7 Shallow Foundations

Abstract

Civil structures such as bridges and buildings are placed on the ground. The particular segment of the structure which interfaces with the ground is called the foundation. In this chapter, we focus on a particular type of foundation called a shallow foundation. Shallow foundations are composed of footings which are plate-type elements placed on the ground. Their function is to transmit the loads in the columns and walls to the ground. In this chapter, we describe the various types of shallow footings and identify the conditions under which each type is deployed. Then, we develop an analytical procedure for establishing the soil pressure distribution under a footing due to an arbitrary column loading. Given the soil pressure distribution, one can generate the shear and moment distribution in the footing and establish the peak values required for design. Lastly, we describe how to determine these design values and also present various strategies for dimensioning shallow footings.

7.1 Introduction

7.1.1 Types of Foundations

Civil structures are viewed as having two parts. That part of the structure which is above ground is called the superstructure; the remaining part in contact with the ground is referred to as either the substructure or the foundation. Up to this point, we have focused on the superstructure. Structural Engineers are responsible for the foundation design as well as the superstructure design. They are aided by Geotechnical Engineers who provide information on the soil properties such as the allowable soil bearing pressure at the site.

Figure 7.1 illustrates the different types of foundations. Shallow foundations are located near ground level. The structural loads are transferred directly to the soil through plate-type elements placed under the columns. These plate elements are called footings. This scheme is feasible only when the soil strength is adequate to resist the applied loading. If the soil near the surface is weak, it is

© Springer International Publishing Switzerland 2016

J.J. Connor, S. Faraji, *Fundamentals of Structural Engineering*,
DOI 10.1007/978-3-319-24331-3_7

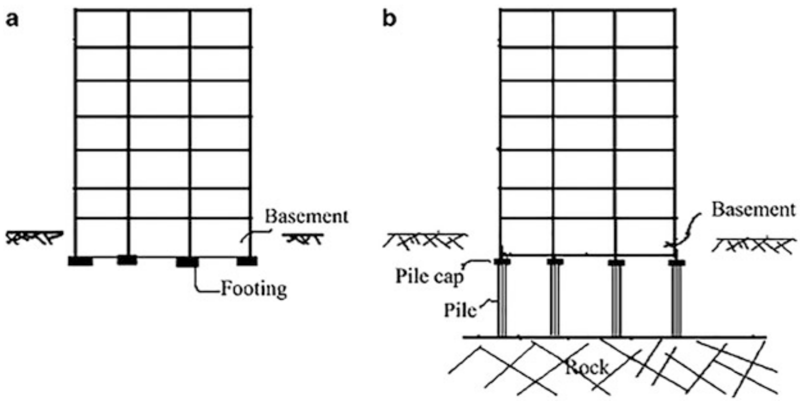


Fig. 7.1 Types of foundations. (**a**) Shallow foundation. (**b**) Deep foundation

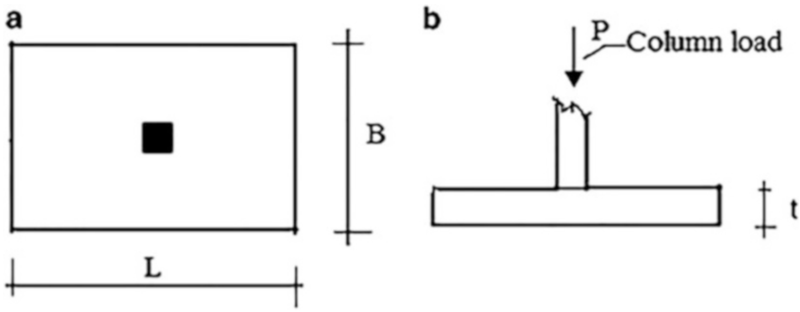


Fig. 7.2 Single footing—axial loading. (**a**) Plan. (**b**) Elevation

necessary to transfer the loads to a deeper soil layer having adequate strength. Piles or caissons are typically used to transmit loads through weak soil media. Basements which serve as underground parking facilities may also be incorporated in foundations.

7.1.2 Types of Shallow Foundations

A spread footing is a reinforced concrete plate-type structural component that rests directly on the ground and supports one or more columns or walls. Different geometrical arrangements of footings are used, depending on the column spacing and soil strength. The simplest scheme is a single footing per column, shown in Fig. 7.2. One usually works with a square area. However, there sometimes are constraints such as proximity to a boundary line which necessitate shifting to a rectangular geometry. We describe later a procedure for determining the "dimensions" of the footing given certain geometric constraints. In what follows, we consider the column load to be an axial force. Later, we extend the analysis to deal with both axial force and bending moment.

When adjacent columns are located too close to each other such that their footings would overlap, or when one of the adjacent columns is located close to a property line, the adjacent footings are combined into a single "mega" footing which is designed to support the multiple column loads. Figure 7.3 illustrates this footing layout which is called a "*combined footing*."

A different strategy is employed when the spacing between columns is large and one of the columns is located too close to a property line to support the entire column load with a single footing. It is necessary to shift some of the column loads over to an adjacent footing by connecting the footings with a strap beam. This scheme is called a "*strap footing*" (see Fig. 7.4).

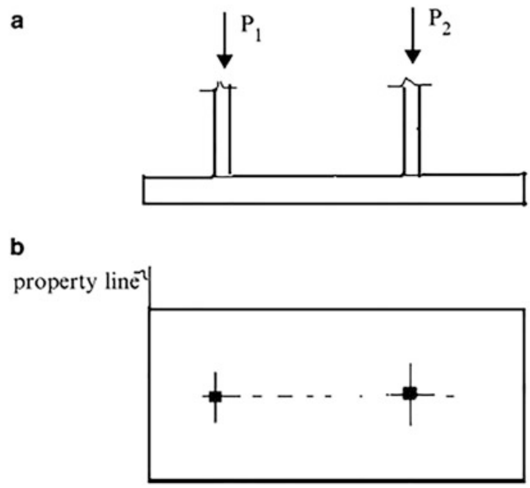


Fig. 7.3 Combined footing layout. (**a**) Elevation. (**b**) Plan view

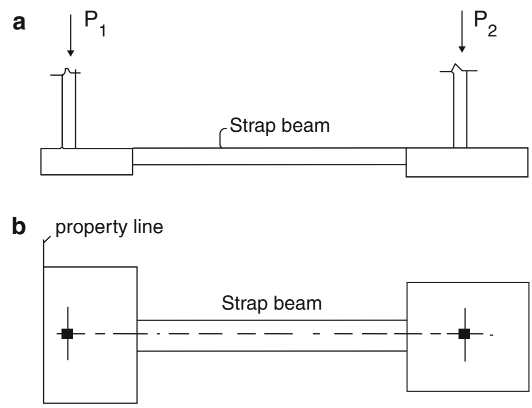


Fig. 7.4 Strap footing layout. (**a**) Elevation. (**b**) Plan view

7.1.3 Soil Pressure Distribution

A vertical loading applied to the footing is resisted by soil pressure acting on the lower surface of the footing. The distribution of pressure depends on the type of soil at the site. Typical distributions for sand and clay type soils are shown in Fig. 7.5. In practice, we approximate the actual pressure distribution due to a concentric load with an "*average uniform*" distribution.

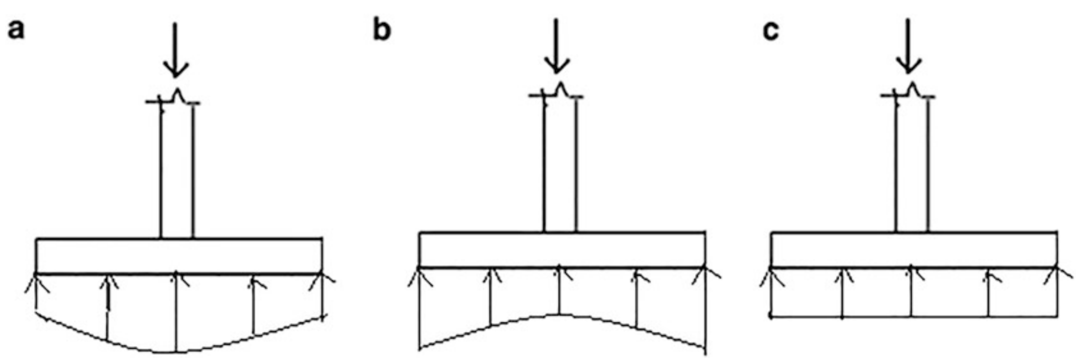


Fig. 7.5 Soil pressure distributions—concentric load. (**a**) Sandy soil. (**b**) Clayey soil. (**c**) Average soil pressure

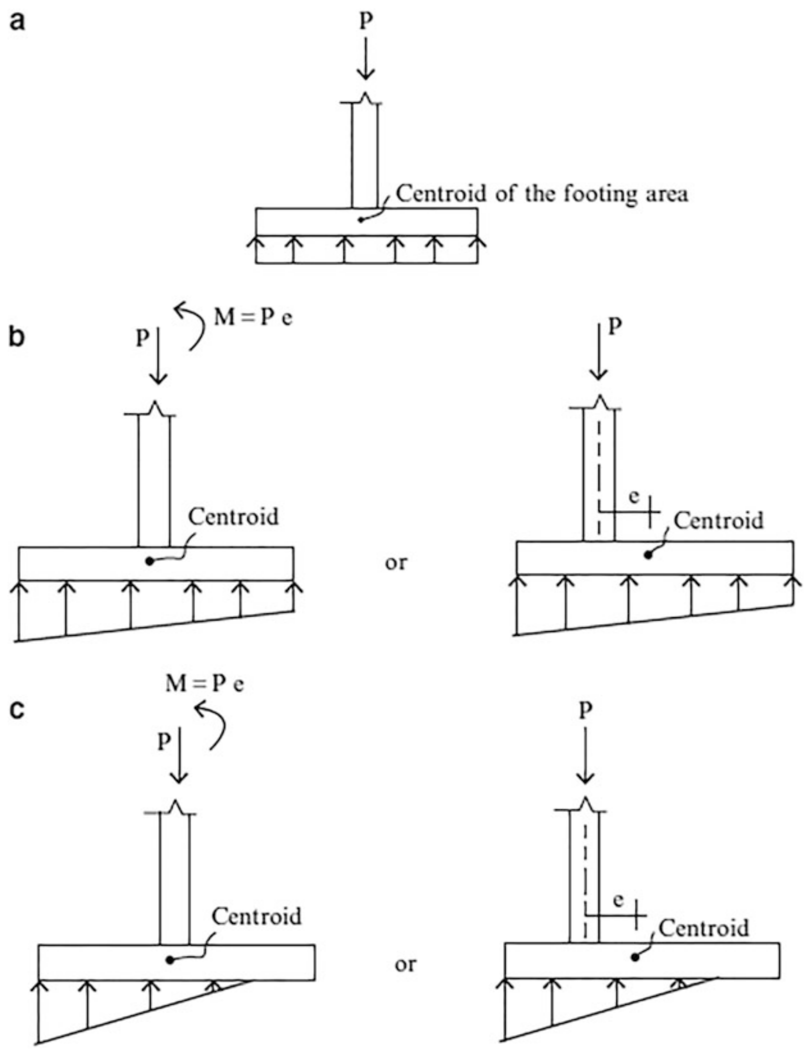


Fig. 7.6 Idealized pressure distributions. (**a**) Uniform. (**b**) Trapezoidal. (**c**) Triangular

Depending on the column loading and the location of the column with respect to the centroid of the footing area, one of the distributions shown in Fig. 7.6 is normally assumed in order to establish the dimensions of the footing. A uniform distribution is the most desirable distribution. Since soil cannot resist tensile stress, one wants to avoid the case illustrated in Fig. 7.6c. We will describe a strategy for selecting the footing dimensions so as to avoid this situation in the following section.

The allowable pressure varies with the type of soil. Soil is a natural material in contrast to steel, which is manufactured with close quality control. Consequently, there is considerable variability in soil properties. Typical allowable soil pressures for various types of soils are listed in Table 7.1. These values are useful for estimating initial footing dimensions.

Table 7.1 Allowable soil pressures—Reference Terzaghi and Peck [1]

Soil type	Allowable bearing pressure [kip/ft^2 (kN/m^2)]
Compact coarse sand	8 (383)
Hard clay	8 (383)
Medium stiff clay	6 (287)
Compact inorganic sand	4 (191)
Loose sand	3 (144)
Soft sand/clay	2 (96)
Loose inorganic sand–silt mixture	1 (48)

7.2 An Analytical Method for Evaluating the Soil Pressure Distribution Under a Footing

We consider the single footing shown in Fig. 7.7. The force P represents the resultant of the column loading. We suppose it has an eccentricity e with respect to the centroid of the footing area. We also suppose the footing area is symmetrical with respect to the x-axis and locate the area such that the column load is on the axis of symmetry. It follows that the pressure loading is symmetrical with respect to this axis. Taking the origin for x at the centroid of the footing area, we express the pressure distribution as a linear function,

$$q(x) = b + ax \tag{7.1}$$

where a and b are unknown parameters. We determine these parameters by enforcing the equilibrium conditions for the footing.

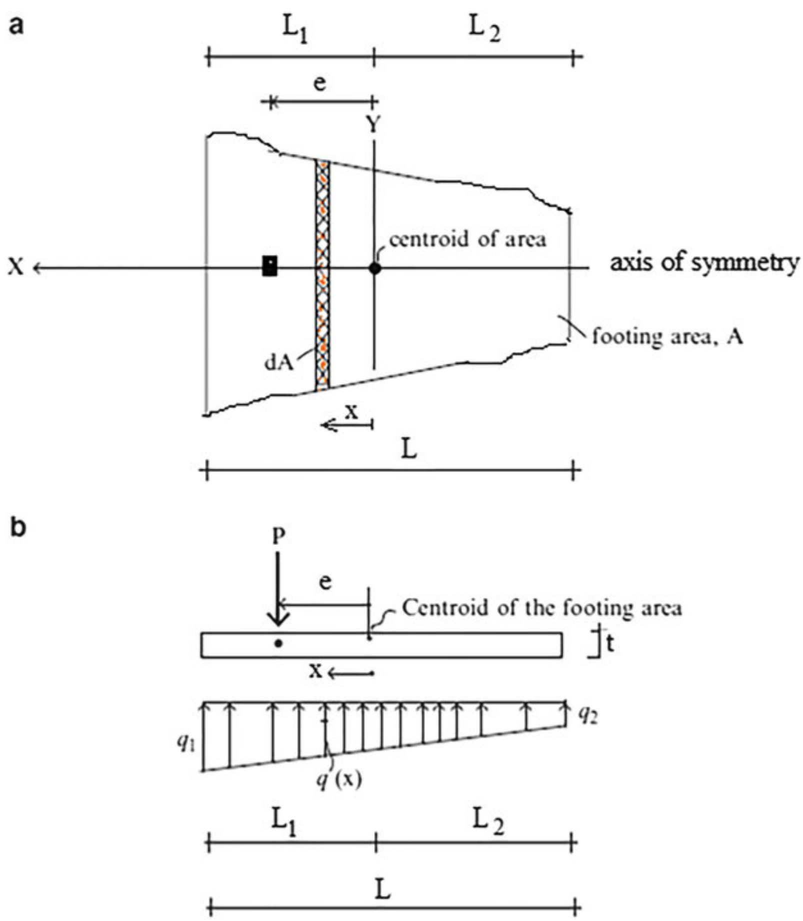


Fig. 7.7 Notation–pressure distribution—single footing. (**a**) Plan. (**b**) Elevation

Since x is measured from the centroid, the first moment of area vanishes. Then, $\int x\,\mathrm{d}A = 0$. Requiring force and moment equilibrium to be satisfied, and noting that the column axial loading has an eccentricity, e, with respect to the centroid of the footing area, leads to the following expressions for b and a.

Vertical force equilibrium:

$$P = \int q(x)\mathrm{d}A = \int (b + ax)\mathrm{d}A = b\int \mathrm{d}A + a\int x\,\mathrm{d}A = bA + 0$$
$$\Downarrow$$
$$b = \frac{P}{A}$$

Moment equilibrium:

$$Pe = \int q(x)x\,\mathrm{d}A$$
$$\Downarrow$$
$$Pe = \int (b + ax)x\,dA = b\int x\,\mathrm{d}A + a\int x^2\,\mathrm{d}A = 0 + aI_y$$
$$\Downarrow$$
$$a = \frac{Pe}{I_y}$$

where I_y is the second moment of the footing area with respect to the Y-axis, $I_y = \int x^2\,\mathrm{d}A$. Substituting for a and b, (7.1) takes the form

$$q(x) = \frac{P}{A} + \frac{Pe}{I_y}x \tag{7.2}$$

The peak pressures are shown in Fig. 7.7b.

$$\begin{aligned} q_1 &= \left\{\frac{P}{A} + \frac{(Pe)L_1}{I_y}\right\} \\ q_2 &= \left\{\frac{P}{A} - \frac{(Pe)L_2}{I_y}\right\} \end{aligned} \tag{7.3}$$

One uses (7.3) to determine the pressure when the footing area is defined.

When the resultant acts at the centroid, $e = 0$ and the pressure distribution reduces to a uniform distribution.

$$q = q_1 = q_2 = \frac{P}{A} \tag{7.4}$$

When $e \neq 0$, the distribution is trapezoidal. As e increases, q_2 decreases. The critical state occurs when $q_2 = 0$. This case is shown in Fig. 7.8.

