8 Vertical Retaining Wall Structures

Abstract

Vertical wall type structures function as barriers whose purpose is to prevent a material from entering a certain space. Typical applications are embankment walls, bridge abutments, and underground basement walls. Structural Engineers are responsible for the design of these structures. The loading acting on a retaining wall is generally due to the soil that is confined behind the wall. Various theories have been proposed in the literature, and it appears that all the theories predict similar loading results. In this chapter, we describe the Rankine theory which is fairly simple to apply. We present the governing equations for various design scenarios and illustrate their application to typical retaining structures. The most critical concerns for retaining walls are *ensuring stability with respect to sliding and overturning, and identifying the regions of positive and negative moment in the wall segments*. Some of the material developed in Chap. 7 is also applicable for retaining wall structures.

8.1 Introduction

8.1.1 Types of Retaining Walls

Vertical retaining wall structures are used to form a vertical barrier that retains a fluid or other material such as soil. Figure 8.1 illustrates different types of vertical retaining wall structures. They are constructed using unreinforced concrete for gravity walls and reinforced concrete for cantilever walls and bridge abutments. The base of the wall/footing is placed below the frost level. The material behind the wall is called backfill and is composed of granular material such as sand.

8.1.2 Gravity Walls

A free body diagram of a gravity structure is shown in Fig. 8.2. The force acting on the structure due to the backfill material is represented by P; the forces provided by the soil at the base are represented

J.J. Connor, S. Faraji, *Fundamentals of Structural Engineering*,
DOI 10.1007/978-3-319-24331-3_8

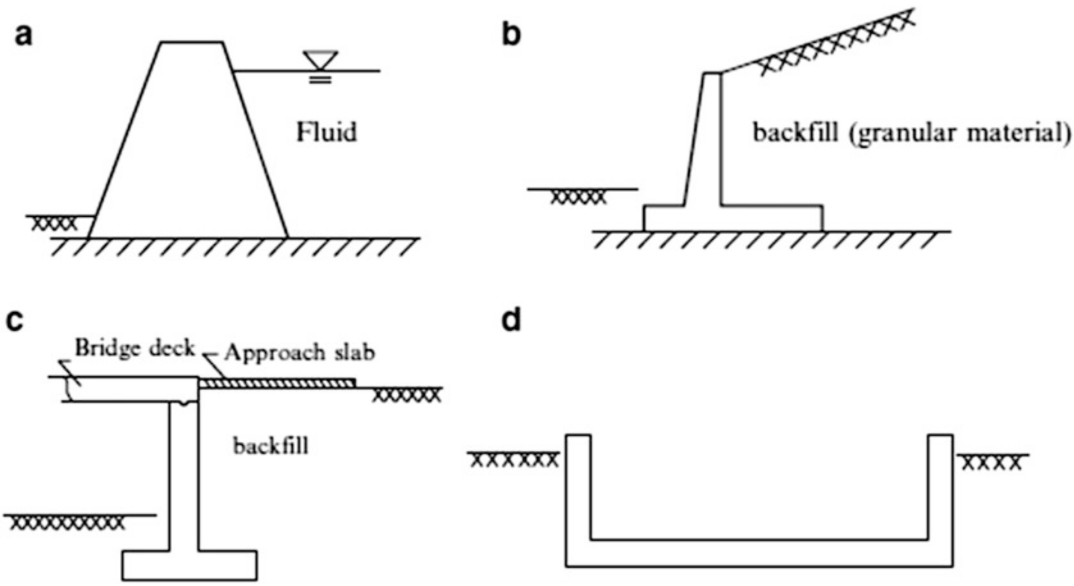


Fig. 8.1 Vertical retaining wall structures. (**a**) Gravity dam. (**b**) Cantilever retaining wall. (**c**) Bridge abutment. (**d**) Underground basement

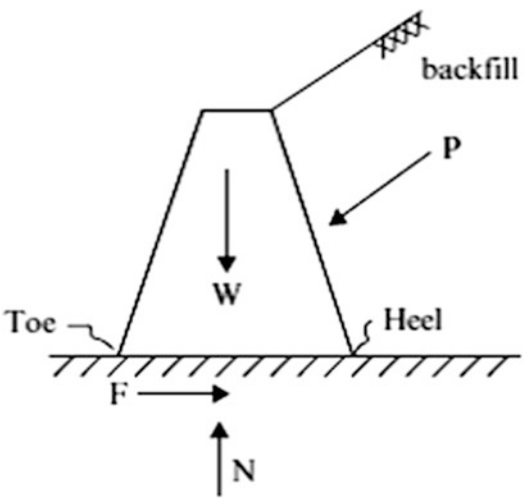


Fig. 8.2 Free body diagram—gravity structure

by the friction force F and the normal force N; lastly, the weight of the structure is represented by W. The end points of the base are called the "toe" and the "heel." We observe that P tends to overturn the wall about its toe and also to slide the structure in the horizontal direction. The overturning tendency is resisted by the gravity force W which has a counterbalancing moment about the toe. Sliding is resisted by the friction which is proportional to the normal force. Therefore, since both resisting mechanisms are due to gravity, this type of structure is called a "Gravity" structure.

Of critical concern are the *sliding and overturning failure modes*. The key design parameter is the length of the base. We need to select this parameter such that the factors of safety for sliding and overturning are sufficient to ensure global stability of the structure.

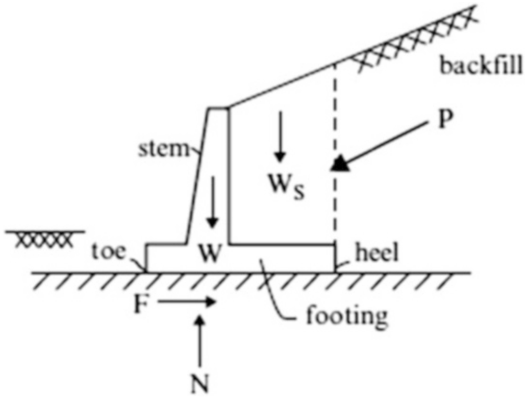


Fig. 8.3 Free body diagram—cantilever structure

8.1.3 Cantilever Walls

The amount of concrete required for a gravity type wall increases with height. Therefore, in order to minimize the concrete volume, the cantilever type retaining wall geometry shown in Fig. 8.3 is used. A portion of the concrete wall is removed and a "footing" extending out from both the heel and toe is added. This change has a stabilizing effect in that the weight of the backfill above the footing, represented by W_s, now contributes to the counterbalancing moment and also to the normal force. The wall stem segment of a cantilever wall carries load through bending action, whereas the gravity wall carries load primarily through *horizontal shear action*. These behavior modes dictate the type of construction.

Cantilever retaining walls, such as shown in Fig. 8.4, are reinforced concrete structures; gravity type walls tend to be unreinforced concrete. The key design issue is the width of the footing. This parameter is controlled by the requirements on the factors of safety with respect to overturning about the toe and sliding of the wall.

Fig. 8.4 Cantilever wall construction

8.2 Force Due to the Backfill Material

8.2.1 Different Types of Materials

8.2.1.1 Fluid

We consider first the case where the backfill material is an ideal fluid. By definition, an ideal fluid has no shear resistance; the state of stress is pure compression. The vertical and horizontal pressures at a point z unit below the free surface are (see Fig. 8.5):

$$p_{\mathrm{v}} = p_{\mathrm{h}} = p = \gamma z \tag{8.1}$$

where γ is the weight density.

