Vertical Loads on Multistory Buildings 15

Abstract

The previous chapter dealt with issues related to the lateral loadings on building systems. In that chapter, we described how one can represent the global lateral loading as loads acting on the individual frames contained in the building system. We focus in this chapter on how one treats vertical loads such as gravity loads. Gravity loads applied to a floor slab are converted to distributed loads acting on the beams which support the slab. Since the floor slab loads involve both dead and live loads, one needs to investigate various floor slab loading patterns in order to establish the maximum values of the design parameters such as bending moment. We apply Müller-Breslau principle for this task. The last section of the chapter contains a case study which illustrates the process of combining lateral and vertical loading, and demonstrates the sensitivity of the structural design to the type of structural system.

15.1 Loads on Frames

Figure 15.1 shows a multistory building system and Figure 15.2 shows a typical makeup of a rectangular building; the structural system is composed of floor slabs that are supported by frames arranged in an orthogonal pattern. The action of wind and earthquake is represented by concentrated lateral loads applied to the nodes. Gravity loads acting on the floor slabs or roof are transferred to the beams and then to the columns. The nature of these beam loads (uniform, concentrated, triangular, trapezoidal) depends on the makeup of the flooring system. In this chapter, we examine first the mechanism by which the floor loads are transferred to the beams and then describe how to establish the critical loading pattern that produces the peak values of moment in an individual beam. Given the peak moments, one can select appropriate beam cross sections.

J.J. Connor, S. Faraji, *Fundamentals of Structural Engineering*,
DOI 10.1007/978-3-319-24331-3_15

Fig. 15.1 Multistory building

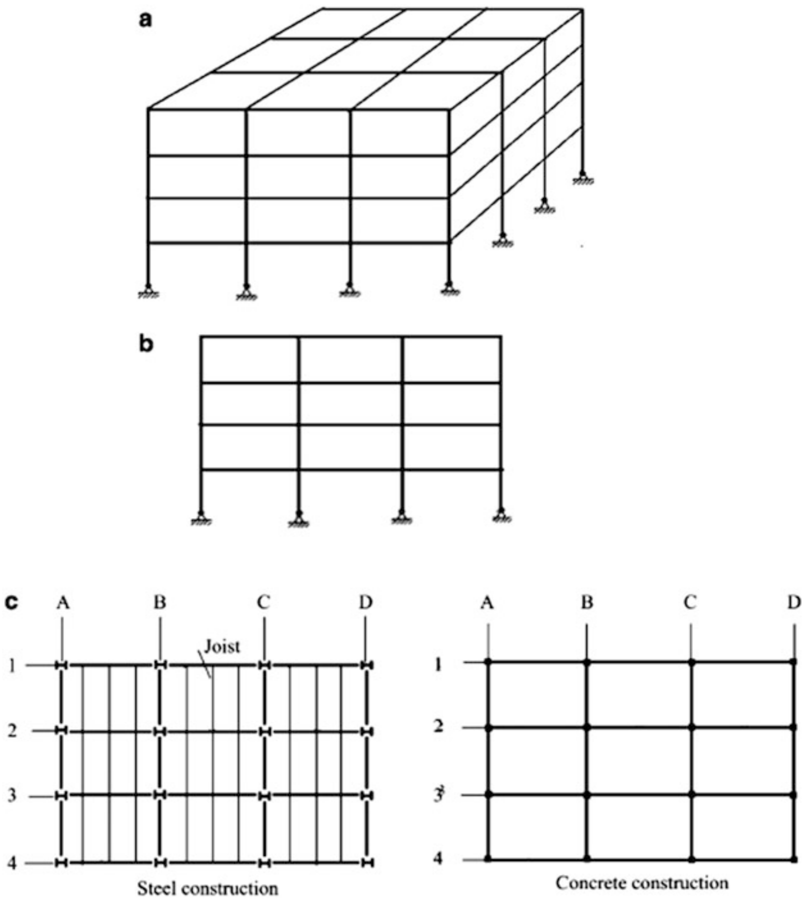


Fig. 15.2 Rectangular building. (**a**) Building frame. (**b**) Elevation view—individual frame. (**c**) Typical plan view—flooring system

15.2 Treatment of Gravity Floor Loads

Figure 15.3 shows a rectangular segment of the floor (abcd) bounded by columns at its corners and beams along its sides. In typical concrete construction, the floor slab and beams are framed simultaneously. The floor slab–beam system functions as a rectangular plate supported on all its sides. If the load is transmitted to all the sides, we refer to this behavior as two-way action. When the load tends to be transmitted primarily in one direction, this behavior is called one-way action.

Whether one-way or two-way action occurs depends on the dimensions and makeup of the floor slab. The most common approach is to work with the tributary areas defined in Fig. 15.4. One constructs 45° lines and computes individual areas. The loading on an area is assigned to the adjacent beam. When the members are located in the interior, these areas are doubled to account for the adjacent panels. In general, the areas are either triangular or trapezoidal.

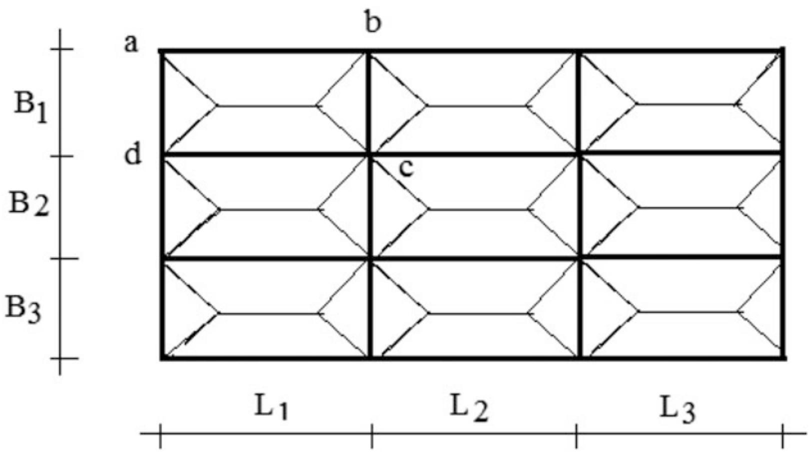


Fig. 15.3 Slab–beam framing scheme—two-way action

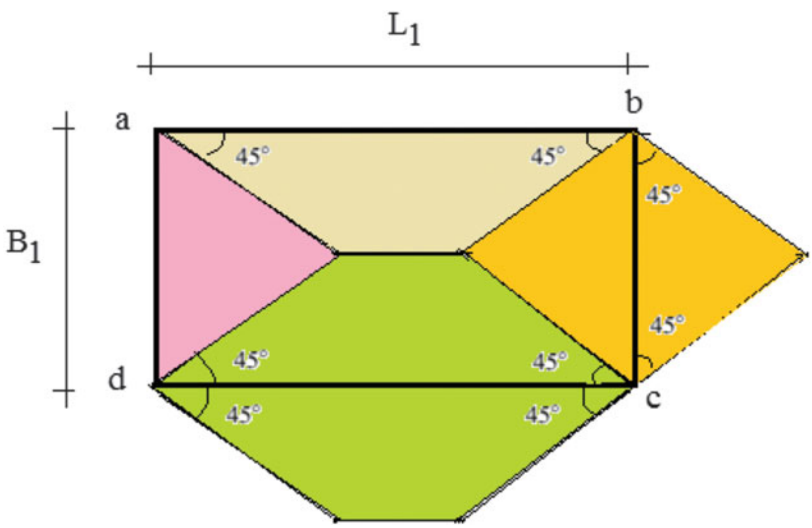


Fig. 15.4 Tributary areas for floor panel abcd

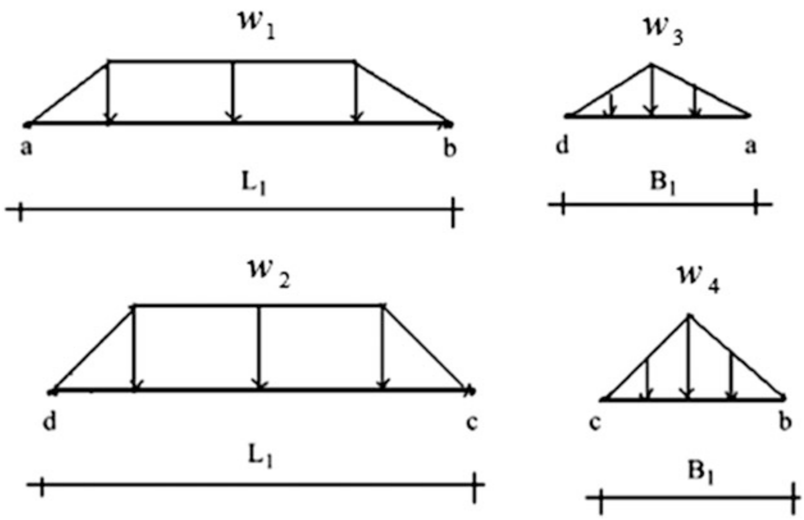


Fig. 15.5 Two-way action perimeter beam loadings for uniform floor loading—panel abcd

The floor loading is represented as a uniform load q (Ib/ft^2 or N/m^2) applied to the floor slab. Using the concept of tributary areas, we convert this loading to a line loading w (Ib/ft or N/m) on the perimeter floor beams. The loading patterns for the beams supporting panel abcd and adjacent panels shown in Fig. 15.5 are listed below.

$$\begin{cases} w_1 = q\left(\dfrac{B_1}{2}\right) \\ w_2 = q\left(\dfrac{B_1}{2} + \dfrac{B_2}{2}\right) \\ w_3 = q\left(\dfrac{B_1}{2}\right) \\ w_4 = qB_1 \end{cases}$$

When steel members are used, the usual approach is to form the floor by first installing joists, then overlaying steel decking, and lastly casting a thin layer of concrete. Loading applied to the floor is transferred through the decking to the joists and ultimately to the beams supporting the joists. For the geometry shown in Fig. 15.6, beams ab and cd carry essentially all the loads applied to the floor panel abcd. The loads on beams ad and bc are associated with the small tributary areas between them and the adjacent joists. Depending on the joist spacing, the beam loads are represented either as concentrated loads or as a uniformly distributed load. The loading patterns are shown in Fig. 15.7 are listed below.

$$\begin{cases} P_1 = q\, a_1\left(\dfrac{B_1}{2}\right) \\ P_2 = q\, a_1\left(\dfrac{B_1}{2} + \dfrac{B_2}{2}\right) \\ w_1 = q\left(\dfrac{B_1}{2}\right) \\ w_2 = q\left(\dfrac{B_1}{2} + \dfrac{B_2}{2}\right) \end{cases}$$

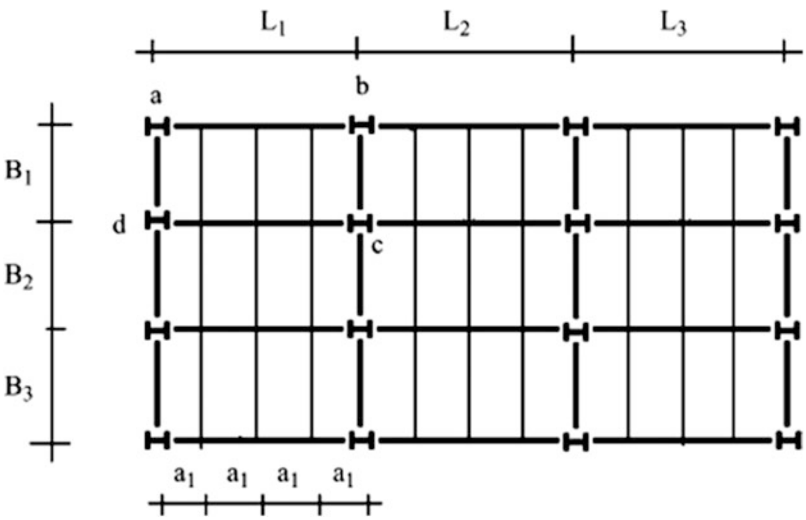


Fig. 15.6 Steel joist/beam framing scheme

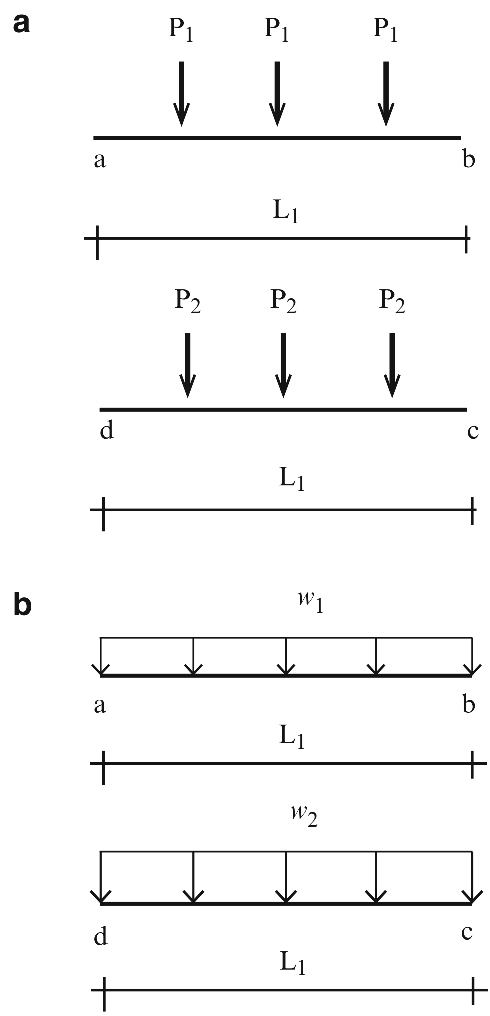


Fig. 15.7 One-way action beam loading for uniform floor loading q. (**a**) Large joist spacing. (**b**) Small joist spacing

15.3 Live Load Patterns for Frame Structures

Gravity type loading is usually the dominant loading for low-rise multistory frames. It consists of both dead and live loading. Given a multistory frame, the first step is to establish the critical loading patterns for the individual members. Once the loading patterns are established, one can carry out an approximate analysis to generate peak force values which are used for the initial design. From then on, one iterates on member properties using an exact analysis method. In this section, we describe how Müller-Breslau's Principle can be employed to establish loading patterns for live gravity loading. We also describe some approximate techniques for estimating the peak positive and negative moments in beams.