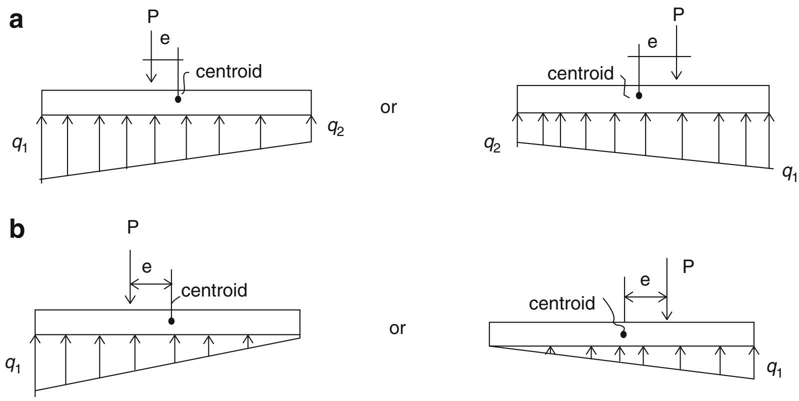


Fig. 7.8 Pressure distributions for $e \le e_{\text{critical}}$. (**a**) $e < e_{\text{critical}}$. (**b**) $e = e_{\text{critical}}$

$$\frac{P}{A} - \frac{(Pe)L_2}{I_y} = 0$$
$$\Downarrow \tag{7.5}$$
$$e_{\text{critical}} = \frac{I_y}{A}\frac{1}{L_2}$$

Applying this reasoning to a rectangular shape of width B and length L, and noting that

$$A = BL \quad I_y = \frac{BL^3}{12} \quad L_1 = L_2 = \frac{L}{2}$$

the expressions for the peak pressures take the form

$$q_1 = \frac{P}{BL} + \frac{6Pe}{BL^2}$$
$$q_2 = \frac{P}{BL} - \frac{6Pe}{BL^2} \tag{7.6}$$

The critical value for e, which corresponds to either q_1 or q_2 equal to 0, is given by

$$e = e_{\text{critical}} = \frac{L}{6} \tag{7.7}$$

In order for the soil pressure to be compressive throughout the footing area, the point of application of the applied loading must be within a zone of width L/3 centered on the centroid. When loaded outside this region, (7.2) does not apply. In this case, the triangular distribution acting on a portion of the surface shown in Fig. 7.9d is used. The soil pressure adjusts its magnitude and extent such that the line of action of the resultant *coincides* with the line of action of the column force. The expressions for q_1 takes the form

$$R = P = q_1\left(\frac{a}{2}B\right)$$
$$\Downarrow$$
$$q_1 = \frac{2P}{Ba}$$

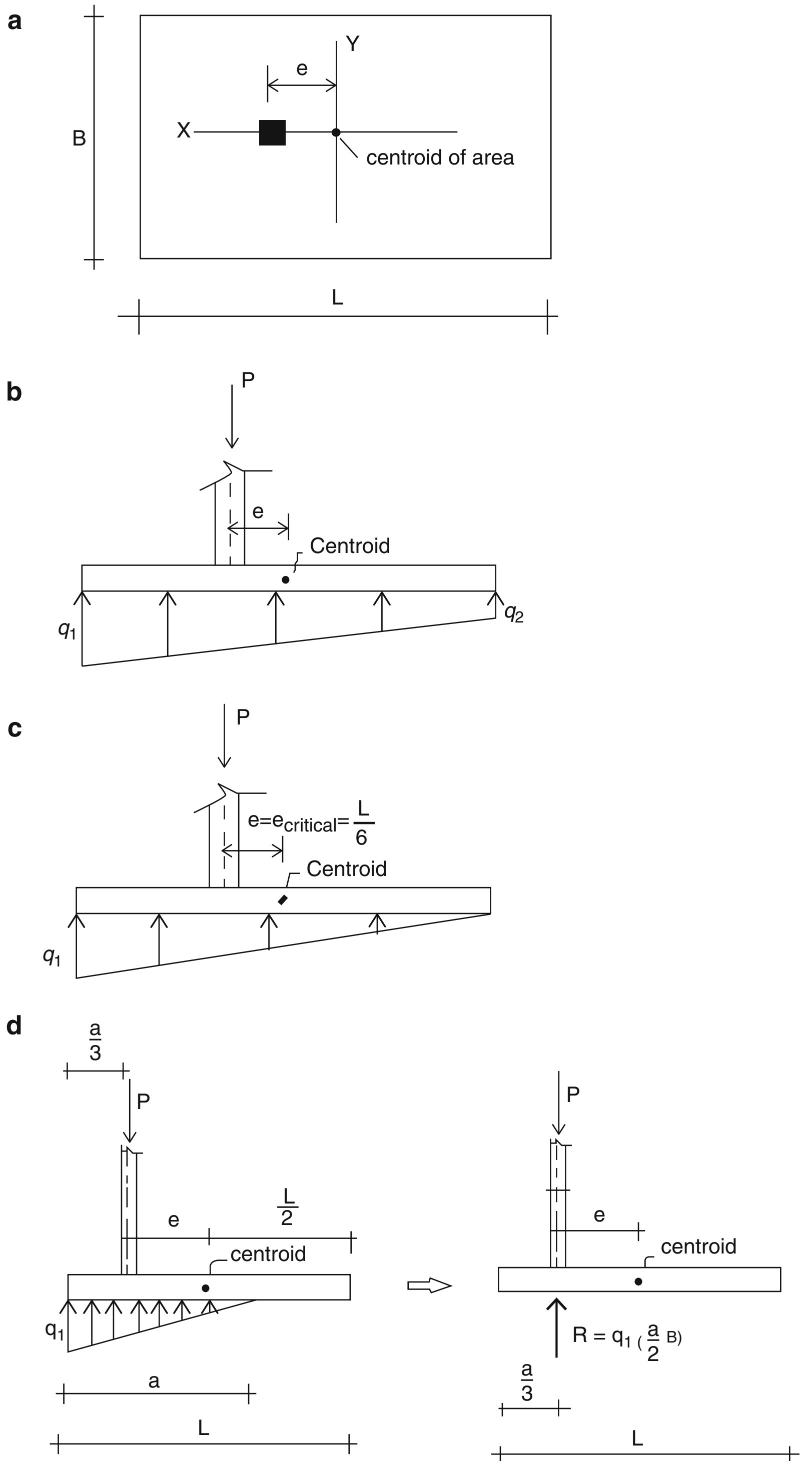


Fig. 7.9 (**a**) Plan—rectangular area. (**b**) Elevation $e < e_{\text{critical}}$. (**c**) Elevation $e = e_{\text{critical}}$. (**d**) Pressure distribution for $e > e_{\text{critical}}$

7.3 Dimensioning a Single Rectangular Footing

Normally, the column position is fixed by the geometry of the structure, and one can only adjust the location of the footing with respect to the column. We consider the case where the design goal is a uniform soil pressure. The optimal dimensions of the footing are achieved by locating the centroid of the footing on the line of action of the column force, i.e., by taking $e = 0$ in Fig. 7.7. The first choice is a square footing. If there is insufficient space in one direction, one can shift to a rectangular footing. If the design is still constrained by space restrictions, one can then follow a different strategy and work with a strap-type footing which is discussed later.

We have shown that the soil pressure distribution is uniform for symmetrically positioned footings. The footing area is *determined using service loads*, P, and the *effective soil pressure*, q_e, which accounts for the weight of the footing and the soil above the footing. This notation is defined in Fig. 7.10. The relevant computations are

$$q_e = q_{allowable} - \gamma_{conc.} t - \gamma_{soil}(h - t) \approx q_{allowable} - \left(\frac{\gamma_{conc.} + \gamma_{soil}}{2}\right) h$$

$$A_{required} \geq \frac{\sum P_{service}}{q_e} \rightarrow L \quad \text{and} \quad B \rightarrow A = LB$$

Current practice estimates the peak values of shear force and moment in the footing using the factored ultimate load P_u and determines the footing thickness and the required reinforcement steel area based on these values. Figure 7.11 illustrates this procedure for a single axial loaded footing. The expressions for the factored ultimate shear and moment are:

$$V_u(x) = Bq_u x$$
$$M_u(x) = \frac{Bq_u x^2}{2} \tag{7.8}$$

where $q_u = \frac{P_u}{A}$. Positive bending moment requires reinforcing steel placed in two directions at the lower surface. One needs to check for two types of shearing actions, one way shear and punching shear.

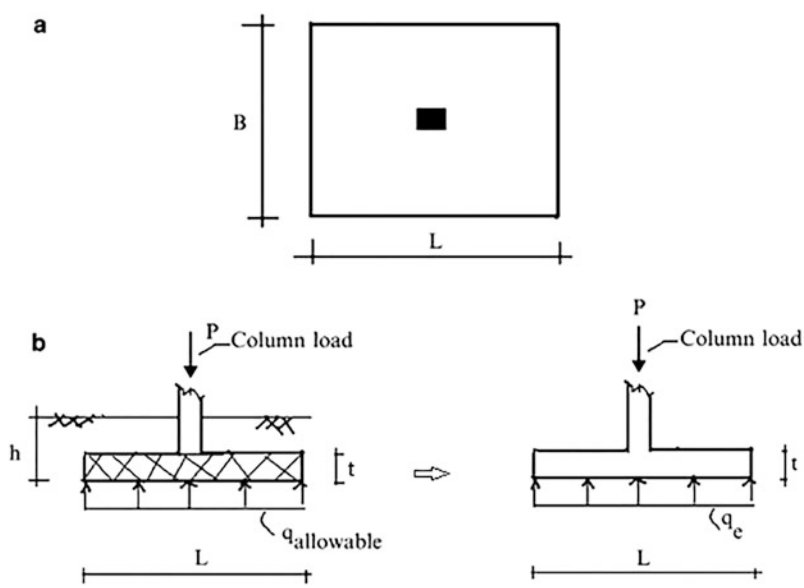


Fig. 7.10 Notation-effective soil pressure (q_e). (**a**) Plan. (**b**) Elevation

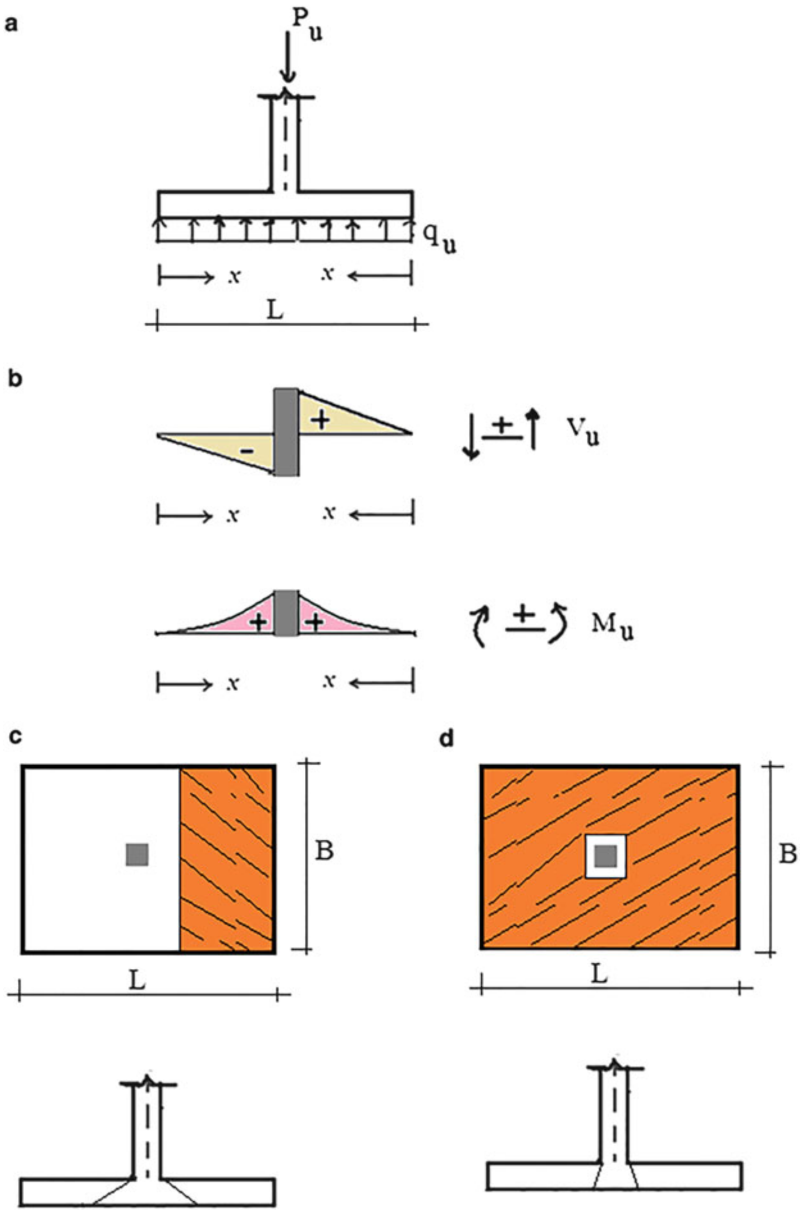


Fig. 7.11 Footing dimensioning process. (**a**) Factored soil pressure. (**b**) Shear and moment diagrams. (**c**) One way shear. (**d**) Punching shear

Figure 7.11 shows the location of the critical sections for shear. The distance parameter depends on the column type (steel, concrete) and the specific design code.

Most footings are constructed using reinforced concrete. The location and magnitude of the steel reinforcement is dictated by the sense of the bending moment distribution (i.e., positive or negative). The function of the reinforcement is to provide the tensile force required by the moment. It follows that the moment diagrams shown in Fig. 7.11b require the reinforcement patterns defined in Fig. 7.12. The actual size/number of the rebar depends on the magnitude of the moment and particular design code used to dimension the member.

Fig. 7.12 Single footing steel details. (**a**) Steel column. (**b**) Reinforced concrete column

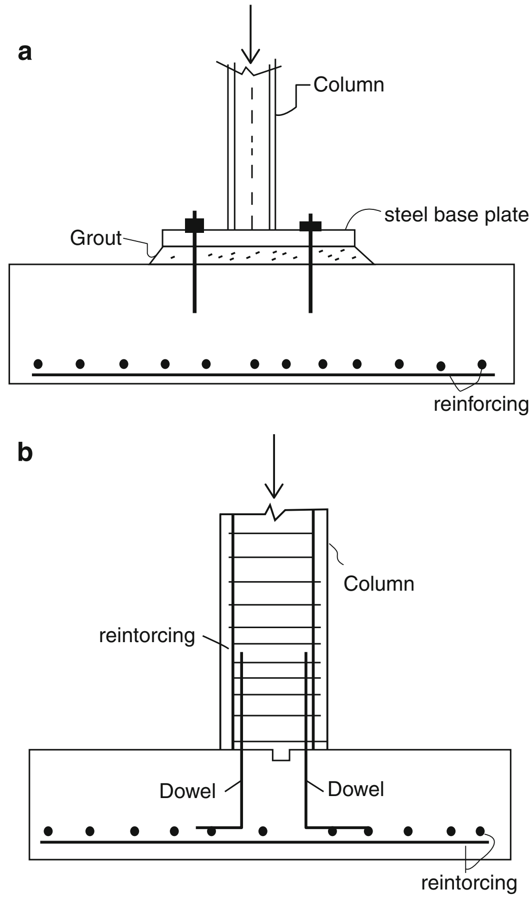


Figure 7.12 illustrates steel reinforcement for steel and concrete columns. A steel plate is welded to the base of a steel column and anchored to the footing with bolts embedded in the concrete. Dowels are used to connect the longitudinal steel in the concrete column to the footing. Usually the column loading is purely axial and the support is considered to be simply supported. However, there are situations where moment as well as axial force is present in the column. The design strategy is the same for both cases.

Example 7.1 Single Footing

Given: A 400 mm × 400 mm concentrically load column with axial dead load ($P_D = 890$ kN), and axial live load ($P_L = 1070$ kN) to be supported on a shallow foundation. The effective soil pressure is $q_e = 165$ kN/m^2 (Fig. E7.1a, b).

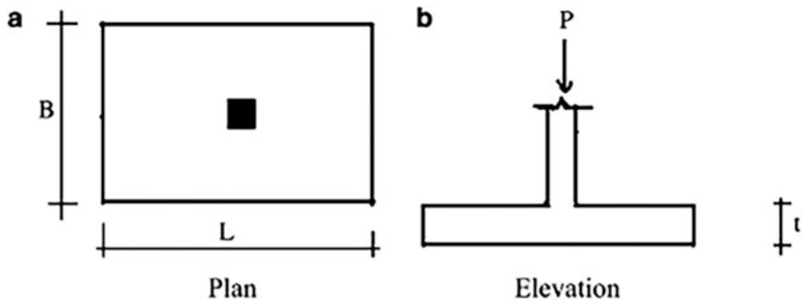


Fig. E7.1 Geometry and loading

Determine: The footing dimensions using service loads. Draw shear and moment diagrams using a factored load of $P_u = 1.2P_D + 1.6P_L$. Consider (a) A square footing, (b) A rectangular footing with $B = 3$ m.

Solution:
Footing dimensions
The required footing area is based on the service load and effective soil pressure.

$$P_{service} = \sum (P_D + P_L) = 890 + 1070 = 1960\,kN$$

$$A_{required} = \frac{P_{service}}{q_e} = \frac{1960}{165} = 11.88\,m^2$$

Assuming a square shape, the required dimension is $\sqrt{11.88} = 3.44\,m$
We use $L = B = 3.5$ m.
Assuming a rectangular footing $B = 3$ m, the required dimension is $L = \frac{11.88}{3} = 3.96\,m$. We use $L = 4$ m

Shear and moment distributions
The factored load is

$$P_u = 1.2P_D + 1.6P_L = 1.2(890) + 1.6(1070) = 2780\,kN$$

The corresponding factored soil pressure, q_u and V_u, M_u are:

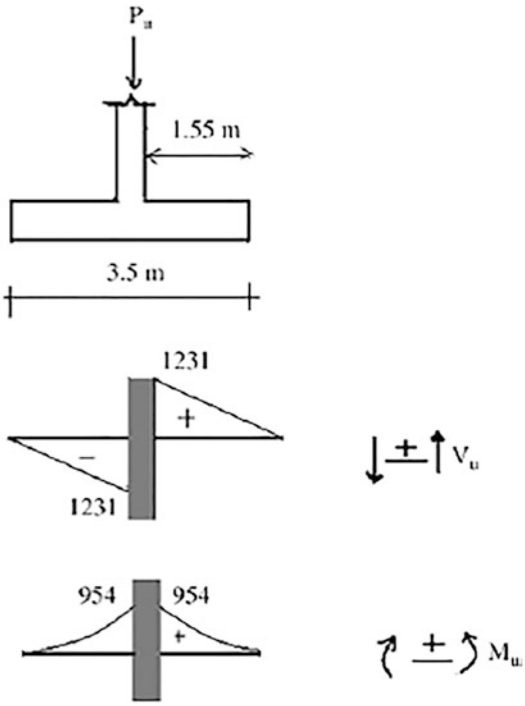
Square shape:

$$q_u = \frac{P_u}{LB} = \frac{2780}{(3.5)(3.5)} = 226.94\,kN/m^2$$

$$V_{u\,max} = 226.94(3.5)(1.55) = 1231\,kN$$

$$M_{u\,max} = 226.94(3.5)\frac{(1.55)^2}{2} = 954\,kN\,m$$

The shear and moment diagrams are plotted below.