We apply this theory to the inclined surface shown in Fig. 8.6. Noting (8.1), the fluid pressure is normal to the surface and varies linearly with depth. The resultant force acts $H/3$ units up from the base and is equal to

$$P = \frac{1}{2}p\frac{H}{\sin\theta} = \frac{1}{2}\gamma\left(\frac{H^2}{\sin\theta}\right) \tag{8.2}$$

Resolving P into horizontal and vertical components leads to

$$\begin{aligned} P_{\mathrm{h}} &= P\sin\theta = \frac{1}{2}\gamma H^2 \\ P_{\mathrm{v}} &= P\cos\theta = \frac{1}{2}\gamma H^2\frac{1}{\tan\theta} \end{aligned} \tag{8.3}$$

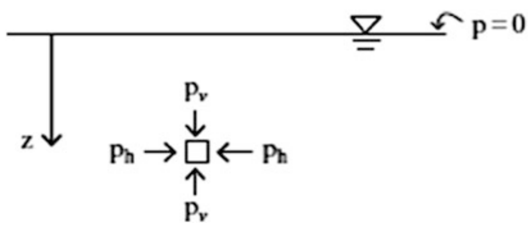
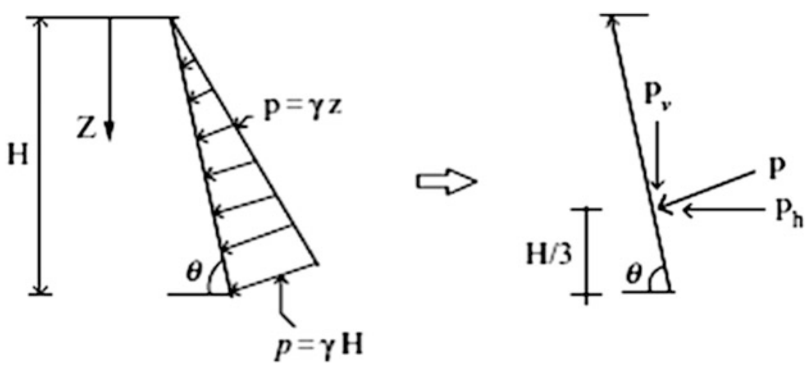


Fig. 8.5 Hydrostatic pressure

Fig. 8.6 Hydrostatic forces on an inclined surface



8.2.1.2 Granular Material

We consider next the case where the backfill behind the wall is composed of a granular material such as dry loose sand (Fig. 8.7). Loose sand behaves in a different manner than a fluid in that sand can resist shearing action as well as compressive action. The maximum shear stress for a sandy soil is expressed as

$$\tau = \sigma_{\rm n} \tan \varphi$$

where $\sigma_{\rm n}$ is the normal stress and φ is defined as the internal friction angle for the soil. A typical value of φ for loose sand is approximately 30°. One can interpret φ as being related to the angle of repose that a volume of sand assumes when it is formed by dumping the sand loosely on the pile. Figure 8.8 illustrates this concept.

The presence of shear stress results in a shift in orientation of the resultant force exerted on the wall by the backfill. A typical case is shown in Fig. 8.9; P is assumed to act at an angle of φ' with respect to the horizontal, where φ' ranges from 0 to φ.

Fig. 8.7 Granular material-stress state

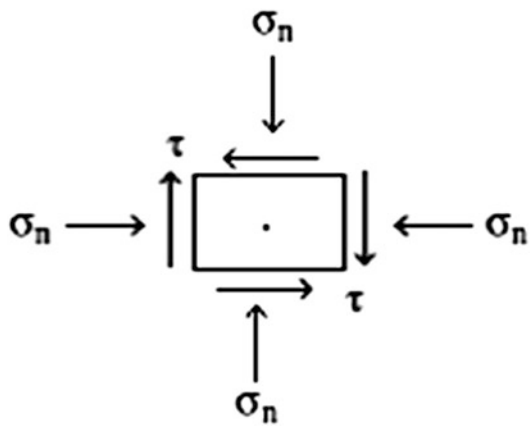


Fig. 8.8 Angle of repose

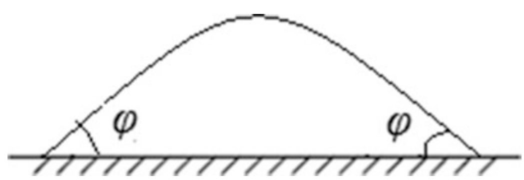
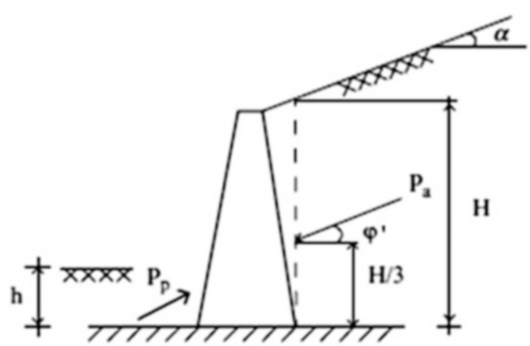


Fig. 8.9 Active and passive failure states



The magnitude of the soil pressure force depends on how the wall moved when the backfill was placed. If the wall moved away from the backfill (to the left in Fig. 8.9), the soil is said to be in an *active failure state*. The other extreme case is when the wall is pushed into the soil; the failure state is said to be in the *passive mode*. There is a significant difference in the force magnitudes corresponding to these states.

In general, the active force is *an order of magnitude* less than the passive force. For the applications that we are considering, the most likely case is when the wall moves away from the soil, and therefore we assume "*active*" conditions. The downward component tends to increase the stability with respect to overturning about the toe and also increases the friction force.

Different theories for the soil pressure distribution have been proposed which relate to the choice of φ'. The Rankine theory assumes $\varphi' = 0$ (i.e., no shear stress), and the Coulomb theory assumes $\varphi' = \varphi$. Considering that there is significant variability in soil properties, both theories predict pressure distributions which are suitable for establishing the wall dimensions.

In what follows we present the key elements of the Rankine theory. There are many textbooks that deal with mechanics of soil. In particular, we suggest Lamb and Whitman [1], Terzaghi and Peck [2], and Huntington [3].

8.2.2 Rankine Theory: Active Soil Pressure

Figure 8.10 defines the geometry and the soil pressure distribution. The pressure is applied to vertical surfaces through the heel and toe and is assumed to vary linearly with depth as shown. The magnitudes of the forces acting on a strip of unit width in the longitudinal direction of the wall are:

$$\begin{aligned} P_{\mathrm{a}} &= \frac{1}{2}\gamma H^2 k_{\mathrm{a}} \\ P_{\mathrm{p}} &= \frac{1}{2}\gamma h^2 k_{\mathrm{p}} \end{aligned} \tag{8.4}$$

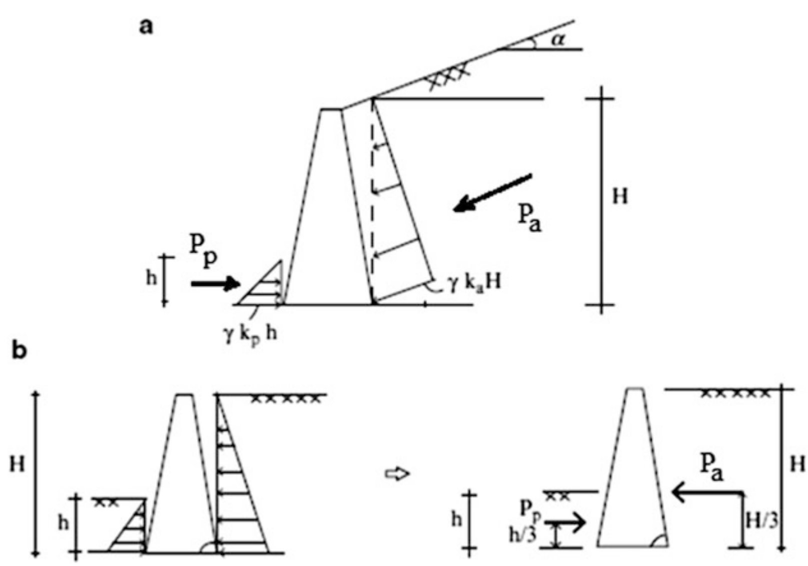


Fig. 8.10 (**a**) Soil pressure distribution for Rankine theory $\alpha \neq 0$. (**b**) Soil pressure distribution for Rankine theory $\alpha = 0$

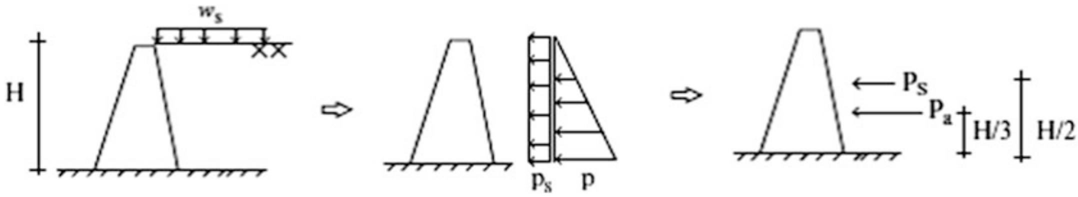


Fig. 8.11 Pressure distributions due to surcharge and active soil pressure

where γ is the unit weight of the soil backfill, k_a and k_p are defined as the active and passive soil pressure coefficients,

$$k_a = \cos\alpha \left\{ \frac{\cos\alpha - \sqrt{(\cos\alpha)^2 - (\cos\varphi)^2}}{\cos\alpha + \sqrt{(\cos\alpha)^2 - (\cos\varphi)^2}} \right\}$$
$$k_p = \frac{1 + \sin\varphi}{1 - \sin\varphi} \tag{8.5}$$

where φ is the internal friction angle, and α is the angle of inclination for the backfill.

