substituting for the forces in (14.13) leads to

$$\begin{aligned} P_x &= u(k_A + k_C) + \frac{B}{2}(k_A - k_C)\omega \\ P_y &= v(k_B + k_D) + \frac{L}{2}(k_B - k_D)\omega \\ M &= \frac{B}{2}(k_A - k_C)u + \frac{L}{2}(k_B - k_D)v + \left\{\frac{B^2}{4}(k_A + k_C) + \frac{L^2}{4}(k_B + k_D)\right\}\omega \end{aligned} \tag{14.15}$$

We see that the response depends on the relative stiffness of the braces. If $k_A \neq k_C$ or $k_B \neq k_D$, the floor will experience rotation when only P_x or P_y is applied at the geometric center. Given the stiffness of the braces, one solves (14.15) for u, v, ω and evaluates the braces forces using (14.14).

Example 14.3

Given: The floor plan, dimensions and layout of the braces, and the brace stiffnesses shown in Fig. E14.3a.

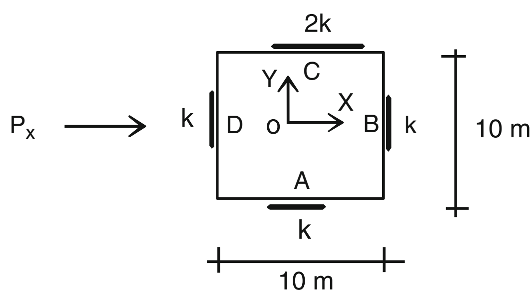


Fig. E14.3a Plan view

Determine: The response due to P_x.

Solution: We note that the braces at A, B, D have equal stiffnesses, and the brace at C is twice as stiff as the others. For convenience, we show these values as just k and $2k$. We set $P_y = M = 0$ in (14.15).

$$P_x = (3k)u + 5(-k)\omega = P$$
$$P_y = (2k)v = 0$$
$$M = 5(-k)u + \left(\frac{500}{4}k\right)\omega = 0$$

Solving these equations, we obtain

$$u = \frac{25P}{70k}$$
$$v = 0$$
$$\omega = \frac{P}{70k}$$

Finally, the brace forces are

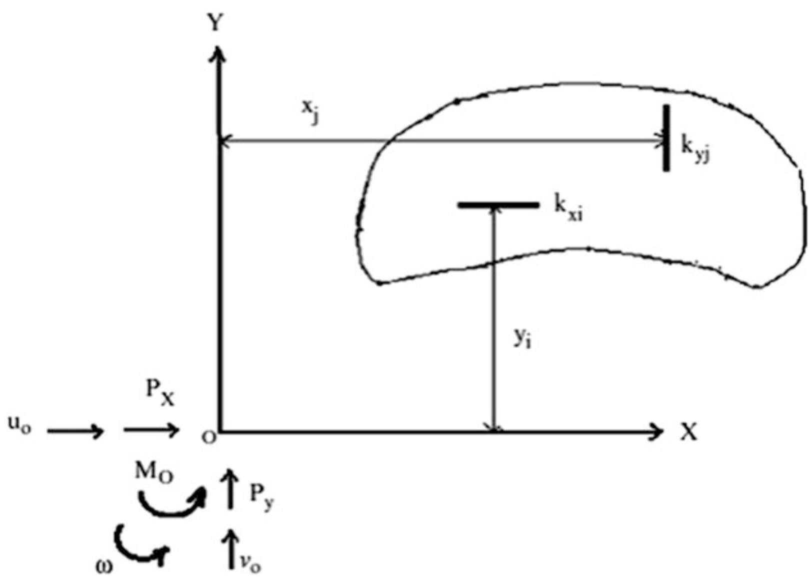
$$F_{Ax} = k_A\left(u + \frac{B}{2}\omega\right) = P\left(\frac{25}{70} + \frac{5}{70}\right) = \frac{3}{7}P$$
$$F_{Cx} = k_C\left(u - \frac{B}{2}\omega\right) = 2P\left(\frac{25}{70} - \frac{5}{70}\right) = \frac{4}{7}P$$
$$F_{By} = k_B\left(v + \frac{L}{2}\omega\right) = P\left(\frac{5}{70}\right) = \frac{1}{14}P$$
$$F_{Dy} = k_D\left(v - \frac{L}{2}\omega\right) = P\left(-\frac{5}{70}\right) = -\frac{1}{14}P$$

To avoid rotation, which is undesirable, one needs to modify either the stiffness at A or C. Taking $k_A = k_C = k$, the response is

$$u = \frac{P}{2k}$$
$$v = \omega = 0$$
$$F_{Ax} = F_{Cx} = \frac{P}{2}$$

The formulation described above can be generalized to deal with an arbitrary number of braces or shear walls oriented in either the X or Y direction. We shift to the notation shown in Fig. 14.12 to

Fig. 14.12 Notation



identify the various braces. Each brace is characterized by a stiffness magnitude (k) and the perpendicular distance from the tangent to the origin.

Assuming the floor is rigid, the tangential motion for the X oriented braces due to a rigid body motion of the origin o is

$$+\rightarrow u_i = u_\text{o} - y_i\omega \tag{14.16}$$

Similarly, the Y motion for brace j is given by

$$+\uparrow v_j = v_\text{o} + x_j\omega \tag{14.17}$$

These motions produce shear forces which act to *oppose* the motion of the floor. The individual forces are

$$\begin{aligned} &+\rightarrow F_{xi} = -k_{xi}u_i = -k_{xi}u_o + k_{xi}y_i\omega \\ &+\uparrow F_{yj} = -k_{yj}v_j = -k_{yj}v_o - k_{yj}x_j\omega \end{aligned} \tag{14.18}$$

Summing forces and moments with respect to the origin leads to the equilibrium equations for the floor

$$\begin{aligned} &+\rightarrow P_x - u_0\left(\sum k_{xi}\right) + \omega\left(\sum y_i k_{xi}\right) = 0 \\ &+\uparrow P_y - v_0\left(\sum k_{yj}\right) - \omega\left(\sum x_j k_{yj}\right) = 0 \\ &M_0 + u_0\left(\sum y_i k_{xi}\right) - v_0\left(\sum x_j k_{yj}\right) - \omega\left\{\sum y_i^2 k_{xi} + \sum x_j^2 k_{yj}\right\} = 0 \end{aligned} \tag{14.19}$$

where P_x, P_y, and M_0 are the external loads on the floor.

We define the following terms:

$$\begin{aligned} \sum k_{xi} &= K_{xx} \\ \sum k_{yj} &= K_{yy} \\ \sum y_i k_{xi} &= K_{xz} \\ \sum x_j k_{yj} &= K_{yz} \\ \sum y_i^2 k_{xi} + \sum x_j^2 k_{yj} &= K_o \end{aligned} \tag{14.20}$$

With this notation, (14.19) takes the following form:

$$\begin{aligned} P_x &= K_{xx}u_0 - K_{xz}\omega \\ P_y &= K_{yy}v_0 + K_{yz}\omega \\ M_0 &= -K_{xz}u_0 + K_{yz}v_0 + K_\text{o}\omega \end{aligned} \tag{14.21}$$

Equation (14.21) applies for an arbitrary choice of origin. Note that forces applied at the origin will produce rotation when either $K_{xz} \neq 0$ or $K_{yz} \neq 0$. If the stiffness distribution is symmetrical, these terms vanish. Rotation of the floor is a torsional mode of response, which introduces an undesirable anti-symmetric deformation in the perimeter facades. Therefore, one approach is to always choose a symmetrical stiffness layout. Another approach is to shift the origin to some other point in the floor. Obviously, the most desirable point corresponds to $K_{xz} = K_{yz} = 0$.

Consider the floor geometry shown in Fig. 14.13. Point o denotes the initial origin and C some arbitrary point in the floor. We locate a new set of axes at C and express the forces in terms of the coordinates with respect to C.

Fig. 14.13 Floor geometry

$$\begin{aligned} u_i &= u_c - y_i' \omega \\ v_i &= v_c + x_i' \omega \end{aligned} \tag{14.22}$$

$$\begin{aligned} F_{xi} &= -k_{xi} u_c + k_{xi} y_i' \omega \\ F_{yi} &= -k_{yi} v_c - k_{yi} x_i' \omega \end{aligned} \tag{14.23}$$

The equilibrium equations referred to point C have the following form:

$$\begin{aligned} & P_{xc} - u_C\left(\sum k_{xi}\right) + \omega\left(\sum y_i' k_{xi}\right) = 0 \\ & P_{yc} - v_C\left(\sum k_{yj}\right) - \omega\left(\sum x_j' k_{yj}\right) = 0 \\ & M_c + u_C\left(\sum y' k_{xi}\right) - v_c\left(\sum x' k_{yj}\right) + \omega\left(-\sum y'^2_i k_{xi} - \sum x'^2_j k_{yj}\right) = 0 \end{aligned} \tag{14.24}$$

We choose point C such that

$$\sum y_i' k_{xi} = \sum x_j' k_{yj} = 0 \tag{14.25}$$

These conditions define the coordinates of point C. Substituting for x' and y' using

$$\begin{aligned} x' &= -x_C + x \\ y' &= -y_C + y \end{aligned}$$

leads to

$$\begin{aligned} x_C &= \frac{\sum x_j k_{yj}}{\sum k_{yj}} \\ y_C &= \frac{\sum y_i k_{xi}}{\sum k_{xi}} \end{aligned} \tag{14.26}$$

The equilibrium equations referred to these new axes simplify to

$$\begin{aligned} P_x &= K_{xx} u_C = R_x \\ P_y &= K_{yy} v_C = R_y \\ M_C &= K_C \omega_C = M_z \end{aligned} \tag{14.27}$$

where

$$K_C = \sum y_i'^2 k_{xi} + \sum x_j'^2 k_{yj}$$

The external and internal forces are shown in Fig. 14.14.

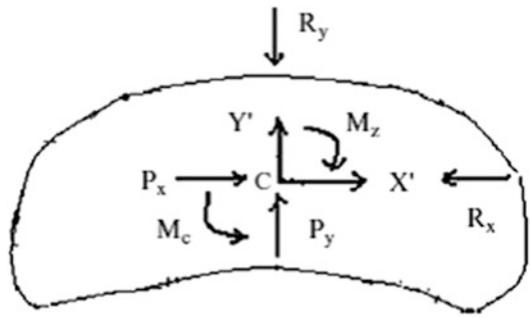


Fig. 14.14 External and internal forces

Point C is called the "center of twist." Figure 14.14 *shows that the resultant of the* "*resisting*" *forces acts at the center of twist*. External forces applied at the center of twist produce only translation; an external moment applied to the floor produces twist about the center of twist. We point out that the coordinates of the center of twist depend on the stiffness of the components located in the story. The location of the center of twist changes when either the position or magnitude of the stiffness components is changed.

Example 14.4

Given: The floor plan shown in Fig. E14.4a. The two shear walls are orthogonal and are located on the *X* and *Y* axes.

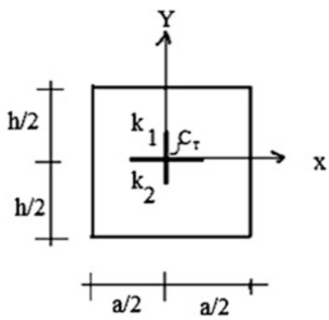


Fig. E14.4a

Determine: The center of twist.

Solution: The center of stiffness lies on an axis of symmetry. In this case, there are two axes of symmetry and therefore the center of twist is at the origin.

Example 14.5

Given: The stiffness distribution shown in Fig. E14.5a.

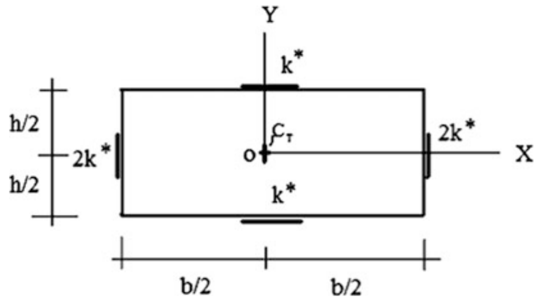


Fig. E14.5a

Determine: The center of twist.

Solution: The stiffness distributions are symmetrical with respect to the X and Y axes. Therefore,

$$X_{\mathrm{CT}} = Y_{\mathrm{CT}} = 0.$$

Example 14.6

Given: The stiffness distribution shown in Fig. E14.6a.

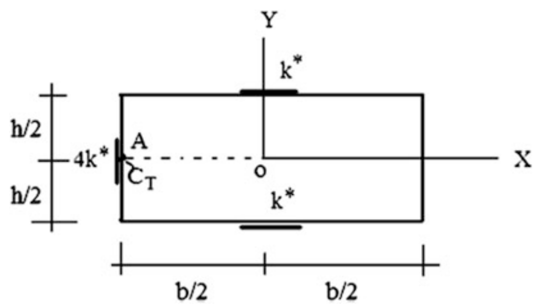


Fig. E14.6a

Determine: The center of twist.

Solution: The center of twist (C_{T}) lies on the X-axis because the stiffness is symmetrical with respect to the X-axis. Summing moments about the origin leads to

$$x_{\mathrm{CT}} = \frac{\sum x_i k_{yi}}{\sum k_{yi}} = \frac{4k^*(-(b/2))}{4k^*} = -\frac{b}{2}$$

Example 14.7

Given: The stiffness distribution and loading shown in Fig. E14.7a.

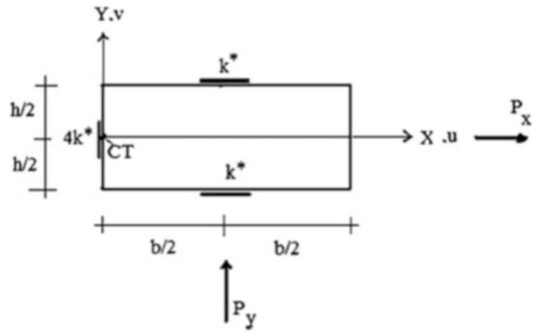


Fig. E14.7a

Determine: The rigid body motion.

Solution: The floor will experience rotation as well as translation since there is a net moment with respect to the center of twist. We determine the motion measures using (14.27). The stiffness measures are

$$K_{xx} = \sum k_{xi} = 2k^*$$
$$K_{yy} = \sum k_{yj} = 4k^*$$
$$K_C = \sum y_i^2 k_{xi} + \sum x_j^2 k_{yj} = 2\left\{\left(\tfrac{h}{2}\right)^2 k^*\right\} = \tfrac{h^2}{2} k^*$$

Then,

$$u = \frac{P_x}{K_{xx}} = \frac{P_x}{2k^*}$$
$$v = \frac{P_y}{K_{yy}} = \frac{P_y}{4k^*}$$
$$\omega = \frac{P_y\left(\frac{b}{2}\right)}{K_C} = P_y \frac{\frac{b}{2}}{\frac{h^2}{2}k^*} = \frac{P_y}{k^*}\frac{b}{h^2}$$

The deformed configuration of the floor is shown in Fig. E14.7b.