Rectangular shape:

$$q_u = \frac{P_u}{LB} = \frac{2780}{(3)(4)} = 231.67\,\text{kN/m}^2$$

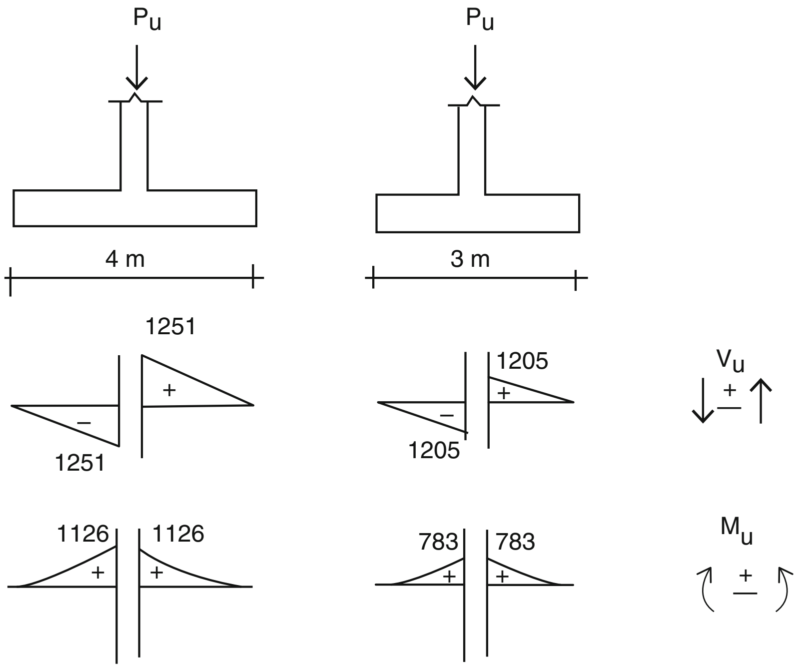
$$V_{u_{max}\,\text{long}} = 231.67(3)(1.8) = 1251\,\text{kN}$$

$$M_{u_{max}\,\text{long}} = 231.67(3)\frac{(1.8)^2}{2} = 1126\,\text{kN m}$$

$$V_{u_{max}\,\text{short}} = 231.67(4)(1.3) = 1205\,\text{kN}$$

$$M_{u_{max}\,\text{short}} = 231.67(4)\frac{(1.3)^2}{2} = 783\,\text{kN m}$$

The shear and moment diagrams are plotted below.



Example 7.2 Dimensioning a Footing Under a Column with Eccentric Loading

Given: A 12 in. × 12 in. column supporting the following loads: $P_D = 120$ kip, $P_L = 80$ kip, $M_D = 60$ kip ft, and $M_L = 40$ kip ft. The effective soil pressure is $q_e = 4.5$ kip/ft^2.

$$P = P_D + P_L = 120 + 80 = 200\,\text{kip}$$
$$M = M_D + M_L = 60 + 40 = 100\,\text{kip ft}$$

Determine: Dimension square/rectangular footings for the following cases.

Case (a): the center line of the column coincides with the centroid of the footing. M is counterclockwise (Fig. E7.2a).

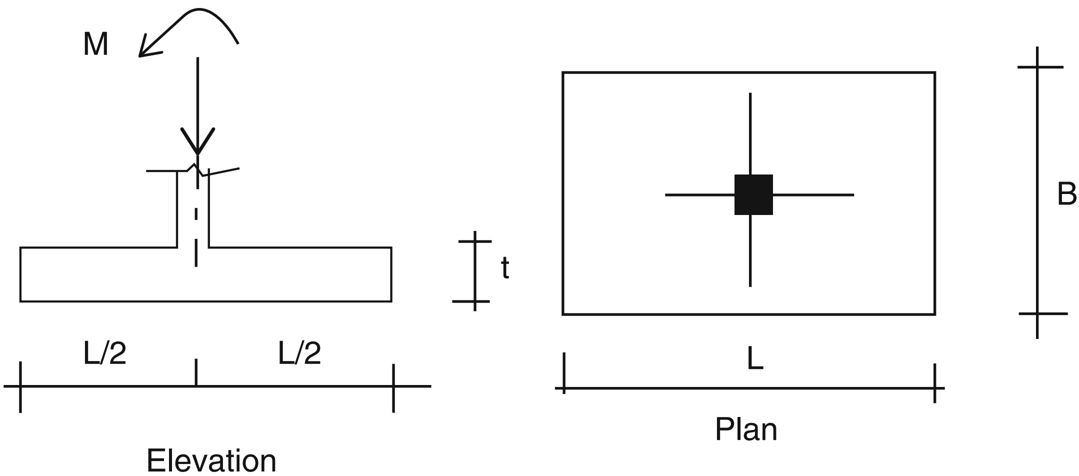


Fig. E7.2a Geometry and loading

Solution: Case (a)
Square footing ($L = B$)**:** We use (7.6) and set $q_1 = q_e$

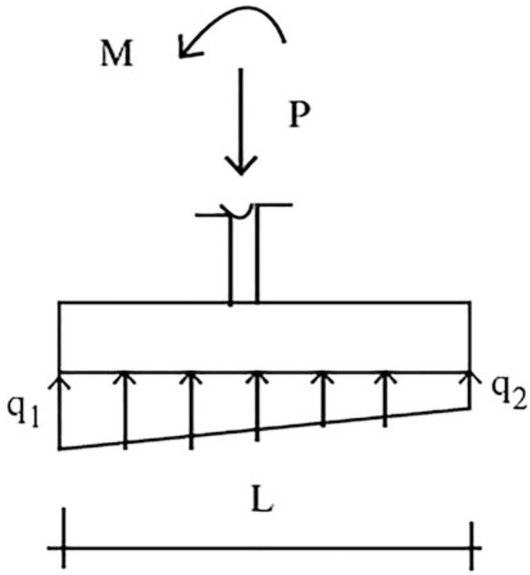
$$\frac{200}{L^2} + \frac{6(100)}{L^3} = 4.5 \Rightarrow L = 7.9\,\text{ft}$$

$$\text{For } L = B = 8\,\text{ft} \Rightarrow q_1, q_2 = 3.125 \pm 1.17 = \left| \begin{array}{l} 4.3\,\text{kip/ft}^2 \\ 1.95\,\text{kip/ft}^2 \end{array} \right.$$

Rectangular footing: We take $B = 6$ ft. The pressure equation has the form

$$\frac{P}{6L} + \frac{6M}{6L^2} = q_e$$
$$\Downarrow$$
$$\frac{200}{6L} + \frac{6(100)}{6L^2} = 4.5 \Rightarrow L = 9.7\,\text{ft}$$

$$\text{For } L = 10\,\text{ft} \quad \text{and} \quad B = 6\,\text{ft} \Rightarrow q_1, q_2 = 3.33 \pm 1.0 = \begin{vmatrix} 4.33\,\text{kip/ft}^2 \\ 2.33\,\text{kip/ft}^2 \end{vmatrix}$$



Case (b): the center line of the column is 3 ft from the property line. *M* is clockwise (Fig. E7.2b).

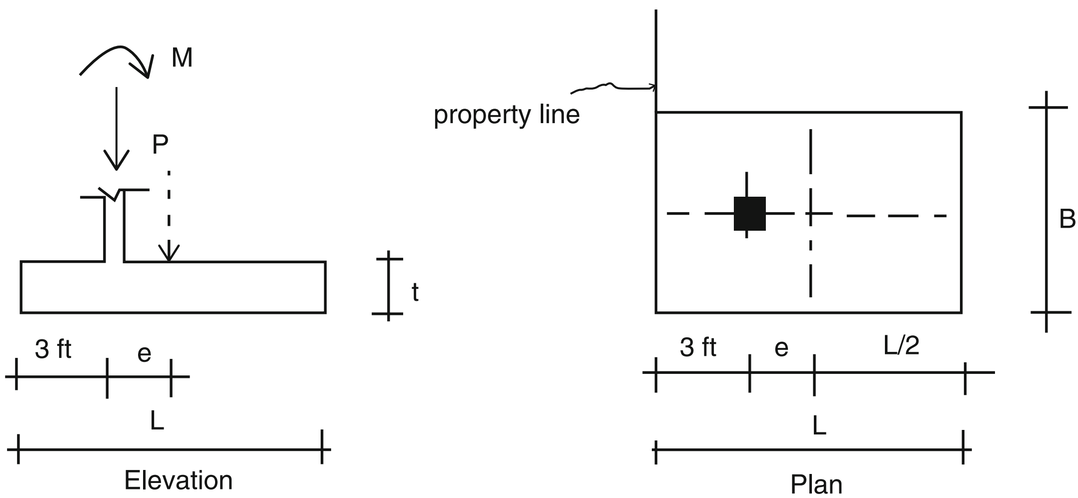


Fig. E7.2b Geometry and loading

Solution: Case (b) We position the centroid of the footing so that it is on the line of action of the resultant force. The location of the resultant is

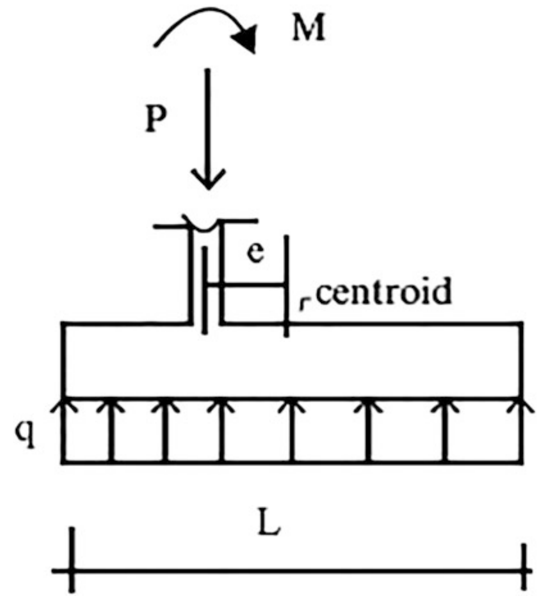
$$M_{\text{total}} = M - Pe = 100 - 200(e) = 0 \Rightarrow e = 0.5\,\text{ft}$$

Then

$$3 + e = \frac{L}{2} \Rightarrow L = 2(3 + e) = 7\,\text{ft}$$

$$q_1 = q_2 = \frac{P}{LB} \leq q_e \rightarrow B \geq \frac{200}{7(4.5)} = 6.35$$

$$\text{For } L = 7\,\text{ft} \quad \text{and} \quad B = 6.5\,\text{ft} \Rightarrow q_1 = q_2 = q = \frac{p}{LB} = \frac{200}{7(6.5)} = 4.39\,\text{kip/ft}^2$$



Case (c): the center line of the column is 3 ft from the property line. M is zero (Fig. E7.2c).

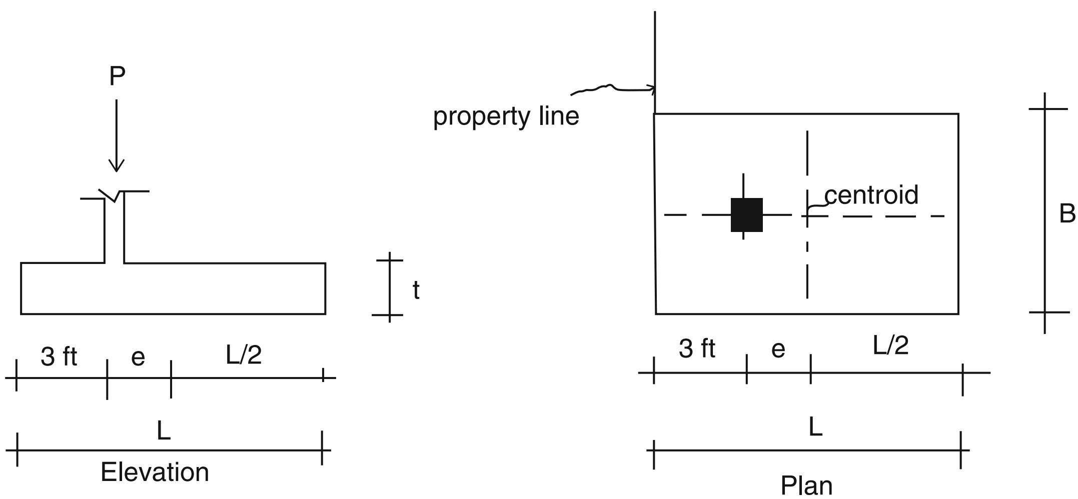


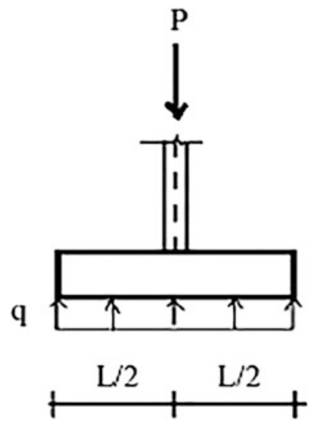
Fig. E7.2c Geometry and loading

Solution: Case (c) For this case, we locate the centroid on the center of the column. Then $e = 0$ and $L = 6.0$ ft. The area is determined with

$$q = q_1 = q_2 = \frac{P}{LB} \leq q_e \rightarrow B \geq \frac{200}{6(4.5)} = 7.4$$

Use $L = 6$ ft and $B = 7.5$ ft

$$q = \frac{P}{LB} = \frac{200}{7.5(6)} = 4.44\,\text{kip/ft}^2$$



Case (d): the center line of the column is 3 ft from the property line. M is counterclockwise (Fig. E7.2d).

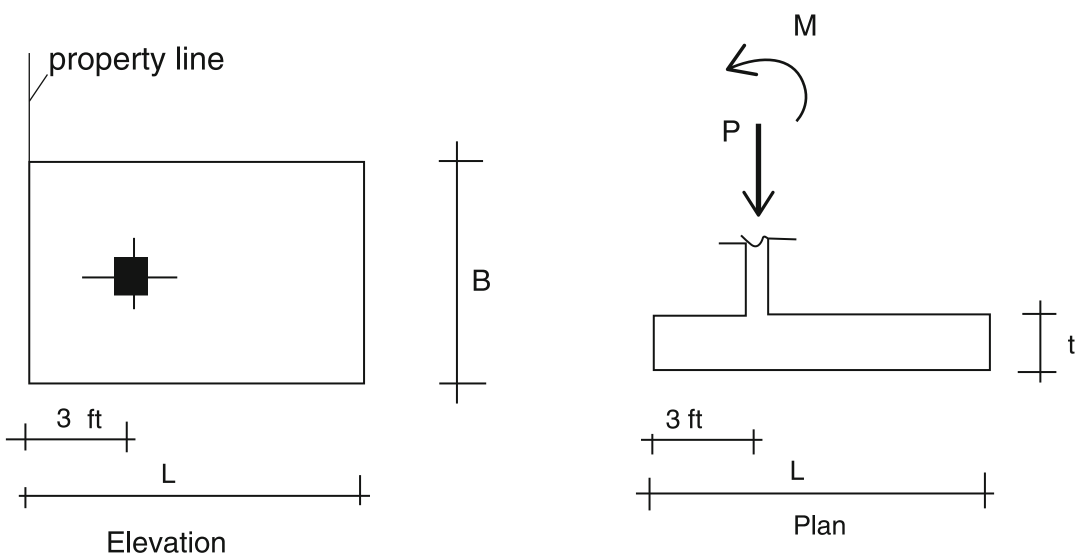
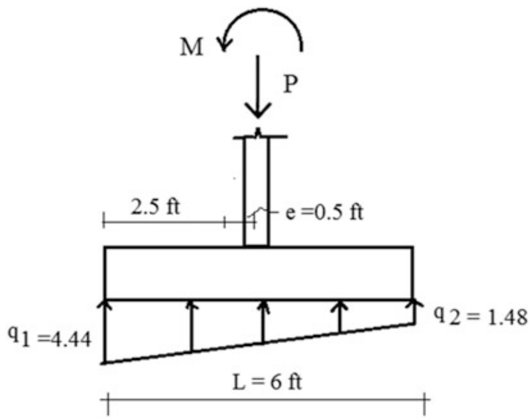


Fig. E7.2d Geometry and loading

Solution: Case (d) We decide to locate the centroid on the column line. Then $e = 0.5$ ft. This leads to the trapezoidal pressure distribution shown below. Taking $L = 6$ ft, and noting (7.3),

$$q_1 = \frac{200}{6B} + \frac{6(100)}{B(6)^2} \leq 4.5 \Rightarrow B \geq 11.1$$

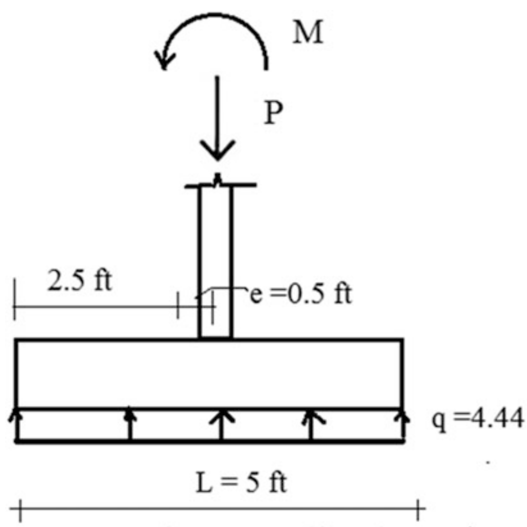
$$\text{For } L = 6\,\text{ft} \quad \text{and} \quad B = 11.25\,\text{ft} \rightarrow q_1, q_2 = 2.96 \pm 1.48 = \left| \begin{array}{l} 4.44\,\text{kip/ft}^2 \\ 1.48\,\text{kip/ft}^2 \end{array} \right.$$



Another option is to take the centroid on the line of action of the resultant force. Then, $e = 0$ ft, $L = 5$ ft, and (7.6) yields

$$\frac{200}{5B} = 4.5 \rightarrow B = 8.9$$

For $L = 5$ ft and $B = 9$ ft $q = q_1 = q_2 = \frac{200}{5(9)} = 4.44\,\text{kip/ft}^2$



7.4 Dimensioning Combined Footings

A combined footing has multiple column loads acting on a single area. This design is adopted when the column spacing is too small to allow for separate footings. Figure 7.13 illustrates the case of two columns. The analytical method described in Sect. 7.2 is also applicable here. One just has to *first replace the column loads with their resultant force*, and then apply (7.2) to determine the pressure distribution.