When the backfill is level, $\alpha = 0$ and k_a reduces to

$$k_a = \frac{1 - \sin\varphi}{1 + \sin\varphi} \tag{8.6}$$

In this case, both resultants are horizontal forces.

8.2.2.1 Soil Pressure Due to Surcharge

When a surcharge is applied to the top of a backfill, additional soil pressure is developed. This pressure is assumed to be uniform over the depth. In the case of a uniform surcharge applied to a horizontal backfill, the *added* pressure is estimated as

$$\begin{aligned} p_s &\approx k_a w_s \\ P_s &\approx k_a w_s H \end{aligned} \tag{8.7}$$

where k_a is defined by (8.6). The soil pressure distributions due to the surcharge and the active soil pressure are illustrated in Fig. 8.11.

8.3 Stability Analysis of Retaining Walls

The key concerns for a retaining wall are overturning about the toe and sliding. In order to address these issues, one needs to determine the forces acting on the wall. This step requires that we carry out an equilibrium analysis.

Consider the typical gravity wall shown in Fig. 8.12. The weights of the wall and soil segments are denoted by W_j; P_a and P_p represent the lateral soil pressure forces; N and F are the normal and tangential (friction) forces due to the soil pressure acting on the base. $\bar{x}$ defines the line of action of the normal force acting on the base.

Summing forces in the vertical direction leads to

$$N = \sum W_j \tag{8.8}$$

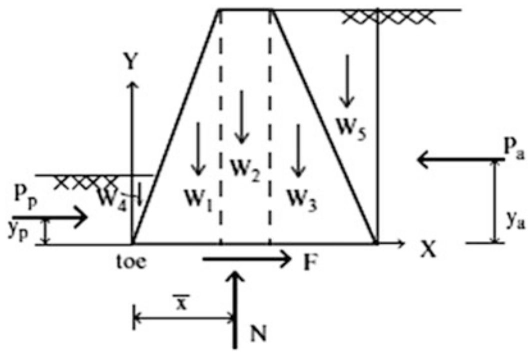


Fig. 8.12 Typical gravity wall

Similarly, horizontal force summation yields

$$F = \sum P_i \tag{8.9}$$

The maximum horizontal force is taken as $F_{\max} = \mu N$. where μ is a friction coefficient for the soil/base interface. This quantity is used to define the factor of safety for sliding:

$$\text{F.S.}_{\text{sliding}} = \frac{F_{\max}}{F} = \frac{\mu N}{F} \tag{8.10}$$

The line of action of N is found by summing the moments about the toe.

$$\begin{gathered} N\bar{x} = P_{\mathrm{p}} y_{\mathrm{p}} - P_{\mathrm{a}} y_{\mathrm{a}} + \sum W_j x_j = M_{\mathrm{net}} \\ \Downarrow \\ \bar{x} = \frac{M_{\mathrm{net}}}{N} \end{gathered} \tag{8.11}$$

For stability with respect to overturning, $\bar{x}$ must be positive. A negative value of $\bar{x}$ implies that the line of action of N lies outside the base. The safety measure for overturning is defined as the ratio of the resisting moment about the toe to the overturning moment.

$$\text{F.S.}_{\text{overturning}} = \frac{M_{\text{resisting}}}{M_{\text{overturning}}} \tag{8.12}$$

Noting Fig. 8.12, this definition expands to

$$\text{F.S.}_{\text{overturning}} = \frac{P_{\mathrm{p}} y_{\mathrm{p}} + \sum W_j x_j}{P_{\mathrm{a}} y_{\mathrm{a}}} \tag{8.13}$$

Typical desired values are $\text{F.S.}_{\text{sliding}} > 1.5$ and $\text{F.S.}_{\text{overturning}} > 2$.

In order to increase the factors of safety against sliding and overturning, either one can increase the width of the concrete wall or one can add a footing extending out from the original base. These schemes are illustrated in Fig. 8.13.

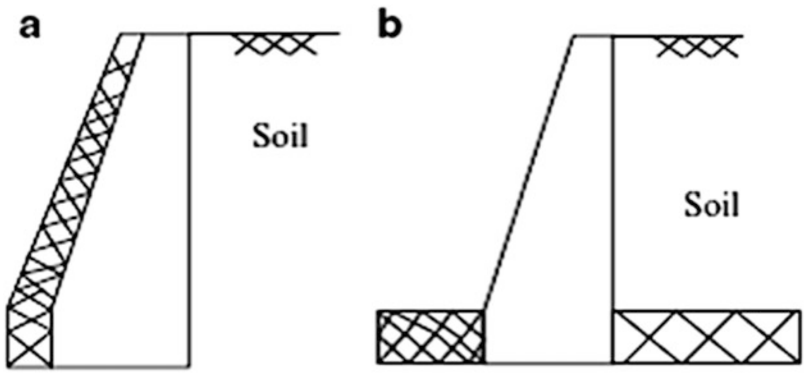


Fig. 8.13 (**a**) Gravity wall and (**b**) cantilever retaining wall

8.4 Pressure Distribution Under the Wall Footing

We consider the pressure acting on the footing is assumed to vary linearly. There are two design constraints: firstly, the peak pressures must be less than the allowable bearing pressure for the soil and secondly the pressure cannot be negative, i.e., tension. Noting the formulation presented in Sect. 7.2, the peak pressures are given by (7.6) (we work with a unit width strip of the footing along the length of the wall, i.e., we take $B = 1$ and N as the resultant) which we list below for convenience. Figure 8.14 shows the soil pressure distributions for various values of e.

$$q_1 = \frac{N}{L}\left\{1 + \frac{6e}{L}\right\}$$
$$q_2 = \frac{N}{L}\left\{1 - \frac{6e}{L}\right\} \tag{8.14}$$

The second design constraint requires $|e| \leq L/6$ or equivalently, the line of action of N must be located within the middle third of the footing width, L. The first constraint limits the maximum peak pressure,

$$|q|_{\max} \leq q_{\text{allowable}}$$

where $q_{\text{allowable}}$ is the allowable soil pressure at the base of the wall. We note that the pressure distribution is uniform when N acts at the centroid of the footing area which, for this case, is the midpoint. Since e depends on the wall height and footing length, we define the optimal geometry as that combination of dimensions for which the soil pressure is *uniform*. Note that the line of action of the resultant N always coincides with the line of action of the applied vertical load.