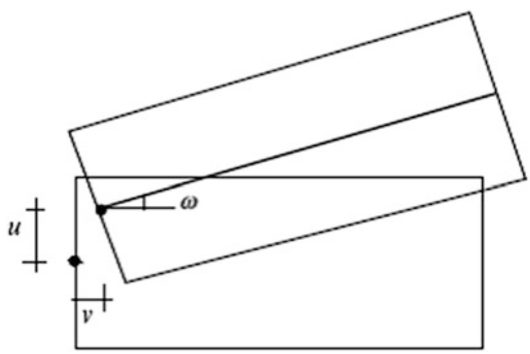


Fig. E14.7b

Example 14.8

Given: The stiffness distribution shown in Fig. E14.8a.

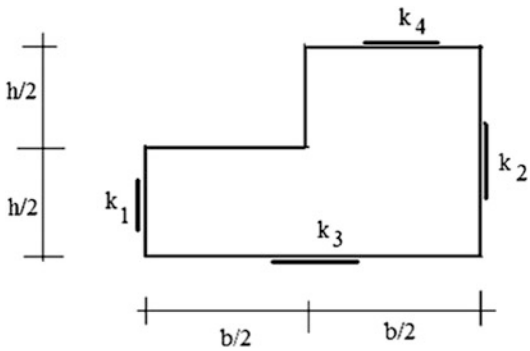


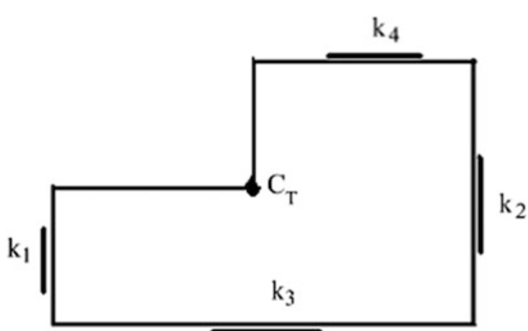
Fig. E14.8a

Determine: The center of twist for the following combination of stiffness factors:

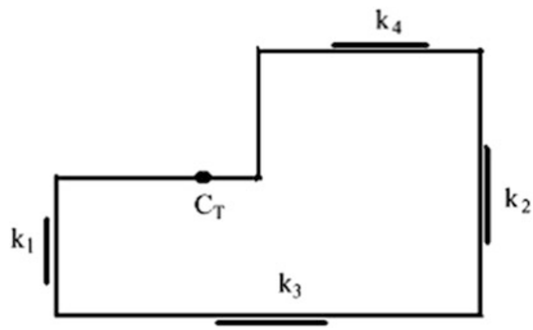
(a) $k_1 = k_2 = k_3 = k_4$
(b) $k_4 = k_3\ k_1 > k_2$
(c) $k_4 > k_3\ k_1 = k_2$
(d) $k_4 > k_3\ k_1 > k_2$

Solution: The problem can be viewed as being equivalent to finding the centroid of a set of areas, with area replaced by stiffness. One can use qualitative reasoning to estimate the location of the center of twist.

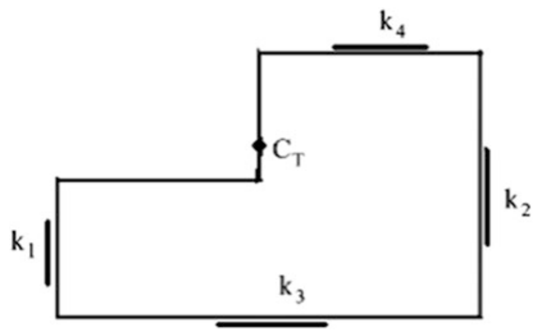
Case (a) $k_1 = k_2 = k_3 = k_4$



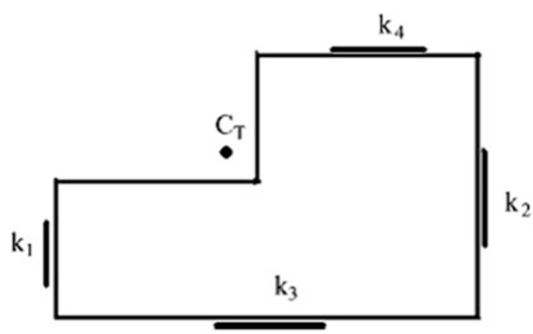
Case (b) $k_4 = k_3\ k_1 > k_2$



Case (c) $k_4 > k_3\ k_1 = k_2$



Case (d) $k_4 > k_3\ k_1 > k_2$

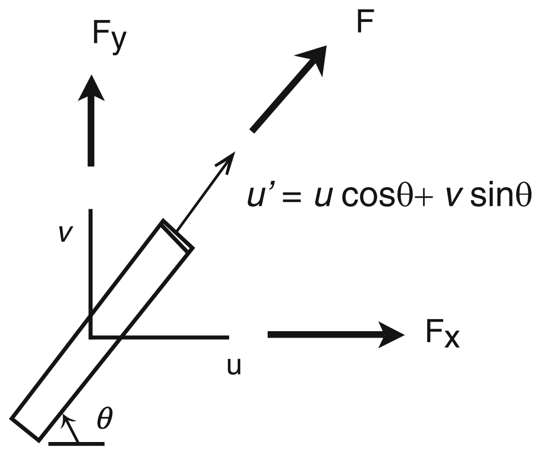


We consider next the single inclined brace shown in Fig. 14.15. Introducing displacements u and v produces a longitudinal force F equal to ku'. Projecting F on the x and y axes leads to

$$\begin{aligned} F_x &= F\cos\theta = k\left[u(\cos\theta)^2 + v\cos\theta\sin\theta\right] \\ F_y &= F\sin\theta = k\left[u\sin\theta\cos\theta + v(\sin\theta)^2\right] \end{aligned} \tag{14.28}$$

Summing these forces over the number of braces, the resultants are given by

Fig. 14.15 Inclined brace



$$R_x = \sum F_x = u\sum k_i \cos^2\theta + v\sum k_i \cos\theta \sin\theta$$
$$R_y = \sum F_y = u\sum k_i \sin\theta \cos\theta + v\sum k_i \sin^2\theta \tag{14.29}$$

We write these equations as

$$R_x = uK_{cc} + vK_{cs}$$
$$R_y = uK_{cs} + vK_{ss} \tag{14.30}$$

where

$$K_{cc} = \sum k_i \cos^2\theta$$
$$K_{cs} = \sum k_i \cos\theta \sin\theta$$
$$K_{ss} = \sum k_i \sin^2\theta$$

Note that when $\theta = 0°$ or $90°$, these expressions reduce to (14.27).

The line of action is determined by summing moments about O. Working first with the X direction (u) and then the Y direction (v) leads to the following pair of equations for x^* and y^*, the coordinates of the center of twist.

$$y^* K_{cc} - x^* K_{cs} = \sum y_i k_i \cos^2\theta - \sum x_i k_i \sin\theta \cos\theta$$
$$y^* K_{cs} - x^* K_{ss} = \sum y_i k_i \sin\theta \cos\theta - \sum x_i k_i \sin^2\theta \tag{14.31}$$

When the stiffness elements are parallel to either x or y, these equations reduce to

$$K_{cc} = \sum k_x \quad K_{cs} = 0 \quad K_{ss} = \sum k_y$$
$$y^* = \frac{\sum y k_x}{\sum k_x} \quad x^* = \frac{\sum x k_y}{\sum k_y}$$

Example 14.9

Given: The stiffness distribution shown in Fig. E14.9a. Two of the braces are inclined with respect to the x-axis.

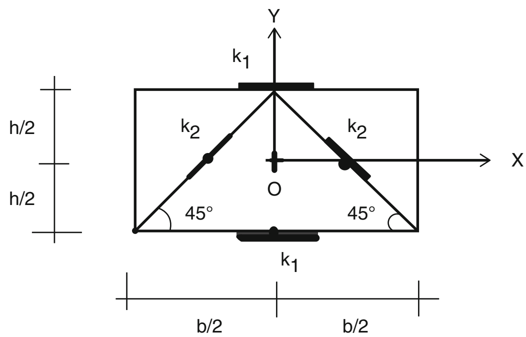


Fig. E14.9a

Determine: The center of twist.

Solution: Evaluating (14.31) leads to

$$K_{cc} = \sum k_i \cos^2\theta = 2k_1 + 2\left(\frac{1}{2}k_2\right) = 2k_1 + k_2$$

$$K_{cs} = \sum k_i \cos\theta \sin\theta = \left(\frac{1}{2}k_2\right) + \left(-\frac{1}{2}k_2\right) = 0$$

$$K_{ss} = \sum k_i \sin^2\theta = 2\left(\frac{1}{2}k_2\right) = k_2$$

$$\sum y_i k_i \cos^2\theta = 0$$

$$\sum x_i k_i \sin^2\theta = 0$$

$$\sum x_i k_i \sin\theta\cos\theta = k_2\frac{b}{4}\sum(-1)\frac{1}{2} + (+1)\left(-\frac{1}{2}\right) = -k_2\frac{b}{4}$$

$$\sum y_i k_i \sin\theta\cos\theta = 0$$

Then,

$$y^* = \frac{\frac{bk_2}{4}}{2k_1+k_2} = \frac{b}{4}\left(\frac{k_2}{k_2 + 2k_1}\right)$$

$$x^* = 0$$

14.3.2 Center of Mass: One-Story Frame

The center of twist for a one-story frame is a property of the stiffness components located in the story below the floor. It defines the point of application of the inter-story resistance forces acting on the floor. These forces depend on the translation and rotation of the floor produced by the applied loading, i.e., they are due to inter-story deformation.

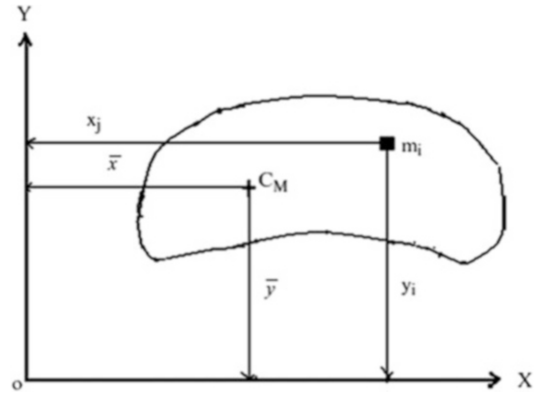


Fig. 14.16 Plan view of floor

When the loading is dynamic, additional inertia forces are generated due to the acceleration of the masses located on the floor. In order to study the equilibrium of the floor, we need to establish the magnitude and location of the resultant of these inertia forces. In what follows, we describe the procedure for locating this resultant.

Figure 14.16 shows a typical plan view of a floor. We locate the origin at some arbitrary point in the floor, and suppose that there are masses located at discrete points in the floor. The center of mass is a particular point in the floor defined by the coordinates $\bar{x}$ and $\bar{y}$, where

$$\bar{x} = \frac{\sum x_i m_i}{\sum m_i} \qquad \bar{y} = \frac{\sum y_i m_i}{\sum m_i} \tag{14.32}$$

Example 14.10

Given: The floor mass layout shown in Fig. E14.10a.

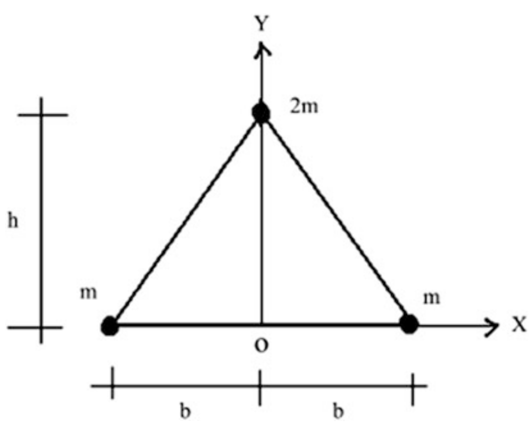


Fig. E14.10a

Determine: The center of mass.

Solution: The center of mass is on the *y*-axis. In general, if the mass distribution is symmetrical, the center of mass lies on the axis of symmetry. We determine the *y* coordinate by summing moments about the *x*-axis (Fig. E14.10b).

$$\bar{y} = \frac{\sum y_i m_i}{\sum m_i} = \frac{(2m)h}{4m} = \frac{h}{2}$$

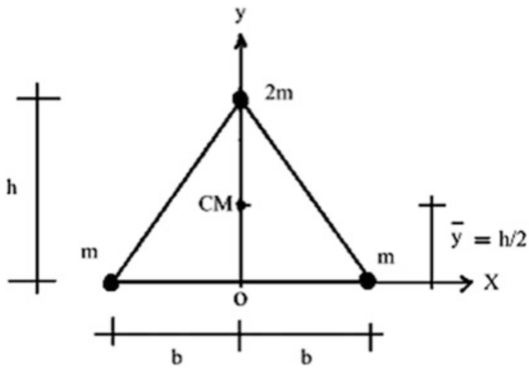


Fig. E14.10b

Example 14.11

Given: The floor mass layout shown in Fig. E14.11a.

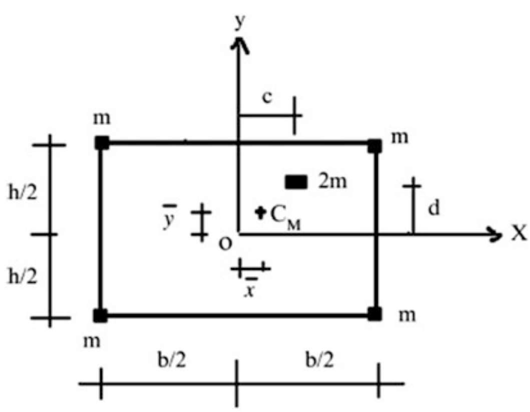


Fig. E14.11a

Determine: The center of mass.

Solution: Summing the moments leads to

$$\sum m_i = 6m$$
$$\bar{x} = \frac{2mc + 2m(b/2) + 2m(-b/2)}{6m} = \frac{c}{3}$$
$$\bar{y} = \frac{2md + 2m(h/2) + 2m(-h/2)}{6m} = \frac{d}{3}$$

Example 14.12

Given: The floor mass layout shown in Fig. E14.12a.

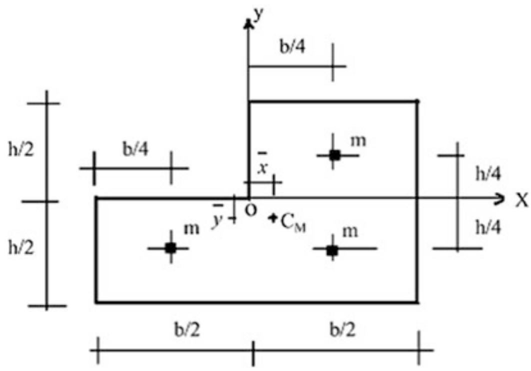


Fig. E14.12a

Determine: The center of mass.