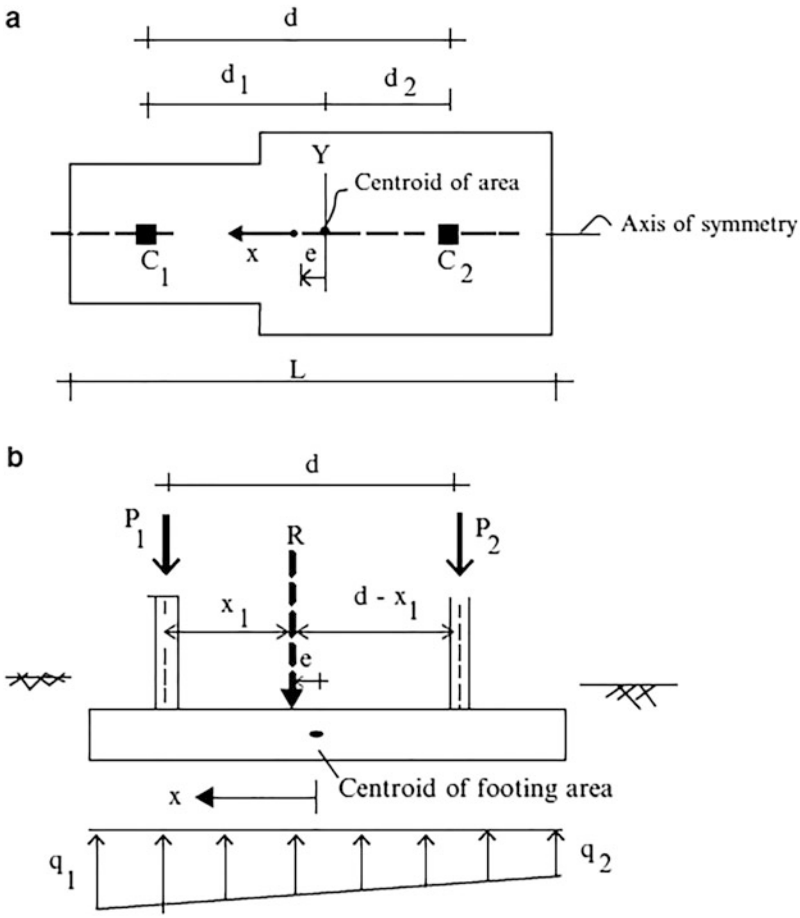


Fig. 7.13 Notation-combined footing. (**a**) Plan. (**b**) Elevation

Specializing (7.2) for this case, and noting the notations defined in Fig. 7.13, the pressure distribution is given by

$$q(x) = \frac{R}{A} + \left(\frac{Re}{I_y}\right)x \qquad (7.9)$$
$$R = P_1 + P_2$$

where e is positive when R is located to the left of the centroid of the footing area.

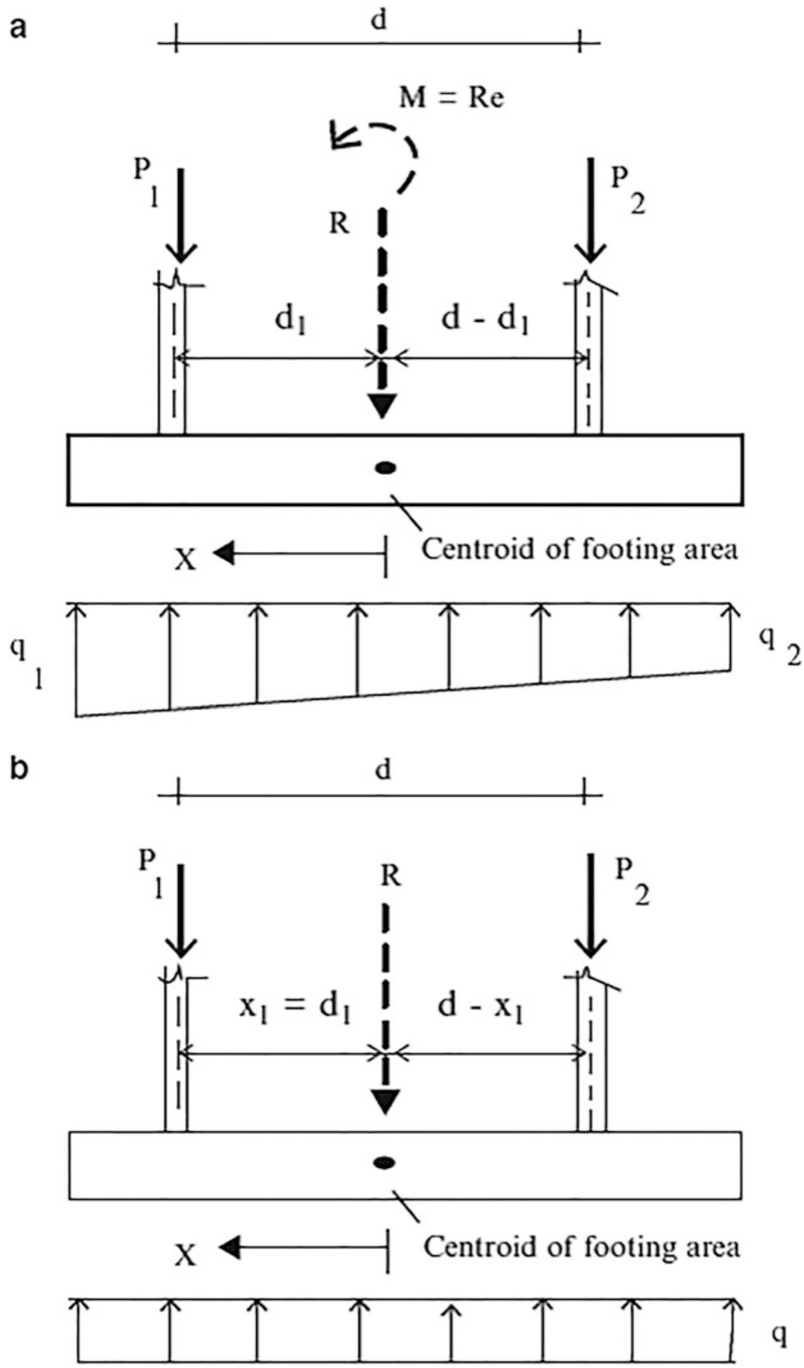
The location of the resultant force can be determined by summing moments about the line of action of P_1

$$x_1 = \frac{P_2 d}{(P_1 + P_2)} \qquad (7.10)$$
$$e = d_1 - x_1$$

It follows that *the soil pressure distribution is uniform when the centroid of the footing area is located on the line of action of the resultant force* (Fig. 7.14). For this case, $e = 0$ and $q = R/A$.

We compute the peak shear and moment, using factored loads. The position of the resultant with respect to the centroid may change when factored loads are used.

Fig. 7.14 Conditions for soil pressure distribution. (**a**) $e > 0$. (**b**) $e = 0$



$$R_u = P_{1u} + P_{2u}$$

Then,

$$x_{1_u} = d_1 - e_u = \frac{P_{2u}}{R_u} d \tag{7.11}$$

If $e_u \neq 0$, the distribution of pressure is trapezoidal, and we use (7.9) to find the corresponding peak pressures due to the factored loads.

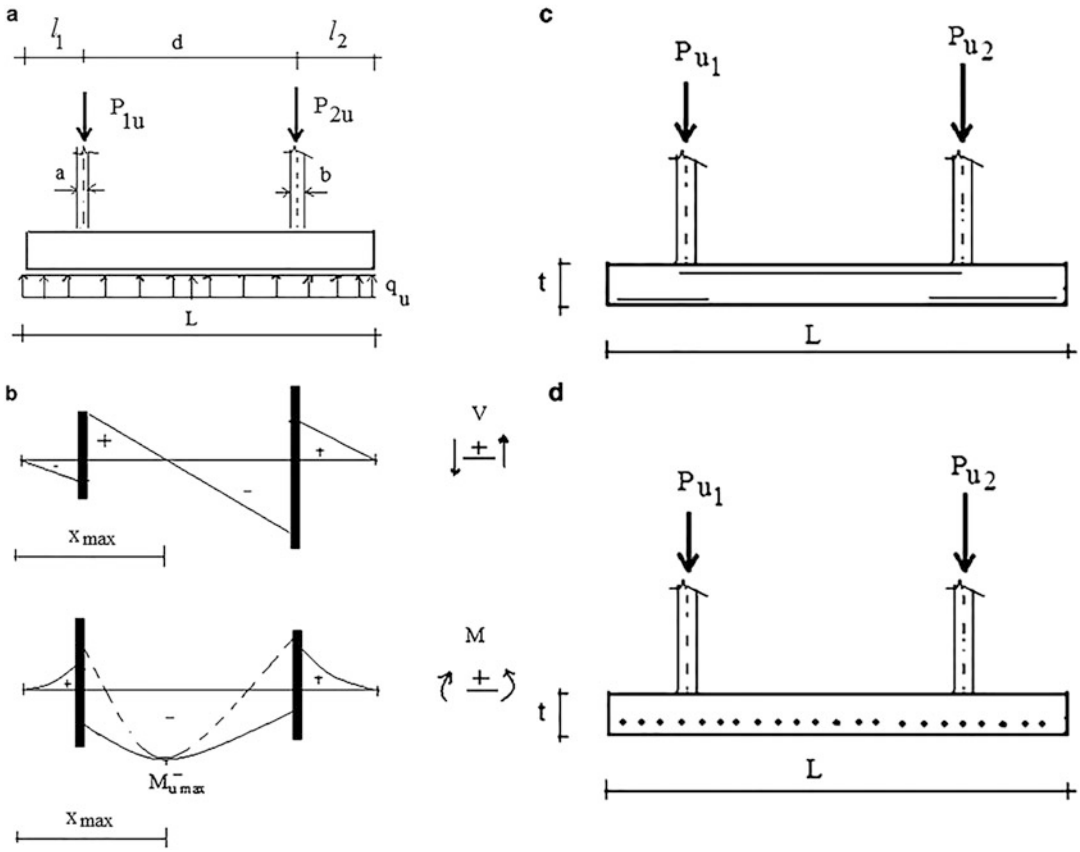


Fig. 7.15 (**a**) Rectangular footing with uniform ultimate soil pressure $e_u = 0$. (**b**) Shear and Moment diagrams $e_u = 0$. (**c**) Steel reinforcing pattern for longitudinal bending. (**d**) Steel reinforcing pattern for transverse bending

The shear and moment diagrams corresponding to uniform soil pressure are plotted in Fig. 7.15b. Note that for this type of footing, the bending moment distribution in the footing in the longitudinal direction has *both positive and negative regions*. The peak moment values are

$$V_u = 0 \rightarrow Bq_u x_{max} - P_{u1} = 0 \rightarrow x_{max} = \frac{P_{u1}}{Bq_u}$$

$$\therefore M^-_{u\,max} = Bq_u \frac{x^2_{max}}{2} - P_{u1}(x_{max} - l_1)$$

$$\text{At edges of columns } V_u \begin{cases} -Bq_u\left(l_1 - \dfrac{a}{2}\right) \\ -Bq_u\left(l_1 + \dfrac{a}{2}\right) + P_{u1} \\ Bq_u\left(l_2 - \dfrac{b}{2}\right) \\ Bq_u\left(l_2 + \dfrac{b}{2}\right) - P_{u2} \end{cases}$$

$$\text{At edges of columns } M_u \begin{cases} \dfrac{Bq_u}{2}\left(l_1 - \dfrac{a}{2}\right)^2 \\ \dfrac{Bq_u}{2}\left(l_1 + \dfrac{a}{2}\right)^2 - P_{u1}\left(\dfrac{a}{2}\right) \\ \dfrac{Bq_u}{2}\left(l_2 - \dfrac{b}{2}\right)^2 \\ \dfrac{Bq_u}{2}\left(l_2 + \dfrac{b}{2}\right)^2 - P_{u2}\left(\dfrac{b}{2}\right) \end{cases}$$

Since the moment diagram for a combined footing generally has both positive and negative values, the steel placement pattern for a combined footing involves placing steel in the top zone as well as the bottom zone of the cross section. The required steel reinforcing patterns are shown in Fig. 7.15c, d. In general, the reinforcement pattern is two way. For the transverse direction, we treat the footing similar to the single footing and the steel for tension is placed at the lower surface.

Example 7.3 Dimensioning a Combined Footing

Given: A combined footing supporting two square columns. Column A is 400 mm × 400 mm and carries a dead load of 700 kN and a live load of 900 kN. Column B is 500 mm × 500 mm and carries a dead load of 900 kN and a live load of 1000 kN. The effective soil pressure is $q_e = 160$ kN/m^2 (Figs. E7.3a and E7.3b).

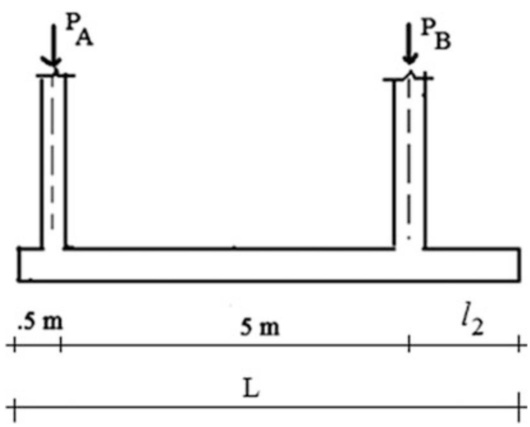


Fig. E7.3a Elevation

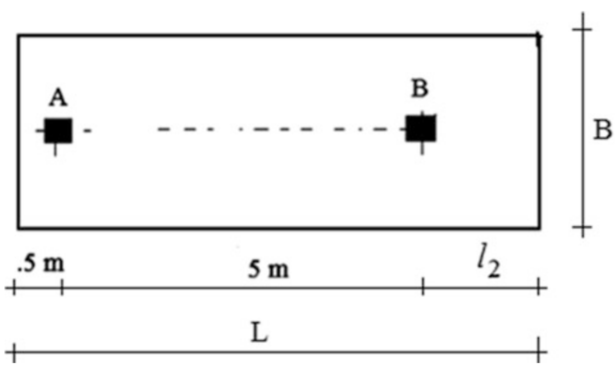


Fig. E7.3b Plan

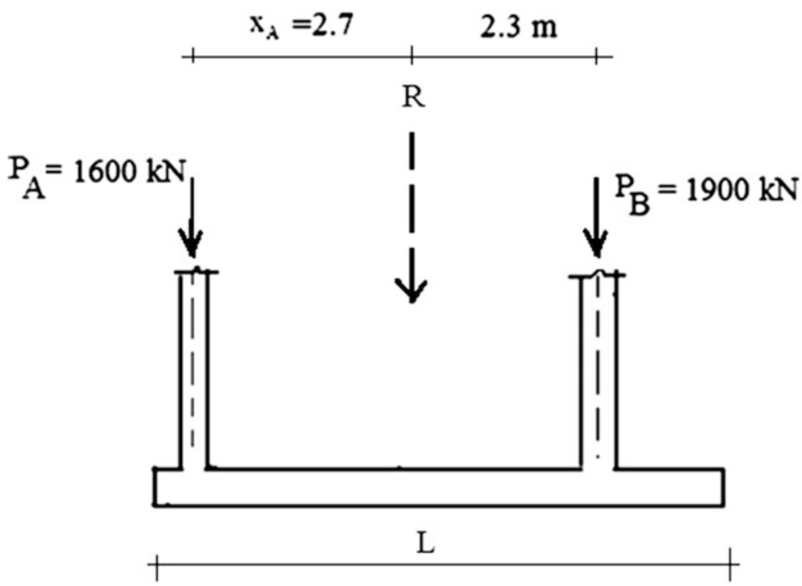
Determine: The dimensions B and L for service load of $P = P_D + P_L$, assuming the soil pressure distribution is uniform. Draw shear and moment diagrams for factored load of $P_u = 1.2P_D + 1.6P_L$.

Solution:

Step I: Locate the resultant force

$$P_A = P_D + P_L = 700 + 900 = 1600\,\text{kN}$$
$$P_B = P_D + P_L = 900 + 1000 = 1900\,\text{kN}$$
$$R = P_A + P_B = 3500\,\text{kN}$$
$$x_A = \frac{1900(5)}{3500} = 2.71\,\text{m}$$

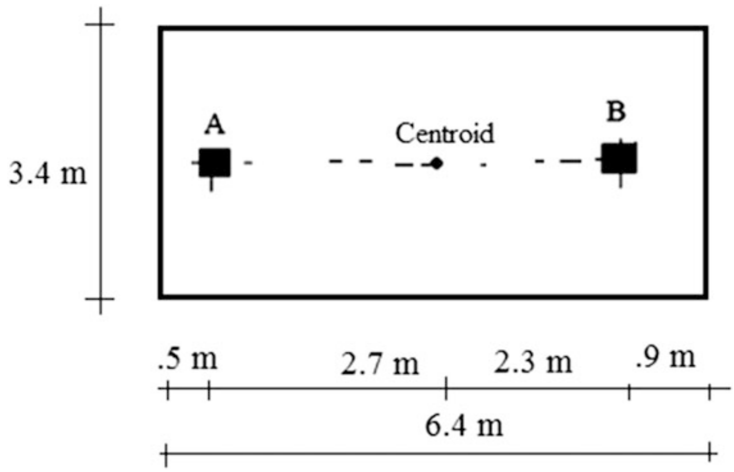
The resultant equals 3500 kN located 2.71 m from column A.