Consider the frame shown in Fig. 15.8. We suppose the gravity live loading is a uniformly distributed load, w, that can act on a portion of any member. Our objective here is to determine the loading patterns that produce the maximum positive moment at A and maximum negative moment at B.

To determine the positive moment at A, we insert a moment release at A and apply self-equilibrating couples as indicated in Fig. 15.9. According to the Müller-Breslau Principle, one applies a downward load to those spans where the beam deflection is upward to produce the maximum positive moment at A. The corresponding loading pattern is shown in Fig. 15.10.

Referring back to Fig. 15.8, we establish the loading pattern for the negative moment at B by inserting a moment release at B and applying a negative moment. In this case, there are two possible deflected shapes depending upon whether one assumes the inflection points are in either the columns

Fig. 15.8 Multistory frame example

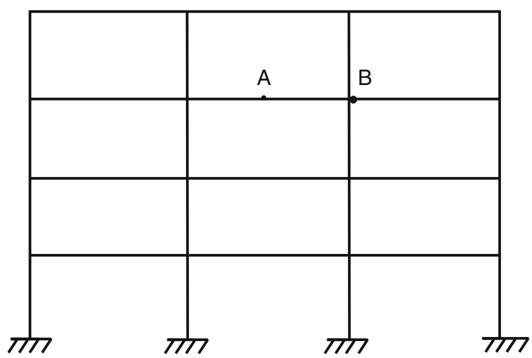


Fig. 15.9 Deflection pattern for positive moment at A

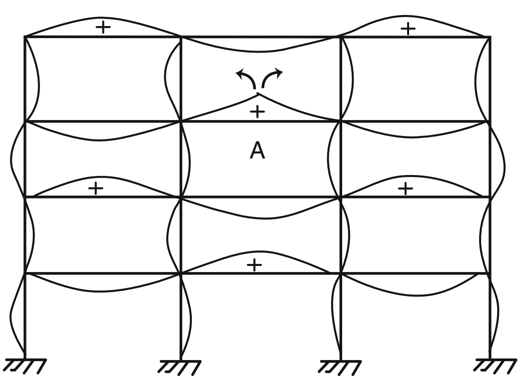


Fig. 15.10 Loading pattern for maximum positive moment at A

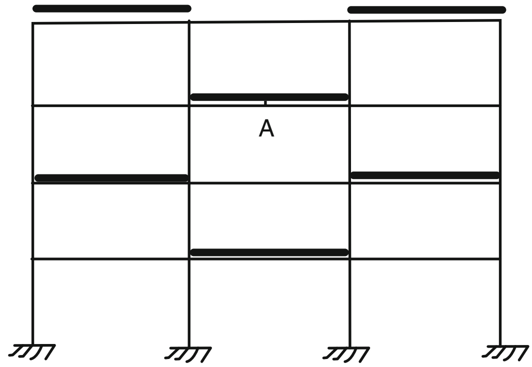


Fig. 15.11 Deflection pattern for negative moment at B—inflection points in beams

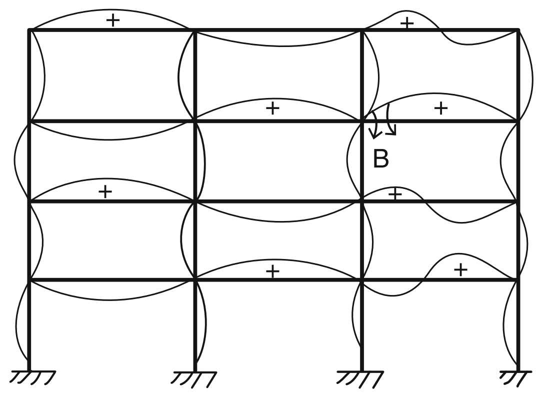
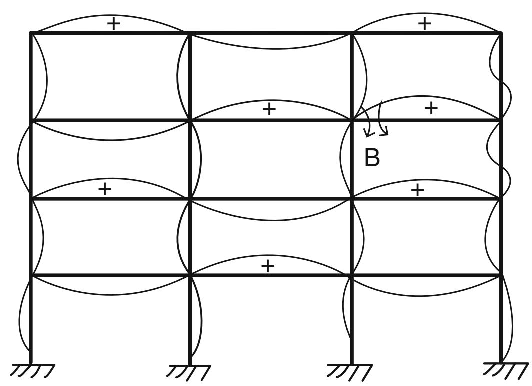
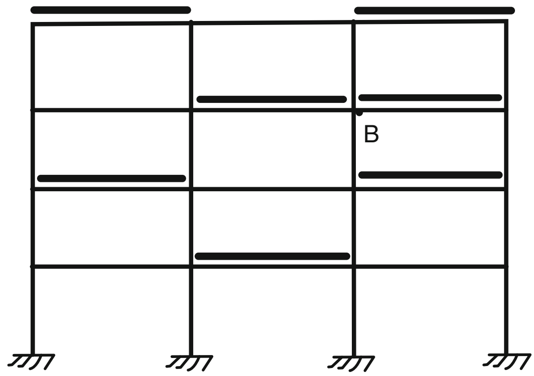


Fig. 15.12 Deflection pattern for negative moment at B—inflection points in columns



or the beams. These shapes are plotted in Figs. 15.11 and 15.12. The exact shape depends on the relative stiffness of the beams and columns which is not known at the preliminary design phase. Although there are cases where there is some ambiguity in the deflected shape, the Müller-Breslau Principle is a very useful tool for generating a qualitative first estimate of the loading pattern (Fig. 15.13). One can refine the estimate later using a structural analysis software system.

Fig. 15.13 Loading pattern for maximum negative moment at B—inflection points in columns



Example 15.1

Given: The rigid steel frame defined in Fig. E15.1a. Assume the member loading is a uniformly distributed live load.

Determine:

(a) Critical loading patterns for gravity live loading using Müller-Breslau's Principle that produces the peak value of moments at mid-spans and end points of the beams.

(b) Use a computer software package to compare the maximum moment corresponding to the critical pattern loading to the results for a uniform loading on all members. Consider all the girders to be the same size and all the columns to be the same size.

Assume $L_1 = 6$ m, $L_2 = 9$ m, $h = 4$ m, $w = 10$ kN/m, and $I_G = 3.5I_C$

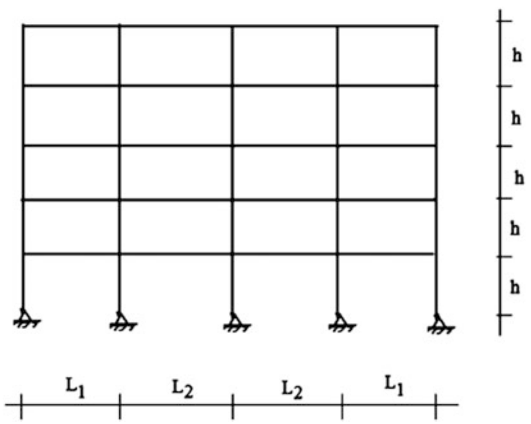


Fig. E15.1a Rigid steel frame

Solution:

Part (**a**): The process followed to determine the critical loading patterns for bending moment in the beams is described below.

Step 1: Positive Moment at mid-span of the beams.

There are two live load patterns for positive moment at the midpoint of the beams. They are listed in Fig. E15.1b.

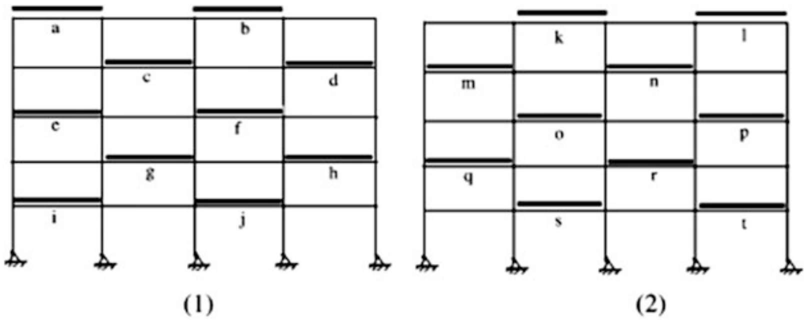


Fig. E15.1b Positive moment loading patterns

Step 2: Negative moment at the end point of the beams

There are 15 patterns of uniform loading for negative moment. Typical patterns are shown in Fig. E15.1c. One carries out analyses for the 15 different loading patterns, and then represents the results by a discrete moment envelope. Figure E15.1d shows the final results.

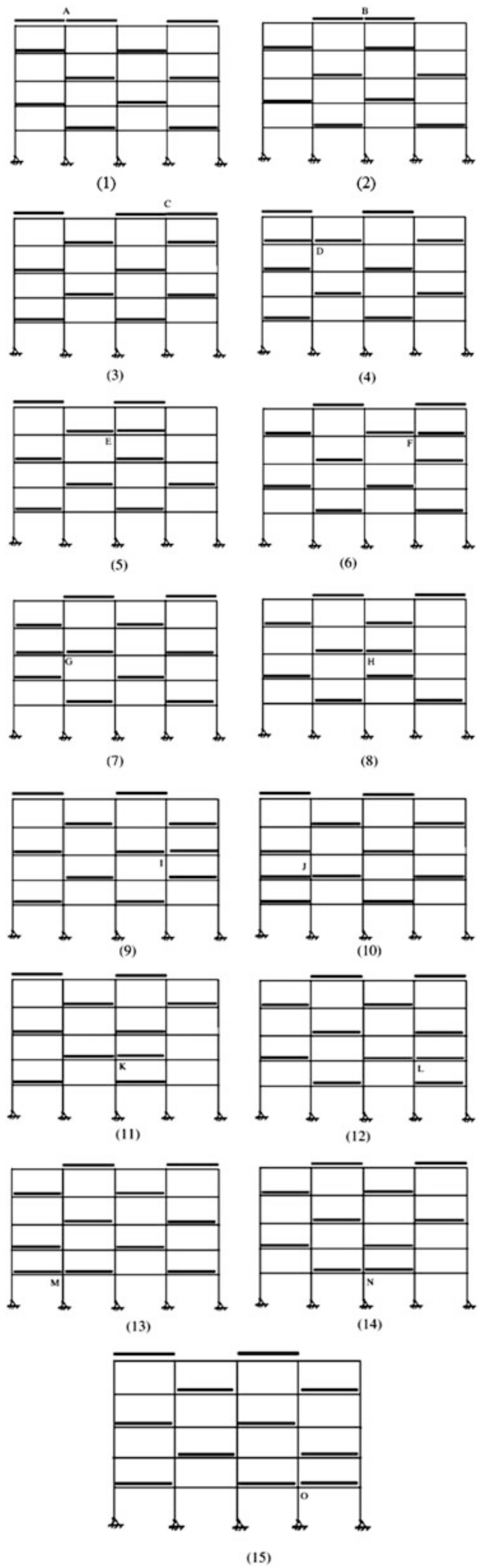


Fig. E15.1c Live load patterns for negative moment

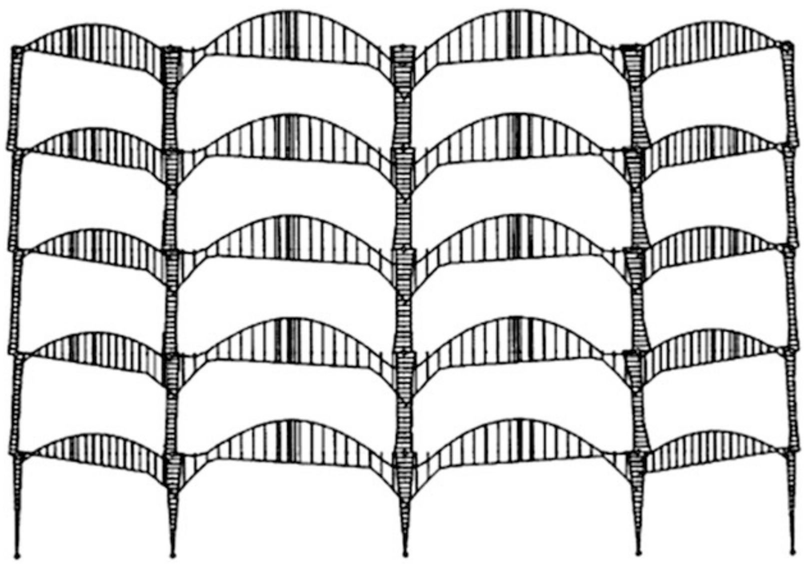


Fig. E15.1d Discrete moment envelope for pattern loading

Part (b): The moment results for uniform loading are plotted in Fig. E15.1e. We note that the uniform loading produces results which underestimate the peak values (30 % for positive moment and 11 % for negative moment). However, since the uniform loading case is easy to implement and provides reasonable results, it frequently is used to generate a first estimate.

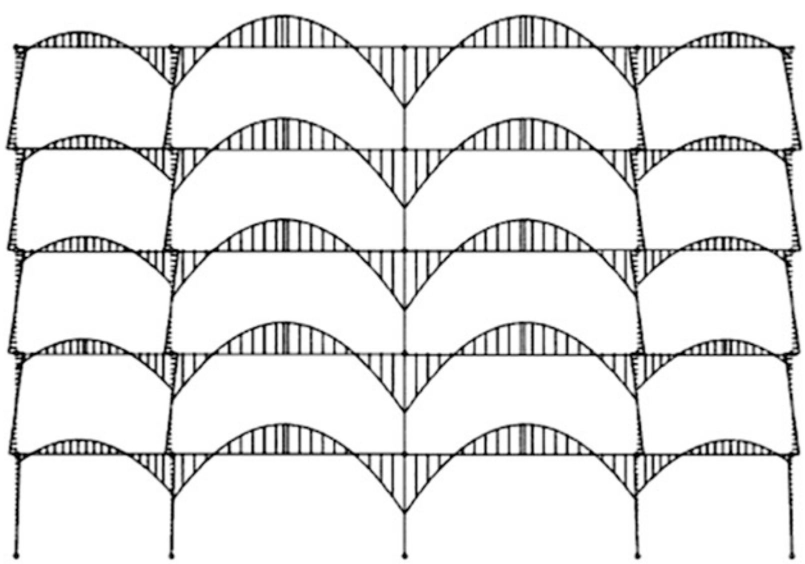


Fig. E15.1e Moment diagram for uniform loading

Example 15.2

Given: The five-story symmetrical rigid frame building shown in Figs. E15.2a, E15.2b, and E15.2c. Assume the building is subjected to uniform gravity dead loading and an earthquake loading in the north–south direction. Consider the floor load to be transmitted to all sides (two-way action). Assume the floors are rigid with respect to lateral motion.