Solution: We sum moments about the x and y axes and obtain

$$\bar{x} = \frac{m(b/4)}{3m} = \frac{b}{12}$$
$$\bar{y} = \frac{-m(h/4)}{3m} = -\frac{h}{12}$$

14.3.3 One-Story Frame: General Response

We have shown that there are two key points in the floor, *the center of twist and the center of mass*. For quasi-static loading, we work with quantities referred to the center of twist. Noting (14.27), the response of the center of twist due to an arbitrary static loading is (Fig. 14.17)

$$u_c = \frac{P_x}{K_{xx}}$$
$$v_c = \frac{P_y}{K_{yy}} \qquad (14.33)$$
$$\omega_c = \frac{M_C}{K_C}$$

Note that twist occurs only when there is an external moment with respect to the center of twist; forces applied at the center of twist produce only translation.

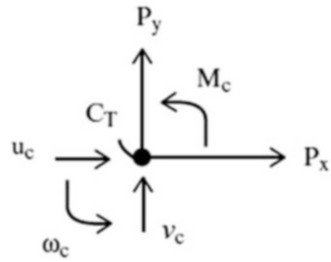


Fig. 14.17 Forces acting at the center of twist

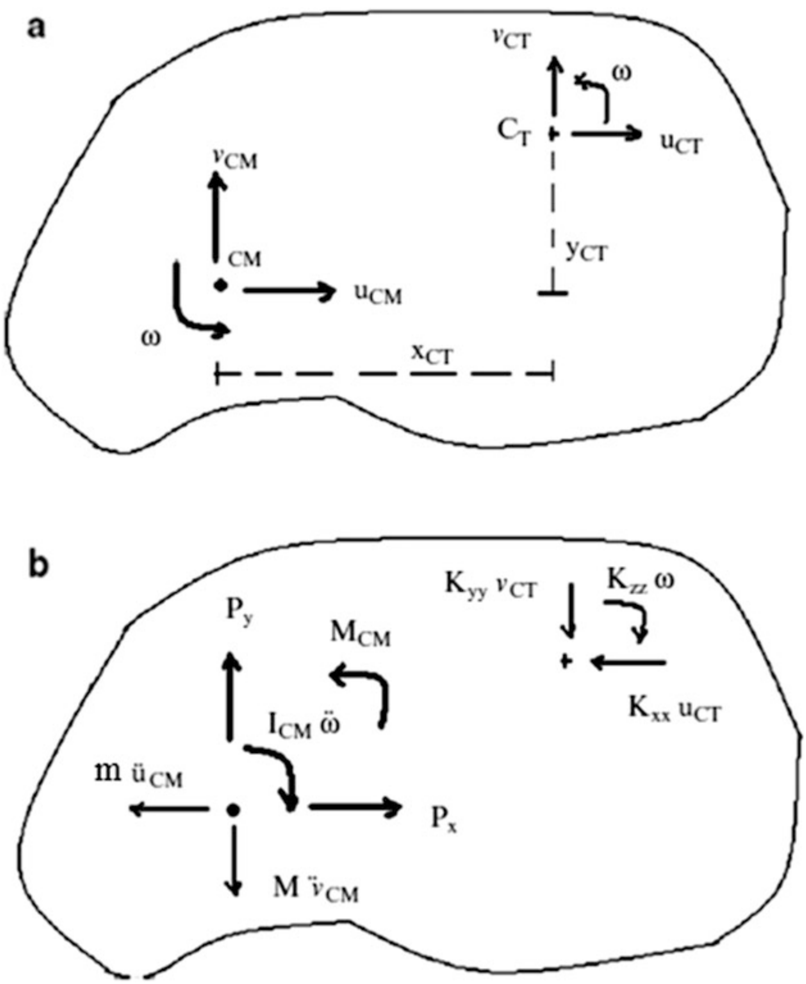


Fig. 14.18 Forces acting at the center of mass (**a**) Displacements. (**b**) Forces

When the loading is dynamic, one needs to include the inertia forces. In this case, it is more convenient to place the origin at the center of mass and work with force and displacement quantities referred to the axes centered at the center of mass. Figure 14.18a illustrates this choice. Note that the resistance forces act at the center of twist and produce a moment about the center of mass. The displacements of the two centers are related by

$$\begin{aligned} u_{\mathrm{CT}} &= u_{\mathrm{CM}} - y_{\mathrm{CT}}\omega \\ v_{\mathrm{CT}} &= v_{\mathrm{CM}} + x_{\mathrm{CT}}\omega \\ \omega_{\mathrm{CT}} &= \omega \end{aligned} \tag{14.34}$$

The equilibrium equations referred to the center of mass have the following form:

$$\begin{aligned} P_x &= m\ddot{u}_{\mathrm{CM}} + K_{xx}(u_{\mathrm{CM}} - y_{\mathrm{CT}}\omega) \\ P_y &= m\ddot{v}_{\mathrm{CM}} + K_{yy}(v_{\mathrm{CM}} + x_{\mathrm{CT}}\omega) \\ M_{\mathrm{CM}} &= I_{\mathrm{CM}}\ddot{\omega} + K_{\mathrm{C}}\omega + x_{\mathrm{CT}}K_{\mathrm{yy}}(v_{\mathrm{CM}} + x_{\mathrm{CT}}\omega) - y_{\mathrm{CT}}K_{xx}(u_{\mathrm{CM}} - y_{\mathrm{CT}}\omega) \\ &= I_{\mathrm{CM}}\ddot{\omega} + \omega\left\{K_{\mathrm{C}} + x_{\mathrm{CT}}^2 K_{\mathrm{yy}} + y_{\mathrm{CT}}^2 K_{xx}\right\} + x_{\mathrm{CT}}K_{yy}v_{\mathrm{CM}} - y_{\mathrm{CT}}K_{xx}u_{\mathrm{CM}} \end{aligned} \tag{14.35}$$

Equation (14.35) shows that the motion is coupled when the center of twist *does not* coincide with the center of mass. The center of mass is usually fixed by the mass distribution on the floor and one usually does not have any flexibility in shifting masses. Therefore, the most effective strategy is to adjust the location of the braces in the story below the floor such that the centers of mass and twist coincide, i.e., to take $x_{\mathrm{CT}} = y_{\mathrm{CT}} = 0$.

Example 14.13

Given: The mass and stiffness layout shown in Fig. E14.13a.

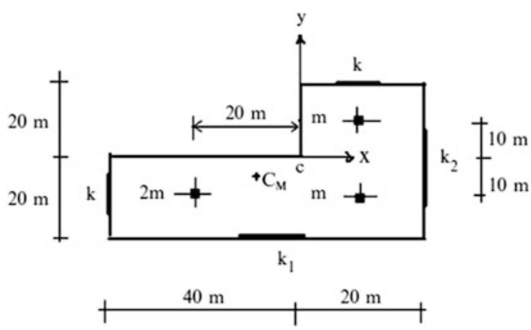


Fig. E14.13a

Determine: The magnitude of the stiffness elements k_1 and k_2 such that the centers of mass and twist coincide.

Solution: First, we locate the center of mass.

$$\begin{aligned} \sum m_i &= 4\,\mathrm{m} \\ \sum x_i m_i &= 10(2\,\mathrm{m}) - 20(2\,\mathrm{m}) = -20\,\mathrm{m} \\ \bar{x} &= \frac{-20\,\mathrm{m}}{4\,\mathrm{m}} = -5\,\mathrm{m} \end{aligned}$$

Similarly,

$$\sum y_i m_i = 10(\text{m}) - 10(3\text{m}) = -20\,\text{m}$$
$$\bar{y} = \frac{-20\,\text{m}}{4\,\text{m}} = -5\,\text{m}$$

Next, we determine k_1 and k_2 by requiring the center of twist to coincide with the center of mass. The steps are

Step 1:

$$x_{\text{CT}} = \frac{\sum x_i k_y}{\sum k_y} = \frac{20k_2 - 40k}{k_2 + k} = -5\,\text{m}$$
$$\Downarrow$$
$$20k_2 - 40k = -5k - 5k_2$$
$$25k_2 = 35k$$
$$k_2 = 1.4k$$

Step 2:

$$y_{\text{CT}} = \frac{\sum y_i k_x}{\sum k_x} = \frac{20(k - k_1)}{k_1 + k} = -5\,\text{m}$$
$$\Downarrow$$
$$25k = 15k_1$$
$$k_1 = 1.67k$$

14.3.4 Multistory Response

A typical floor in a multistory structure is connected to the adjacent floors by stiffness elements such as columns, shear walls, and braces. When the floors displace, inter-story deformation due to the relative motion between the floors is developed, resulting in self-equilibrating story forces which act on the adjacent floors. Figure 14.19 illustrates this mode of behavior. Floors i and $i + 1$ experience lateral displacements which produce shear deformations in the braces

$$\gamma = u_{i+1} - u_i$$

and corresponding shear forces

$$F = k\gamma = k(u_{i+1} - u_i)$$

These forces act on both floors $i + 1$ and floor i; the sense is reversed for the lower floor (this follows from Newton's law of action equal reaction).

In order to express these resistance forces in terms of displacements, we need to specify a common reference frame for all the floors. We suppose the floors translate and rotate with respect to this common reference frame. We consider floor i. We determine the inter-story displacement measures for the two centers of twist associated with the stories above and below floor i and apply (14.27). The resulting expressions for the resultant forces acting on floor i are listed below. Their sense is

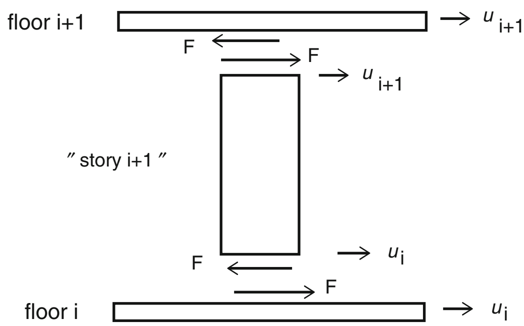


Fig. 14.19 Forces due to inter-story deformation

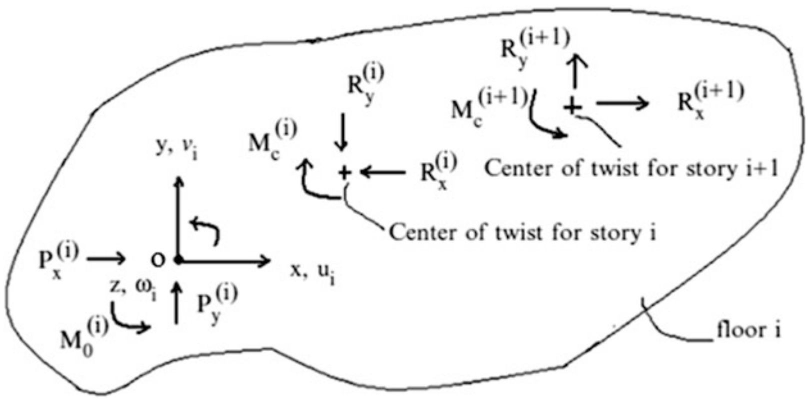


Fig. 14.20 Forces acting on floor *i*

defined in Fig. 14.20. Note that the direction of the forces due to story $i + 1$ are opposite to those due to story i.

For story i:

$$\begin{aligned} R_x^{(i)} &= K_{xx}^{(i)}\left[\left\{u_i - y_{\mathrm{CT}}^{(i)}\omega_i\right\} - \left\{u_{i-1} - y_{\mathrm{CT}}^{(i)}\omega_{i-1}\right\}\right] \\ R_y^{(i)} &= K_{yy}^{(i)}\left[\left\{v_i - x_{\mathrm{CT}}^{(i)}\omega_i\right\} - \left\{v_{i-1} - x_{\mathrm{CT}}^{(i)}\omega_{i-1}\right\}\right] \\ M_{\mathrm{C}}^{(i)} &= K_{\mathrm{C}}^{(i)}[\omega_i - w_{i-1}] \end{aligned} \tag{14.36}$$

For story $i + 1$:

$$\begin{aligned} R_x^{(i+1)} &= K_{xx}^{(i+1)}\left[\left\{u_{i+1} - y_{\mathrm{CT}}^{(i+1)}\omega_{i+1}\right\} - \left\{u_i - y_{\mathrm{CT}}^{(i+1)}\omega_i\right\}\right] \\ R_y^{(i+1)} &= K_{yy}^{(i+1)}\left[\left\{v_{i+1} + x_{\mathrm{CT}}^{(i+1)}\omega_{i+1}\right\} - \left\{v_i - x_{\mathrm{CT}}^{(i+1)}\omega_i\right\}\right] \\ M_{\mathrm{C}}^{(i+1)} &= K_{\mathrm{C}}^{(i+1)}[\omega_{i+1} - \omega_i] \end{aligned} \tag{14.37}$$

The inertia forces for a floor depend on the mass distribution and acceleration of the floor. They act at the center of mass, a property of the floor. Figure 14.21 shows the inertia forces for floor i.

We require floor i to be in equilibrium. Summing forces with respect to the origin at O results in the following equilibrium equations:

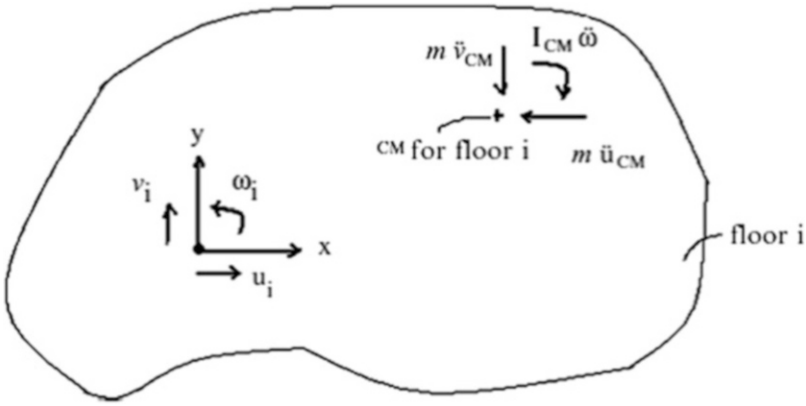


Fig. 14.21 Inertia forces for floor i

$$
\begin{aligned}
&P_x^{(i)} - m^{(i)}\ddot{u}_{i,\mathrm{CM}} - R_x^{(i)} + R_x^{(i+1)} = 0 \\
&P_y^{(i)} - m^{(i)}\ddot{v}_{i,\mathrm{CM}} - R_y^{(i)} + R_y^{(i+1)} = 0 \\
&M_0^{(i)} - I_{\mathrm{CM}}^{(i)}\ddot{\omega} - M_{\mathrm{C}}^{(i)} + M_{\mathrm{C}}^{(i+1)} + m^{(i)}y_{\mathrm{CM}}^{(i)}\ddot{u}_{i,\mathrm{CM}} - m^{(i)}x_{\mathrm{CM}}^{(i)}\ddot{v}_{i,\mathrm{CM}} + y_{\mathrm{CT}}^{(i)}R_x^{(i)} \\
&-x_{\mathrm{CT}}^{(i)}R_y^{(i)} - y_{\mathrm{CT}}^{(i+1)}R_x^{(i+1)} + x_{\mathrm{CT}}^{(i+1)}R_y^{(i+1)} = 0
\end{aligned}
\tag{14.38}
$$

The form of (14.36) and (14.37) shows that the equilibrium equations for floor i involve the displacements for floor $i - 1$, i, and $i + 1$. Assuming there are n floors, there are n sets of equations similar in form to (14.38).