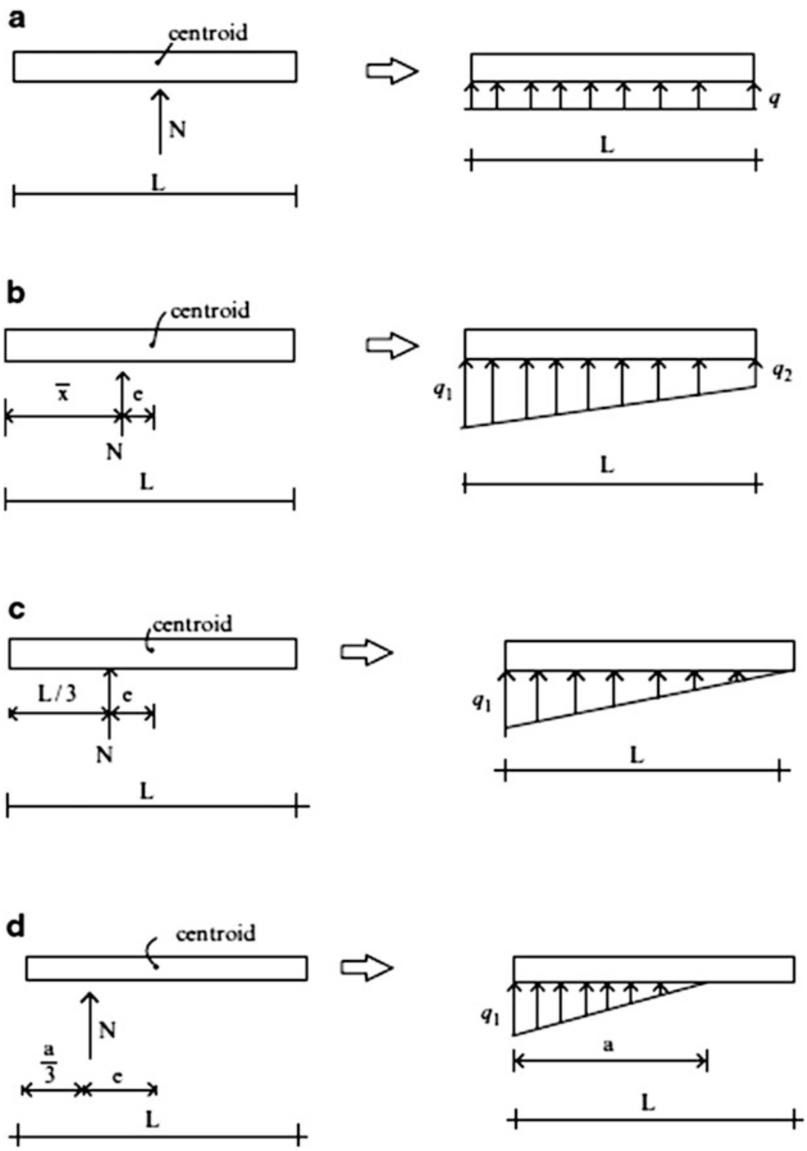


Fig. 8.14 Pressure distributions on footing/wall base. (**a**) $e = 0$. (**b**) $e < L/6$. (**c**) $e = L/6$. (**d**) $e > L/6$

Example 8.1 Gravity Retaining Wall Analysis

Given: The concrete gravity wall and soil backfill shown in Fig. E8.1a.

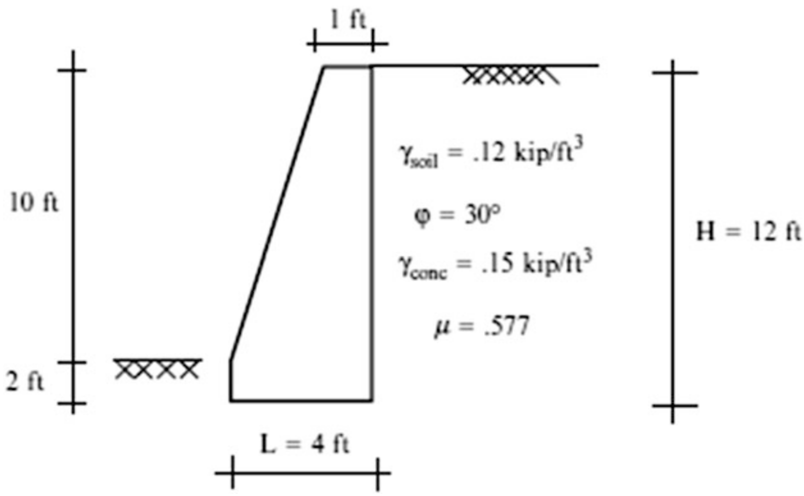


Fig. E8.1a Wall geometry

Determine: The factor of safety against sliding; the factor of safety against overturning; the line of action of the resultant. Use the Rankine theory for soil pressure computations. Neglect the passive pressure acting on the toe.

Solution:

For $\varphi = 30 k_a = \dfrac{1 - \sin\varphi}{1 + \sin\varphi} = \dfrac{1}{3}$

Then $P_a = \frac{1}{2}(0.12)(12)^2\left(\frac{1}{3}\right)(1\,\text{ft}) = 2.88\,\text{kip/ft of wall}$

Next, we compute the weight of the concrete wall segments per foot of wall. Noting Fig. E8.1b,

$$W_1 = (0.150)(10)(1)(1\,\text{ft}) = 1.5\,\text{kip}$$
$$W_2 = (0.150)\left(\frac{10}{2}\right)(3)(1\,\text{ft}) = 2.25\,\text{kip}$$
$$W_3 = (0.150)(4)(2)(1\,\text{ft}) = 1.2\,\text{kip}$$

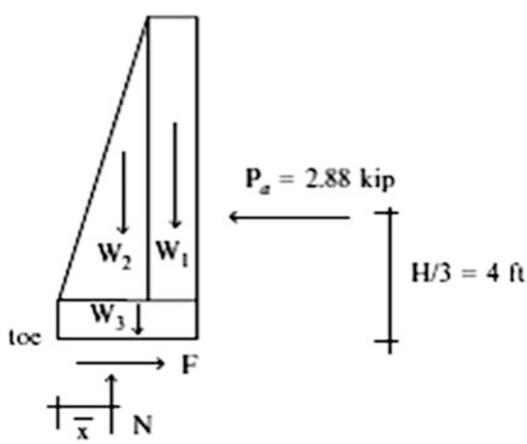


Fig. E8.1b Free body diagram

Applying vertical force equilibrium yields

$$N = W_1 + W_2 + W_3 = 1.5 + 2.25 + 1.2 = 4.95\,\text{kip}$$

The factor of safety with respect to sliding is defined as the ratio of the maximum available friction force $F_{\max}$ to the actual horizontal force.

$$F_{\max} = \mu N = N\tan\phi = 0.577(4.95) = 2.86\,\text{kip}$$

$$\text{F.S.}_{\text{sliding}} = \frac{\mu N}{P_a} = \frac{2.86}{2.88} = 0.99$$

The line of action of N is determined by summing moments about the toe. The factor of safety with respect to overturning is defined as the ratio of the resisting moment to the overturning moment, both quantities with respect to the toe.

$$M_{B_{\text{overturning}}} = P_a\left(\frac{H}{3}\right) = 2.88(4) = 11.52\,\text{kip ft}$$

$$\begin{aligned} M_{B_{\text{resisting}}} &= W_1(3.5) + W_2(2) + W_3(2) \\ &= 1.5(3.5) + 2.25(2) + 1.2(2) = 12.15\,\text{kip ft} \end{aligned}$$

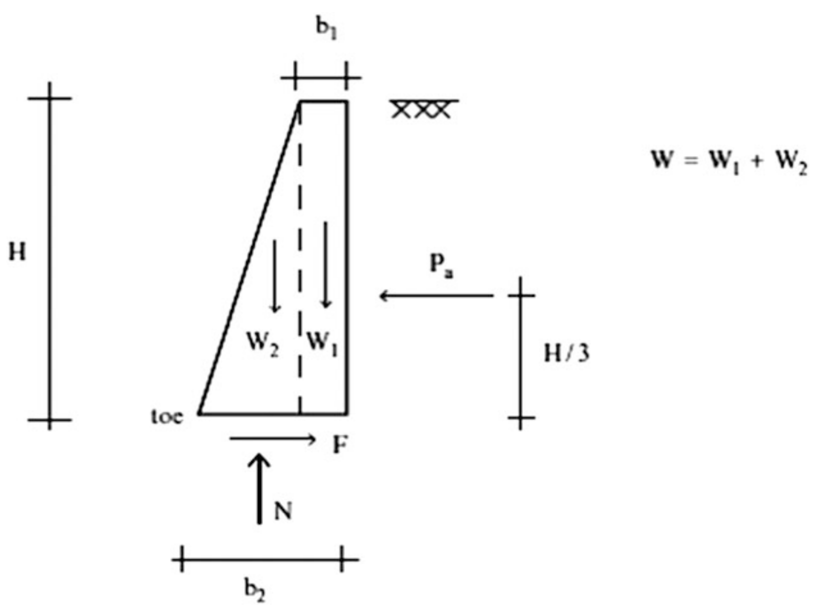
$$\text{F.S.}_{\text{overturning}} = \frac{M_{B_{\text{resisting}}}}{M_{B_{\text{overturning}}}} = \frac{12.15}{11.52} = 1.05$$

$$M_{\text{net}} = M_{B_{\text{overturning}}} - M_{B_{\text{resisting}}} = 0.63\,\text{kip ft clockwise}$$

$$\bar{x} = \frac{M_{\text{net}}}{N} = \frac{0.63}{4.95} = 0.13\,\text{ft}$$

In order to increase the factors of safety, the geometry needs to be modified.