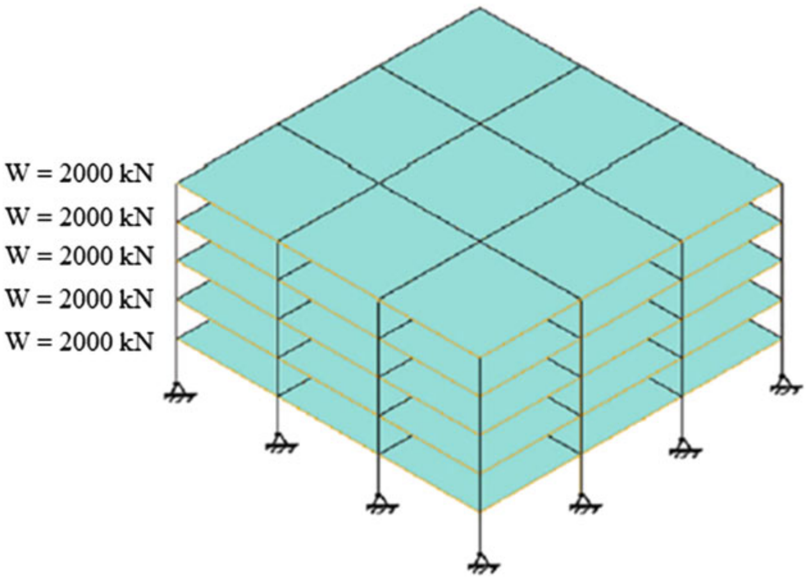


Fig. E15.2a 3D model

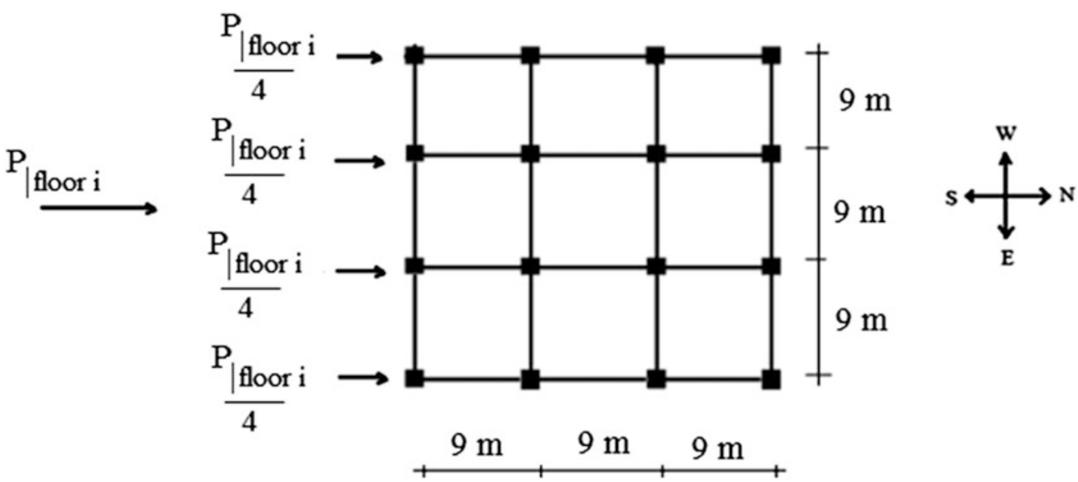


Fig. E15.2b Typical floor plan

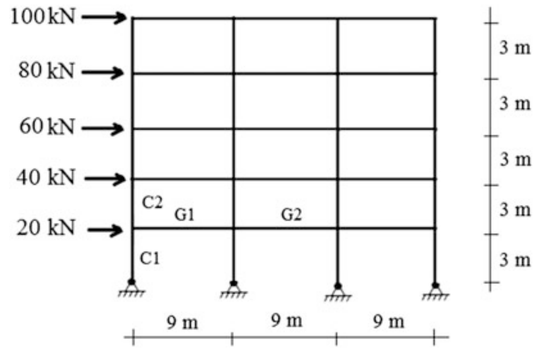


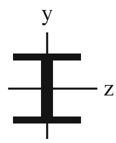
Fig. E15.2c Earthquake in N–S direction—specified floor loads on a typical frame

Determine: The maximum forces in the columns and beams for a typical interior bay and the lateral displacement of the floors using computer software. Assume all the beams to be the same size and all the columns to be the same size.

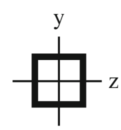
The corresponding cross-sectional properties are specified for two cases:

The second case corresponds to doubling the column inertias for case one

$$I\text{ shape beams}\begin{cases} I_z = 445{,}146{,}750\,\text{mm}^4 \\ I_y = 22{,}798{,}170\,\text{mm}^4 \\ I_x = 1{,}135{,}750\,\text{mm}^4 \\ A = 12{,}320\,\text{mm}^2 \end{cases}$$



$$\text{Hollow square columns}\begin{cases} \text{case (1)} \begin{cases} I_z = I_y = 148{,}520{,}925\,\text{mm}^4 \\ I_x = 233{,}390{,}025\,\text{mm}^4 \\ A = 10{,}677\,\text{mm}^2 \end{cases} \\ \text{case (2)} \begin{cases} I_z = I_y = 309{,}106{,}575\,\text{mm}^4 \\ I_x = 486{,}749{,}250\,\text{mm}^4 \\ A = 15{,}867\,\text{mm}^2 \end{cases} \end{cases}$$



Solution: The floor loading is uniformly applied to the floor slab. Using the concept of tributary areas, we convert this loading to line loadings w on the perimeter floor beams. Note that the N–S and E–W loading are identical because of the geometry.

$$W_{\text{floor total}} = 2000\,\text{kN}$$

$$\frac{2000}{(27)(27)} = 2.744\,\text{kN/m}^2$$

$$\Downarrow$$

$$w_{1\,\text{gravity}} = 2.744(4.5) = 12.35\,\text{kN/m}$$

$$w_{2\,\text{gravity}} = 2.744(9) = 24.7\,\text{kN/m}$$

The gravity line loading patterns for the perimeter floor beams are listed below.

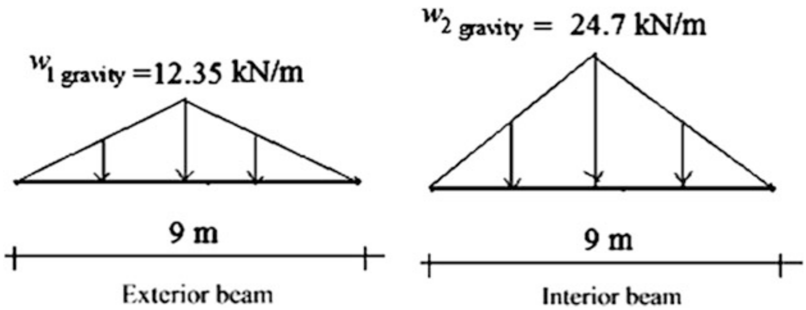


Fig. E15.2d

Using computer software, we analyze a 2D model of the rigid frame for gravity and earthquake loading. This approach is possible because the geometry and stiffness properties are symmetrical.

The critical values for the column forces (axial, shear, moment) occur in the first story. Results for the beams, columns, and lateral displacement corresponding to the two choices for column properties are listed and plotted below.

$$\text{Column}\begin{cases}\text{Gravity} & \text{Case}(1)\begin{cases}F_{\max}=574\,\text{kN}\\ V_{\max}=36\,\text{kN}\\ M_{\max}=62\,\text{kN m}\end{cases} \quad \text{Case}(2)\begin{cases}F_{\max}=567\,\text{kN}\\ V_{\max}=42\,\text{kN}\\ M_{\max}=76\,\text{kN m}\end{cases}\\ \text{Earthquake} & \text{Case}(1)\begin{cases}F_{\max}=130\,\text{kN}\\ V_{\max}=95\,\text{kN}\\ M_{\max}=252\,\text{kN m}\end{cases} \quad \text{Case}(2)\begin{cases}F_{\max}=127\,\text{kN}\\ V_{\max}=99\,\text{kN}\\ M_{\max}=258\,\text{kN m}\end{cases}\end{cases}$$

$$\text{Beam}\begin{cases}\text{Gravity} & \text{Case}(1)\begin{cases}F_{\max}=36\,\text{kN}\\ V_{\max}=61\,\text{kN}\\ M_{\max}=115\,\text{kN m}\end{cases} \quad \text{Case}(2)\begin{cases}F_{\max}=42\,\text{kN}\\ V_{\max}=60\,\text{kN}\\ M_{\max}=113\,\text{kN m}\end{cases}\\ \text{Earthquake} & \text{Case}(1)\begin{cases}F_{\max}=82\,\text{kN}\\ V_{\max}=51\,\text{kN}\\ M_{\max}=247\,\text{kN m}\end{cases} \quad \text{Case}(2)\begin{cases}F_{\max}=84\,\text{kN}\\ V_{\max}=48\,\text{kN}\\ M_{\max}=224\,\text{kN m}\end{cases}\end{cases}$$

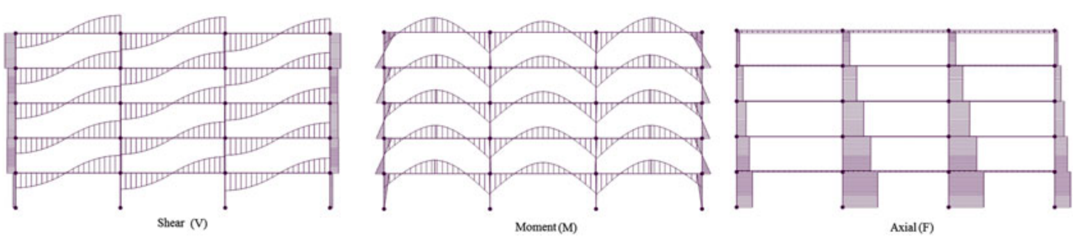


Fig. E15.2e Shear, moment, axial force diagrams—gravity loading

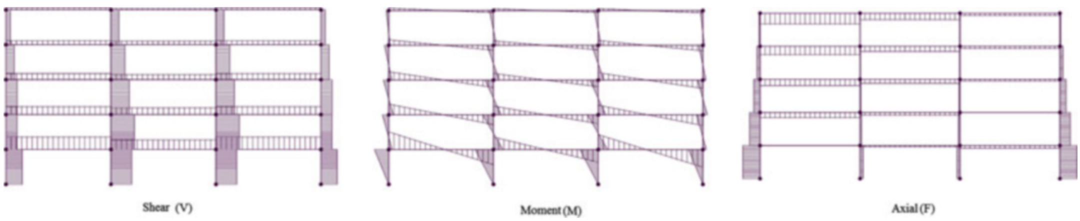


Fig. E15.2f Shear, moment, axial force diagrams—earthquake loading

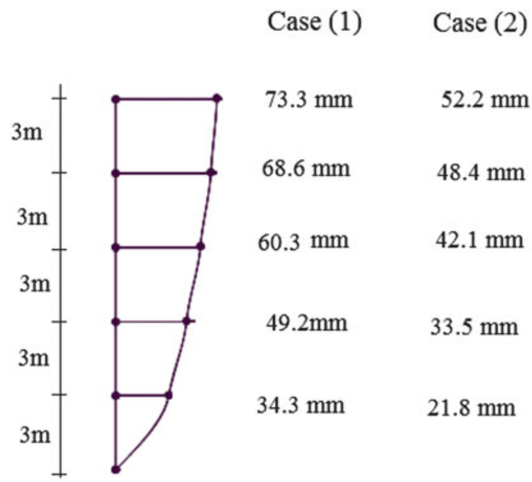


Fig. E15.2g Lateral displacement of the floors—earthquake

Note that there is only a small difference in the force magnitudes when the column inertia values are doubled. The main effect is on the lateral displacement which is to be expected.

15.4 A Case Study: Four-Story Building

In this section, we illustrate the computation of the design parameters for two typical structural systems, a rigid frame and a partially braced frame, having the same loading and geometry. We also use the same code-based procedures to estimate the structural properties. Our objective is to compare the required design parameters which provide an estimate of the relative efficiency of the two systems.

15.4.1 Building Details and Objectives

The building is a four-story steel frame building with a green roof. Figure 15.14 shows the typical floor plan and elevation views. The rigid flooring system transmits the gravity load primarily in the E–W direction to the floor beams oriented in the N–S direction (one-way action).