Step II: Select a rectangular geometry. We position the rectangle so that its centroid is on the line of action of the resultant. The design requirement is

$$\frac{L}{2} = x_A + 0.5 = (2.71) + 0.5 = 3.21$$
$$A_{\text{required}} = \frac{R}{q_e} = \frac{3500}{165} = 21.2\,\text{m}^2$$

Take $L = 6.4$ m and $B = 3.4$ m $\rightarrow A = B \times L = 21.76$ m^2
The final geometry is shown below



Step III: Draw the shear and moment diagrams corresponding to the factored loads $P_u = 1.2P_D + 1.6P_L$. We work with the soil pressure integrated over the width of the footing. This leads to the "total" shear and "total" moment. These distributions are plotted below. Note that we treat the column loads as concentrated forces. One can also model them as distributed loads over the width of the column.

$$P_{Au} = 1.2P_D + 1.6P_L = 1.2(700) + 1.6(900) = 2280\,\text{kN}$$
$$P_{Bu} = 1.2P_D + 1.6P_L = 1.2(900) + 1.6(1000) = 2680\,\text{kN}$$
$$R_u = P_{Au} + P_{Bu} = 4960\,\text{kN}$$

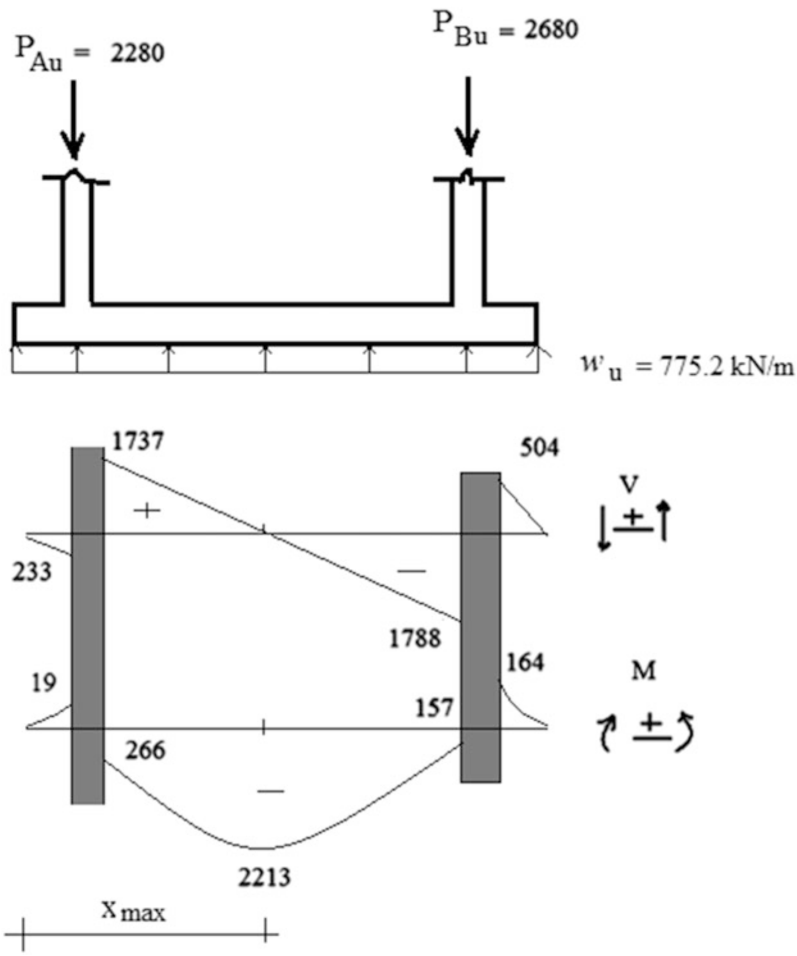
$$x_{Au} = \frac{2680(5)}{4960} = 2.701\,\text{m}$$

The factored resultant acts 2.701 m from column A. It follows that $e = 12$ mm. We neglect this eccentricity and assume the pressure is uniform.

$$q_u = \frac{R_u}{A} = \frac{4960}{6.4(3.4)} = 228\,\text{kN/m}^2$$

Then for $B = 3.4$ m, $w_u = 228(3.4) = 775.2$ kN/m.

The shear and moment diagrams are plotted below.



Example 7.4

Given: A combined footing supporting two square columns. Column C_1 is 16 in. × 16 in. and carries a service load of 220 kip. Column C_2 is 18 in. × 18 in. and carries a service load of 440 kip (Figs. E7.4a and E7.4b).

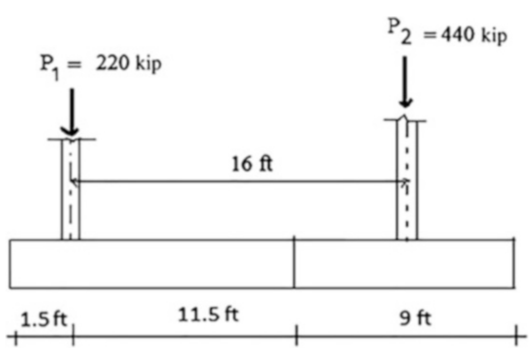


Fig. E7.4a Elevation

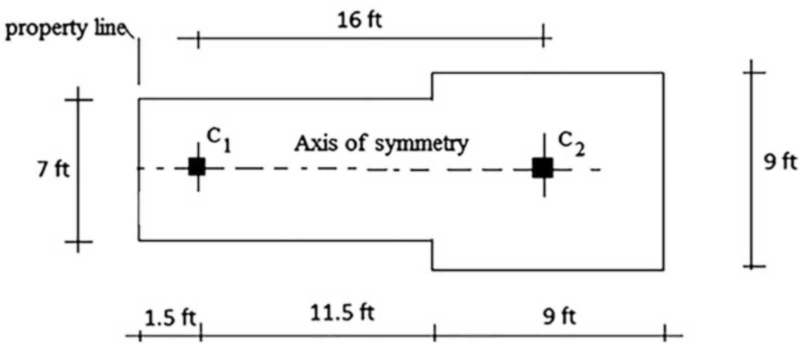


Fig. E7.4b Plan

Determine: The soil pressure distribution caused by the service loads P_1 and P_2.
Solution:
Locate the centroid of the area

$$A = (9)(9) + (13)(7) = 172\,\text{ft}^2$$
$$L_1 = \frac{81(17.5) + 91(6.5)}{172} = 11.68\,\text{ft}$$
$$d_1 = 11.68 - 1.5 = 10.18\,\text{ft}$$

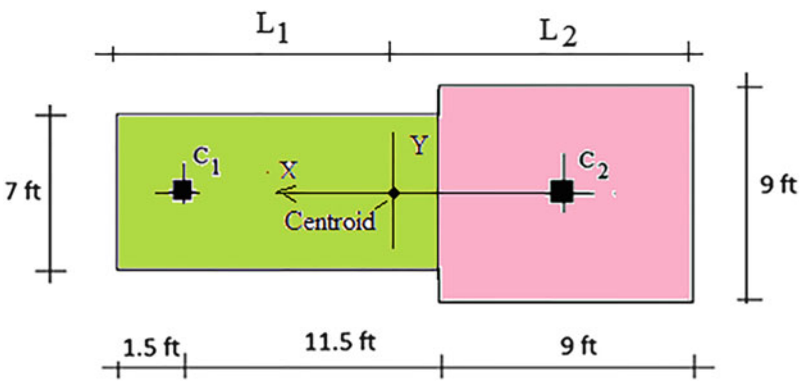


Fig. E7.4c

Locate the resultant force

$$R = P_1 + P_2 = 220 + 440 = 660\,\text{kip}$$
$$x_1 = \frac{P_2 d}{R} = \frac{440(16)}{660} = 10.67\,\text{ft}$$
$$e = -0.49\,\text{ft}$$

The peak pressures are

$$q_1 = \left\{\frac{R}{A} + \frac{(Re)L_1}{I_y}\right\} = \frac{660}{172} + \frac{660(-0.49)11.68}{7014} = 3.3\,\text{kip/ft}^2$$

$$q_2 = \left\{\frac{R}{A} - \frac{(Re)L_2}{I_y}\right\} = \frac{660}{172} - \frac{660(-0.49)10.32}{7014} = 4.3\,\text{kip/ft}^2$$

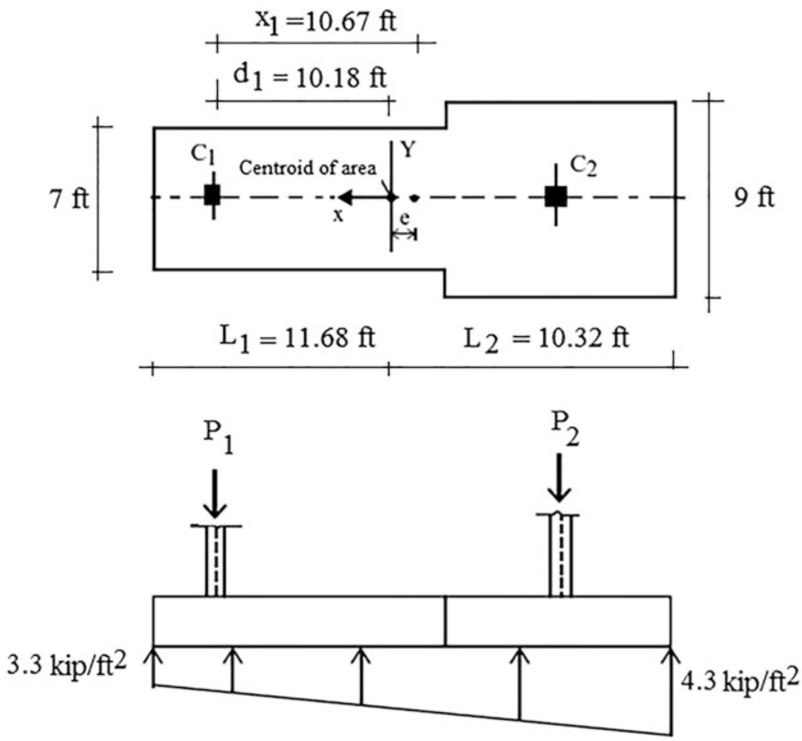


Fig. E7.4d

7.5 Dimensioning Strap Footings

Strap footings consist of individual footings placed under each column and connected together with a rigid beam to form a single unit. Figure 7.16 illustrates the geometric arrangement for two columns supported by two rectangular footings. The centroid for the interior footing (footing #2) is usually taken to be on the line of action of the interior column. The zone under the rigid beam is generally filled with a geofoam material that has essentially no stiffness and provides negligible pressure on the beam. Therefore, all of the resistance to the column loads is generated by the soil pressure acting on the individual footing segments.

We suppose the axis connecting the columns is an axis of symmetry for the area segments. The approach follows essentially the same procedure as employed for combined footings. Figure 7.17 defines the notation for this method.

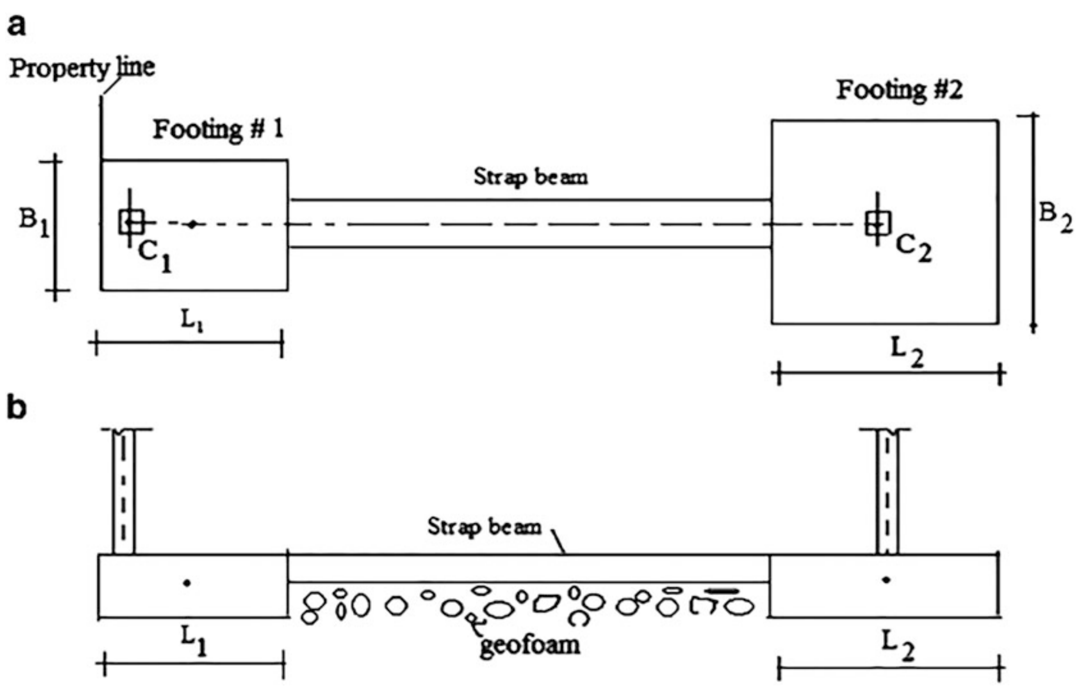


Fig. 7.16 Strap footing. (**a**) Plan. (**b**) Elevation

First, we locate the resultant of the column loads.

$$x_1 = \frac{P_2}{R}d$$
$$R = P_1 + P_2$$
$$e = d_1 - x_1$$

Note that when e is negative, R is located to the right of the centroid (see Fig. 7.17b).

Next, we take footing #2 to be located such that its centroid coincides with the line of action of load P_2.

We locate the origin of the x-axis at an arbitrary point on the axis of symmetry and use (7.1) to determine the soil pressure acting on the individual footings. We assume there is no soil pressure acting on the link member. Noting (7.1), the soil pressure is taken as

$$\begin{aligned} q(x) &= b + ax \quad \text{for footings } \#1 \text{ and } \#2 \\ q(x) &= 0 \quad \text{for the strap beam.} \end{aligned}$$

The coefficients are evaluated by *integrating over the footing areas*. Enforcing equilibrium leads to

$$\begin{aligned} R &= \int q(x)\mathrm{d}A = b(A_1 + A_2) + a\left[\int_{A_1} x\,\mathrm{d}A + \int_{A_2} x\,\mathrm{d}A\right] \\ Re &= \int x\,q(x)\mathrm{d}A = b\left[\int_{A_1} x\,\mathrm{d}A + \int_{A_2} x\,\mathrm{d}A\right] + a\left[\int_{A_1} x^2\mathrm{d}A + \int_{A_2} x^2\mathrm{d}A\right] \end{aligned} \tag{7.12}$$

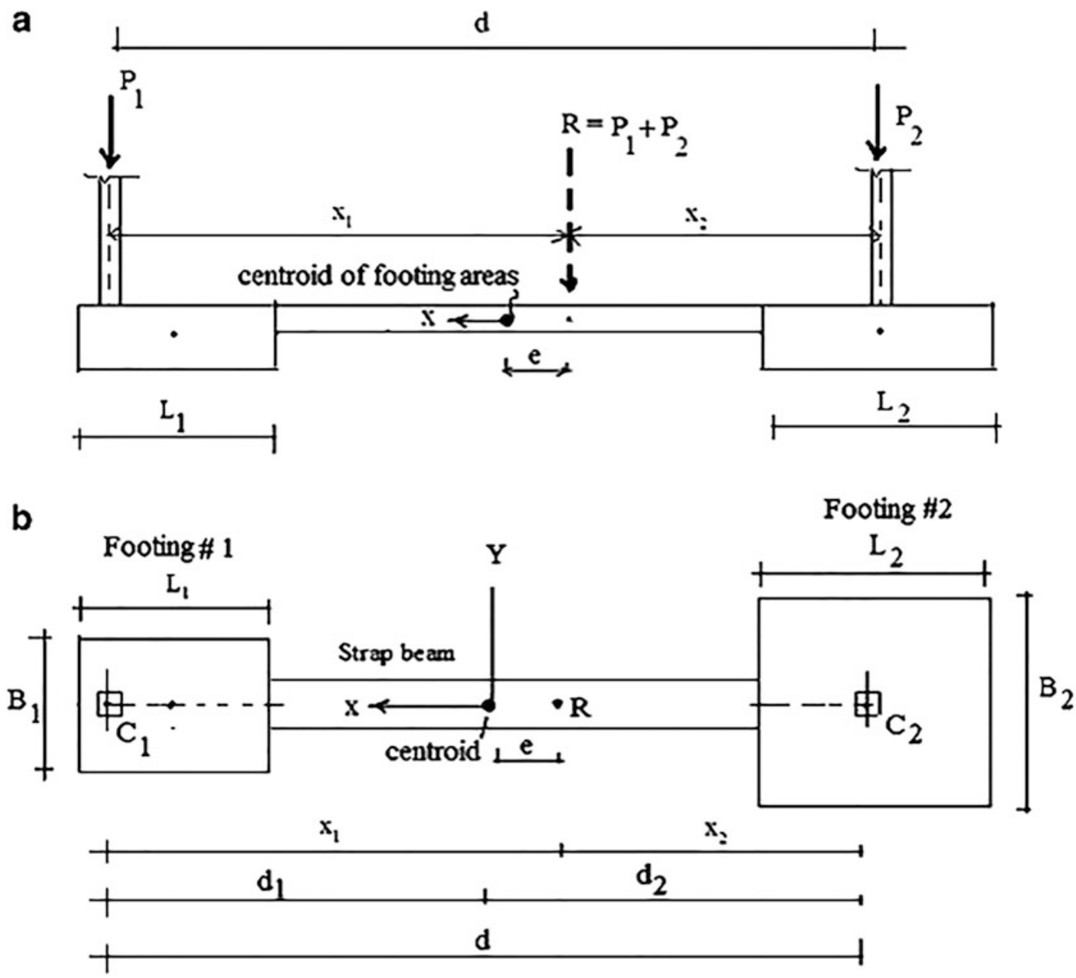


Fig. 7.17 Notation and pressure distribution for strap footing. (**a**) Elevation. (**b**) Plan

Now, we take the origin for x at the centroid of the combined section. Then $\int_{A_1} x\,\mathrm{d}A + \int_{A_2} x\,\mathrm{d}A = 0$ and (7.12) reduces to

$$R = b(A_1 + A_2)$$
$$Re = a(I_{Y1} + I_{Y2}) \tag{7.13}$$

where $(I_{Y1} + I_{Y2})$ is sum of the second moments of area of the two footing cross sections about the Y-axis through the centroid. The I_Ys are computed using the following equations:

$$I_{Y1} = \bar{I}_{Y1} + A_1\bar{d}_1^2$$
$$I_{Y2} = \bar{I}_{Y2} + A_2\bar{d}_2^2$$

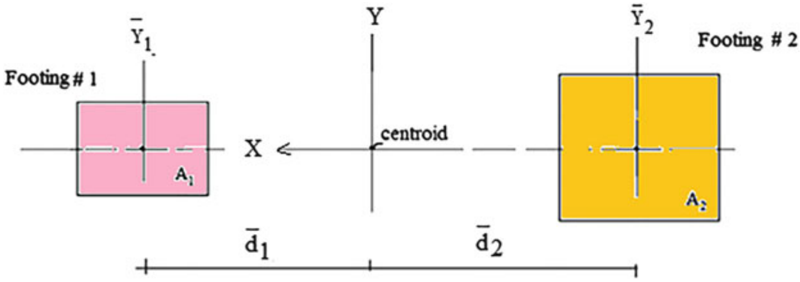


Fig. 7.18 Geometry–strap footing

Lastly, the pressure equation takes the form:

$$q(x) = \frac{R}{(A_1 + A_2)} + \frac{Re}{(I_{Y1} + I_{Y2})}x \tag{7.14}$$

We use (7.14) to determine the pressure for a given geometry and loading.