The following procedure is useful for estimating appropriate values for b_1 and b_2. Given the wall height, one can derive expressions for the factors of safety. The details are listed below.



$$W = \frac{(b_1 + b_2)}{2} H\gamma_c = N$$

$$P_a = k_a \left\{ \frac{1}{2}\gamma_s H^2 \right\}$$

$$\text{F.S.}_{\text{sliding}} = \frac{\mu N}{P_a}$$

$$\Downarrow$$

$$\text{F.S.}_{\text{sliding}} = \left(\frac{\mu\gamma_c}{k_a\gamma_s}\right)\left(\frac{b_2}{H}\right)\left(1 + \frac{b_1}{b_2}\right)$$

$$M_{\text{overturning}} = \frac{H}{3}P_a = \frac{1}{6}k_a\gamma_s H^3$$

$$M_{\text{resisting}} = \frac{2}{3}(b_2 - b_1)W_2 + \left(b_2 - \frac{b_1}{2}\right)W_1$$

$$= \frac{H\gamma_c b_2^2}{3}\left\{1 - \frac{1}{2}\left(\frac{b_1}{b_2}\right)^2 + \frac{b_1}{b_2}\right\}$$

$$\text{F.S.}_{\text{overturning}} = \frac{M_{B_{\text{resisting}}}}{M_{B_{\text{overturning}}}} = \frac{\frac{H\gamma_c b_2^2}{3}\left\{1 - \frac{1}{2}\left(\frac{b_1}{b_2}\right)^2 + \frac{b_1}{b_2}\right\}}{\frac{1}{6}k_a\gamma_s H^3}$$

$$\Downarrow$$

$$\text{F.S.}_{\text{overturning}} = \frac{2\gamma_c}{k_a\gamma_s}\left(\frac{b_2}{H}\right)^2\left\{1 - \frac{1}{2}\left(\frac{b_1}{b_2}\right)^2 + \frac{b_1}{b_2}\right\}$$

One specifies the factor of safety with respect to overturning, and the ratio b_1/b_2, and then computes the value for b_2/H. With b_2/H known, one checks for sliding and if necessary modifies the value of b_2/H.

Example 8.2

Given: The concrete gravity wall and soil backfill shown in Fig. E8.2a.

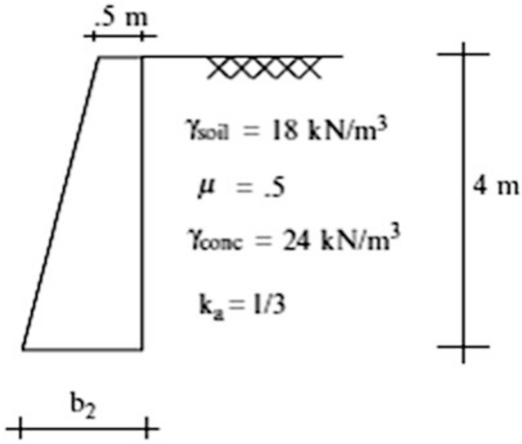


Fig. E8.2a

Determine: The required value for b_2. Take the factors of safety for overturning and sliding to be equal to 2 and 1.5, respectively.

Solution: Given $b_1 = 0.5$ m, $H = 4$ m, F.S.$_{\text{overturning}} = 2$, and F.S.$_{\text{sliding}} = 1.5$, we determine b_2 corresponding to the two stability conditions.

$$\text{F.S.}_{\text{overturning}} = \frac{2\gamma_c}{k_a\gamma_s}\left(\frac{b_2}{H}\right)^2\left\{1 - \frac{1}{2}\left(\frac{b_1}{b_2}\right)^2 + \frac{b_1}{b_2}\right\}$$

$$\frac{2(24)}{\left(\frac{1}{3}\right)(18)}\left(\frac{b_2}{4}\right)^2\left\{1 - \frac{1}{2}\left(\frac{0.5}{b_2}\right)^2 + \frac{0.5}{b_2}\right\} = 2$$

$$\therefore \quad b_2^2 + 0.5b_2 - 4.125 = 0 \quad b_{2\,\text{required}} = 1.8\,\text{m}$$

$$\text{F.S.}_{\text{sliding}} = \left(\frac{\mu\gamma_c}{k_a\gamma_s}\right)\left(\frac{b_2}{H}\right)\left(1 + \frac{b_1}{b_2}\right)$$

$$1.5 = \frac{0.5(24)}{\left(\frac{1}{3}\right)(18)}\left(\frac{b_2}{4}\right)\left(1 + \frac{0.5}{b_2}\right) \quad b_{2\,\text{required}} = 2.5\,\text{m}$$

Use $b_2 = 2.5$ m

Example 8.3 Retaining Wall with Footing

Given: The walls defined in Figs. E8.3a, E8.3b, and E8.3c. These schemes are modified versions of the wall analyzed in Example 8.1. We have extended the footing to further stabilize the wall.

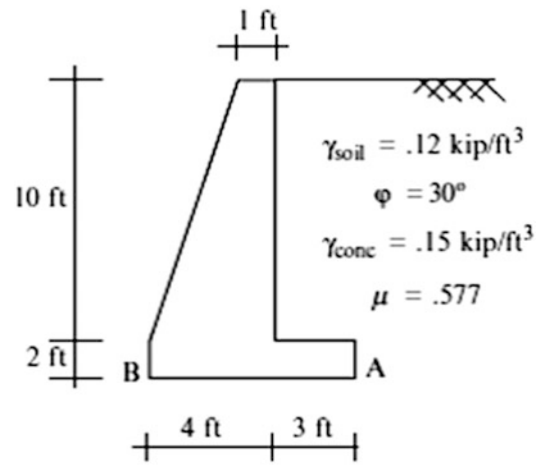


Fig. E8.3a Case "A"

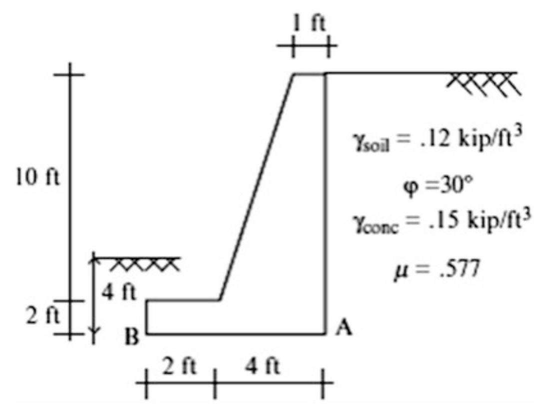


Fig. E8.3b Case "B"

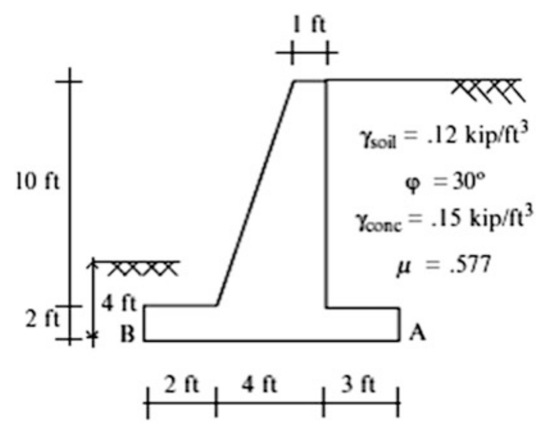


Fig. E8.3c Case "C"

Determine: The factor of safety against sliding; the factor of safety against overturning; the base pressure distribution. Use the Rankine theory for soil pressure computations. Neglect the passive pressure acting on the toe.

Solution:
Case "**A**": We work with the free body diagram shown in Fig. E8.3d. The vertical surface is taken to pass through the heel.

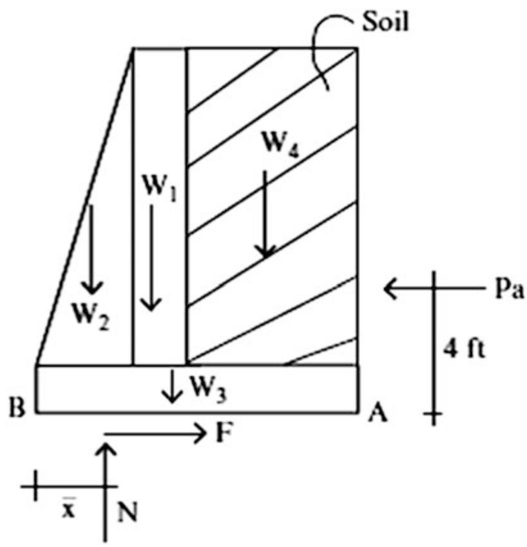


Fig. E8.3d

From Example 8.1:

$$W_1 = 1.5\,\text{kip} \quad W_2 = 2.25\,\text{kip} \quad P_a = 2.88\,\text{kip} \quad M_{B_{\text{overturning}}} = 11.52\,\text{kip ft}$$