The loading and member data are as follows:

- Floor dead load = 0.055 kip/ft^2
- Floor live load = 0.07 kip/ft^2
- Roof dead load = 0.18 kip/ft^2

- Roof live load = 0.02 kip/ft^2
- Global wind loads acting in the N–S and E–W direction are defined in Fig. 15.15. They correspond to a peak wind speed of 80 mph for a building located in Boston, Massachusetts.
- The weight of exterior walls will be carried by the edge beams.
- Self-weight of exterior walls = 1.1 kip/ft
- Based on economic considerations related to fabrication and construction, the choice of member sizes is restricted to the following:
  - All the roof beams in the N–S direction are the same size.
  - All the floor beams in the N–S direction are the same size.
  - All the floor/roof beams in the E–W direction are the same size.
  - All the columns have the same size.
  - All the braces have the same size.
- The following combinations of loads for strength design are to be considered:

$$w_u = \begin{cases} 1.4w_D \\ 1.2w_D + 1.6w_{L_{floor}} + 0.5w_{L_{roof}} \\ 1.2w_D + 0.5w_{L_{floor}} + 0.5w_{L_{roof}} + 1.6w_{wind} \end{cases}$$

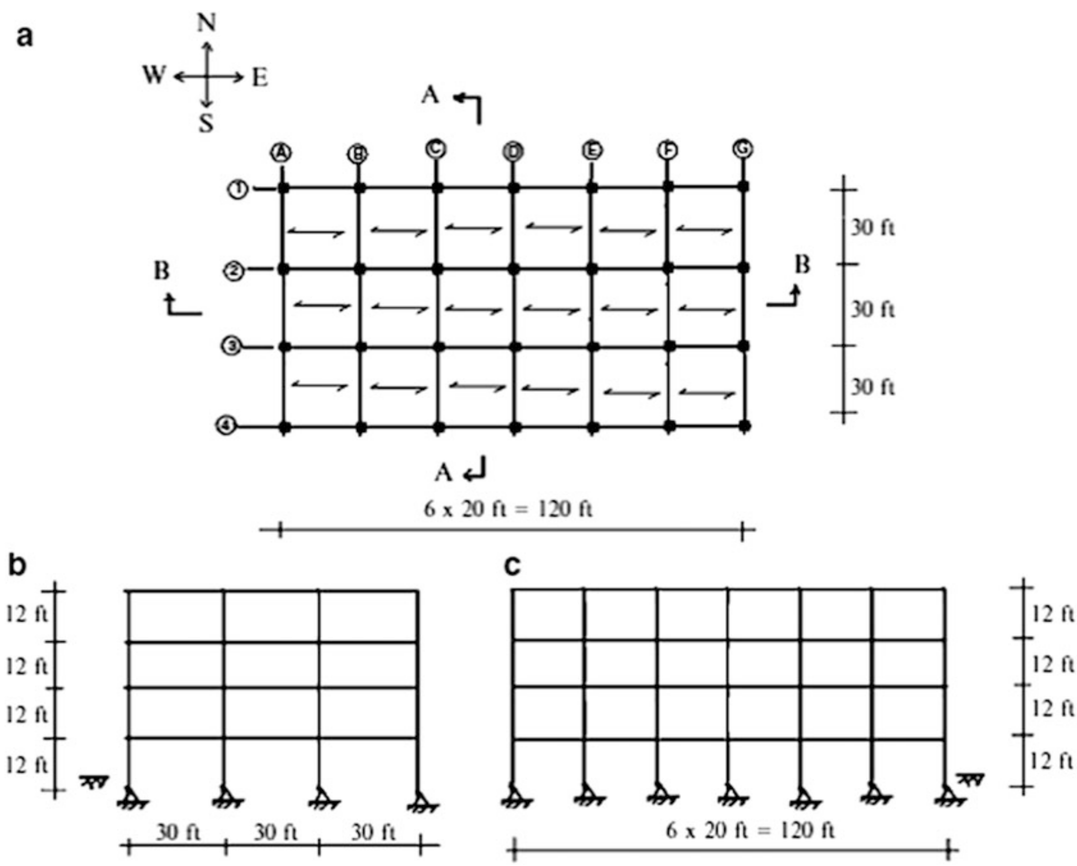


Fig. 15.14 Floor plan and elevation views—case study. (**a**) Plan. (**b**) N–S elevation—Section A-A. (**c**) E–W elevation—Section B-B

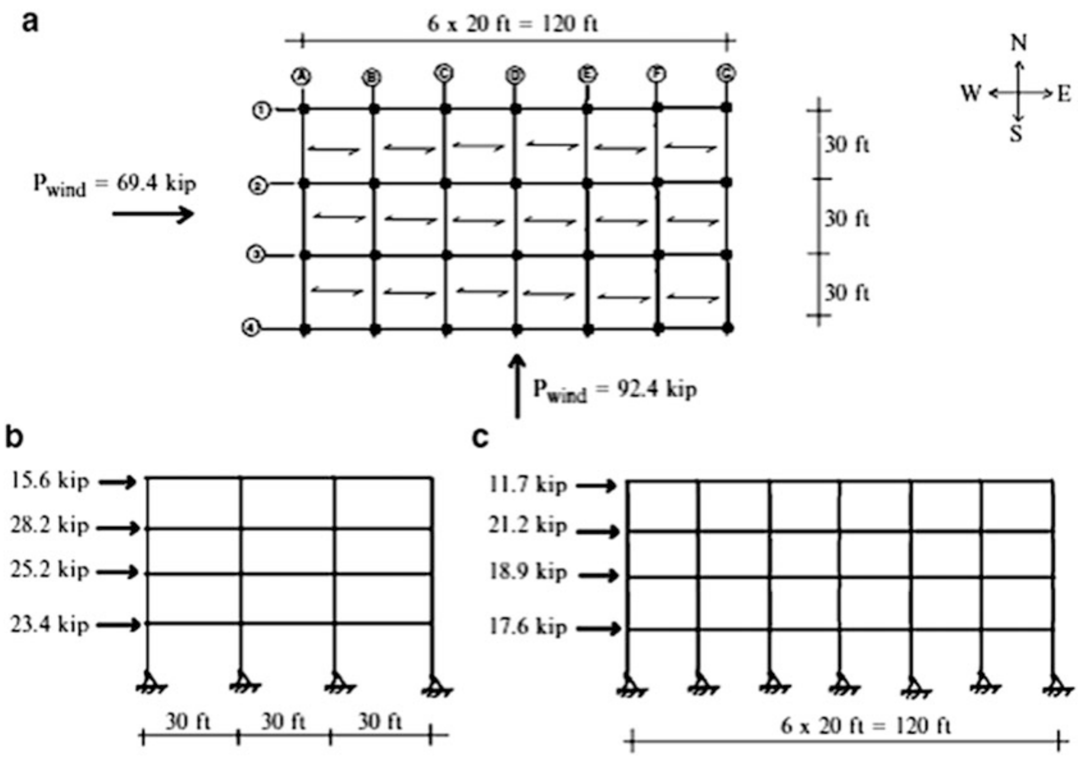


Fig. 15.15 Global Wind loads. (**a**) Plan. (**b**) N–S. (**c**) E–W

- The following limits are required by the serviceability constraint:

$$\text{Limit beam deflection to} \begin{cases} \dfrac{L_{\text{beam}}}{240} & \text{for (DL + LL)} \\ \dfrac{L_{\text{beam}}}{360} & \text{for LL} \end{cases}$$

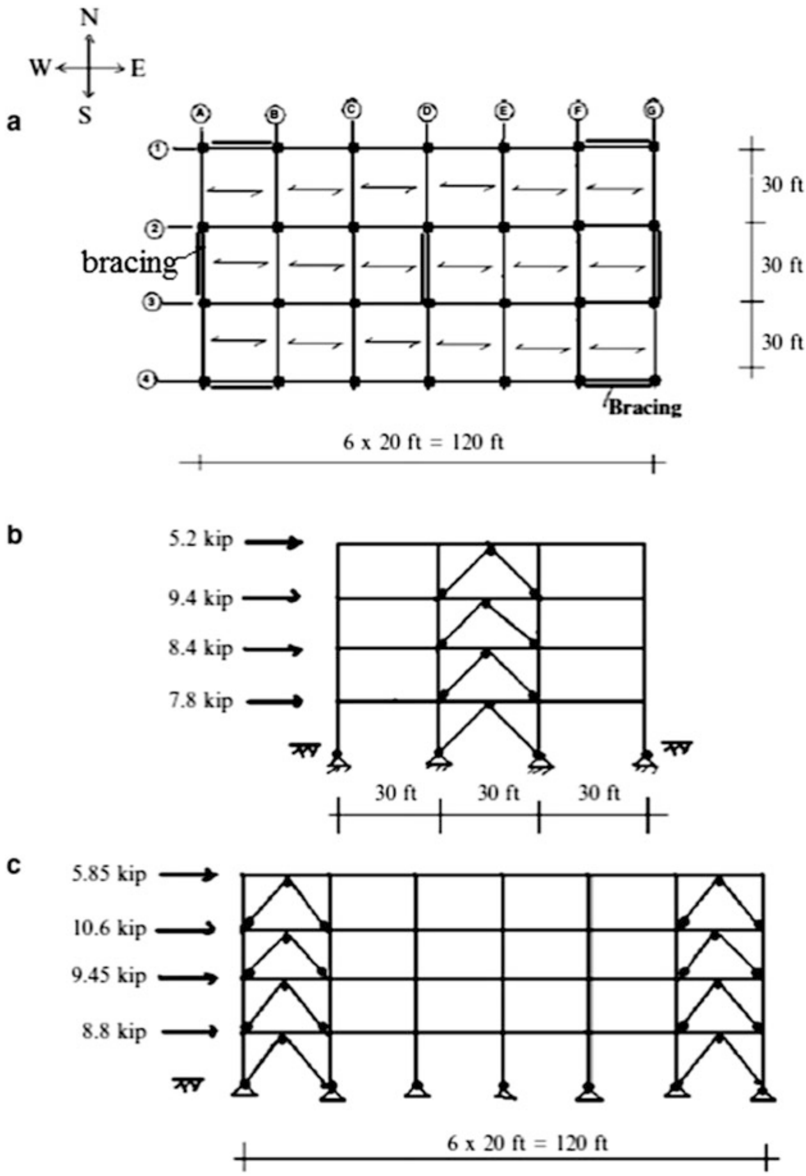
$$\text{Limit building drift to } \frac{H_{\text{building}}}{300}$$

Case (**1**): The structure is a braced frame, i.e., all the connections between beams and columns are pinned.
Case (**2**): The structure is a rigid frame in the N–S direction and a braced frame in the E–W direction. All the connections between beams and columns in the N–S direction are moment (rigid) connections; in the E–W direction, they remain pinned.

15.4.2 Case (1) Frames Are Braced in Both N–S and E–W Directions: Computation Details

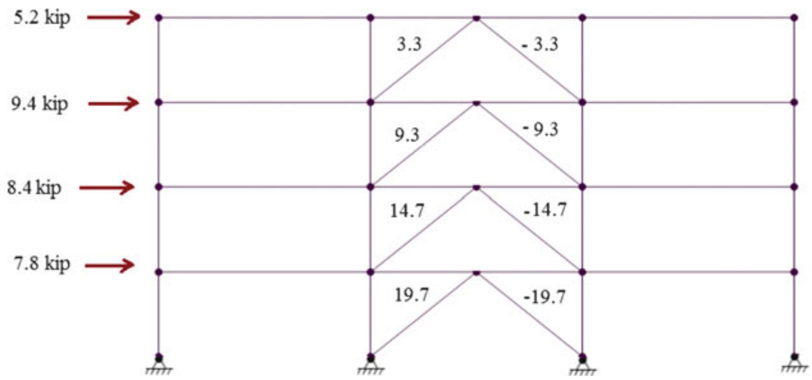
We suppose that since the connections between the beams and the columns are pin connections in both the N–S and E–W directions, the lateral load (wind) is carried by the bracing. The brace layout is governed by architectural considerations. We use the *K* bracing schemes shown in Fig. 15.16.

Fig. 15.16 Braced frame configuration. (**a**) Plan—braced in both directions. (**b**) N–S elevation—braced frames A-A, D-D, G-G. (**c**) E–W elevation braced frames 1-1, 4-4

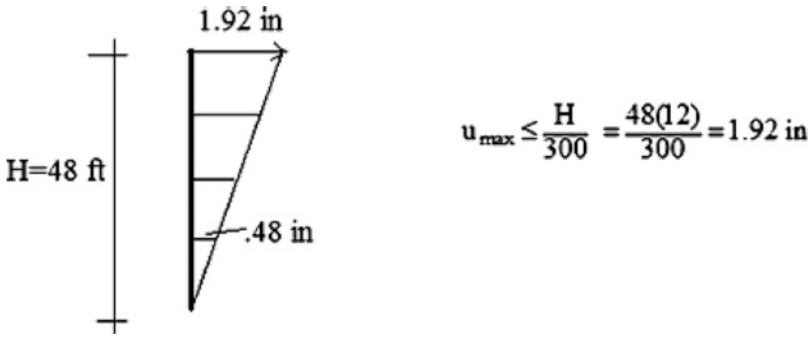


The braces have equal stiffnesses and the floors are rigid. Therefore the global wind load will be distributed equally between braces (see Chap. 14). The column load is purely axial since the members are pinned. We establish the column load per floor working with the tributary floor area associated with the column. The beams are simply supported, and the beam loading is based on one-way action (uniformly loaded).

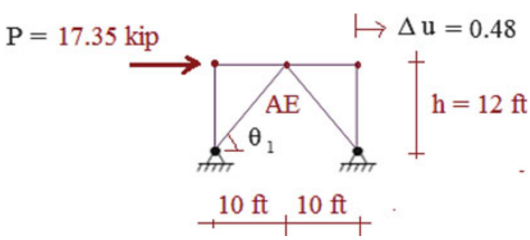
Since all the members are pinned, the total lateral wind load on a floor is carried by the bracing systems. The axial forces in a typical brace are shown on the sketch below. We assume the shear is equally distributed between the diagonals.



The constraint on the maximum lateral deflection at the top floor is $u_{max} \le H/300$:



We assume the inter-story displacement is constant for the stories and focus on the first story which has the maximum shear force.



Noting the equations presented in Sect. 11.4.3, we solve for the required area.

$$k_{brace} = \frac{2AE}{h}\left(\sin\theta_1 \cos^2\theta_1\right)$$
$$P = k_{brace}\Delta u$$

$$\therefore P = \frac{2AE}{h}\left(\sin\theta_1 \cos^2\theta_1\right)\Delta u \quad \Rightarrow \quad A = \frac{Ph}{2E\left(\sin\theta_1 \cos^2\theta_1\right)\Delta u}$$

leads to

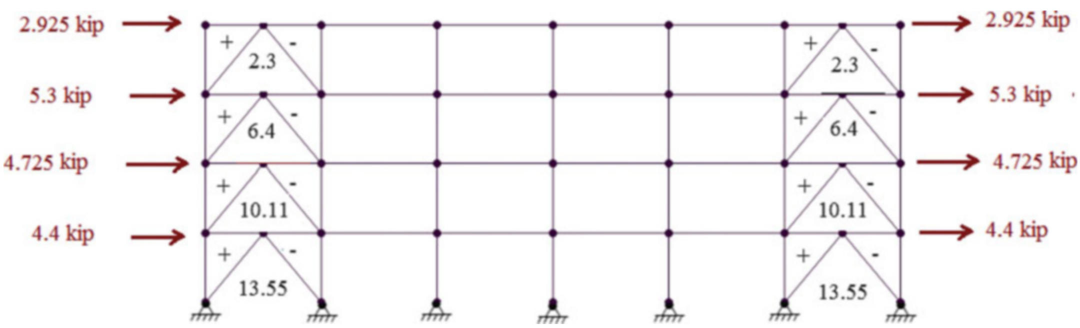
$$A_{\text{required}} = \frac{(30.8)(12 \times 12)}{2(29,000)(\sin(38.66)\cos^2(38.66))(0.48)} = 0.42\,\text{in.}^2$$

The diagonal elements may be subjected to either tension or compression loading depending on the direction of the wind. The maximum axial force due to wind in the bracing is 19.7 kip. Applying the appropriate load factor, the design value is P_u = 19.7(1.6) = 31.5 kip.