When the location of the center of mass is the same for all the floors, we take the origin at the "common" center of mass. If the center of twist also coincides with the center of mass, the equations simplify to

$$
\begin{aligned}
P_x^{(i)} &= m^{(i)}\ddot{u}_i + K_{xx}^{(i)}(u_i - u_{i-1}) - K_{xx}^{(i+1)}(u_{i+1} - u_i) \\
P_y^{(i)} &= m^{(i)}\ddot{v}_i + K_{yy}^{(i)}(v_i - v_{i-1}) - K_{yy}^{(i+1)}(v_{i+1} - v_i) \\
M_0^{(i)} &= I^{(i)}\ddot{\omega}_i + K_{\mathrm{C}}^{(i)}(\omega_i - \omega_{i-1}) + K_{\mathrm{C}}^{(i+1)}(\omega_{i+1} - \omega_i)
\end{aligned}
\tag{14.39}
$$

where u, v, and ω are the displacement measures for the center of mass.

These equations are useful for qualitative reasoning about the behavior. In general, we want to avoid torsion, if possible. Therefore, we distribute the inter-story stiffness elements such that the location of the center of twist is constant for all stories. In regions where the seismic loading is high, such as California, one needs to consider dynamic response. In this case, the goal in seismic design is to have the center of mass and center of twist coincide throughout the height of the structure.

The formulation obtained above can be interpreted as a "shear beam" formulation for a building system in the sense that the assumptions we introduced concerning the behavior of a floor are similar to those for a beam subjected to shearing and torsional action. These assumptions are applicable for low-rise buildings, where the aspect ratio, defined as the ratio of height to width, is of order 1. Most buildings are in this category. For tall buildings and for those structures having flexible floors, one creates idealized models consisting of 3D frame structures composed of columns, beams, shear walls, and floor plates. These models generally involve a large number of variables and require computer-based analysis methods to generate solutions. The advantage of simple models is that one can reason about behavior through examination of analytical solutions. Both approaches are necessary and each has a role.

14.3.5 Matrix Formulation: Shear Beam Model

In what follows, we introduce matrix notation and express the equations defined in the previous section in a form similar to the equations for a member system that are presented in Chap. 12. We number the floor and stories consecutively, and work with the common X–Y–Z reference frame shown in Fig. 14.22. The following notation is used for floor i:

$$\begin{aligned} \underline{U}_i &= \{u_i, v_i, \omega_i\} = \text{Floor displacement vector} \\ P_i &= \left\{P_{xi}, P_{yi}, M_{zi}\right\} = \text{External load vector} \end{aligned} \tag{14.40}$$

These quantities are referred to the common global reference frame located at point O.

The inter-story displacements at the center of twist for story i are expressed as a matrix product.

$$\Delta \underline{U}_{\text{CT},i} = \underline{T}_{\text{CT},i}\{\underline{U}_i - \underline{U}_{i-1}\} \tag{14.41}$$

where $\underline{T}_{\text{CT}}$ has the following general form:

$$\underline{T}_{\text{CT},\,i} = \begin{bmatrix} 1 & 0 & -y_{\text{CT}}^{(i)} \\ 0 & 1 & x_{\text{CT}}^{(i)} \\ 0 & 0 & 1 \end{bmatrix} \tag{14.42}$$

The corresponding story resistance force matrices acting at the centers of twist are related to these inter-story displacements by

$$\begin{aligned} \underline{R}_{\text{CT},i} &= K_i \Delta \underline{U}_{\text{CT},i} \\ \underline{R}_{\text{CT},i+1} &= K_{i+1} \Delta \underline{U}_{\text{CT},i+1} \end{aligned} \tag{14.43}$$

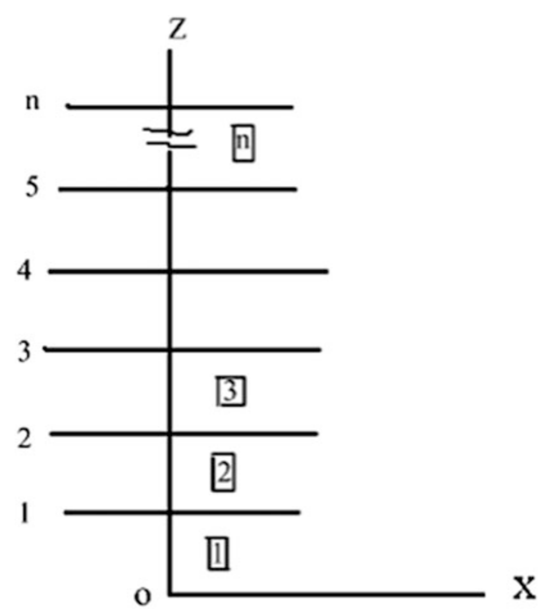


Fig. 14.22 Numbering scheme for floors and stories

where $\underline{\boldsymbol{K}}_j$ depends on the stiffness properties for story j.

$$\underline{\boldsymbol{K}}_j = \begin{bmatrix} \boldsymbol{K}_{cc}^{(j)} & \boldsymbol{K}_{cs}^{(j)} & 0 \\ \boldsymbol{K}_{cs}^{(j)} & \boldsymbol{K}_{ss}^{(j)} & 0 \\ 0 & 0 & \boldsymbol{K}_{\mathrm{C}}^{(j)} \end{bmatrix} \tag{14.44}$$

We need to transfer these forces from the center of twist to the origin of the common reference frame. This operation involves the transpose of $\underline{T}_{\mathrm{CT}}$.

$$\begin{aligned} \underline{R}_{\mathrm{o},i} &= \underline{T}_{\mathrm{CT},i}^{\mathrm{T}} \underline{R}_{\mathrm{CT},i} \\ \underline{R}_{\mathrm{o},i+1} &= \underline{T}_{\mathrm{CT},i+1}^{\mathrm{T}} \underline{R}_{\mathrm{CT},i+1} \end{aligned} \tag{14.45}$$

Using (14.41) and (14.43), (14.45) expands to

$$\begin{aligned} \underline{R}_{\mathrm{o},\, i} &= \underline{K}_{\mathrm{o},\, i}\left(\underline{U}_i - \underline{U}_{i-1}\right) \\ \underline{R}_{\mathrm{o},\, i+1} &= \underline{K}_{\mathrm{o},\, i+1}\left(\underline{U}_{i+1} - \underline{U}_i\right) \end{aligned} \tag{14.46}$$

where $\underline{K}_{\mathrm{o}}$ is the stiffness matrix referred to the common origin, O.

$$\underline{K}_{\mathrm{o},j} = \underline{T}_{\mathrm{CT},j}^{\mathrm{T}} \underline{K}_j \underline{T}_{\mathrm{CT},\,\mathrm{j}} \tag{14.47}$$

One starts with the properties of the center of twist namely, K_{xx}, K_{yy}, K_{C}, x_{CT}, y_{CT}, and then generates $\underline{K}_{\mathrm{o}}$ for each story.

We consider next the inertia forces which act at the center of mass of the floor. The displacements are related by

$$\begin{aligned} \underline{U}_{\mathrm{CM},\, i} &= T_{\mathrm{CM},\, i} U_i \\ T_{\mathrm{CM},\, i} &= \begin{bmatrix} 1 & 0 & -y_{\mathrm{CM}} \\ 0 & 1 & x_{\mathrm{CM}} \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{14.48}$$

The inertia force matrix acting at the center of mass is related to the acceleration matrix by

$$\underline{F}_{\mathrm{CM},\, i} = -\underline{m}_i \ddot{U}_{\mathrm{CM},\, i} = \underline{m}_i \underline{T}_{\mathrm{CM},\, i} \ddot{U}_i \tag{14.49}$$

where

$$\underline{m}_i = \begin{bmatrix} m^{(i)} & & \\ & m^{(i)} & \\ & & I_{\mathrm{CM}}^{(i)} \end{bmatrix} \tag{14.50}$$

Translating these forces from the center of mass to the origin leads to

$$\begin{aligned} \underline{F}_{\mathrm{o},\, i} &= m_{\mathrm{o},\, i} \ddot{\underline{U}}_i \\ \underline{m}_{\mathrm{o},\, i} &= \underline{T}_{\mathrm{CM},\, i}^{T} \underline{m}_i \underline{T}_{\mathrm{CM},i} \end{aligned} \tag{14.51}$$

We interpret $\underline{m}_{\mathrm{o},i}$ as the effective mass matrix for floor i.

Finally, summing forces for floor i, the matrix equilibrium equation referred to the common reference frame has the form.

$$\underline{P}_{o,i} = m_{o,i}\ddot{\underline{U}}_i + \underline{R}_{o,i} - \underline{R}_{o,i+1} = 0 \tag{14.52}$$

Substituting for the internal resistance matrices, the expanded form for floor i is

$$\underline{P}_{o,i} = m_{o,i}\ddot{\underline{U}}_i + \underline{K}_{o,i}(U_i - U_{i-1}) - \underline{K}_{o,i+1}(U_{i+1} - U_i) \tag{14.53}$$

We suppose there are N floors and express the complete set of N equations as a single matrix equation,

$$\underline{P} = \underline{m\ddot{U}} + \underline{KU} \tag{14.54}$$

We assemble $\underline{m}$ and $\underline{K}$ in partitioned form (N rows and N columns). The entries follow from (14.53).

$$\begin{aligned}
& i = 1, 2, \ldots, N \\
& \underline{m}_{o,i} \text{ in partitioned row } i \text{ and column } i \text{ of } \underline{m} \\
& \quad + \underline{K}_{o.i}\left\{\begin{array}{l}\text{in row } i \text{ and column } i \\ \text{in row } i-1 \text{ and column } i-1\end{array}\right\} \text{of } \underline{K} \\
& \quad - \underline{K}_{o.i}\left\{\begin{array}{l}\text{in row } i \text{ and column } i-1 \\ \text{in row } i-1 \text{ and column } i\end{array}\right\} \text{of } \underline{K} \\
& \underline{P}_{o,i} \text{ in row } i \text{ of } P
\end{aligned} \tag{14.55}$$

Note that this approach is *identical* to the procedure that we followed in Chap. 12 to assemble the system matrices for a member system. The following example illustrates the steps for a three-story structure.

Example 14.14

Given: The three-story structure shown in Fig. E14.14a. Assume the transformed mass and stiffness properties are known for each floor.

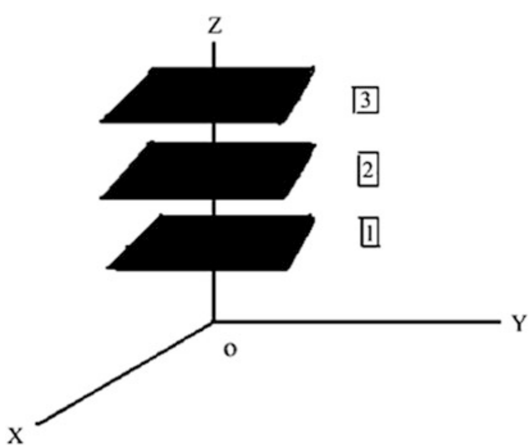


Fig. E14.14a

Determine: The non-zero entries in the system load, mass, and stiffness matrices.

Solution: $N = 3$ for this example. The partitioned form of the equations is listed below.

$$\begin{Bmatrix} \underline{P}_{o,1} \\ \underline{P}_{o,2} \\ \underline{P}_{o,3} \end{Bmatrix} = \begin{bmatrix} \underline{m}_{o,1} & & \\ & \underline{m}_{o,2} & \\ & & \underline{m}_{o,3} \end{bmatrix} \begin{Bmatrix} \underline{\ddot{U}}_1 \\ \underline{\ddot{U}}_2 \\ \underline{\ddot{U}}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} \left(\underline{K}_{o,1} + \underline{K}_{o,2}\right) & -\underline{K}_{o,2} & 0 \\ -\underline{K}_{o,2} & \left(\underline{K}_{o,2} + \underline{K}_{o,3}\right) & -\underline{K}_{o,3} \\ 0 & -\underline{K}_{o,3} & \underline{K}_{o,3} \end{bmatrix} \begin{Bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_3 \end{Bmatrix}$$

14.4 Response of Symmetrical Buildings

We consider the symmetrical structural system shown in Fig. 14.23. We locate the global reference frame on the symmetry axis. By definition, the center of mass and center of twist for all the floors are located on the Z-axis.

We suppose the external floor loading is applied in the X direction. This loading is resisted by the frames supporting the floors. Each frame displaces in the X direction and develops resistance through shearing action between the floors.

A typical frame is modeled as a set of discrete masses supported by shear springs. Figure 14.24 illustrates this idealization. The shear spring stiffness for a story in a frame is determined by summing the contribution of the columns contained in the story. Using the approximate method for estimating lateral stiffness for frames developed in Chap. 11, the equivalent shear stiffness for a story in a frame is estimated as

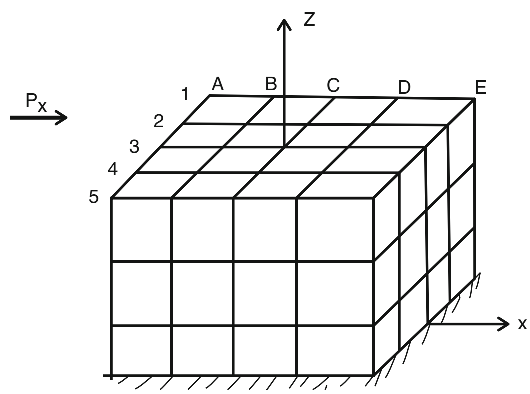
$$k_{\text{story}\,i} = \frac{12E}{h^3} \sum_{\text{inter col}} I_c \frac{1}{(1 + (r/2))} + \frac{12E}{h^3} \sum_{\text{exter col}} I_c \frac{1}{(1 + r)} \tag{14.56}$$

where r is the ratio of relative stiffness factors for the column and girder.

$$r = \frac{I_{\text{col}}/h}{I_{\text{grider}}/L}$$

We evaluate the story shear stiffness factors for each frame. When shear walls or braces are present in a story, we combine the stiffness terms corresponding to the braces with the terms due to the columns.