The weight of the footing is

$$W_3 = (0.150)(7)(2)(1\,\text{ft}) = 2.1\,\text{kip}$$

The weight of soil is $W_4 = (0.120)(10)(3)(1\text{ ft}) = 3.6$ kip
Then

$$N = \sum W_i = 1.5 + 2.25 + 2.1 + 3.6 = 9.45\text{kip}$$
$$W_4 + W_3 + W_2 + W_1$$
$$F_{\max} = \mu N = 0.577(9.45) = 5.45\,\text{kip}$$
$$\text{F.S.}_{\text{sliding}} = \frac{F_{\max}}{P_a} = \frac{5.45}{2.88} = 1.89$$

We sum moments about the toe:

$$\begin{aligned} M_{B_{\text{resisting}}} &= W_1(3.5) + W_2(2) + W_3(3.5) + W_4(5.5) \\ &= 1.5(3.5) + 2.25(2) + 2.1(3.5) + 3.6(5.5) = 36.69\,\text{kip ft} \\ M_{B_{\text{overturning}}} &= 11.52\,\text{kip ft} \end{aligned}$$

Using these moments, the factor of safety is

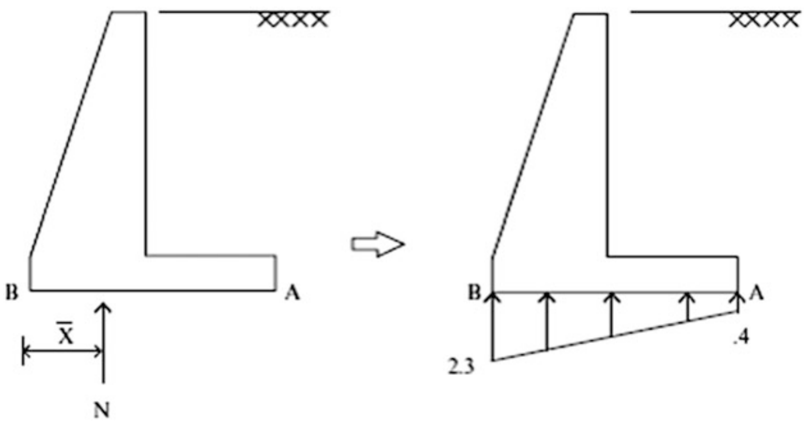
$$\text{F.S.}_{\text{overturning}} = \frac{M_{\text{B}_{\text{resisting}}}}{M_{\text{B}_{\text{overturning}}}} = \frac{36.9}{11.52} = 3.2$$

Next, we determine the line of action of the resultant

$$M_{\text{net}} = M_{\text{B}_{\text{overturning}}} - M_{\text{B}_{\text{resisting}}} = 25.38\,\text{kip ft}$$
$$\bar{x} = \frac{M_{\text{net}}}{N} = \frac{25.38}{9.45} = 2.68\,\text{ft}$$
$$e = \frac{L}{2} - \bar{x} = 3.5 - 2.68 = 0.82\,\text{ft} < \frac{L}{6} = 1.167\,\text{ft}$$

Lastly, we compute the pressure loading acting on the base.

$$q = \frac{N}{L}\left(1 \pm \frac{6e}{L}\right) = \frac{9.45}{7}\left(1 \pm \frac{6(0.82)}{7}\right) \Rightarrow q_1 = 2.3\,\text{kip/ft}^2, q_2 = 0.4\,\text{kip/ft}^2$$



Case "**B**": For this case, we work with the free body diagram shown in Fig. E8.3e. The dimensions are defined in Fig. E8.3b. $W_3 = (0.150)(6)(2)(1\text{ ft}) = 1.8$ kip. $W_5 = (0.120)(2)(2)(1\text{ ft}) = 0.48$ kip

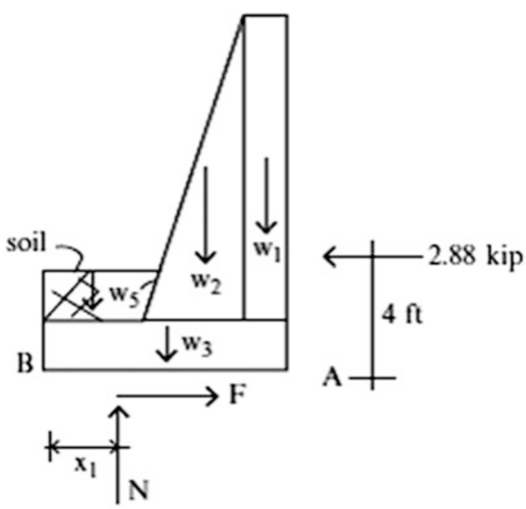


Fig. E8.3e

The calculations proceed as follows:

$$N = W_1 + W_2 + W_3 + W_5 = 1.5 + 2.25 + 1.8 + 0.48 = 6.03\,\text{kip}$$

$$F_{\max} = \mu N = 0.577(6.03) = 3.48\,\text{kip}$$

$$\text{F.S.}_{\text{sliding}} = \frac{F_{\max}}{P_{\text{a}}} = \frac{3.48}{2.88} = 1.2$$

We sum moments about the toe:

$$M_{\text{B}_{\text{resisting}}} = W_1(5.5) + W_2(4) + W_3(3) + W_5(1)$$
$$= 1.5(5.5) + 2.25(4) + 1.8(3) + 0.48(1) = 23.13\,\text{kip ft}$$

$$M_{\text{B}_{\text{overturning}}} = 11.52\,\text{kip ft}$$

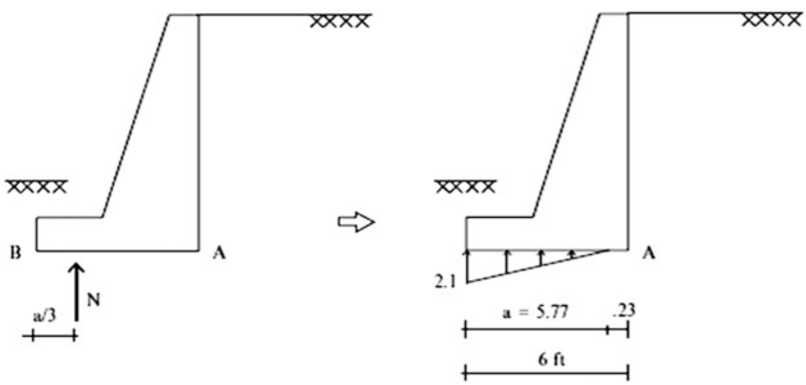
$$\text{F.S.}_{\text{overturning}} = \frac{M_{\text{B}_{\text{resisting}}}}{M_{\text{B}_{\text{overturning}}}} = \frac{23.13}{11.52} = 2.0$$

$$M_{\text{net}} = M_{\text{B}_{\text{overturning}}} - M_{\text{B}_{\text{resisting}}} = 11.61\,\text{kip ft}$$

$$\bar{x} = \frac{M_{\text{net}}}{N} = \frac{11.61}{6.03} = 1.925\,\text{ft}$$

$$e = \frac{L}{2} - \bar{x} = 3 - 1.925 = 1.07\,\text{ft} > \frac{L}{6} = 1.0\,\text{ft} \quad \therefore \bar{x} = \frac{a}{2} \quad a = 5.77\,\text{ft}$$

$$q_1 = \frac{2N}{a} = \frac{2(6.03)}{5.77} = 2.1\,\text{kip/ft}^2$$



Note that the line of action of the normal force is within the base but the pressure is negative at the heel.