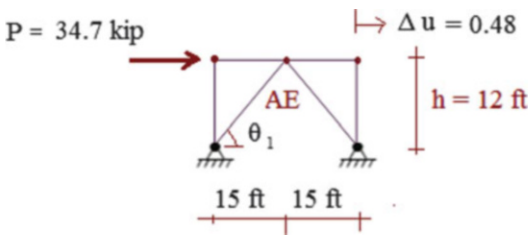
$$\text{NS bracing}\begin{cases} P_{u_{\max}} = 19.7(1.6) = 31.5\,\text{kip} \\ L_{\text{bracing}} = \sqrt{12^2 + 15^2} = 19.2\,\text{ft} \\ \Delta u = 0.48\,\text{in.} \rightarrow A_{\text{req}} = 0.42\,\text{in.}^2 \end{cases}$$

Based on the design axial load P_u = 31.5 kip, an effective length of 19.2 ft, and the required area based on the lateral sway of 0.48 in., one selects a cross-sectional area and uses this section for all the brace members in the N–S direction.

We repeat the same type of analysis for the E–W bracing except that now the bracing system is indeterminate. We assume each of the braces carries ½ the lateral load, and estimate the forces in the brace members by hand computations or use computer analysis. The force results are listed below.



The maximum factored axial force in the bracing is P_u = 13.55(1.6) = 21.7 kip.



We compute the required brace area following the same approach used for the N–S bracing system. The required area is given by

$$A_{\text{required}} = \frac{(17.35)(12 \times 12)}{2\,(29,000)\,(\sin(50.19)\cos^2(50.19))\,(0.48)} = 0.285\,\text{in.}^2$$

15.4.2.1 Interior Columns

The column load is purely axial since the members are pinned. We establish the column load per floor working with the tributary floor areas for dead and live loads, and the brace forces due to wind. The column on the first floor has the maximum axial force.

The loads in an interior column located in the first story are

$$P_{\text{D}} = 20(30)\{0.18 + .055(3)\} = 207\,\text{kip}$$

$$P_{\text{L roof}} = 20(30)\{0.02\} = 12\,\text{kip}$$

$$P_{\text{L floor}} = 20(30)\{0.07(3)\} = 126\,\text{kip}$$

$$P_{\text{N-S Wind}} = 29.4\,\text{kip}$$

Evaluating the following load combinations

$$P_{\text{u}} = \begin{cases} 1.4P_{\text{D}} = 290\,\text{kip} \\ 1.2P_{\text{D}} + 1.6P_{\text{L}} + 0.5P_{\text{L}_r} = 456\,\text{kip} \leftarrow \\ 1.2P_{\text{D}} + 0.5P_{\text{L}} + 0.5P_{\text{L}_r} + 1.6P_{\text{Wind}} = 364\,\text{kip} \end{cases}$$

leads to the design value of P_u = 456 kip. One selects a cross section based on P_u = 456 kip and an effective length of 12 ft.

15.4.2.2 Interior Beams

Interior Floor Beams (30 ft span):

$$w_{\text{D}_{\text{floor}}} = 0.055(20) = 1.1\,\text{kip/ft} \;\Rightarrow\; M_{\text{D}_{\text{floor}}} = \frac{w_{\text{D}}L^2}{8} = \frac{1.1(30)^2}{8} = 124\,\text{kip ft}$$

$$w_{\text{L}_{\text{floor}}} = 0.07(20) = 1.4\,\text{kip/ft} \;\Rightarrow\; M_{\text{L}_{\text{floor}}} = \frac{w_{\text{L}}L^2}{8} = \frac{1.4(30)^2}{8} = 157.5\,\text{kip ft}$$

$$w_{\text{u}} = \begin{cases} 1.4w_{\text{D}} = 1.54\,\text{kip/ft} \\ 1.2w_{\text{D}} + 1.6w_{\text{L}} = 3.56\,\text{kip/ft} \leftarrow \end{cases}$$

$$M_{\text{u}} = \frac{w_{\text{u}}L^2}{8} = \frac{3.56(30)^2}{8} = 400.5\,\text{kip ft}$$

$$V_{\text{u}} = \frac{w_{\text{u}}L}{2} = \frac{3.56(30)}{2} = 53.4\,\text{kip}$$

Interior Roof Beams (30 ft span):

$$w_{D_{roof}} = 0.18(20) = 3.6\,\text{kip/ft} \;\Rightarrow\; M_{D_{roof}} = \frac{w_D L^2}{8} = \frac{3.6(30)^2}{8} = 405\,\text{kip ft}$$

$$w_{L_{roof}} = 0.02(20) = 0.4\,\text{kip/ft} \;\Rightarrow\; M_{L_{roof}} = \frac{w_L L^2}{8} = \frac{0.4(30)^2}{8} = 45\,\text{kip ft}$$

$$M_{L_{roof}} = \frac{w_L L^2}{8} = \frac{0.4(30)^2}{8} = 45\,\text{kip ft}$$

$$w_u = \begin{cases} 1.4 w_D = 5.04\,\text{kip/ft} \leftarrow \\ 1.2 w_D + 1.6 w_L = 4.96\,\text{kip/ft} \end{cases}$$

$$M_u = \frac{w_u L^2}{8} = \frac{5.04(30)^2}{8} = 567\,\text{kip ft}$$

$$V_u = \frac{w_u L}{2} = \frac{5.04(30)}{2} = 75.6\,\text{kip}$$

The design is constrained by the deflection at mid-span.

$$v_{max} = \frac{5wL^4}{384EI}$$
$$\leq \frac{L}{240} = \frac{30(12)}{240} = 1.5\,\text{in} \quad \text{for}\,(w = w_D + w_L)$$
$$\leq \frac{L}{360}\frac{30(12)}{360} = 1.0\,\text{in} \qquad \text{for}\,(w = w_L)$$

These constraints lead to the following conditions on the required I.

$$\text{Floor}\begin{cases} I_{(D+L)_{req}} = \dfrac{5(w_D + w_L)(30)^4(12)^3}{384(29,000)(1.5)} = 418.96(w_D + w_L) = 1,047\,\text{in.}^4 \leftarrow \\ I_{L_{req}} = \dfrac{5(w_L)(30)^4(12)^3}{384(29,000)(1.0)} = 628.45(w_L) = 880\,\text{in.}^4 \end{cases}$$

$$\text{Roof}\begin{cases} I_{(D+L)_{req}} = 418.96(3.6 + 0.4) = 1,676\,\text{in.}^4 \leftarrow \\ I_{Lreq} = 628.45(0.4) = 251\,\text{in.}^4 \end{cases}$$

15.4.2.3 Summary for Case (1)

The relevant design parameters for the braced frame are listed below.

N–S roof beams	$M_u = 567\,\text{kip ft}$	$I_{req} = 1676\,\text{in}^4$
N–S floor beams	$M_u = 400\,\text{kip ft}$	$I_{req} = 1047\,\text{in}^4$
E–W beams	$M_u = 77\,\text{kip ft}$	$I_{req} = 137\,\text{in}^4$
Columns	$P_u = 456\,\text{kip}$	
E–W braces	$P_u = 21.7\,\text{kip}$	$A_{req} = .285\,\text{in}^2$
N–S braces	$P_u = 31.5\,\text{kip}$	$A_{req} = .42\,\text{in}^2\text{e}$

15.4.3 Case (2) Frames Are Rigid in the N–S Direction But Remain Braced in the E–W Direction

Figure 15.17a shows a plan view of this structural scheme. Our objective here is to generate the response of an individual rigid frame and to compare the design values for the braced vs. rigid frame structural concepts.

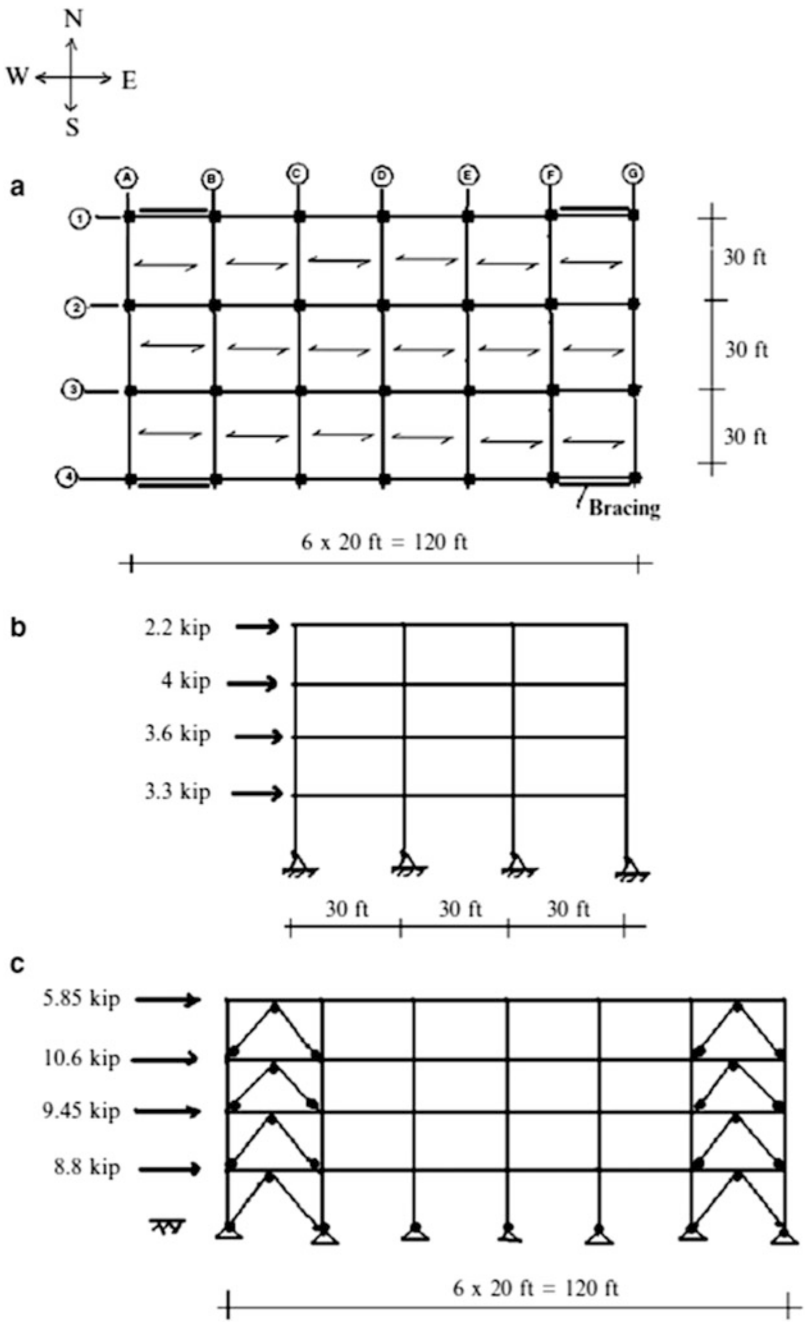
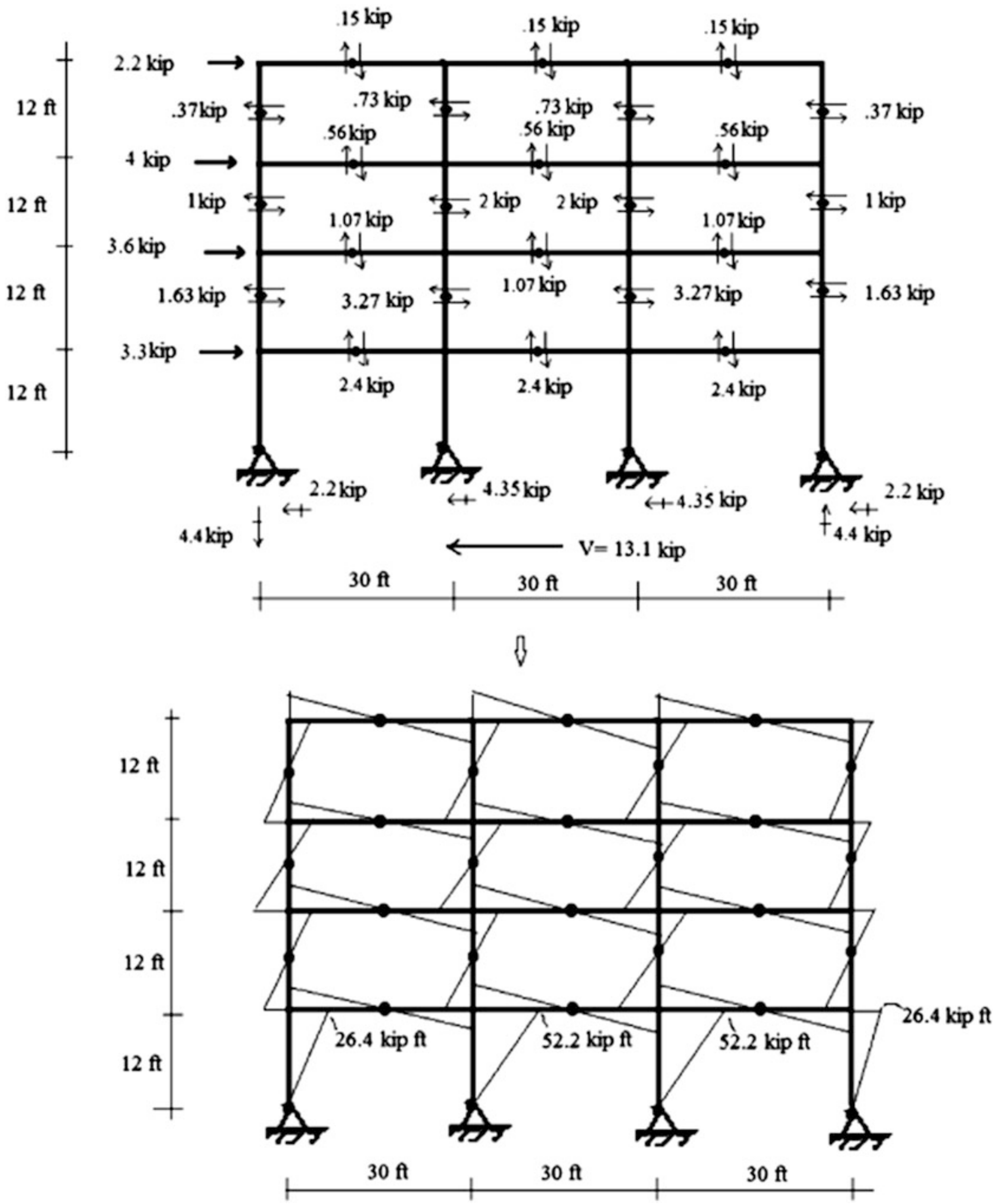