Fig. 14.23 Symmetrical building structure



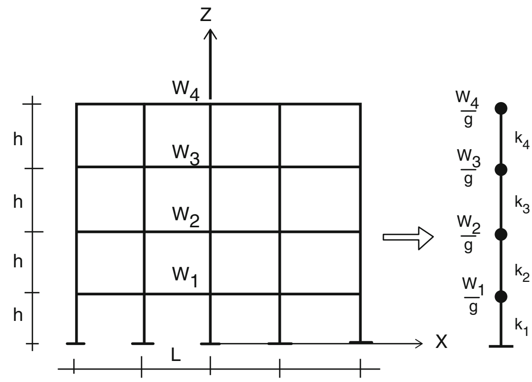


Fig. 14.24 Shear model of typical frame

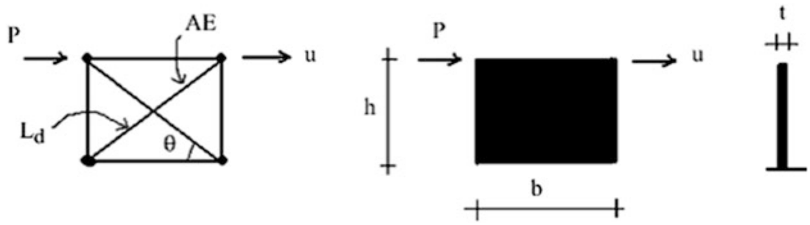


Fig. 14.25 Shear stiffness elements. (**a**) Steel brace. (**b**) Concrete shear wall

$$k\big|_{\text{story } i} = k_{\text{col}}\big|_{\text{story } i} + k_{\text{brace}}\big|_{\text{story } i} \tag{14.57}$$

The shear stiffness factors for the shear elements defined in Fig. 14.25 are

$$k_{\text{shearwall}} = \frac{h}{Gbt} \tag{14.58}$$

$$k_{\text{brace}} = \frac{2AE}{L_{\text{d}}}(\cos\theta)^2 \tag{14.59}$$

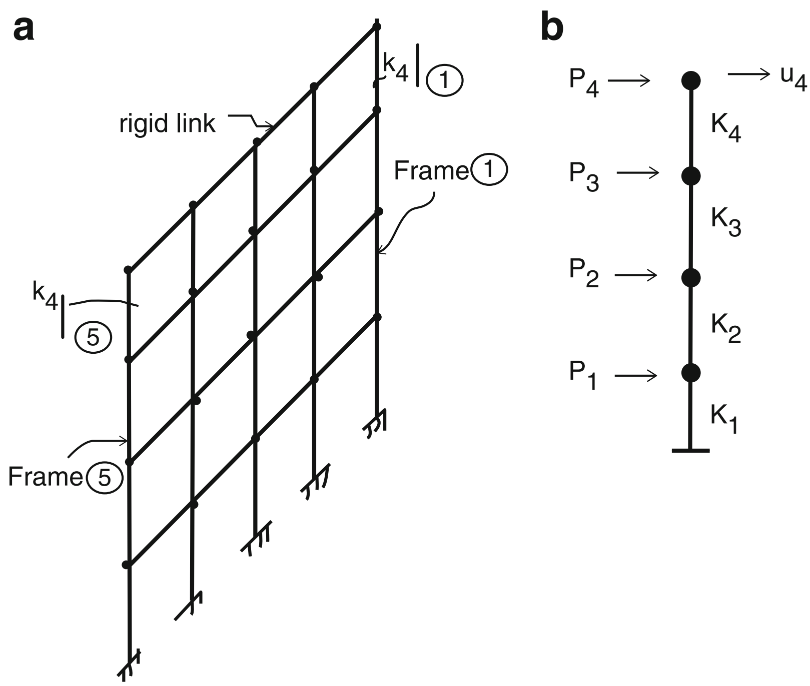
The complete building system is represented as a set of frames in parallel linked through the "rigid" floor slab. Figure 14.26 illustrates this idealization. At each story level, *all frames experience the same lateral displacement*. It follows that the story shear force in a particular frame is proportional to the ratio of the frame story shear stiffness to the global story shear stiffness which is defined as

$$\begin{aligned} K_{\text{global, floor } i} &= K_i = \sum_{\text{frames}} k_i\Big|_{\text{frame } j} \\ V_{\text{frame } j} &= \frac{k_i|_{\text{frame } j}}{K_i} V_i\big|_{\text{global}} \end{aligned} \tag{14.60}$$

Generalizing this result, we can state that the lateral global loads are distributed to the individual parallel frames in proportion to their relative stiffness.

Noting Fig. 14.26, the global shear for a story is equal to the sum of the loads acting on the floors above the particular floor. For example,

Fig. 14.26 Idealized building model. (**a**) Set of frames with rigid link. (**b**) Global loads and global story stiffnesses



$$\begin{aligned} V_1\big|_{\text{global}} &= P_1 + P_2 + P_3 + P_4 \\ V_2\big|_{\text{global}} &= P_2 + P_3 + P_4 \end{aligned} \tag{14.61}$$

One first evaluates these global shear forces and then determines the individual frame story shears with (14.60).

Suppose the ratio of story stiffness to global story stiffness is constant for all stories in frame j

$$\frac{k_i|_{\text{frame}\,j}}{K_i} = \alpha_j \tag{14.62}$$

Then, it follows that frame j carries a fraction equal to α_j of the total applied load. This result is useful since it allows one to reason in a qualitative way about how global floor loads are distributed into the frames. For example, suppose that there are n frames having equal stiffness. Then, each frame carries $(1/n)$ of the total lateral load.

Example 14.15

Given: The symmetrical rigid frame structure shown in Fig. E14.15a. Assume the frame properties are constant throughout the building height and also assume the structure is uniformly loaded. (a) The columns in frames 2 and 3 are twice as stiff as the columns in frames 1 and 4 and the floor slab is rigid. (b) Assume equal frame stiffnesses and rigid floor slab. (c) Assume equal frame stiffnesses and a flexible floor slab.

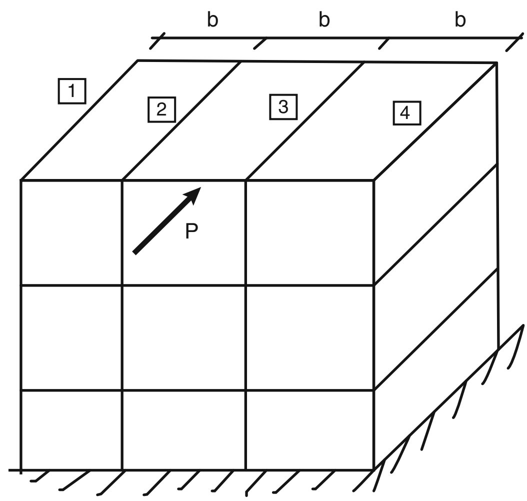


Fig. E14.15a

Determine: The distribution of the total lateral load to the individual frames.

Solution:

Part (**a**): A typical floor is shown in Fig. E14.15b. The equivalent story shear stiffness factors are defined as k^* and $2k^*$. The resultant global shear force acts at the midpoint of the side, and there is no twist since the stiffness distribution is symmetrical.

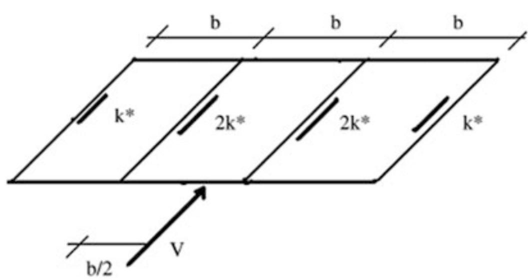


Fig. E14.15b Typical floor

The total story stiffness is

$$\sum k_j = (k^*)(1+2+2+1) = 6k^*$$

According to (14.60), the fraction of the total story shear carried by an individual frame is equal to the ratio of the frame story stiffness to the total story stiffness. Then,

For frames 1 and 4

$$V_1 = V_4 = V\frac{k^*}{6k^*} = \frac{1}{6}V$$

For frames 2 and 3

$$V_2 = V_3 = V\frac{2k^*}{6k^*} = \frac{1}{3}V$$

In this case, *the interior frames carry twice as much load as the exterior frames*.

Part (b): If the floor slab is rigid and equal frame stiffnesses are used, the frame load distribution shown in Fig. E14.15c is now applicable; the shear is assigned uniformly to the frames.

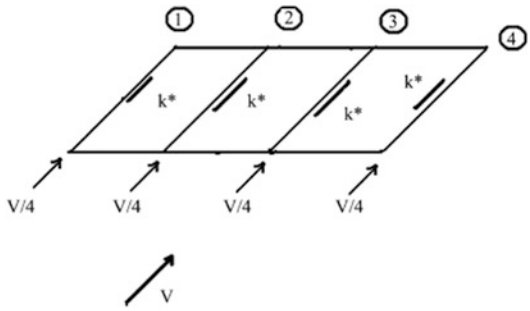


Fig. E14.15c Typical floor

Part (c): Suppose one generates an estimate for the global loading on an individual frame using the tributary areas for the frames. Consider the structure shown in Fig. E14.15a. We divide the façade area into area segments and associate these segmental areas with the frames adjacent to the areas as illustrated in Fig. E14.15d.

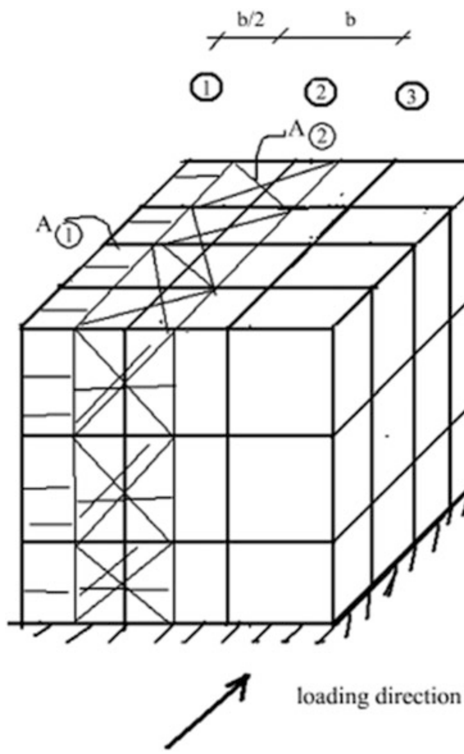


Fig. E14.15d

We note that the width for the segmental areas 1 and 4 is ½ the width for the interior tributary areas. Therefore, assuming the external loading is constant over the width, it follows that the magnitude of the loads for frames 1 and 4 is ½ the load for the interior frames. This breakdown is shown in Fig. E14.15e. This distribution is based on the assumption that the frames act independently, i.e., the floor slabs are flexible.

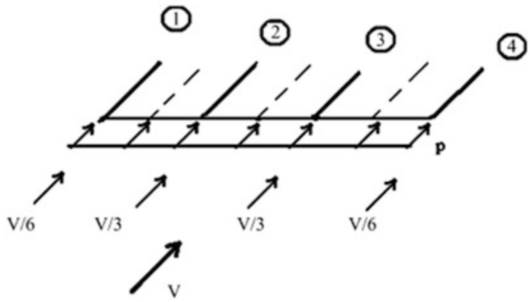


Fig. E14.15e

Example 14.16

Given: The five-story symmetrical rigid frame building shown in Figs. E14.16a and E14.16b. Assume the building can be subjected to an earthquake in either the North–South or East–West directions. Take the spectral acceleration as $S_a = 0.15g$. Consider all the beams to be the same size and all the columns to be the same size. Assume $I_B = 4I_C$.

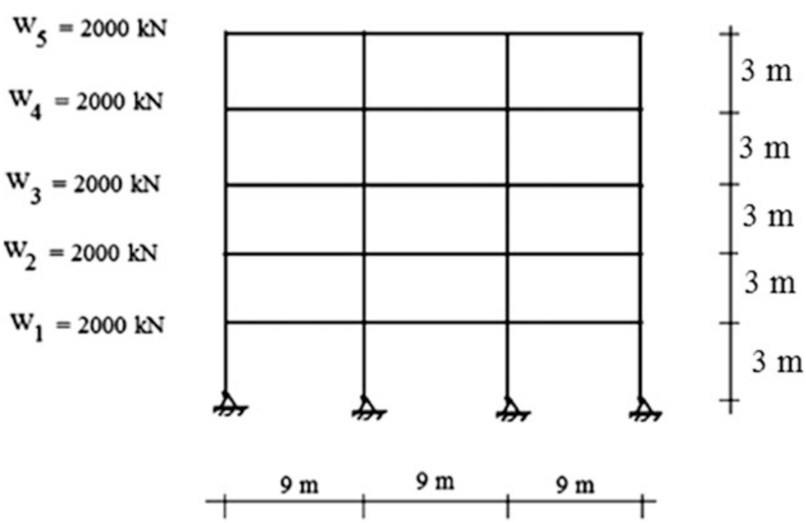


Fig. E14.16a Elevation

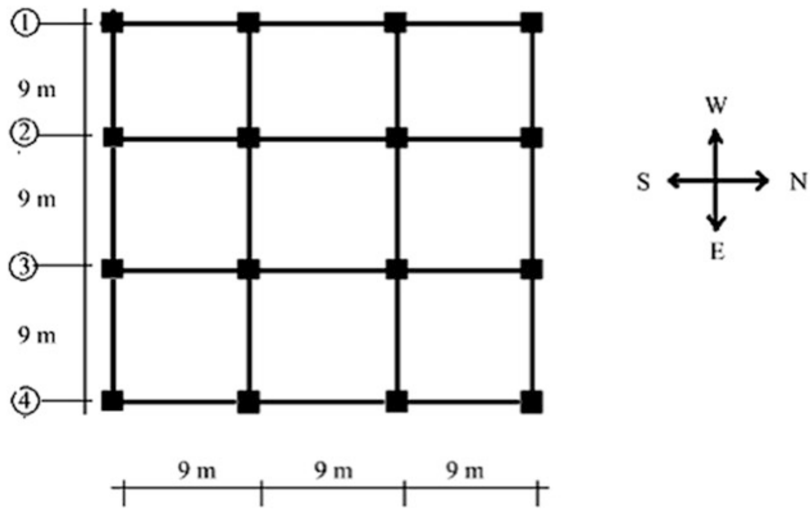


Fig. E14.16b Typical floor plan

Determine: The maximum moments in the columns (a) for rigid floors and (b) for flexible floors.