When dimensioning the footing, *we locate the centroid of the combined footing area on the line of action of the resultant*. This step results in a uniform pressure,

$$e = 0 \quad \rightarrow \quad q = \frac{R}{(A_1 + A_2)} = \frac{P_1 + P_2}{(A_1 + A_2)} \tag{7.15}$$

Given the effective soil pressure, we determine the total area with

$$A_1 + A_2 \geq \frac{P_1 + P_2}{q_e} \tag{7.16}$$

The solution procedure is as follows:

We assume the magnitude of either A_1 or A_2 and compute the other area with (7.16). Since we are locating footing #2 such that its centroid coincides with the line of action of P_2, it follows from Fig. 7.17 that $x_2 \equiv d_2$. Then noting Fig. 7.18, $\bar{d}_2 \equiv d_2$. Lastly, we determine $\bar{d}_1$ with (7.17)

$$A_1\bar{d}_1 = A_2\bar{d}_2 \tag{7.17}$$

This equation corresponds to setting $e = 0$.

An alternative design approach proceeds as follows. Consider Fig. 7.19. The resultants of the pressure distributions acting on the footings are indicated by R_1 and R_2. Summing moments about the line of action of R_1 leads to

$$R_2 = P_2 - \frac{P_1 e_1}{d - e_1} \tag{7.18}$$

Summing forces leads to

$$R_1 + R_2 = P_1 + P_2$$

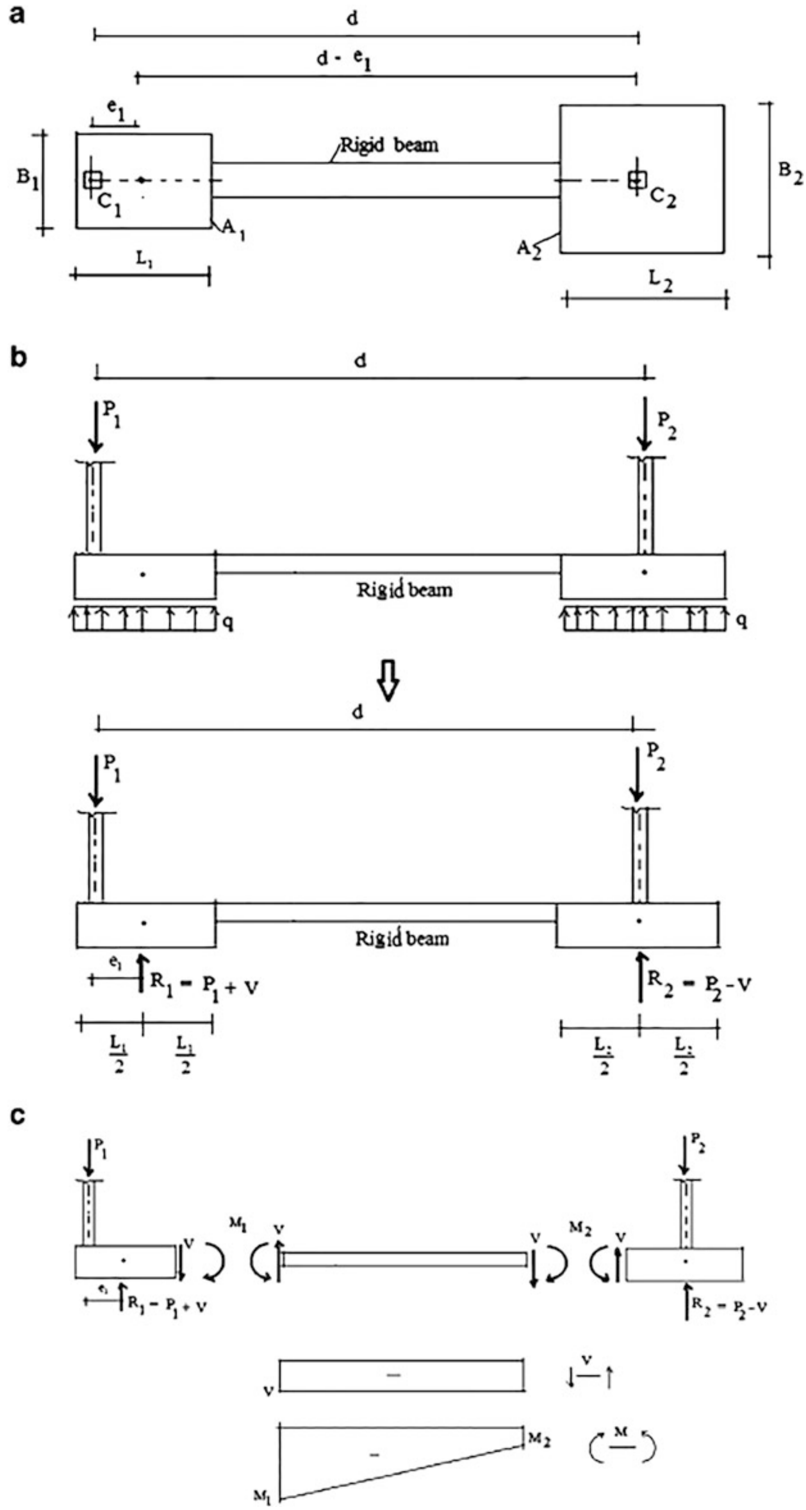


Fig. 7.19 Approximate strap footing analysis. (**a**) Plan (**b**) Elevation (**c**) Components of footing

Then

$$R_1 = P_1 + \frac{P_1 e_1}{d - e_1} \tag{7.19}$$

Let

$$V = \frac{P_1 e_1}{d - e_1} \tag{7.20}$$

then

$$\begin{aligned} R_1 &= P_1 + V \\ R_2 &= P_2 - V \\ R &= R_1 + R_2 = P_1 + P_2 \end{aligned} \tag{7.21}$$

The quantity, V, is the shear force in the strap beam.

Once e_1 is specified, one can determine R_1 and R_2. We also *assume* the soil pressure acting on the footing is constant and equal to the effective soil pressure (q_e). Then,

$$\begin{aligned} A_{1\,\text{required}} &= \frac{R_1}{q_e} \\ A_{2\,\text{required}} &= \frac{R_2}{q_e} \end{aligned} \tag{7.22}$$

Typical reinforcing patterns required for bending in strap footings are illustrated in Fig. 7.20.

Fig. 7.20 Typical reinforcing patterns

Example 7.5

Given: The eccentrically loaded footing A connected to the concentrically loaded footing B by strap beam as shown below. Assume the strap is placed such that it does not bear directly on the soil (Figs. E7.5a and E7.5b).

Determine: The soil pressure profile under the footings.

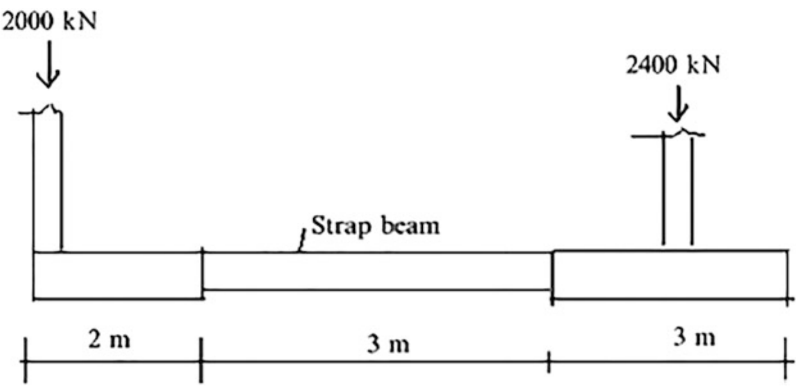


Fig. E7.5a Elevation

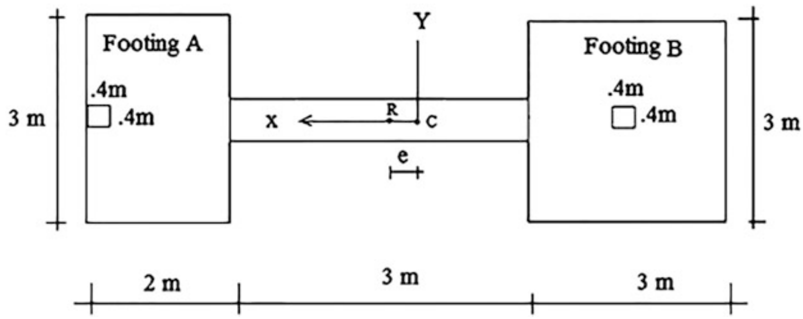


Fig. E7.5b Plan view

Solution: Noting Fig. 7.17, the various measures are

$$d = 6.3\,\mathrm{m}$$

$$x_1 = \frac{2400(6.3)}{4400} = 3.436\,\mathrm{m}$$

$$A_1 = 2(3) = 6\,\mathrm{m}^2$$

$$A_2 = 3(3) = 9\,\mathrm{m}^2$$

$$R = 2000 + 2400 = 4400\,\mathrm{m}^2$$

$$6\overline{d}_1 = 9\left(5.5 - \overline{d}_1\right) \Rightarrow \overline{d}_1 = 3.3\,\mathrm{m} \quad \overline{d}_2 = 2.2\,\mathrm{m}$$

$$d_1 = \overline{d}_1 + 0.8 = 4.1\,\mathrm{m}$$

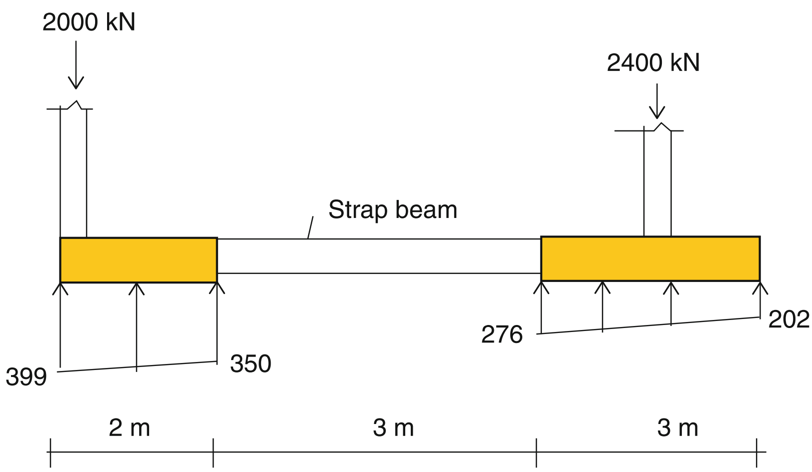
$$e = d_1 - x_1 = 0.66\,\mathrm{m}$$

$$I_{Y1} + I_{Y2} = \frac{3(2)^3}{12} + 6(3.3)^2 + \frac{3(3)^3}{12} + 9(2.2)^2 = 117.65\,\mathrm{m}^4$$

Note that e is positive when R is located to the left of the centroid

$$q(x) = \frac{R}{(A_1 + A_2)} + \frac{Re}{(I_{Y1} + I_{Y2})}x$$
$$= \frac{4400}{15} - \frac{4400(0.66)}{117.65}x = 293.3 + 24.7x$$
$\therefore$
$$q(4.3) = 399\,\mathrm{kN/m^2}$$
$$q(2.3) = 350\,\mathrm{kN/m^2}$$
$$q(-0.7) = 276\,\mathrm{kN/m^2}$$
$$q(-3.7) = 202\,\mathrm{kN/m^2}$$

The corresponding soil pressure is shown below.



Example 7.6 Dimensioning a Strap Footing

Given: The exterior column C_1 is 12 in. × 12 in. and carries a dead load of 160 kip and a live load of 130 kip. The interior column C_2 is 16 in. × 16 in. and carries a dead load of 200 kip and a live load of 185 kip. The property line is at the edge of column #1 and the distance between the center lines of the columns #1 and #2 is 20 ft. The effective soil pressure is $q_e = 4.625$ kip/ft^2 (Figs. E7.6a and E7.6b).

Determine: The dimensions of the footings for both columns using the two methods described above.

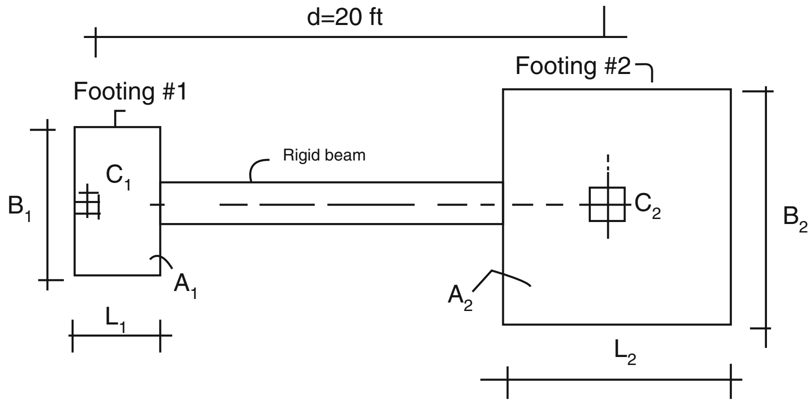


Fig. E7.6a Plan

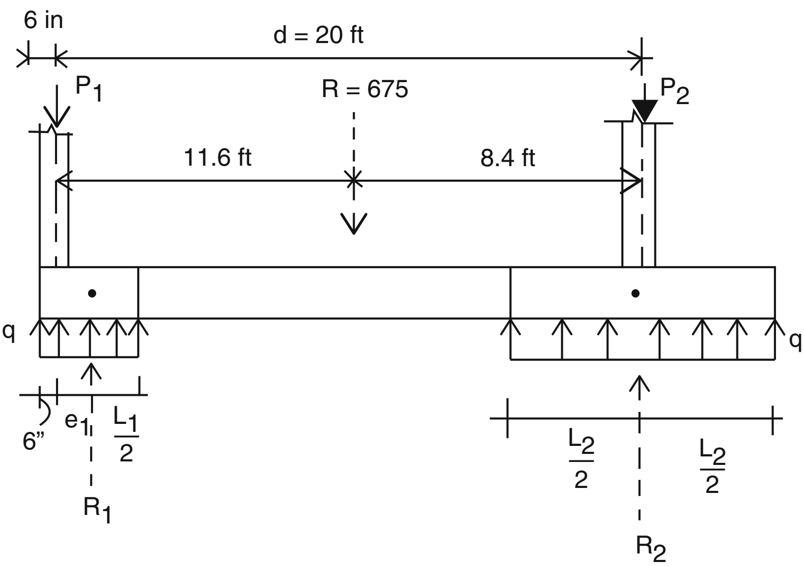


Fig. E7.6b Elevation

Solution:

Procedure #1: The individual column loads are:

$$P_1 = 160 + 130 = 290\,\text{kip}$$

$$P_2 = 200 + 185 = 385\,\text{kip}$$

Next, we locate the resultant of the column loads.

$$R = P_1 + P_2 = 675\,\text{kip}$$

$$d_1 = x_1 = \frac{385}{675}(20) = 11.407\,\text{ft}$$

$$d_2 = x_2 = 20 - 11.407 = 8.593\,\text{ft}$$

Noting (7.16), we obtain:

$$A_1 + A_2 = \frac{675}{4.625} \geq 146\,\text{ft}^2$$

We estimate A_2 knowing R_2, the resultant of the pressure distribution acting on footing # 2, is less than P_2.

$$A_2 \lesssim \frac{385}{4.625} = 83\,\text{ft}^2$$

We take $L_2 = B_2 = 8.5\,\text{ft}\left(A_2 = 72.25\,\text{ft}^2\right)$.

Then $A_1 \geq 73.75\,\text{ft}^2$

Noting (7.17),

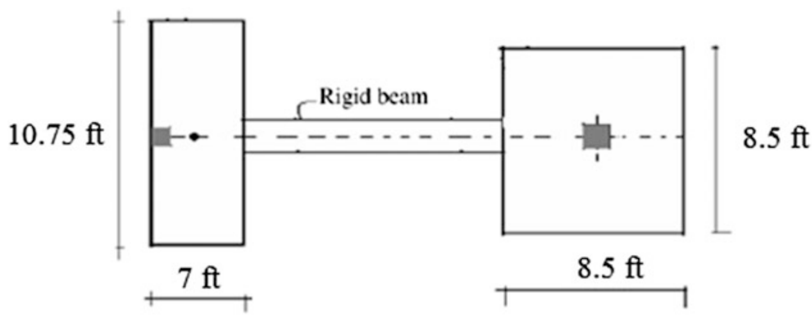
$$73.75\overline{d}_1 = 72.25(8.593) \quad \Rightarrow \quad \overline{d}_1 = 8.418\,\text{ft}$$

Then

$$\frac{L_1}{2} \approx e_1 + \left(d_1 - \overline{d}\right) = 0.5 + (11.407 - 8.418) = 3.49\,\text{ft}$$

Take $L_1 = 7\,\text{ft}$ and $B_1 = 10.75\,\text{ft}\left(A_1 = 75.25\,\text{ft}^2\right)$

The final dimensions are shown below.