Fig. 15.17 Rigid frame N–S, braced frame E–W. (**a**) Plan. (**b**)Typical rigid frame elevation—N–S; wind loading. (**c**) E–W elevation braced frames 1-1, 4-4; wind loading

15.4.3.1 Strategy for N–S Beams and Columns

We specify moment connections between the beams and the columns in the N–S direction and assume that the lateral wind load will be carried equally by the seven rigid frames, because the floor slabs are rigid and the rigid frames have equal stiffnesses. The E–W direction remains the same as the beams in this direction are pin ended. Since the beams in the N–S direction are now rigidly connected to the columns, end moments will be developed in the beams. *The net effect is a reduction in the maximum moment in the beams*. For a first estimate, assuming full fixity, the peak moment reduces from $wL^2/8$ to $wL^2/12$, a reduction of 33 %. It follows that the beams will be *lighter*; however, the columns will be *heavier* since they now must be designed for both axial force and moment.

Wind loading introduces end moments in the beams and columns. We use the portal method (see Chap. 11) to estimate these values. The results are shown on the sketch below.



15.4.3.2 Estimated Properties: Beams

We *estimate* the design moment for roof and floor beams based on the following combination of factored moments:

Roof:

$$M_u^- \approx \begin{cases} 1.4\dfrac{w_D L^2}{12} = 378\,\text{kip ft} \leftarrow \\ (1.2w_D + 1.6w_L)\dfrac{L^2}{12} = 372\,\text{kip ft} \\ (1.2w_D + 0.5w_L)\dfrac{L^2}{12} + 1.6M_{\text{wind}} = 343\,\text{kip ft} \end{cases}$$

Floor:

$$M_u^- \approx \begin{cases} 1.4\dfrac{w_D L^2}{12} = 115.5\,\text{kip ft} \\ (1.2w_D + 1.6w_L)\dfrac{L^2}{12} = 267\,\text{kip ft} \leftarrow \\ (1.2w_D + 0.5w_L)\dfrac{L^2}{12} + 1.6M_{\text{wind}} = 209\,\text{kip ft} \end{cases}$$

As a first estimate, we use tributary areas to estimate the axial load in the columns due to dead and live loads. The most critical load combinations for the columns are

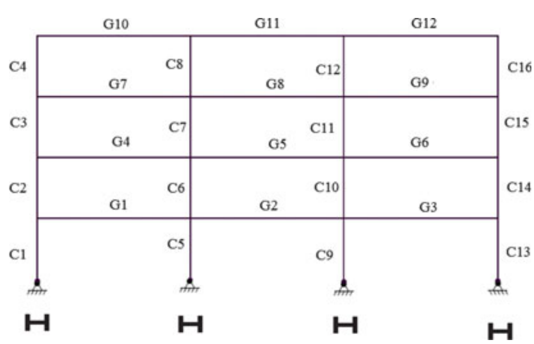
$$\begin{cases} P_u = 1.2P_D + 1.6P_{\text{wind}} + 0.5P_{L_{\text{floors}}} + 0.5P_{L_{\text{roof}}} \approx 317\,\text{kip} \\ M_u \approx 1.6M_{\text{wind}} \approx 1.6(52.2) \approx 84\,\text{kip ft} \end{cases}$$

$$\begin{cases} P_u = 1.2P_D + 1.6P_{L_{\text{floors}}} + 0.5P_{L_{\text{roof}}} \approx 461\,\text{kip} \\ M_u \approx 10\,\text{kip ft} \end{cases}$$

Based on the above estimated force values, we select the following cross-sectional properties.

$$\begin{cases} I_{\text{col}} = 272\,\text{in.}^4 & A_{\text{col}} = 14.4\,\text{in.}^2 \\ I_{\text{beam/floor}} = 1070\,\text{in.}^4 & A_{\text{beam/floor}} = 19.1\,\text{in.}^2 \\ I_{\text{beam/roof}} = 1830\,\text{in.}^4 & A_{\text{beam/roof}} = 24.3\,\text{in.}^2 \end{cases}$$

Determining the actual properties is an iterative process. We expect the beam sizes to decrease, and the column size to increase as the iteration proceeds due to the shift from braced frame to rigid frame. We orient the cross sections such that the bending occurs about the strong axis as indicated on the sketch below.



15.4.3.3 Live Load Patterns

We determine the live load patterns for the maximum positive and negative moments for the beams and for the maximum axial force for columns and then analyze the model under the combined dead, live, and wind loads. The wind loads are defined in Fig. 15.17b. Figure 15.18 shows live load patterns for maximum moments in beams, and axial force in columns.

Live Load Patterns for Positive Moment—Beams:

There are two load patterns for maximum positive moment at mid-span of the beams.

Negative Moment Live Load Patterns—Beams:

There are eight loading patterns for maximum negative end moments of the beams.

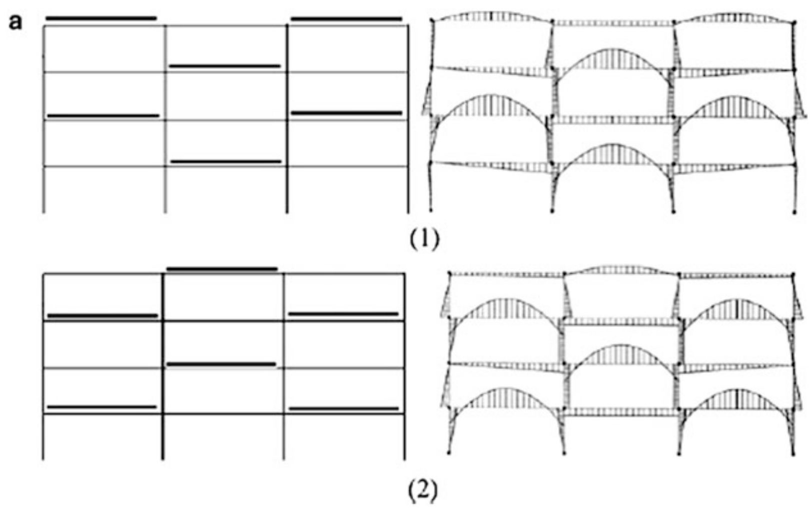


Fig. 15.18 (**a**) Positive live load (LL) moment patterns (*1*)–(*2*). (**b**) Negative live load (LL) moment patterns (*3*)–(*10*). (**c**) Live load patterns for axial force in column (*11*)–(*12*)

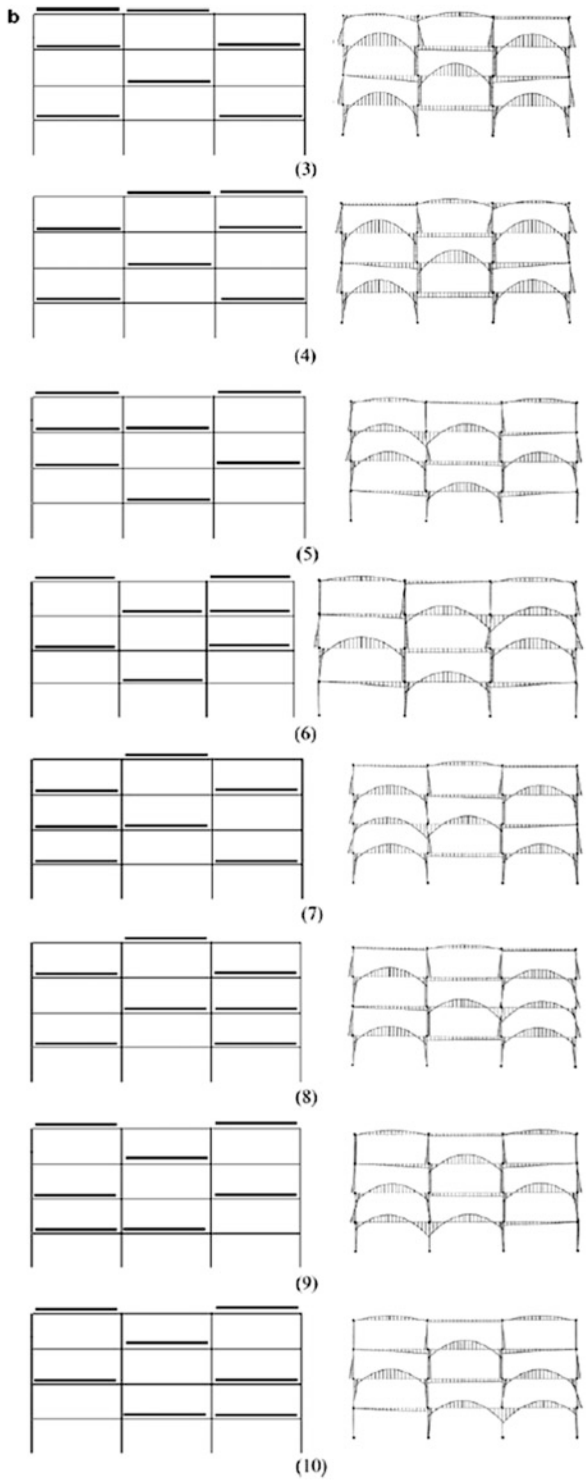


Fig. 15.18 (continued)

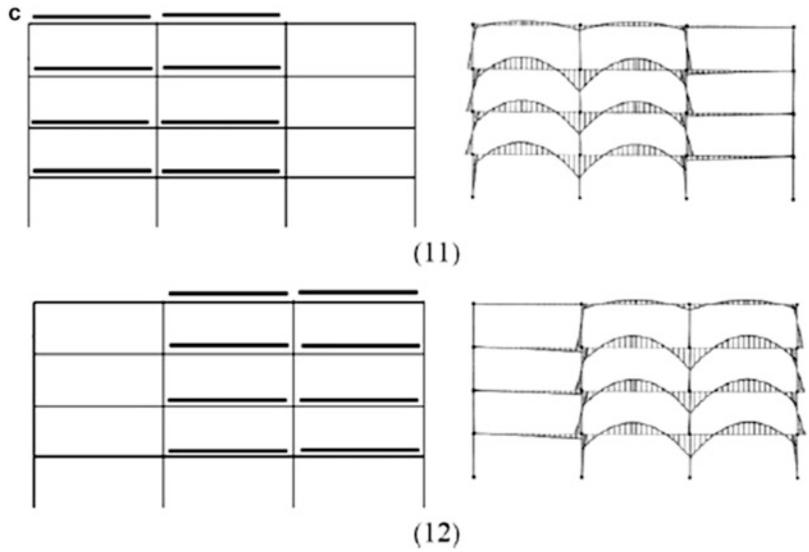


Fig. 15.18 (continued)

Axial Force Live Load Patterns—Columns:
The following two load patterns establish the peak values of the column axial forces.

15.4.3.4 Discrete Moment Envelop Plot-Live Load

Using a computer software system, results of the analyses for the ten live load patterns defined in Fig. 15.18a, b are used to construct the discrete moment envelope plots shown in Fig. 15.19. These plots show the *peak positive and negative moments at various sections along the spans* generated by the ten different loading patterns. The absolute peak values are summarized in Fig. 15.20.

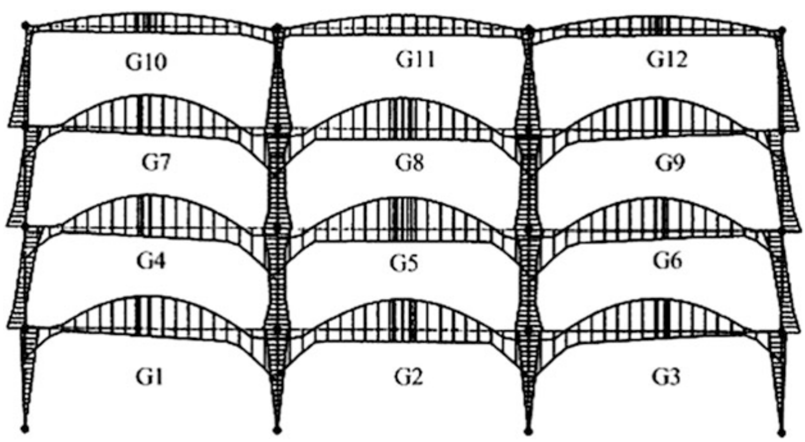


Fig. 15.19 Peak positive and negative discrete moment envelopes due to pattern live loading

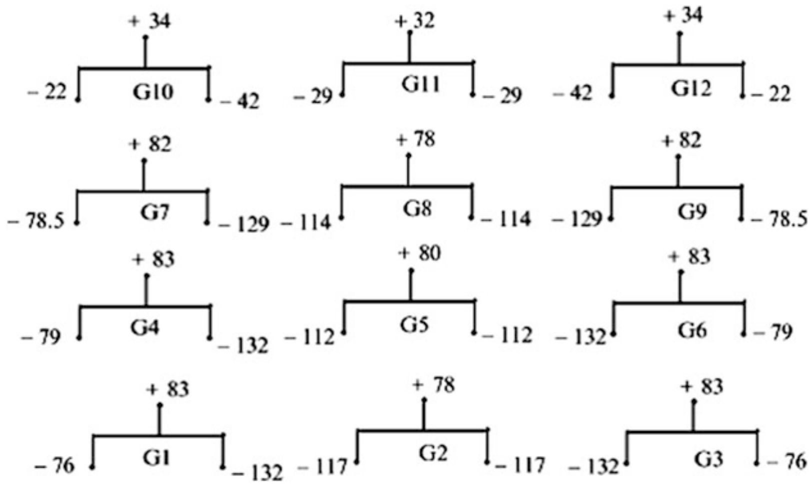


Fig. 15.20 Absolute maximum positive and negative moments in the beams due to pattern live loading (kip ft)

The factored discrete envelopes are plotted in Fig. 15.21. Using this updated information, we determine revised values for the cross-sectional

$$\begin{cases} I_{\text{col}} = 341\,\text{in.}^4 & A_{\text{col}} = 17.7\,\text{in.}^2 \\ I_{\text{beam/floor}} = 890\,\text{in.}^4 & A_{\text{beam/floor}} = 16.2\,\text{in.}^2 \\ I_{\text{beam/roof}} = 1140\,\text{in.}^4 & A_{\text{beam/roof}} = 16.2\,\text{in.}^2 \end{cases}$$

Lastly, using these properties, we generate updated values for the design moments shown in Fig. 15.22.