Case "**C**": We work with the free body diagram shown in Fig. E8.3f. The dimensions are defined in Fig. E8.3c. The revised value of W_3 is $W_3 = (0.15)(9)(2)(1\text{ ft}) = 2.7$ kip

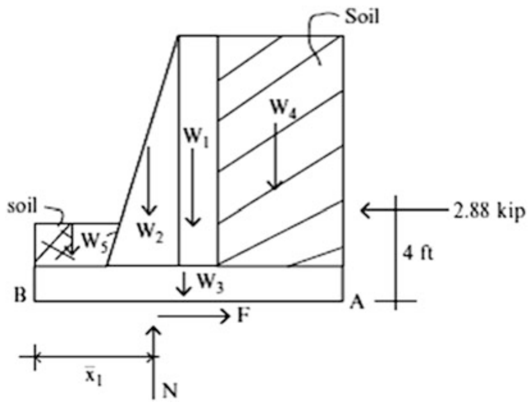


Fig. E8.3f

Then

$$N = W_1 + W_2 + W_3 + W_4 + W_5 = 1.5 + 2.25 + 2.7 + 3.6 + 0.48 = 10.53\,\text{kip}$$

$$F_{\max} = \mu N = 0.577(10.53) = 6.1\,\text{kip}$$

$$\text{F.S.}_{\text{sliding}} = \frac{F_{\max}}{P_a} = \frac{6.1}{2.88} = 2.12$$

We sum moments about the toe:

$$\begin{aligned} M_{B_{\text{balancing}}} &= W_1(5.5) + W_2(4) + W_3(4.5) + W_4(7.5) + W_5(1) \\ &= 1.5(5.5) + 2.25(4) + 2.7(4.5) + 3.6(7.5) + 0.48(1) = 56.88\,\text{kip ft} \end{aligned}$$

$$\text{F.S.}_{\text{overturning}} = \frac{M_{B_{\text{resisting}}}}{M_{B_{\text{overturning}}}} = \frac{56.88}{11.52} = 4.94$$

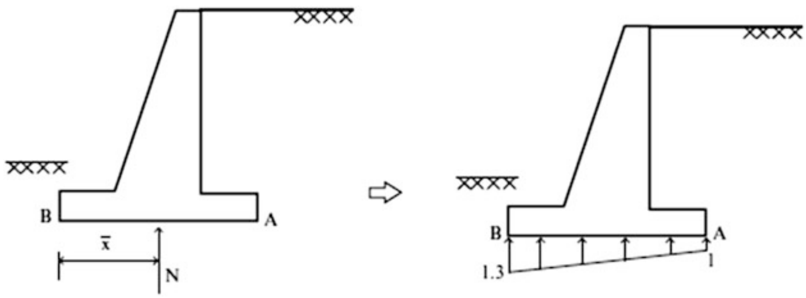
$$M_{\text{net}} = M_{B_{\text{overturning}}} - M_{B_{\text{resisting}}} = 45.36\,\text{kip ft}$$

$$\bar{x} = \frac{M_{\text{net}}}{N} = \frac{45.36}{10.53} = 4.3\,\text{ft}$$

$$e = \frac{L}{2} - \bar{x} = 4.5 - 4.3 = 0.2\,\text{ft}$$

$$|e| < L/6 = 1.5\,\text{ft}$$

$$\therefore q = \frac{N}{L}\left(1 \pm \frac{6e}{L}\right) = \frac{10.53}{9}\left(1 \pm \frac{6(0.2)}{9}\right) \Rightarrow q_1 = 1.3\,\text{kip/ft}^2, q_2 = 1.0\,\text{kip/ft}^2$$



We point out that case C has the lowest peak pressure. The analysis results are summarized in the table below.

	Case A	Case B	Case C
N	9.45 kip	6.03 kip	10.53 kip
Friction	5.45 kip	3.48 kip	6.1 kip
$F.S._{sliding}$	1.89	1.2	2.12
$M_{balancing}$	36.9 kip ft	23.13 kip ft	56.88 kip ft
$M_{overturning}$	11.52 kip ft	11.52 kip ft	11.52 kip ft
$F.S._{overturning}$	3.2	2.0	4.94
$\bar{x}$	2.68 ft	1.925 ft	4.3 ft
e	0.82 ft $< L/6$	1.07 ft $> L/6$	0.2 ft $< L/6$
q_1	2.3 kip/ft	2.1 kip/ft	1.3 kip/ft^2
q_2	0.4 kip/ft	–	1.0 kip/ft^2

Example 8.4 Cantilever retaining wall

Given: The retaining wall and soil backfill shown in Fig. E8.4a

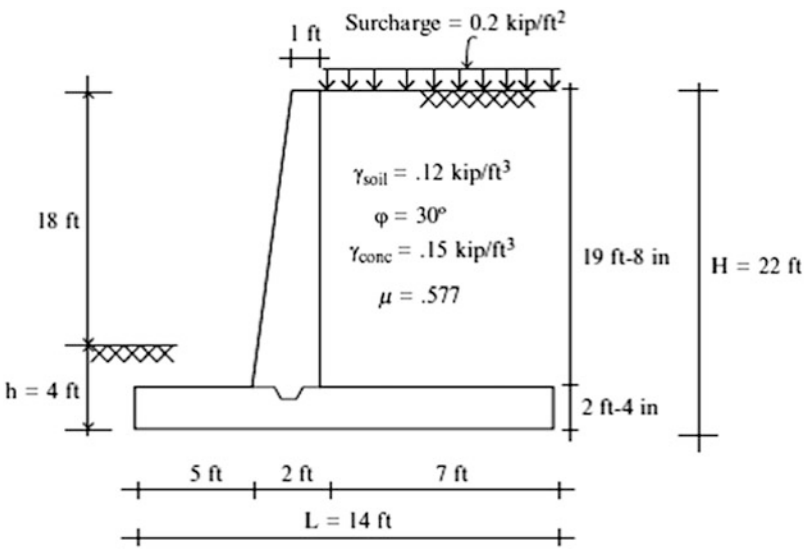


Fig E8.4a

Determine: The factor of safety against sliding; the factor of safety against overturning; the base pressure distribution. Assume the allowable soil pressure = 4 ksf. Use the Rankine theory for soil pressure computations.

Solution:

Noting Fig. E8.4b, the soil pressure and weight forces are

$$P_a = \frac{1}{2}k_a\gamma_s H^2 = \frac{1}{2}\left(\frac{1}{3}\right)(0.12)(22)^2 = 9.68\,\text{kip}$$

$$P_p = \frac{1}{2}k_p\gamma_s H'^2 = \frac{1}{2}(3)(0.12)(4)^2 = 2.88\,\text{kip}$$

$$P_s = k_a w_s H = \frac{1}{3}(0.2)(22) = 1.47\,\text{kip}$$

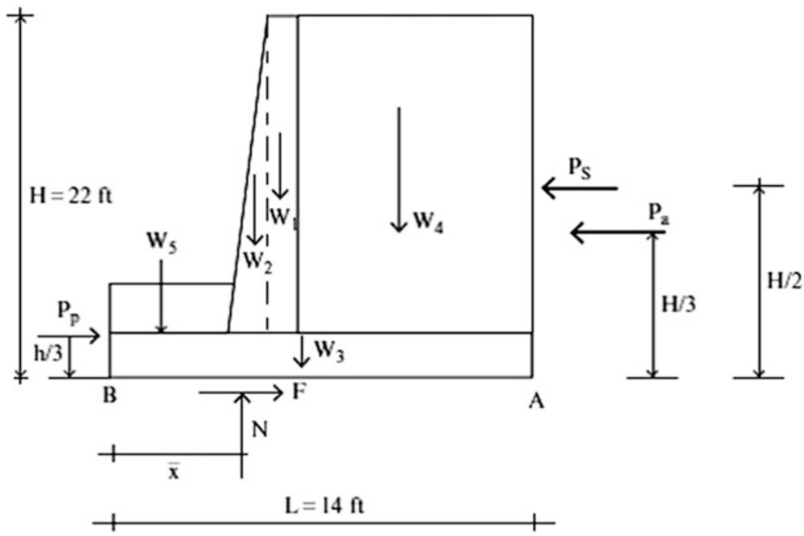


Fig. E8.4b

$$W_1 = 0.15(1)(19.66) = 2.95\,\text{kip}$$

$$W_2 = 0.15(1)\left(\frac{19.66}{2}\right) = 1.47\,\text{kip}$$

$$W_3 = 0.15(2.34)(14) = 4.19\,\text{kip}$$

$$W_4 = 0.12(7)(19.66) = 16.5\,\text{kip}$$

$$W_5 = 0.12(5)(1.67) = 1.0\,\text{kip}$$

The normal and horizontal forces are

$$N = W_1 + W_2 + W_3 + W_4 + W_5 = 26.84\,\text{kip}$$

$$F_{\max} = \mu N = 0.577(26.84) = 15.5\,\text{kip}$$

$$\sum F_{\text{horizontal}} = P_{\text{a}} + P_{\text{s}} - P_{\text{p}} = 9.68 + 1.47 - 2.88 = 8.27\,\text{kip} \leftarrow$$

Next, we compute the factors of safety.

$$\text{F.S.}_{\text{sliding}} = \frac{F_{\max}}{\sum F_{\text{horizontal}}} = \frac{15.5}{8.27} = 1.87$$

$$M_{\text{B}_{\text{overturning}}} = P_{\text{a}}\left(\frac{H}{3}\right) + P_{\text{s}}\left(\frac{\text{H}}{2}\right) = 9.68\left(\frac{22}{3}\right) + 1.47\left(\frac{22}{2}\right) = 87.2\,\text{kip ft}$$

$$\begin{aligned} M_{\text{B}_{\text{resisting}}} &= W_1(6.5) + W_2(5.67) + W_3(7) + W_4(10.5) + W_5(2.5) + P_{\text{p}}(1.33) \\ &= 2.95(6.5) + 1.47(5.67) + 4.91(7) + 16.5(10.5) \\ &\quad + 1.0(2.5) + 2.88(1.33) \\ &= 241.5\,\text{kip ft} \end{aligned}$$

$$\text{F.S.}_{\text{ovreturning}} = \frac{M_{\text{B}_{\text{resisting}}}}{M_{\text{B}_{\text{overturning}}}} = \frac{241.5}{87.2} = 2.77$$

Lastly, we determine the location of the line of action of N.