Solution: We use (14.10). The base shear is given by

$$V|_{\text{base}} = \frac{\left(\sum_{i=1}^{5} Z_i W\right)^2}{\sum_{i=1}^{5} W_i (Z_i)^2} \frac{S_a}{g} \frac{(2000(3) + 2000(6) + 2000(9) + 2000(12) + 2000(15))^2}{2000(3)^2 + 2000(6)^2 + 2000(9)^2 + 2000(12)^2 + 2000(15)^2}(0.15)$$

$$= 1227\,\text{kN}$$

Then, applying (14.11), we obtain the individual floor loads (Fig. E14.16c).

$$P|_{\text{floor } i} = \left(\frac{W_i Z_i}{\sum W_i Z_i}\right) V|_{\text{base}}$$

$$\sum_{i=1}^{5} W_i Z_i = 2000(3) + 2000(6) + 2000(9) + 2000(12) + 2000(15)$$

$$= 90,000\,\text{kN/m}$$

$$P|_{\text{floor 1}} = \frac{3(2000)}{90,000}(1227) = 81.8\,\text{kN}$$

$$P|_{\text{floor 2}} = \frac{6(2000)}{90,000}(1227) = 163.6\,\text{kN}$$

$$P|_{\text{floor 3}} = \frac{9(2000)}{90,000}(1227) = 245.4\,\text{kN}$$

$$P|_{\text{floor 4}} = \frac{12(2000)}{90,000}(1227) = 327.2\,\text{kN}$$

$$P|_{\text{floor 5}} = \frac{15(2000)}{90,000}(1227) = 409\,\text{kN}$$

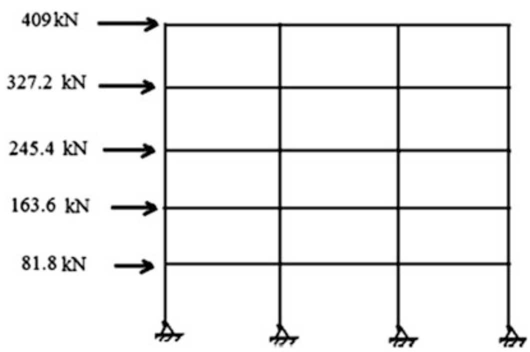


Fig. E14.16c Earthquake floor loads

It remains to distribute the floor loads to the frames. Since the structure is symmetrical, we need to consider only one direction, say the N–S direction (Fig. E14.16d).

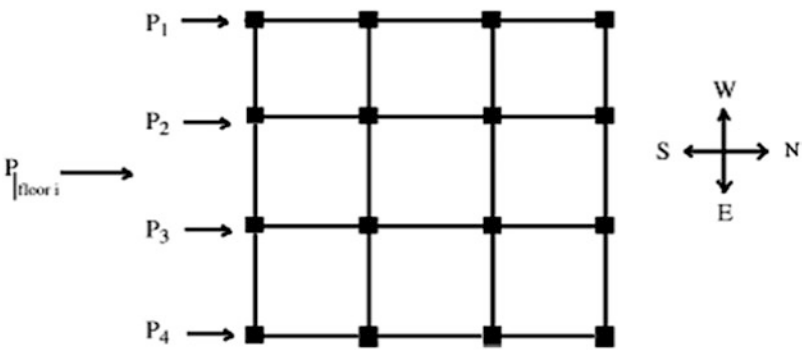


Fig. E14.16d Floor load distribution

Part a:
When the floor slab is rigid, and the frame stiffnesses are equal, the floor load is distributed uniformly to the frames (Fig. E14.16e).

$$P_1 = P_2 = P_3 = P_4 = \frac{1}{4} P_{|\text{floor } i}$$

Therefore,

$$P_1 = P_2 = P_3 = P_4 = \begin{cases} 20.5\,\text{kN} \\ 40.9\,\text{kN} \\ 61.4\,\text{kN} \\ 81.8\,\text{kN} \\ 102.3\,\text{kN} \end{cases}$$

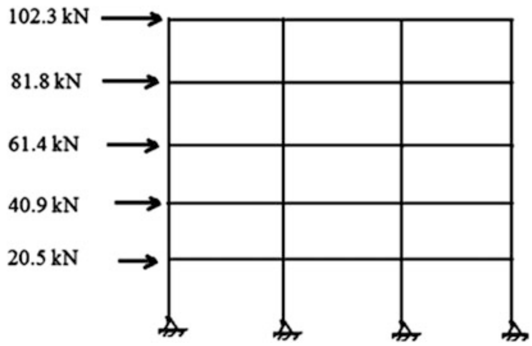


Fig. E14.16e Frame load—rigid floors

Part b:
When the floor slab is flexible, the loads are proportioned to their tributary floor areas. Then, it follows that (Fig. E14.16f)

$$P_2 = P_3 = \frac{1}{3} P_{|\text{floor } i} = \begin{cases} 27.3\,\text{kN} \\ 54.5\,\text{kN} \\ 81.8\,\text{kN} \\ 109\,\text{kN} \\ 136.3\,\text{kN} \end{cases}$$

$$P_1 = P_4 = \frac{1}{6} P_{|\text{floor } i} = \begin{cases} 13.6\,\text{kN} \\ 27.3\,\text{kN} \\ 40.9\,\text{kN} \\ 54.5\,\text{kN} \\ 68.2\,\text{kN} \end{cases}$$

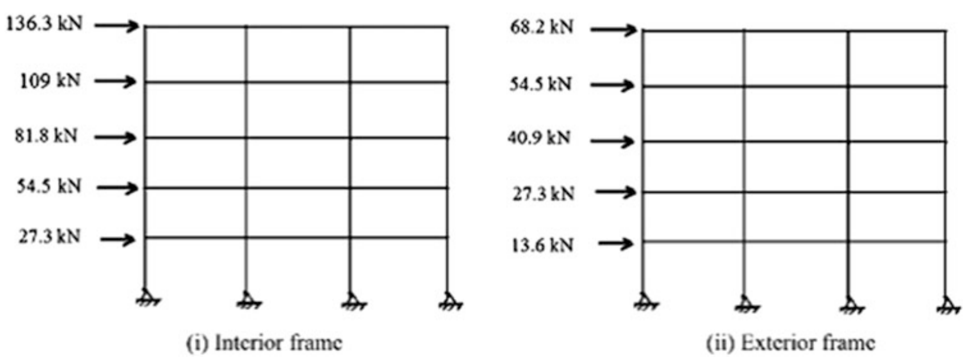


Fig. E14.16f Frame loads—flexible floors

Part c:
We apply the stiffness method described in Chap. 11 to estimate the maximum moments in the exterior and interior columns.

$$I_{C_{ext}} = I_C \text{ and } I_B = 4I_C \Rightarrow \frac{I_{C_{ext}}/h}{I_B/L} = \frac{I_C/h}{I_B/L} = \frac{(I_C/4)}{(4I_C/9)} = 0.5625$$

$$k_E = \frac{3EI_{CE}}{h^3}\left\{\frac{1}{1+\frac{1}{2}\left(\frac{I_{CE}/h}{I_b/L}\right)}\right\} = \frac{2.34EI_C}{h^3} \Rightarrow \frac{k_E}{k_I} = 0.89$$

$$k_I = \frac{3EI_{CI}}{h^3}\left\{\frac{1}{1+\frac{1}{4}\left(\frac{I_{CI}/h}{I_b/L}\right)}\right\} = \frac{2.63EI_C}{h^3}$$

Noting that

$$\frac{V_E}{V_I} = \frac{k_E}{k_I}$$

we express the total shear as

$$V_{Total} = 2V_E + 2V_I = 2\left(\frac{k_E}{k_I} + 1\right)V_I \Rightarrow V_I = 0.265V_{Total}$$

The distributions are shown in Figs. E14.16g and E14.16h.

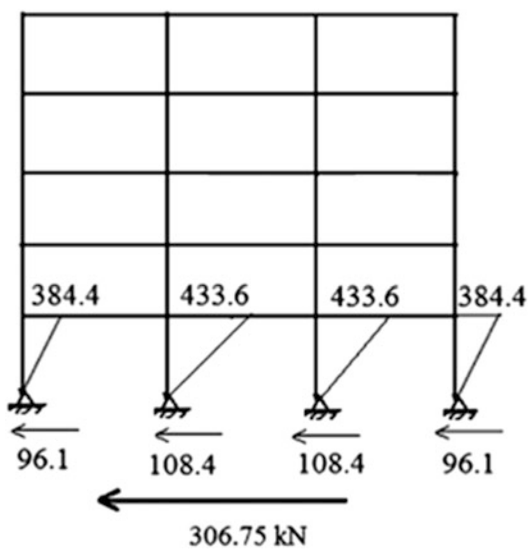


Fig. E14.16g Maximum column moments—rigid floors

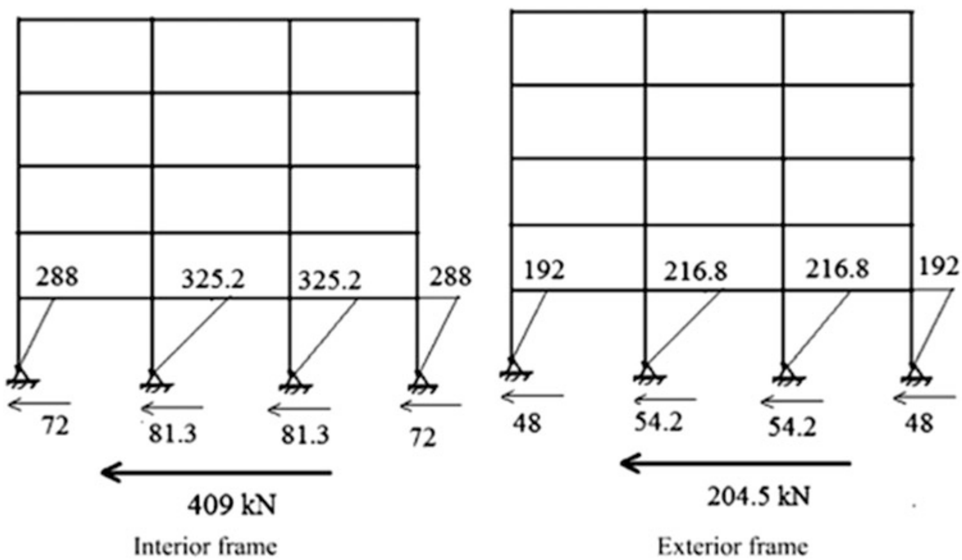


Fig. E14.16h Maximum column moments—flexible floors

Example 14.17

Given: The one-story frame shown in Figs. E14.17a, E14.17b, E14.17c. Assume the cross sections are equal.

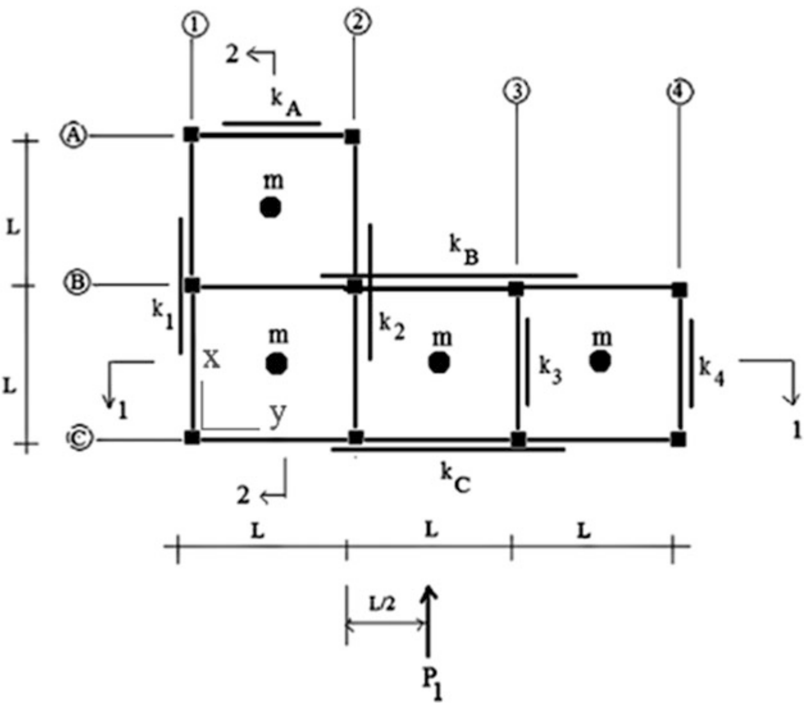


Fig. E14.17a Plan

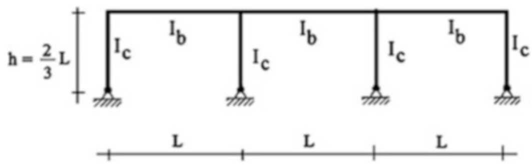


Fig. E14.17b Elevation—section 1-1

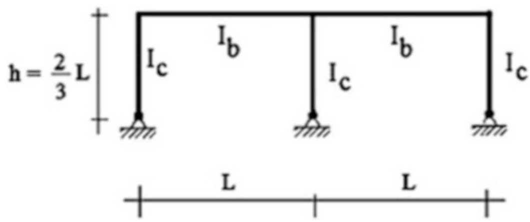


Fig. E14.17c Elevation—section 2-2

Determine:

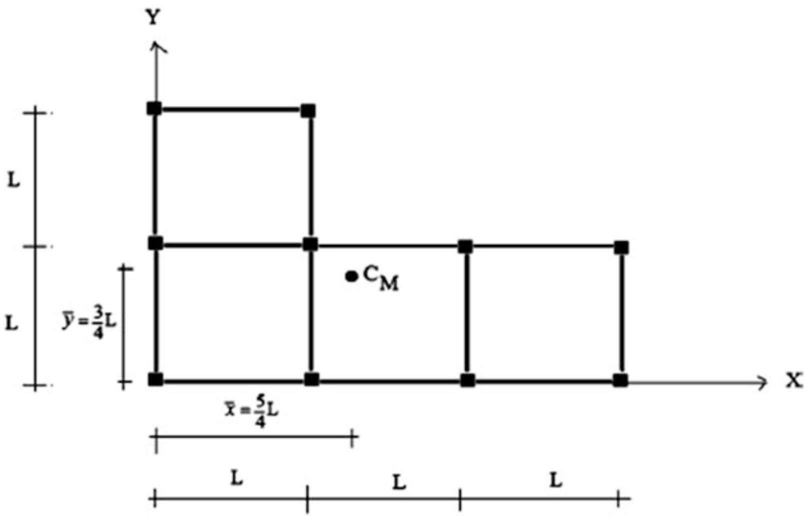
(a) The center of mass.
(b) The center of twist. Take $I_b = 2I_c$.
(c) The revised stiffness required on lines B-B and 2-2 so that the center of stiffness coincides with the center of mass.
(d) The translation and rotation of the center of twist for the structure determined in part (c) due to load P_1.