Since the columns are subjected to both axial action and bending, we need to scan the results for the individual loadings and identify the loadings that produce the maximum axial force and the maximum moment in the columns. Carrying out this operation, we identify the combinations for columns C1 and C5 listed in Fig. 15.23.

Given these design values, one generates new estimates for the cross-sectional properties. If these new estimates differ significantly from the original estimates, the analysis needs to be repeated since the results are based on the relative stiffness of the beams and columns. It turns out for this study that the initial estimates are sufficiently accurate.

15.4.3.5 Summary for Case (2)

N–S roof beams	$M_u = 442\,\text{kip ft}$
N–S floor beams	$M_u = 336\,\text{kip ft}$
E–W beams	same as case(1)
Columns	$\begin{cases} P_u = 490\,\text{kip} \\ M_u = 10\,\text{kip ft} \end{cases}$ or $\begin{cases} P_u = 306\,\text{kip} \\ M_u = 111\,\text{kip ft} \end{cases}$
E–W braces	same as case(1)

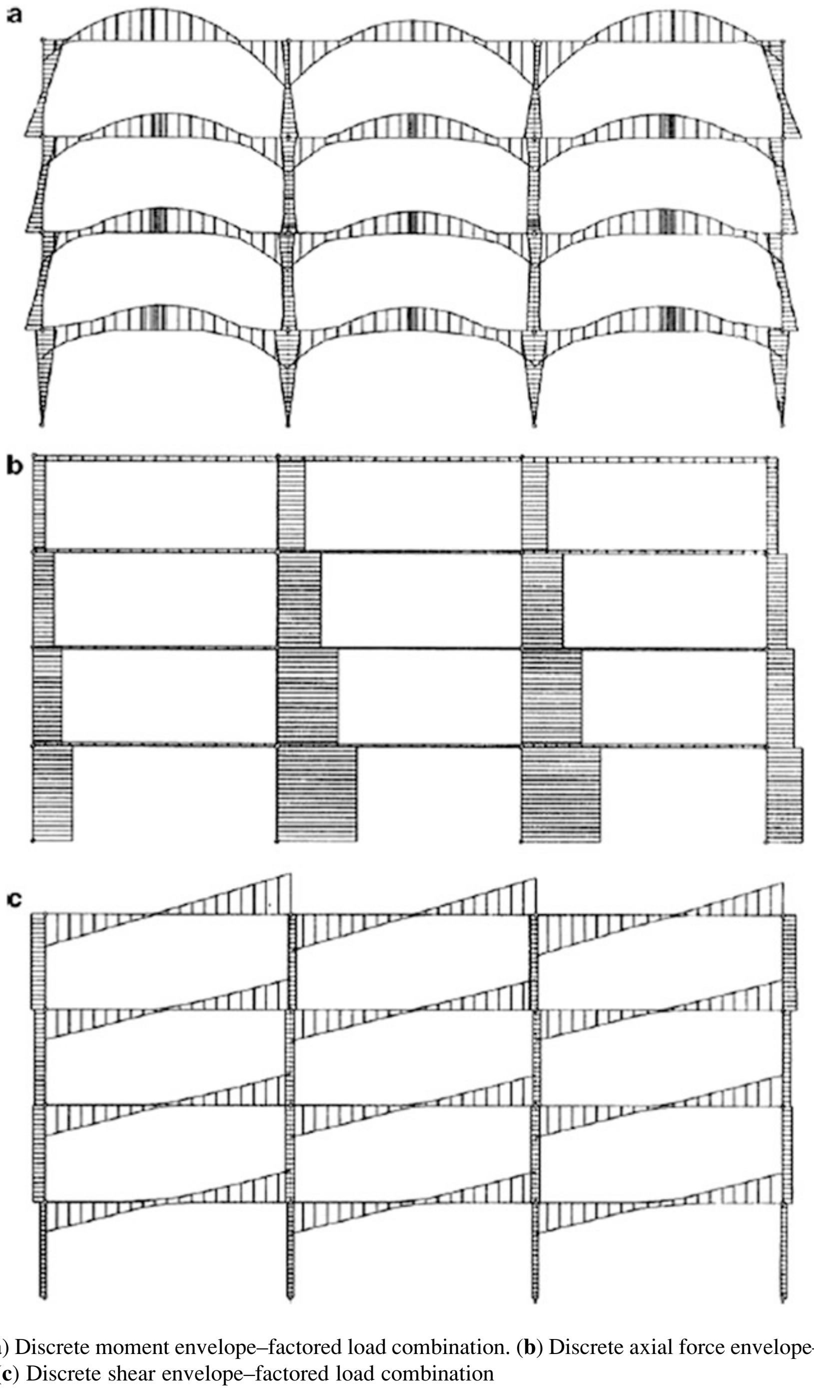


Fig. 15.21 (**a**) Discrete moment envelope–factored load combination. (**b**) Discrete axial force envelope–factored load combination. (**c**) Discrete shear envelope–factored load combination

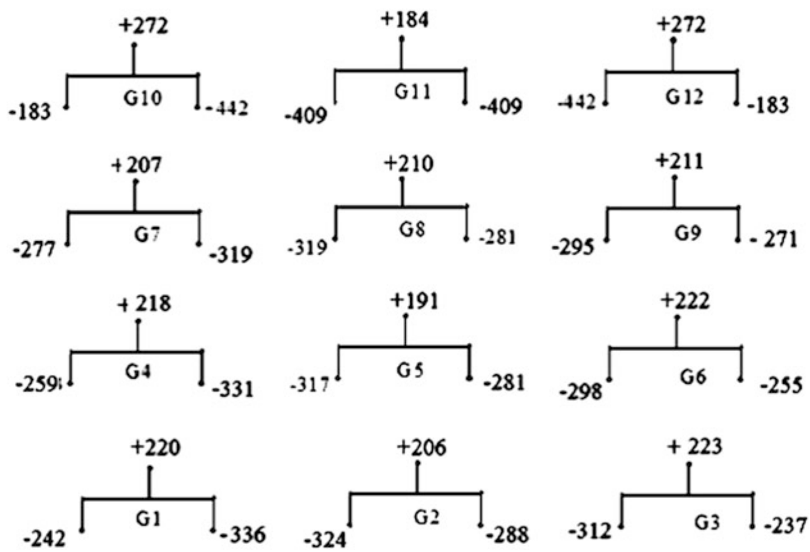
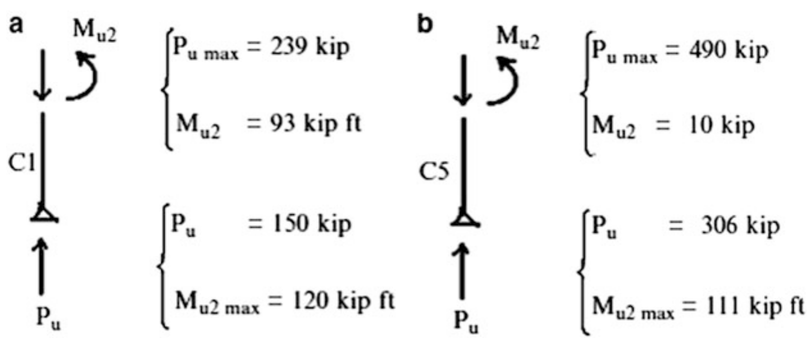


Fig. 15.22 Maximum and minimum design moments (kip ft)

Fig. 15.23 Critical axial load–moment combinations for Columns C1 and C5. (**a**) Exterior column. (**b**) Interior column



15.4.4 Discussion

The following table contains the design values corresponding to the two cases.

	Case (1) Braced	Case(2) Rigid in N–S direction
N–S roof beam	$M_u = 567\,\text{kip ft}$	$M_u = 442\,\text{kip ft}$
N–S floor beam	$M_u = 400\,\text{kip ft}$	$M_u = 336\,\text{kip ft}$
E–W beams	$M_u = 77\,\text{kip ft}$	$M_u = 77\,\text{kip ft}$
Columns	$P_u = 456\,\text{kip}$	$\begin{cases} P_u = 490\,\text{kip} \\ M_u = 10\,\text{kip ft} \end{cases}$ or $\begin{cases} P_u = 306\,\text{kip} \\ M_u = 111\,\text{kip ft} \end{cases}$
E–W beams	$P_u = 24\,\text{kip}$	$P_u = 24\,\text{kip}$
N–S beams	$P_u = 31.5\,\text{kip}$	Not required

Comparing values, we see that the N–S rigid frame structure is more efficient in the sense that its design values are less, and therefore the required cross-sections are lighter. However, the lateral displacements will be greater.

15.5 Summary

15.5.1 Objectives

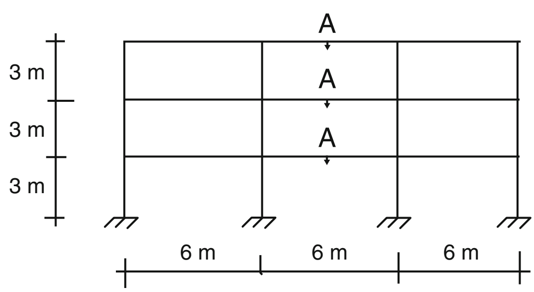
- To describe how gravity floor loading is transformed into distributed loading acting on the supporting beams.
- To show how Müller-Breslau principle can be applied to establish critical patterns of live gravity loading for the peak bending moments in rigid frames.
- To present a case study which integrates all the different procedures for dealing with dead, live, wind, and earthquake loads.

15.5.2 Key Concepts

- The floor slabs in concrete buildings are cast simultaneously with the supporting beams. The type of construction provides two possible load paths for gravity loads. Which path dominates depends on the relative magnitude of the side dimensions. The terms one-way and two-way actions are limiting cases where (1) one side is large with respect to the other side and (2) the sides are of the same order of magnitude.
- Gravity loading produces only positive moment in the beams of a braced frame.
- Positive moment at mid-span in the beams of a rigid frame is due only to gravity loading.
- Negative moment in the beams of a rigid frame is generated by both gravity and lateral loads.

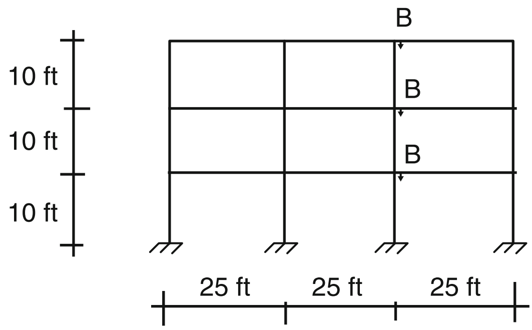
15.6 Problems

Problem 15.1



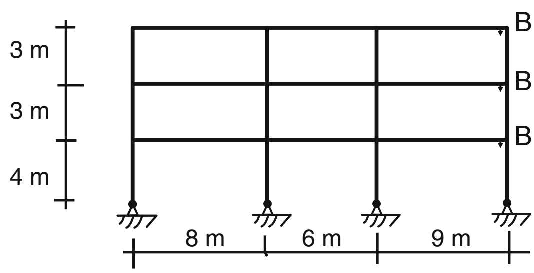
Using Müller-Breslau Principle, determine the loading patterns (uniformly distributed member load) that produce the peak values of positive moment at point A (mid-span) for each story.

Problem 15.2



Using Müller-Breslau Principle, estimate the loading patterns (uniformly distributed member load) that produce the peak value of negative moment at B for each story. Check the results using a software package. Take $I_c = 150$ in.^{4} for all the columns and $I_g = 300$ in.^{4} for all the beams.

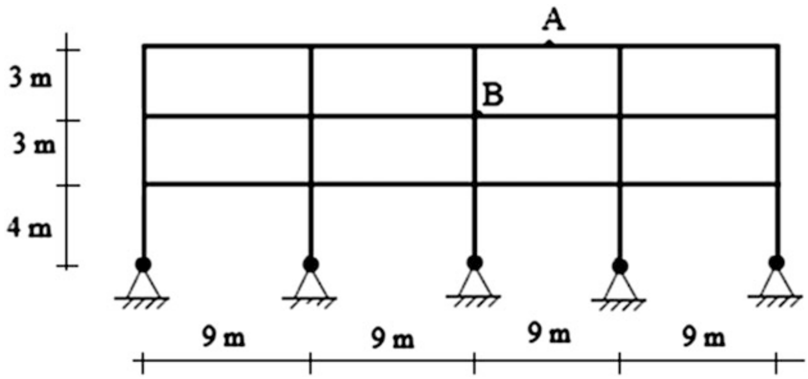
Problem 15.3 Using Müller-Breslau Principle, estimate the loading patterns (uniformly distributed member load) that produce the peak value of negative moment at B for each story. Check the results using a software package. Take $I_b = 300(10)^6$ mm^4 for all the beams and $I_c = 100(10)^6$ mm^4 for all the columns.