Procedure #2: We illustrate the second design approach here**.** We estimate A_1 by requiring the pressure under the footing #1 to be equal to q_e.

$$A_1 > \frac{P_1}{q_e} = \frac{290}{4.625} = 62.7\,\text{ft}^2$$

We take $L_1 = 6$ ft as a first estimate. Then, noting Fig. E7.6b

$$e_1 = \frac{L_1}{2} - 0.5 \approx 2.5\,\text{ft}$$

The remaining steps are listed below

$$V = \frac{P_1 e_1}{d - e_1} = \frac{290(2.5)}{20 - 2.5} = 41.43\,\text{kip}$$

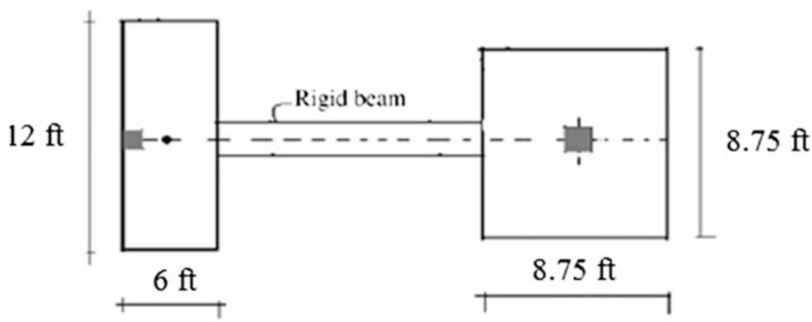
$$R_1 = P_1 + V = 290 + 41.43 = 331.43\,\text{kip}$$

$$R_2 = P_2 - V = 385 - 41.43 = 343.57\,\text{kip}$$

$$A_{1_{\text{required}}} = \frac{331.43}{4.625} = 71.66\,\text{ft}^2 \Rightarrow B_1 = \frac{71.66}{6} = 11.94 \Rightarrow L_1 = 6\,\text{ft} \quad B_1 = 12\,\text{ft}$$

$$A_{2_{\text{required}}} = \frac{343.57}{4.625} = 74.29\,\text{ft}^2 \Rightarrow L_2 = B_2 = \sqrt{74.29} = 8.62 \Rightarrow B_2 = L_2 = 8.75\,\text{ft}$$

The final dimensions are shown below.



Repeating this computation for the ultimate loading case,

$$P_{1u} = 1.2P_D + 1.6P_L = 1.2(160) + 1.6(130) = 400\,\text{kip}$$
$$P_{2u} = 1.2P_D + 1.6P_L = 1.2(200) + 1.6(185) = 536\,\text{kip}$$

and assuming the same value for e_1 leads to

$$V_u = \frac{P_{u1}e_1}{d - e_1} = \frac{400(2.5)}{17.5} = 57.14\,\text{kip}$$

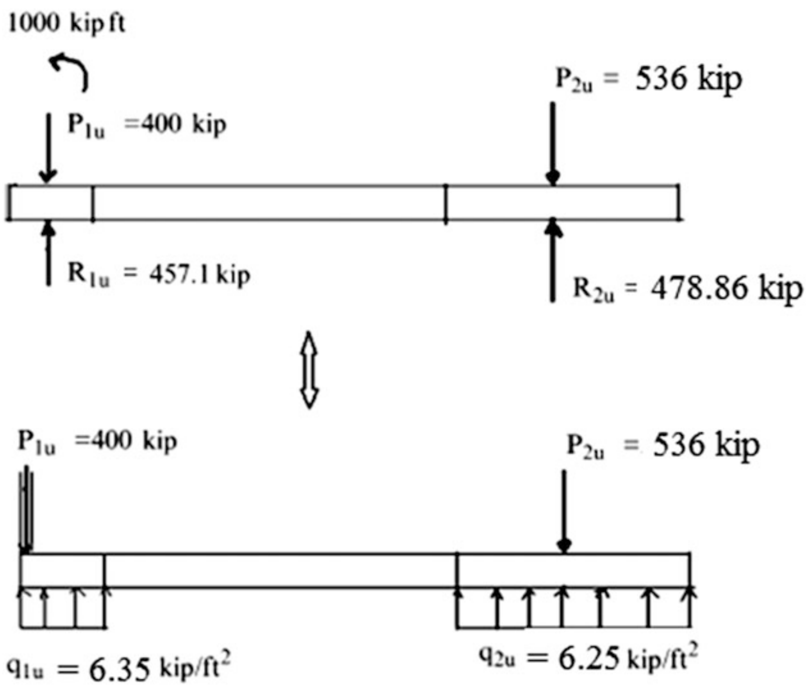
$$R_{1u} = P_{1u} + V_u = 400 + 57.14 = 457.14\,\text{kip}$$

$$q_{1u} = \frac{R_{1u}}{B_1L_1} = \frac{457.14}{6(12)} = 6.35\,\text{kip/ft}^2$$

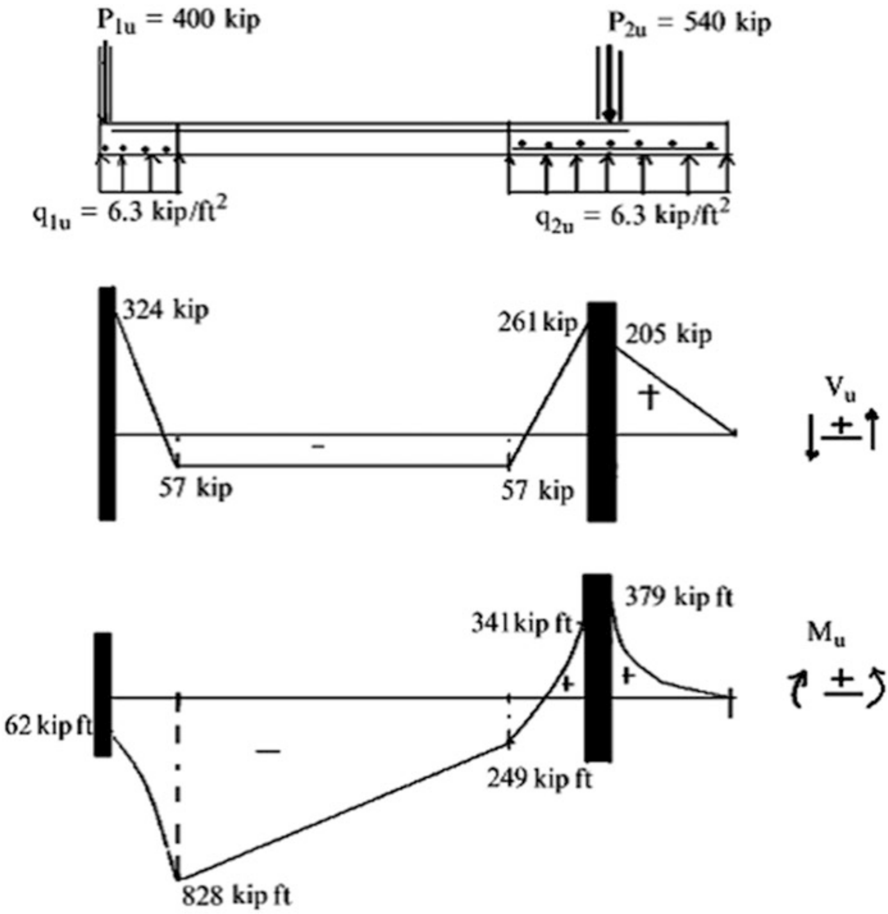
$$R_{2u} = P_{2u} - V_u = 536 - 57.14 = 478.86\,\text{kip}$$

$$q_{2u} = \frac{R_{2u}}{B_2L_2} = \frac{478.86}{8.75(8.75)} = 6.25\,\text{kip/ft}^2$$

The corresponding forces are shown in the sketches below.



The shear and moment diagrams are plotted below.



7.6 Summary

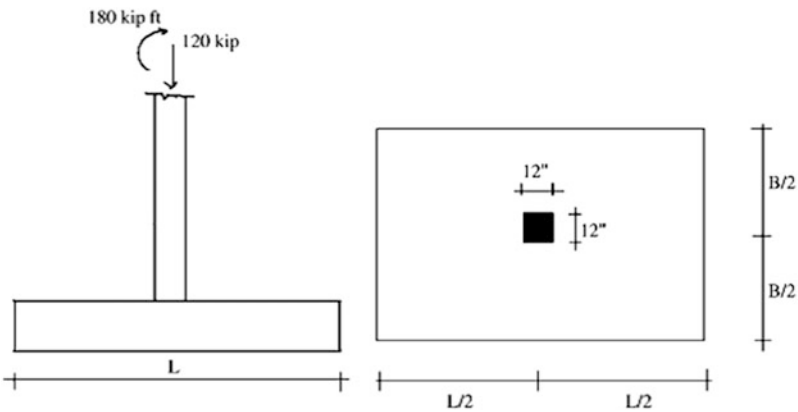
7.6.1 Objectives of the Chapter

- To describe the various types of footings used in shallow foundations.
- To develop an analytical procedure for dimensioning footings.
- To develop a general analytical procedure for generating the shear and moment distribution in footings based on the assumption of a linear soil pressure distribution.
- To identify critical loading conditions which produce pressure loading distributions with high peak magnitudes.

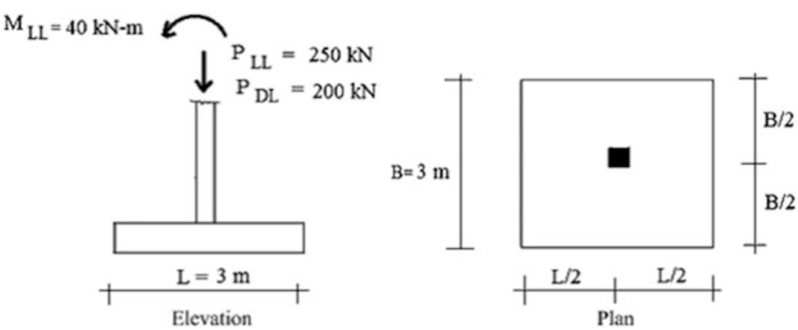
7.7 Problems

Problem 7.1
Consider the footing geometry shown below. Determine the soil pressure distribution corresponding to

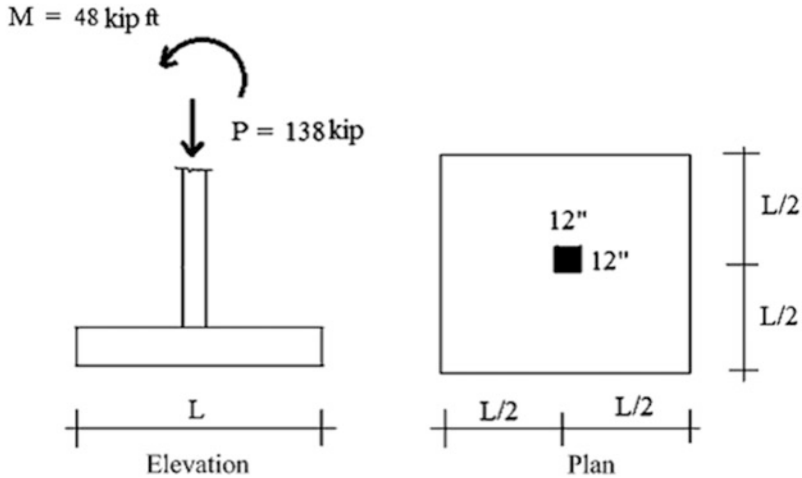
(a) $B = L = 8$ ft
(b) $L = 10$ ft, $B = 5$ ft



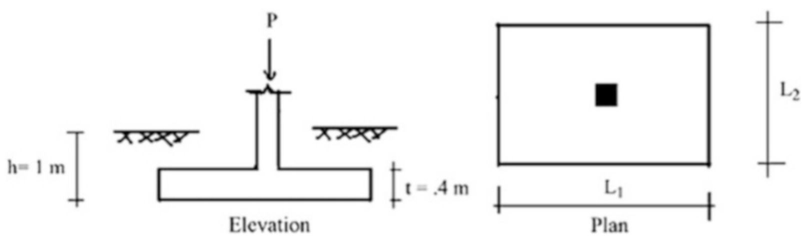
Problem 7.2
The plan view and elevation of a single footing supporting a 300 mm × 300 mm column are shown below. Determine the soil pressure distribution under the footing. Use a factor of 1.2 for DL and 1.6 for LL.



Problem 7.3
The plan view and elevation of a single footing supporting a column are shown below. The effective soil pressure is 4 kip/ft^2. Determine the required value of L.

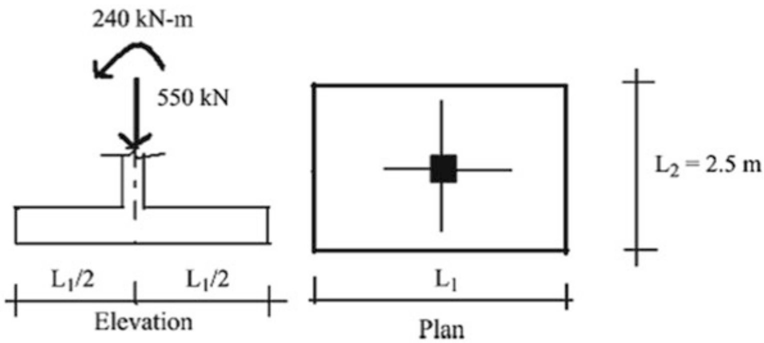


Problem 7.4
A 450 mm × 450 mm concentrically load column is to be supported on a shallow foundation. The base of the footing is 1 m below grade. Estimate the size of the footing using service loads. Draw shear and moment diagrams using a factor load of $P_u = 1.2P_D + 1.6P$. The allowable soil pressure is $q_{allowable} = 250$ kN/m^2, $\gamma_{soil} = 18$ kN/m^3, $\gamma_{conc} = 24$ kN/m^3, $P_D = 1000$ kN, and $P_L = 1400$ kN. Consider: (a) A square footing ($L_1 = L_2 = L$) and (b) A rectangular footing with $L_2 = 2.5$ m.

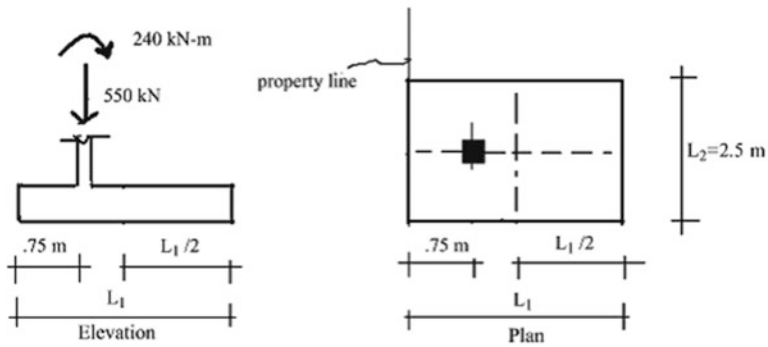


Problem 7.5
A 350 mm × 350 mm column is to be supported on a shallow foundation. Determine the dimensions (either square or rectangular) for the following conditions. The effective soil pressure is $q_{effective} = 180$ kN/m^2.

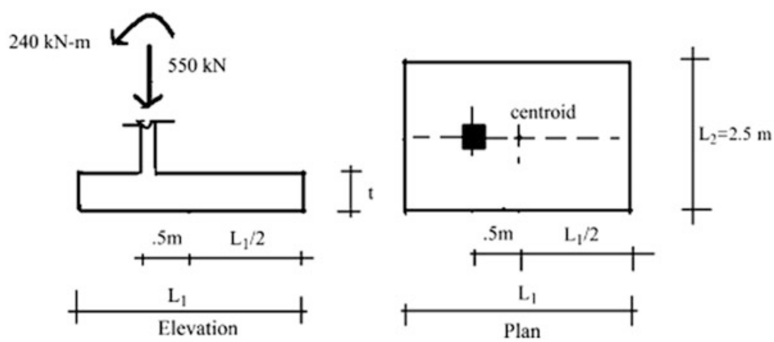
(a) The center line of the column coincides with the center line of the footing.



(b) The center line of the column is 0.75 m from the property line.



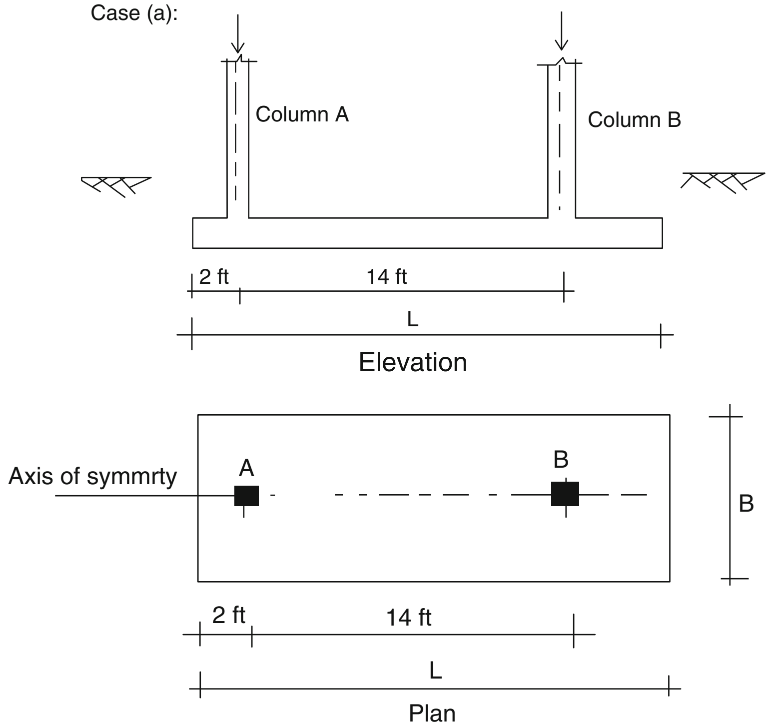
(c) The center line of the column is 0.5 m from the centroid of the footing.