Solution:

(a) The center of mass

$$\bar{x} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(0.5L)2m + (1.5L)m + (2.5L)m}{4m} = \frac{5}{4}L$$

$$\bar{y} = \frac{\sum y_i m_i}{\sum m_i} = \frac{(0.5L)3m + (1.5L)m}{4m} = \frac{3}{4}L$$



(b) The center of twist

$$I_{\mathrm{CE}} = I_{\mathrm{CI}} = I_{\mathrm{c}}$$
$$I_{\mathrm{b}} = 2I_{\mathrm{c}}$$
$$\left(\frac{I_{\mathrm{CI}}/h}{I_{\mathrm{b}}/L}\right) = \left(\frac{I_{\mathrm{CE}}/h}{I_{\mathrm{b}}/L}\right) = \left(\frac{3I_{\mathrm{c}}/2L}{2I_{\mathrm{c}}/L}\right) = 0.75$$

Using the shear stiffness equations (11.11) and (11.12), the relevant stiffness factors are

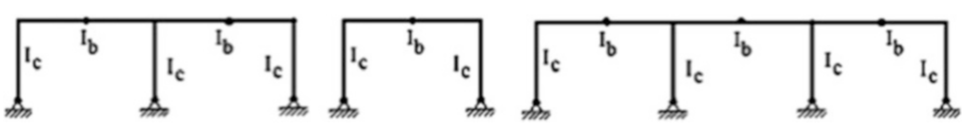
$$f_{\mathrm{BE}} = \left\{\frac{1}{1+\frac{1}{2}\left(\frac{I_{\mathrm{CE}}/h}{I_{\mathrm{b}}/L}\right)}\right\} = 0.73$$

$$f_{\mathrm{BI}} = \left\{\frac{1}{1+\frac{1}{4}\left(\frac{I_{\mathrm{CE}}/h}{I_{\mathrm{b}}/L}\right)}\right\} = 0.84$$

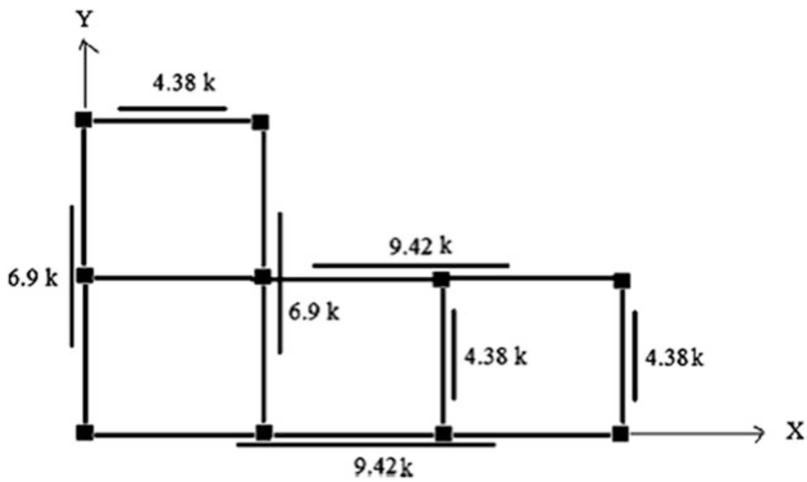
Let $(EI_c/h^3) = k$. Then,

$$k_{\mathrm{BE}} = \frac{3EI_{\mathrm{CE}}}{h^3}\left\{\frac{1}{1+\frac{1}{2}\left(\frac{I_{\mathrm{CE}}/h}{I_{\mathrm{b}}/L}\right)}\right\} = \frac{3EI_{\mathrm{CE}}}{h^3} f_{\mathrm{BE}} = 2.19k$$

$$k_{\mathrm{BI}} = \frac{3EI_{\mathrm{CI}}}{h^3}\left\{\frac{1}{1+\frac{1}{4}\left(\frac{I_{\mathrm{CI}}/h}{I_{\mathrm{b}}/L}\right)}\right\} = \frac{3EI_{\mathrm{CI}}}{h^3} f_{\mathrm{BI}} = 2.52k$$



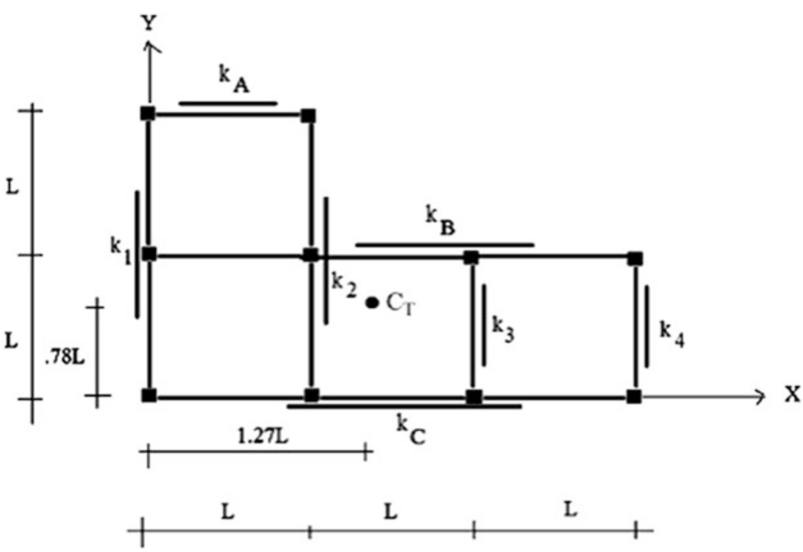
$$k_1 = k_2 = 2k_{\mathrm{BE}} + k_{\mathrm{BI}} = 6.9k$$
$$k_3 = k_4 = k_{\mathrm{A}} = 2k_{\mathrm{BE}} = 4.38k$$
$$k_{\mathrm{B}} = k_{\mathrm{C}} = 2k_{\mathrm{BE}} + 2k_{\mathrm{BI}} = 9.42k$$



Finally, one obtains the coordinates

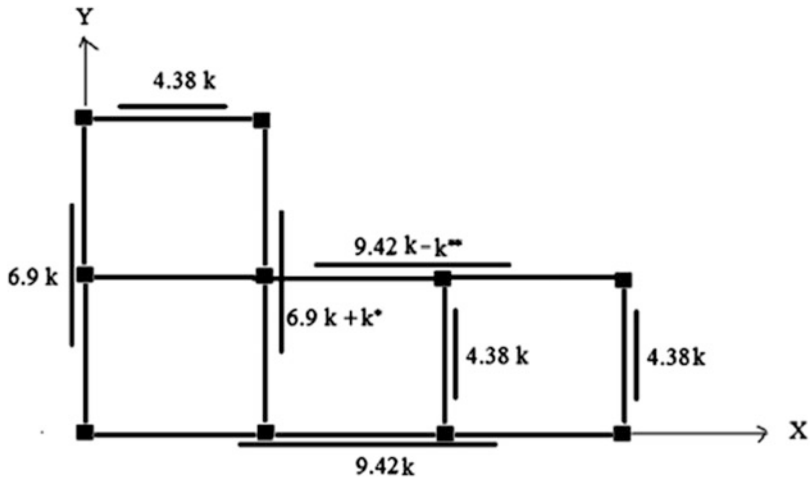
$$x_{\mathrm{CT}} = \frac{\sum x_j k_{yj}}{\sum k_{yj}} = \frac{6.9k(L) + 4.38k(2L) + 4.38k(3L)}{2(6.9k + 4.38k)} = 1.27L$$

$$y_{\mathrm{CT}} = \frac{\sum y_i k_{xi}}{\sum k_{xi}} = \frac{9.42k(L) + 4.38k(2L)}{2(9.42k) + 4.38k} = 0.78L$$

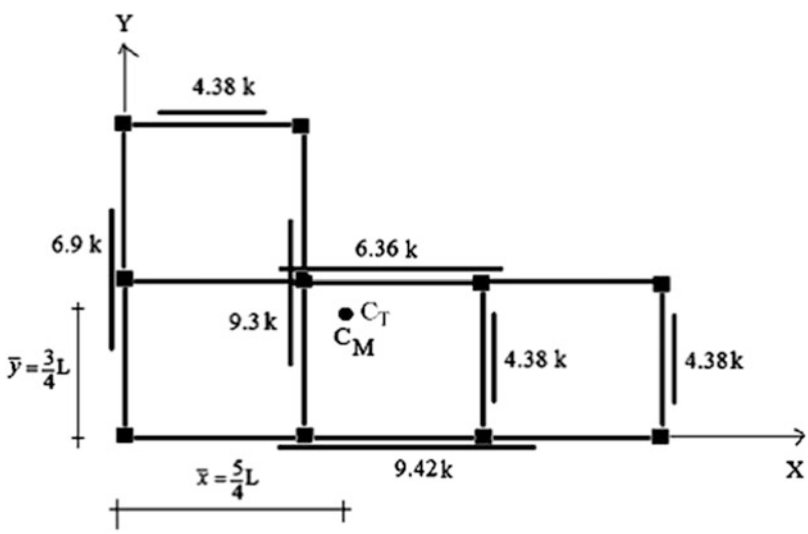


(c) The revised stiffness required on lines B-B and 2-2

$$x_{CT} = \bar{x} = \frac{5}{4}L = \frac{(6.9k + k^*)(L) + 4.38k(2L) + 4.38k(3L)}{6.9k + (6.9k + k^*) + 2(4.38k)}$$
$$y_{CT} = \bar{y} = \frac{3}{4}L = \frac{(9.42k - k^{**})(L) + 4.38k(2L)}{9.42k + (9.42k - k^{**}) + 4.38k}$$
$$\therefore k^* = 2.4k \quad k^{**} = 3.06k$$



(d) The translation and rotation of the center of twist



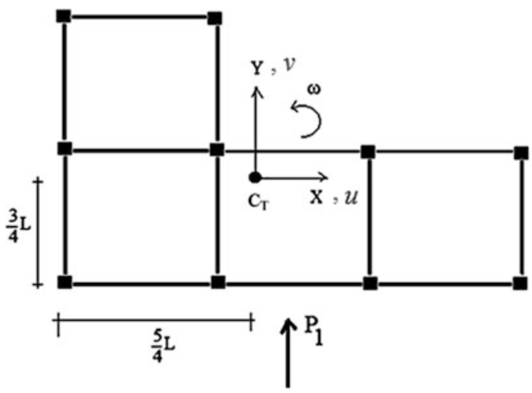
Noting the result for part (c) and (14.20) specialized for the center of twist, the displacements of the center of twist due to P_1 are

$$K_{yy} = \sum k_{yj} = k\{6.9 + 9.3 + 2(4.38)\} = 24.96k$$

$$K_{\text{o}} = \sum y_j^2 k_{xi} + \sum x_j^2 k_{yj} = \left\{4.38k(1.25L)^2 + 9.42k(0.75L)^2 + 6.36k(0.25L)^2\right\}$$
$$+ \left\{6.9k(1.25L)^2 + 9.3k(0.25L)^2 + 4.38k(0.75L)^2 + 4.38k(1.75L)^2\right\} = 39.78kL^2$$

$$M_{\text{o}} = P_1\left(\frac{L}{4}\right)$$

$$u = 0$$
$$v = \frac{P_y}{K_{yy}} = \frac{P_1}{24.9k}$$
$$\omega = \frac{M_0}{K_{\text{o}}} = \frac{P_1(L/4)}{39.78kL^2} = 0.00628\frac{P_1}{kL}$$



14.5 Summary

14.5.1 Objectives

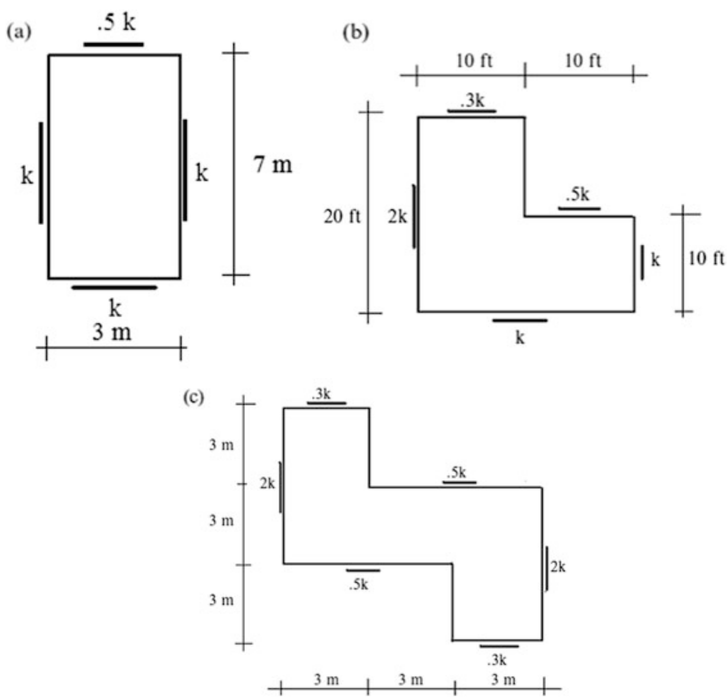
- To describe various idealized models that are used to represent building structures as an assemblage of plane frames and rigid floor slabs.e
- To introduce procedures for generaeting wind and earthquake loads for building structures.
- To introduce the concepts of center of mass and center of stiffness and apply these concepts to typical building structures.
- To formulate the governing equations for a building idealized as a three-dimensional shear beam.
- To represent these equations using matrix notation.
- To specialize the formulation for symmetrical buildings.

14.5.2 Key Facts and Concepts

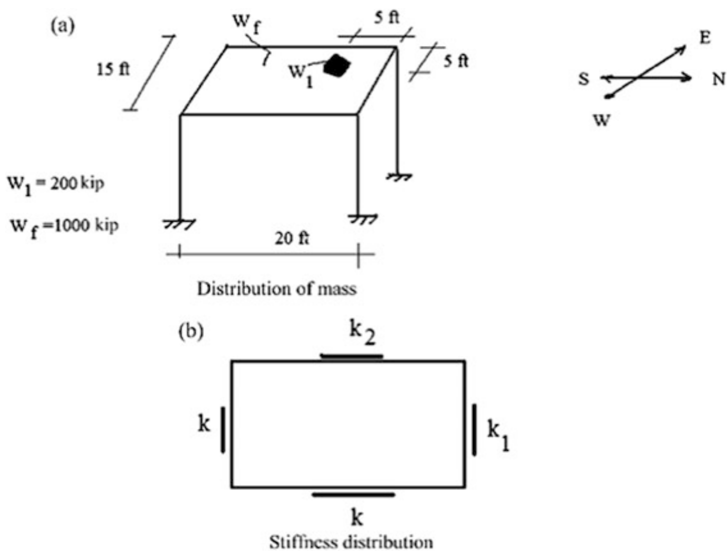
- The normal pressure due to wind varies as a power law ($p \sim z^{1/7}$) in the vertical direction.
- Seismic excitation is represented by a set of inertia forces acting at the floor levels. These forces are defined in terms of certain parameters that depend on the site and are specified by design codes. The vertical force distribution depends on the floor masses and increases with distance from the base.
- The center of mass is a property of a floor, i.e., it depends on the mass distribution within the floor. It is important since the resultant of the inertia forces passes through the center of mass.
- The center of stiffness is a property of the lateral stiffness distribution in a story. Twisting of the floor slab will occur when the resultant force acting on a story does not pass through the center of stiffness. Ideally one positions the center of stiffness to coincide with the center of mass if dynamic loading is one of the design loading conditions.
- Given a set of parallel frames connected by a rigid diaphragm and subjected to a lateral load applied at the center of twist, the load carried by an individual frame is proportional to the relative stiffness of the frame.

14.6 Problems

Problem 14.1 Consider the plan view of one-story rigid frames shown below. Determine the center of twist corresponding to the brace stiffness patterns shown.