Problem 15.4 For the frame shown below

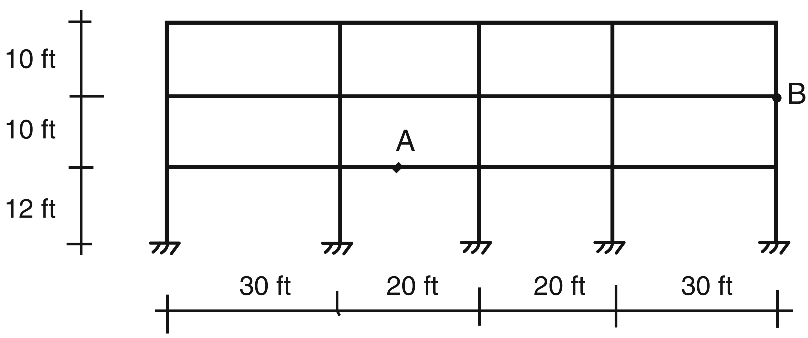
(a) Using Müller-Breslau Principle, sketch the influence lines for the positive moment at A and the negative moment at B.

(b) Use a software package to determine the maximum values of these quantities due to a uniformly distributed live load of 30 kN/m and a uniformly distributed dead load of 20 kN/m. Take $I_c = 100(10)^6$ mm^4 for all the columns and $I_b = 200(10)^6$ mm^4 for all the beams.

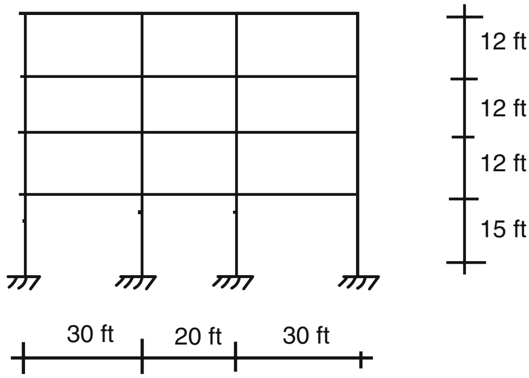


Problem 15.5 For the frame shown below

(a) Using Müller-Breslau Principle, sketch the influence lines for the positive moment at A and the negative moment at B.
(b) Use a software package to determine the maximum values of these quantities due to a uniformly distributed live load of 1.8 kip/ft and a uniformly distributed dead load of 1.2 kip/ft. Take $I_c = 480$ in.^{4} for all the columns and $I_b = 600$ in.^{4} for all the beams.



Problem 15.6

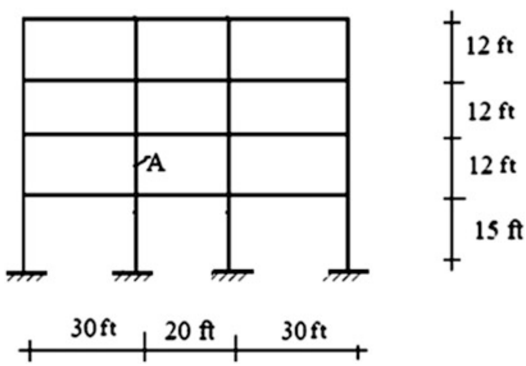


Consider the typical frame defined above. Assume the bay spacing is 20 ft.

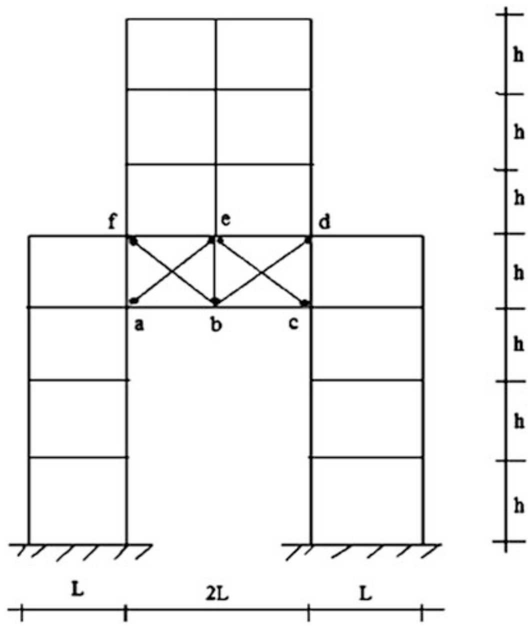
(a) Determine the floor loads per bay due to an earthquake of intensity $S_a = 0.3g$. Assume the following dead weights. Roof load $= 0.08$ kip/ft^2 and floor load $= 0.06$ kip/ft^2.
(b) Estimate the column shear forces due to this earthquake.
(c) Estimate the column shear forces due to both gravity and earthquake.

Problem 15.7 Consider the frame shown below. Assume a uniform gravity live loading for the beams.

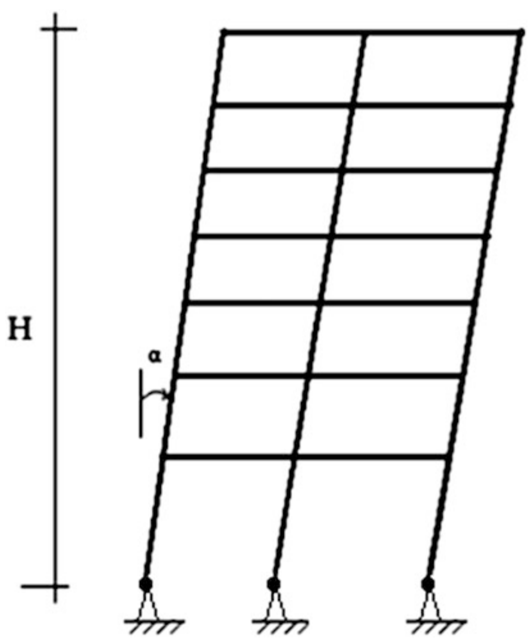
(a) Describe how you would apply Müller-Breslau Principle to establish the loading pattern for the compressive axial load in column A.
(b) Compare the axial load in column A of the pattern loading to the uniform loading on all members. Consider all the girders to be of the same size and all the columns to be of the same size. Assume $I_{\text{beam}} = 2.5I_{\text{column}}$ and $w = 1.2$ kip/ft. Use computer software.



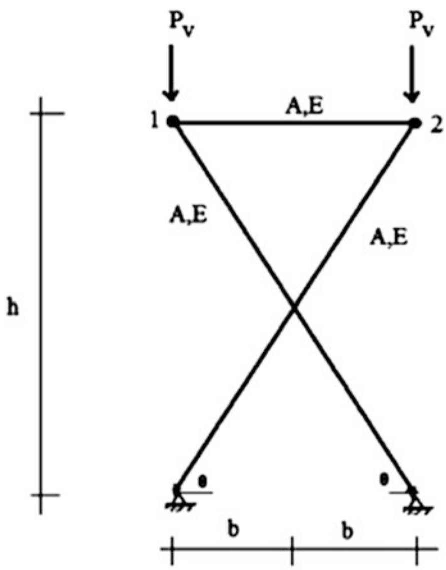
Problem 15.8 Discuss the function of the structure, abcdef. How would you determine the gravity loading acting on it? Assume uniform gravity loading for the beams.

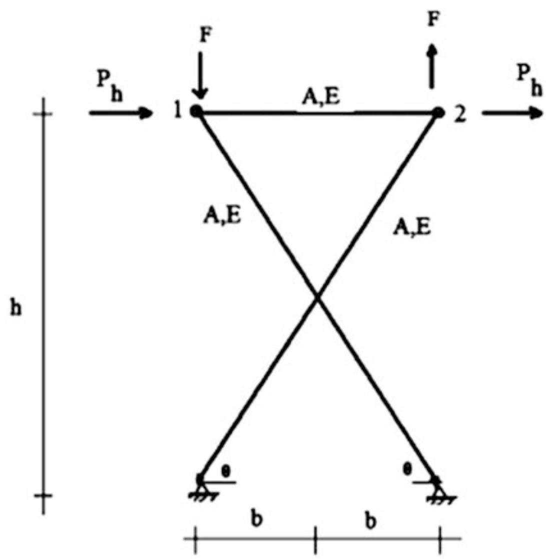


Problem 15.9 Consider a uniform floor gravity loading on the floors of the multistory rigid frame building shown below. Investigate how the internal forces vary with the angle α ranging from 0 to 20°, considering H constant. Is there a limiting value for α?

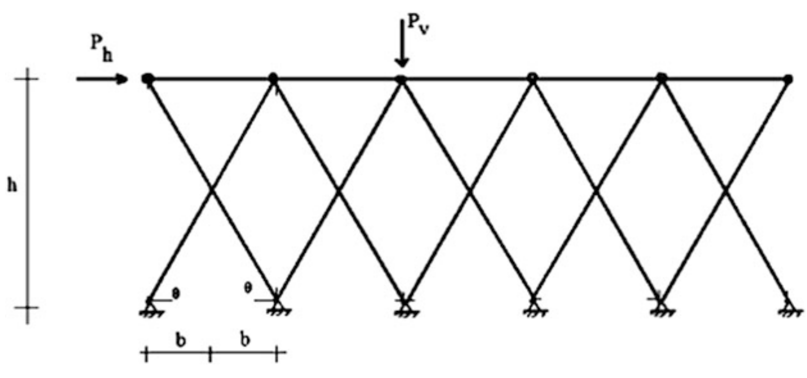


Problem 15.10 Consider the structures shown below. All the members are pinned at their ends.





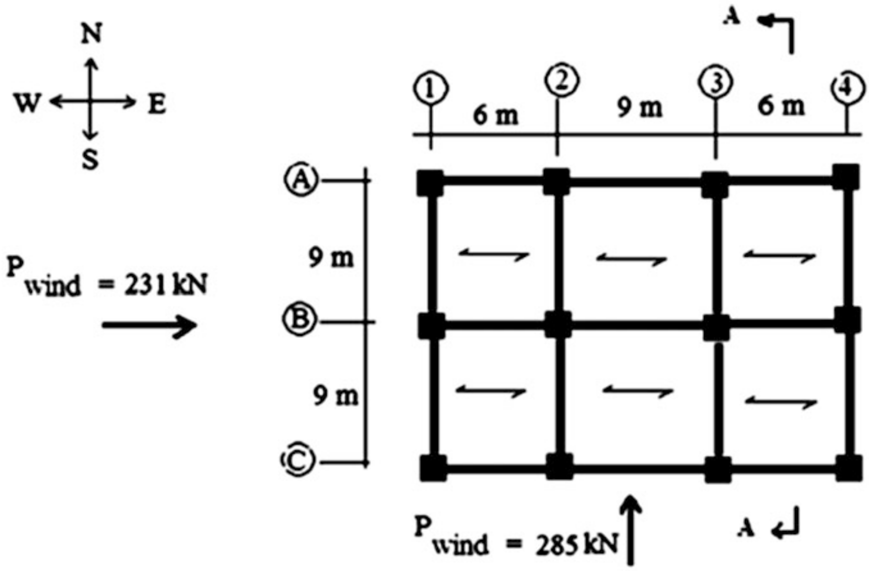
(a) Determine expression for the axial forces in the diagonal members.
(b) Determine the horizontal and vertical displacements of the nodes 1 and 2.
(c) Extend the analysis to the structure shown below. This structure is called a DIAGRID structure.



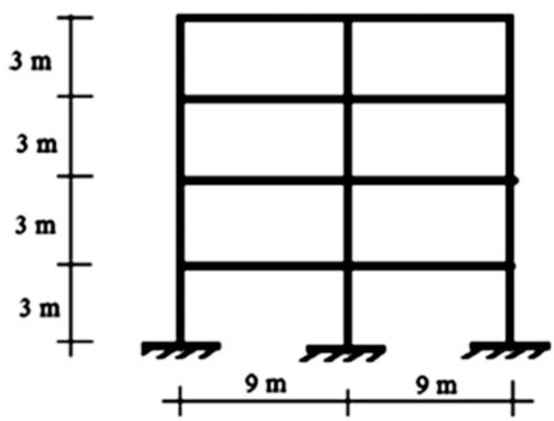
Problem 15.11 For the structure shown below, assume the floors are flexible and the flooring system transits the gravity loading to the floor beams in the N–S direction (one-way action). Assume all beams are the same size and all the columns are the same size. $I_{\text{beam}} = 3I_{\text{column}}$, $\text{Floor}_{\text{gravity}} = 175$ kN/m^2.

Compare the maximum forces in beams and columns caused by combination of gravity and wind for the following cases.

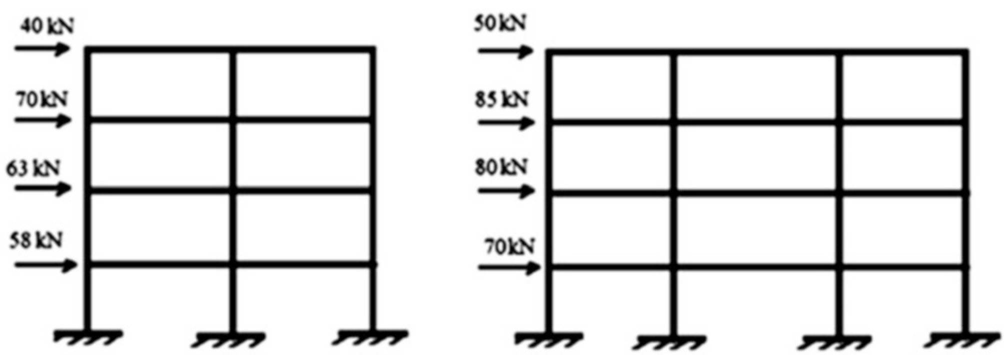
1. The structure is considered to be a braced frame, i.e., all the connections between beams and columns are pinned both in N–S and E–W direction. Assume the frames are suitably braced.
2. The structure is considered to be a rigid frame in the N–S direction and a braced frame in the E–W direction, i.e., all the connections between beams and columns are moment connections in N–S direction, but connections in the E–W direction remain pinned.



Typical plan



Elevation—Section A-A



Global wind