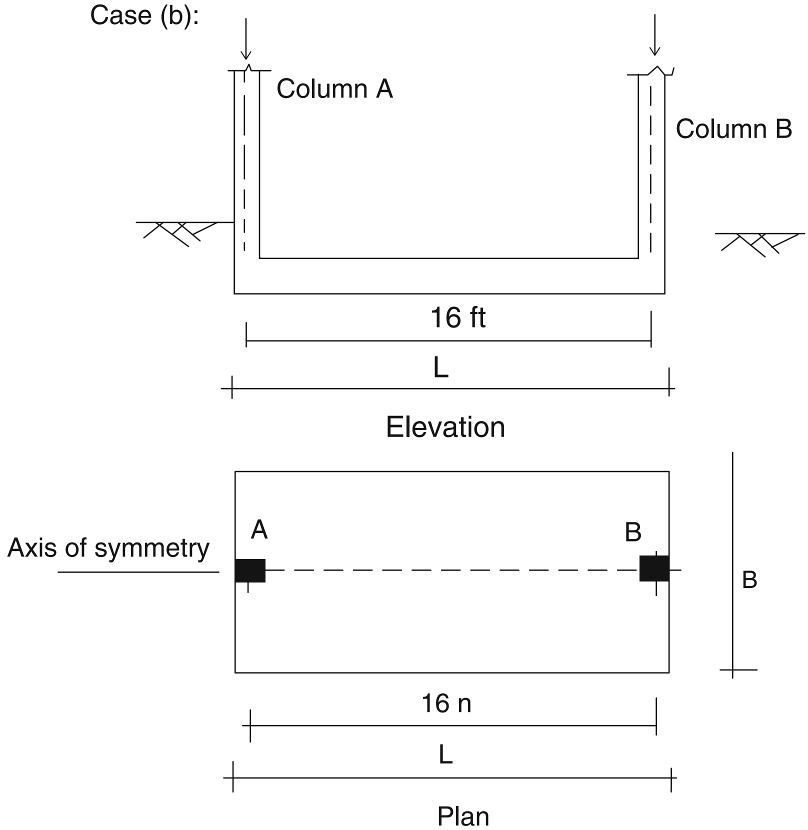
Problem 7.6

A combined footing supports two square columns: Column A is 14 in. × 14 in. and carries a dead load of 140 kip and a live load of 220 kip. Column B is 16 in. × 16 in. and carries a dead load of 260 kip and a live load of 300 kip. The effective soil pressure is $q_e = 4.5$ kip/ft^2. Assume the soil pressure distribution is uniform, except for case (b). Determine the footing dimensions for the following geometric configurations. Establish the shear and moment diagrams corresponding to the factored loading, $P_u = 1.2P_D + 1.6P_L$.

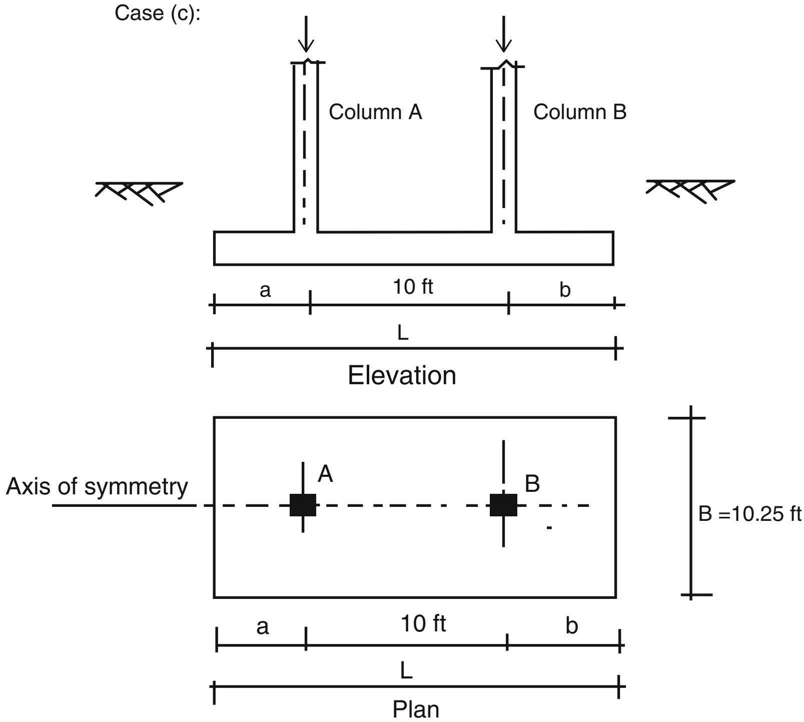
Case (a):


Column A
Column B
2 ft
14 ft
L
Elevation
Axis of symmrty
A
B
B
2 ft
14 ft
L
Plan

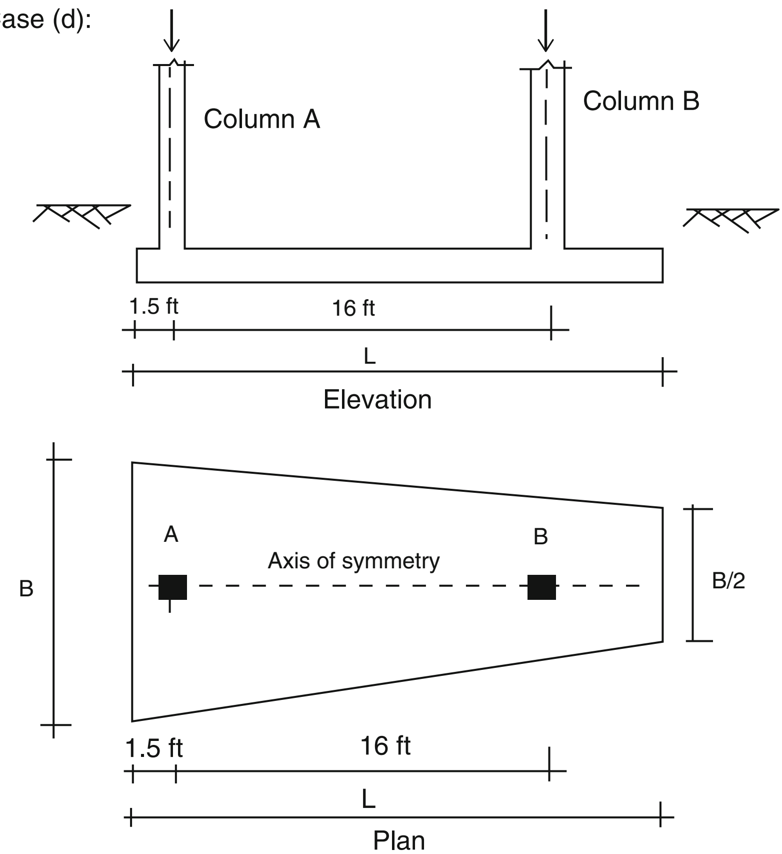
Case (b):

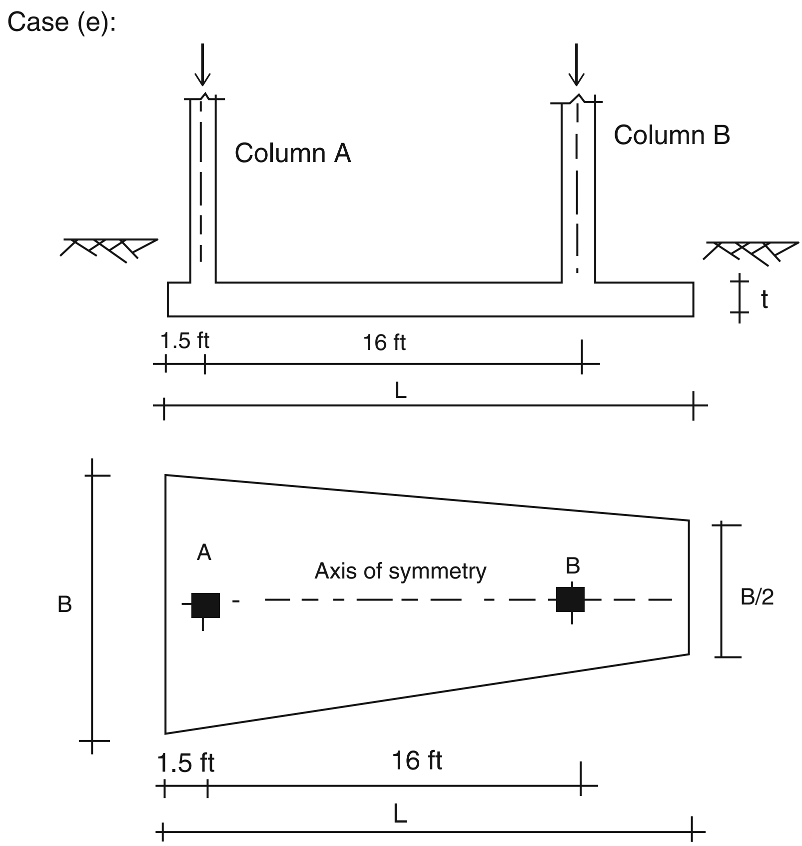

Column A
Column B
16 ft
L
Elevation
Axis of symmetry
A
B
B
16 n
L
Plan

Case (c):


Column A
Column B
a
10 ft
b
L
Elevation
Axis of symmetry
A
B
B =10.25 ft
a
10 ft
b
L
Plan

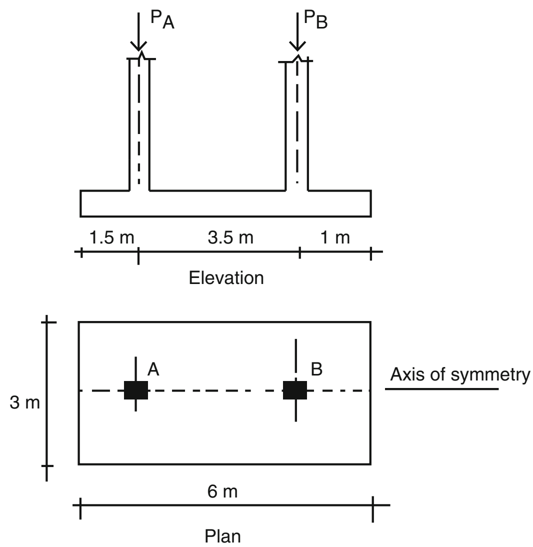
Case (d):


Column A
Column B
1.5 ft
16 ft
L
Elevation
B
A
B
Axis of symmetry
B/2
1.5 ft
16 ft
L
Plan



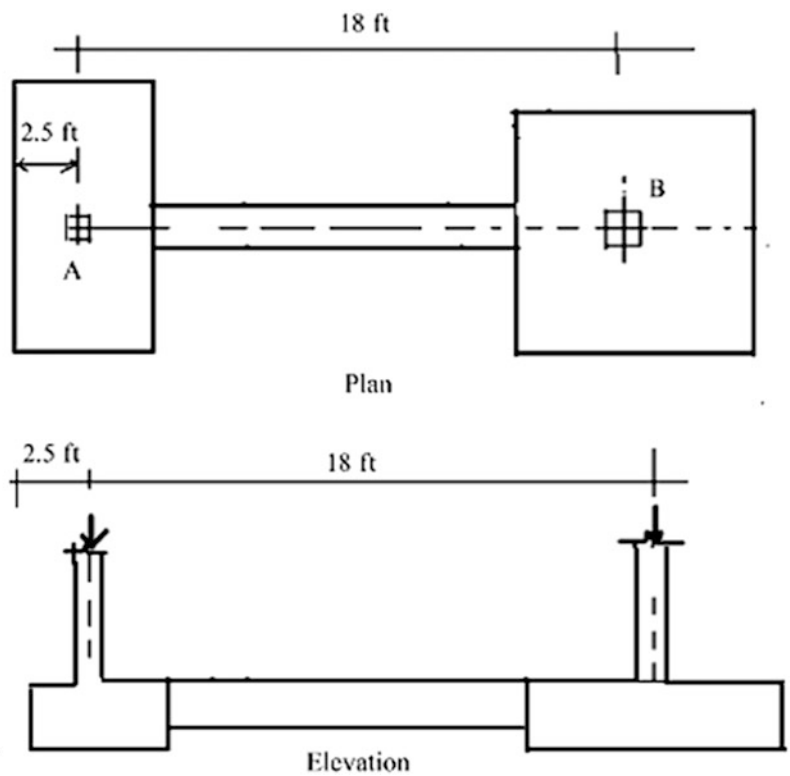
Problem 7.7

Column A is 350 mm × 350 mm and carries a dead load of 1300 kN and a live load of 450 kN. Column B is 450 mm × 450 mm and carries a dead load of 1400 kN and a live load of 800 kN. The combined footing shown below is used to support these columns. Determine the soil pressure distribution and the shear and bending moment distributions along the longitudinal direction corresponding to the factored loading, $P_u = 1.2P_D + 1.6P_L$.



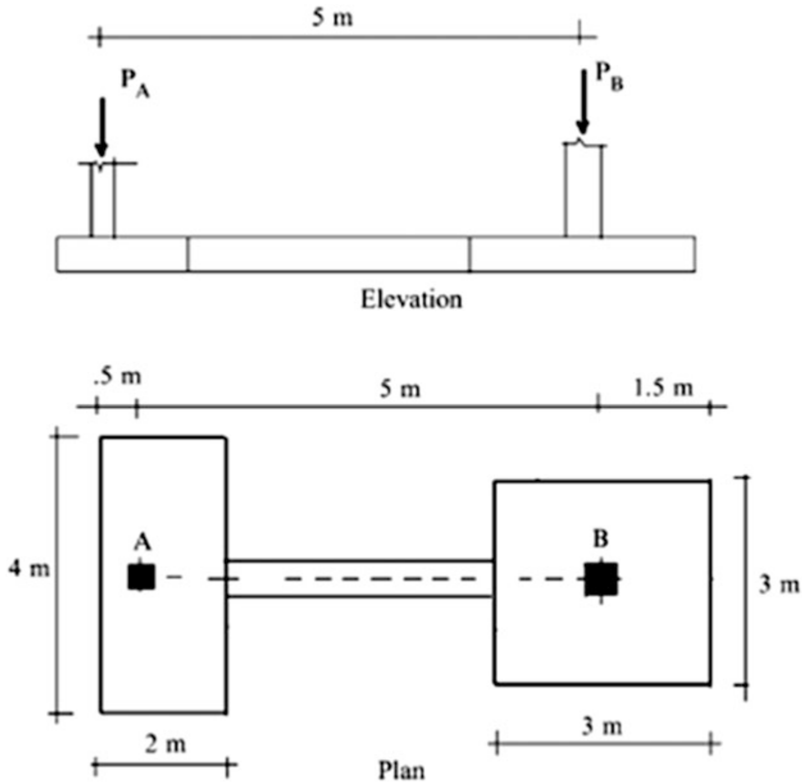
Problem 7.8

Dimension a strap footing for the situation shown. The exterior column A is 14 in. × 14 in. and carries a dead load of 160 kip and a live load of 130 kip; the interior column B is 18 in. × 18 in. and carries a dead load of 200 kip and a live load of 187.5 kip; the distance between the center lines of the columns is 18 ft. Assume the strap is placed such that it does not bear directly on the soil. Take the effective soil pressure as $q_e = 4.5$ kip/ft^2. Draw shear and moment diagrams using a factored load of $P_u = 1.2P_D + 1.6P_L$.



Problem 7.9

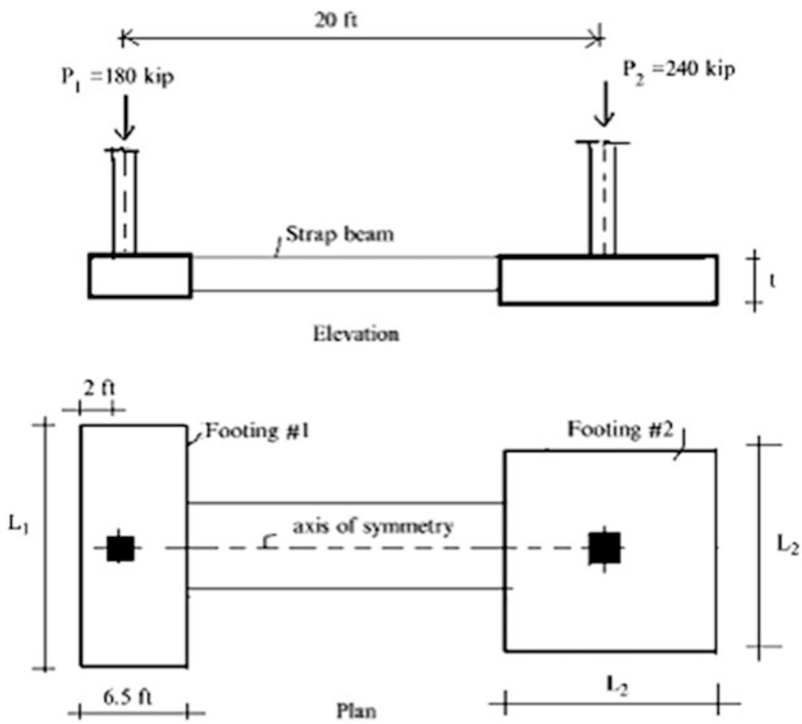
Column A is 350 mm × 350 mm and carries a dead load of 1300 kN and a live load of 450 kN. Column B is 450 mm × 450 mm and carries a dead load of 1400 kN and a live load of 800 kN. A strap footing is used to support the columns and the center line of Column A is 0.5 m from the property line. Assume the strap is placed such that it does not bear directly on the soil. Determine the soil pressure distribution and the shear and bending moment distributions along the longitudinal direction corresponding to the factored loading, $P_u = 1.2P_D + 1.6P_L$.



Problem 7.10

An exterior 18 in. × 18 in. column with a total vertical service load of $P_1 = 180$ kip and an interior 20 in. × 20 in. column with a total vertical service load of $P_2 = 240$ kip are to be supported at each column by a pad footing connected by a strap beam. Assume the strap is placed such that it does not bear directly on the soil.

(a) Determine the dimensions L_1 and L_2 for the pad footings that will result in a uniform effective soil pressure not exceeding 3 kip/ft^2 under each pad footing. Use ¼ ft increments.
(b) Determine the soil pressure profile under the footings determined in part (a) when an additional loading, consisting of an uplift force of 80 kip at the exterior column and an uplift force of 25 kip at the interior column, is applied.



Reference

1. Terzaghi K, Peck RB. Soil mechanics in engineering practice. New York: Wiley; 1967.