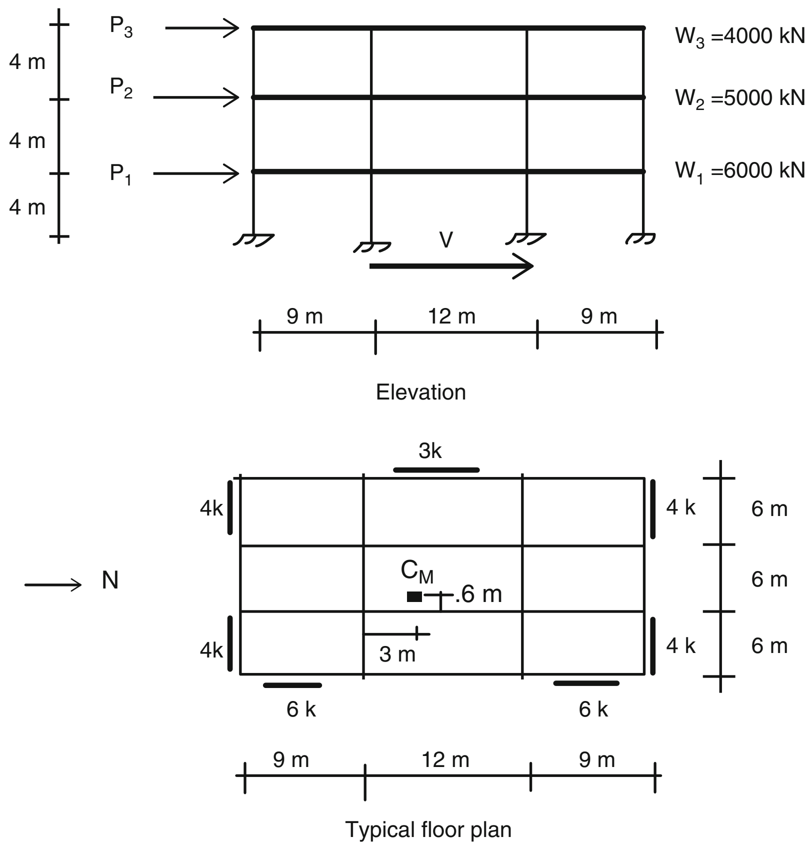
Problem 14.2



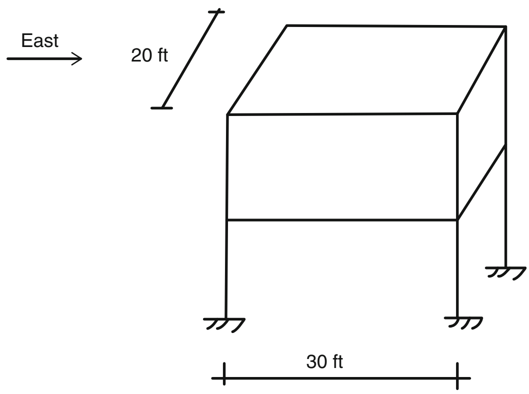
The one-story frame shown has an unsymmetrical mass distribution.

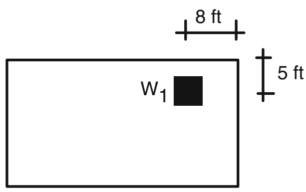
(a) Determine the center of mass.
(b) Determine the stiffness parameters k_1 and k_2 such that the center of stiffness coincides with the center of mass.
(c) Determine the earthquake floor loads corresponding to $S_a = 0.3g$. Consider both direction, i.e., N–S and E–W.

Problem 14.3 For the rigid frame shown below, determine (a) the center of twist (C_T) and (b) the seismic floor loads applied at the center of mass (C_M) for an N–S earthquake with $S_a = 0.3g$. Assume properties are equal for each floor.

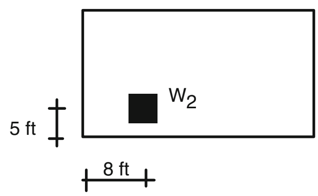


Problem 14.4 Consider the two-story rigid frame defined below. Assume the weight of the floor slabs is equal to w_{floor}. Concentrated masses are located on each floor as indicated.





Plan-Floor 1

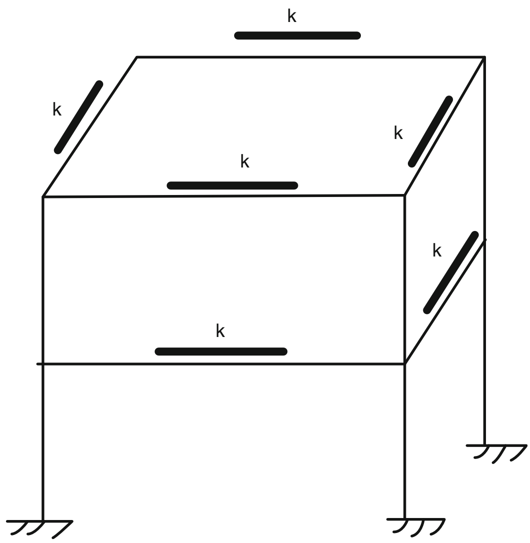


Plan-Floor 2

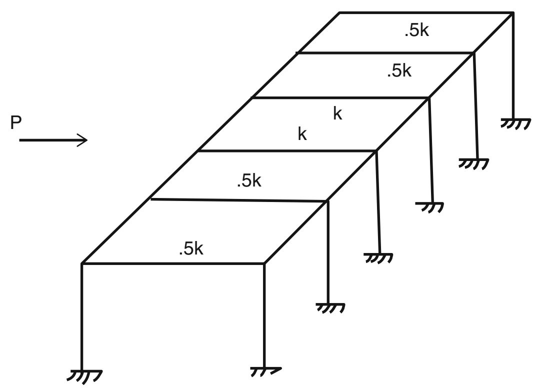
(a) Determine the position of the center of mass for each floor.

(b) Assume the structure is subjected to an earthquake acting in the east direction. Determine the earthquake forces for the individual floors. Assume $w_{\text{floor}} = 1000$ kip, $w_1 = w_2 = 1000$ kip, and $S_a = 0.3g$.

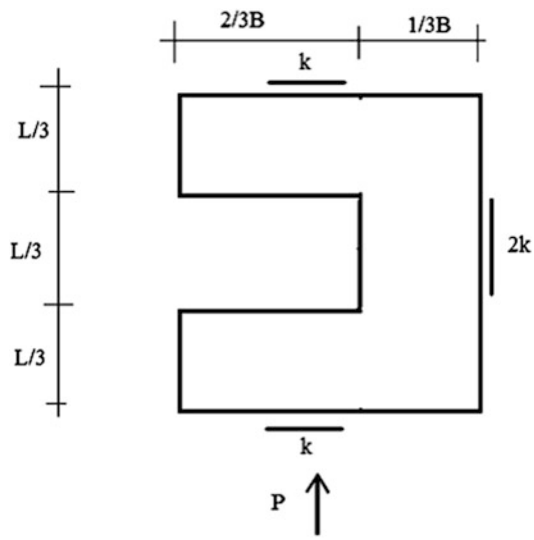
(c) Suppose the story stiffness distribution shown below is used. Describe qualitatively how the structure will displace when subjected to an earthquake. Consider the stiffness distribution to be the same for each floor.



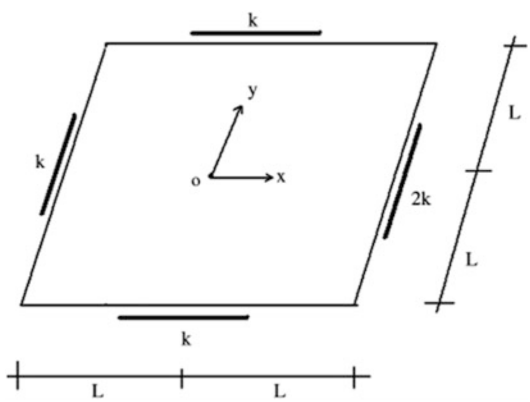
Problem 14.5 Consider the single-story multi-frame structure shown below. Determine the lateral forcef in the frames due to a global load P. Consider both wind and earthquake loading. Assume the slab is rigid.



Problem 14.6 Consider the stiffness distribution for the one-story rigid frame shown below. Determine the displaced configuration under the action of the loading shown. Assume the slab is rigid.

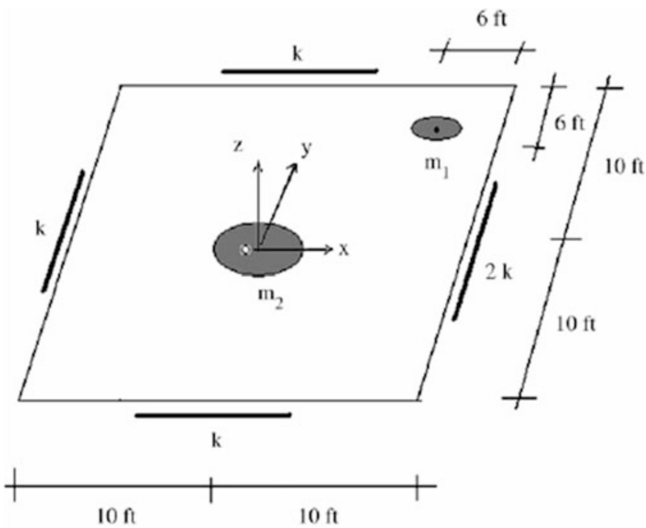


Problem 14.7



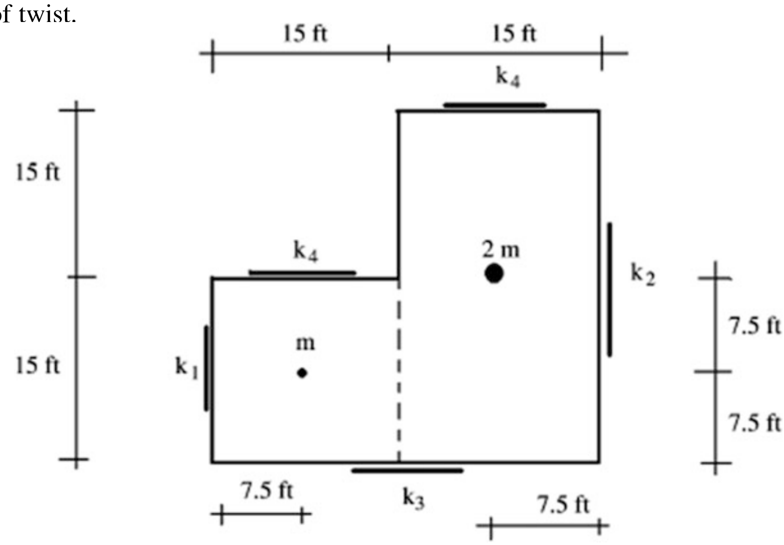
(a) Determine the center of twist.
(b) Using (14.47) determine $\underline{K}_0$.

Problem 14.8 Consider the plan view of a one-story frame shown below. Using the matrix formulation presented in Sect. 14.3.5 generate the equations of motion for the story. Take $m_1 = 1000$ Ib, $m_2 = 500$ Ib, and $k = 10$ kip/in.

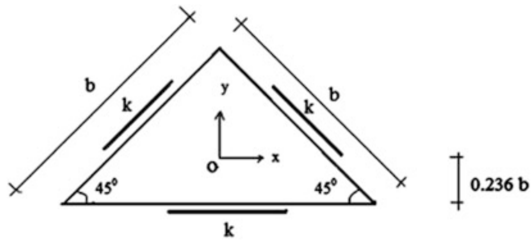


Problem 14.9 Consider the one-story plan view shown below.

(a) Locate the center of mass.
(b) Locate the center of twist. Take $k_1 = k_2 = k_3 = k_4 = k$.
(c) Take $k_1 = k_3 = 10$. Suggest values for k_2 and k_4 such that the center of mass coincides with the center of twist.

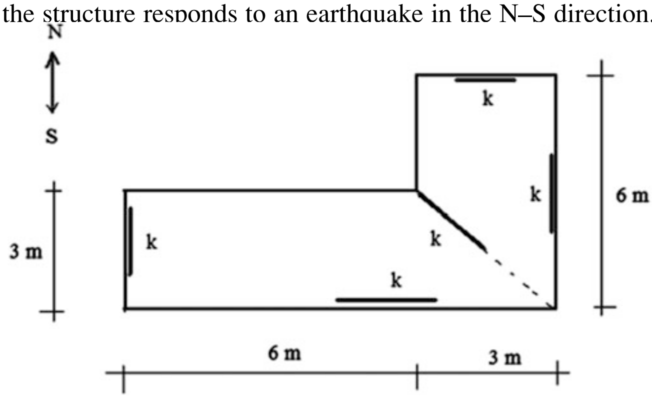


Problem 14.10 Consider the floor plan shown below. Assume the mass is uniformly distributed over the floor area. Establish the equations of motion referred to point O.



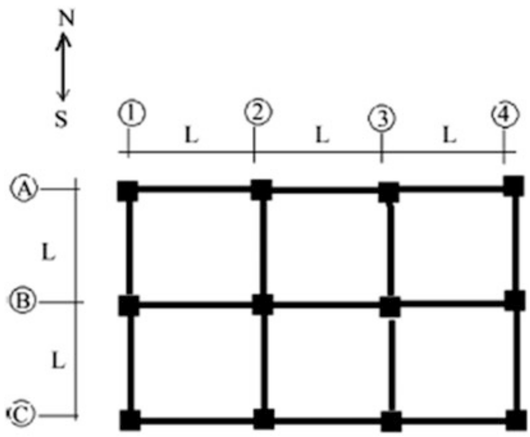
Problem 14.11 Consider the roof plan for a one-story structure shown below. Assume the shear walls have equal stiffness and the roof dead load is uniform.

(a) Determine the center of mass and the center of stiffness.
(b) Describe how the structure responds to an earthquake in the N–S direction.

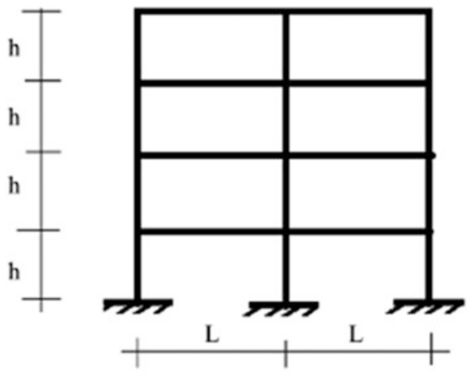


Problem 14.12 The framing shown below has identical rigid frames along column lines 1, 2, 3, and 4, and cross-bracing along lines A, B, and C. Consider all the beams and all the columns to be the same size. Assume $I_b = 3I_c$ and $L = 2h$.

(a) Assuming the roof/floor slab are rigid with respect to the rigid frames, what part of the total seismic load due to a N–S earthquake is carried by the frame 4?
(b) Repeat part (a) considering the roof/floor slab to be flexible.



Plan



Elevation – N-S

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