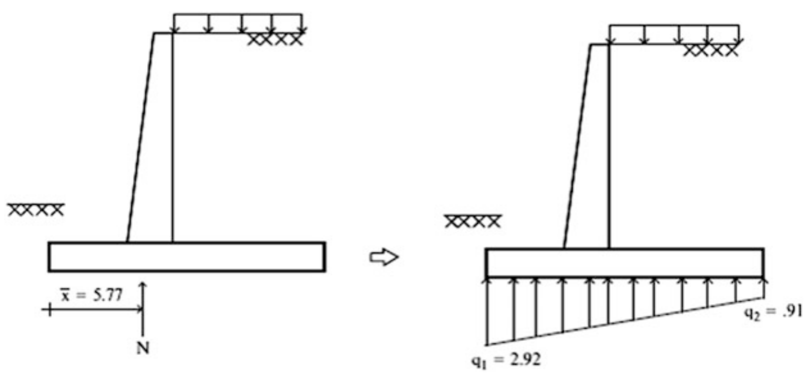
$$M_{\text{net}} = M_{\text{B}_{\text{overturning}}} - M_{\text{B}_{\text{resisting}}} = 154.8\,\text{kip ft}$$

$$\bar{x} = \frac{M_{\text{net}}}{N} = \frac{154.8}{26.84} = 5.77\,\text{ft}$$

$$e = \frac{L}{2} - \bar{x} = \frac{14}{2} - 5.77 = 1.23\,\text{ft} < \frac{L}{6} = 2.33\,\text{ft}$$

Using the above values, the peak pressures are

$$q = \frac{N}{L}\left(1 \pm \frac{6e}{L}\right) = \frac{26.84}{14}\left(1 \pm \frac{6(1.23)}{14}\right) \Rightarrow q_1 = 2.92\,\text{kip/ft}^2 \quad q_2 = 0.91\,\text{kip/ft}^2$$



Example 8.5 Retaining Wall Supported by Concrete Piles

Given: The structure shown in Fig. E8.5a. Assume all the loads acting on the wall are resisted by the axial loads in the concrete piles. Consider the pile spaced at 6 ft on center. Use Rankine theory.

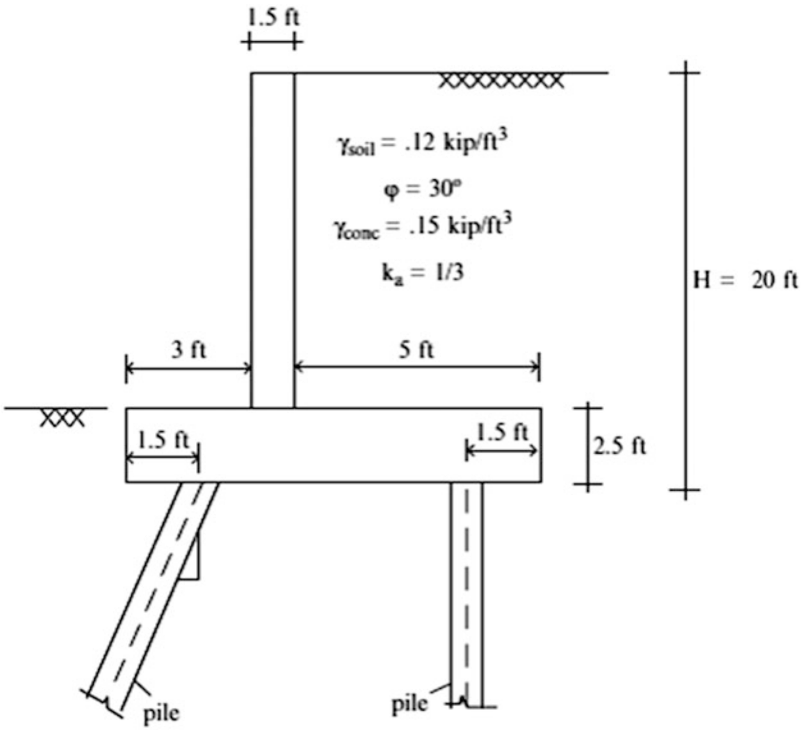


Fig. E8.5a

Determine: The axial loads in the piles.

Solution: We consider a 6 ft segment of the wall. The free body diagram for this segment is shown in Fig. E8.5b. F_1 and F_2 denote the pile forces; P_a is the active lateral soil force; and the W term relates to various weights. We neglect the passive soil force and assume the horizontal load is carried by the inclined pile.

$$P_a = \frac{1}{2}\left(\frac{1}{3}\right)(0.12)(20)^2(6) = 48\,\text{kip}$$

$$W_1 = (5)(17.5)(6)(0.12) = 63\,\text{kip}$$

$$W_2 = (1.5)(17.5)(6)(0.15) = 23.6\,\text{kip}$$

$$W_3 = (2.5)(9.5)(6)(0.15) = 21.4$$

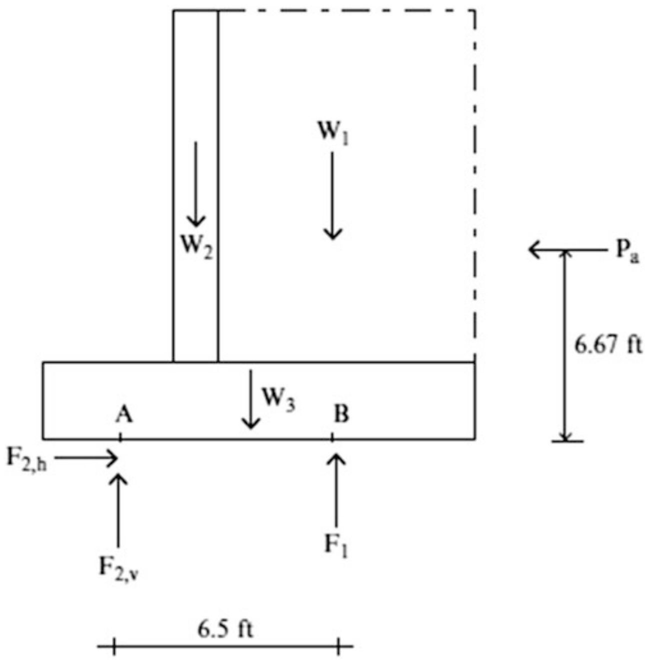


Fig. E8.5b

By summing the moments about A, we determine F_1:

$$\sum M_{\mathrm{A}} = 0 \quad (2.25)W_2 + +(5.5)W_1 = 6.67P_{\mathrm{a}} + 6.5F_1 \Rightarrow F_1 = 22.92\,\mathrm{kips}$$

Summing the vertical forces leads to

$$\sum F_y = 0 \Rightarrow F_{2,\mathrm{v}} = 85.1\,\mathrm{kip}$$

Similarly, the horizontal loads yields

$$\sum F_x = 0 \Rightarrow F_{2,\mathrm{h}} = P_{\mathrm{a}} = 48\,\mathrm{kip}$$

Then, the axial force in the battered pile is

$$F_2 = \sqrt{F_{\mathrm{h}}^2 + F_{\mathrm{v}}^2} = 97.7$$

And the required batter is 48/85.1 = 0.56

8.5 Critical Sections for Design of Cantilever Walls

The different segments of a typical cantilever retaining wall structure are shown in Fig. 8.15. The stem functions as a cantilever beam supported by the footing. Gravity and lateral loading are transmitted by the stem onto the footing which then distributes the loading onto the soil. The footing has two counteracting loadings at the heel; the loading due to the weight of the soil, and the pressure loading. The latter is usually neglected when estimating the peak negative moment in the footing. The

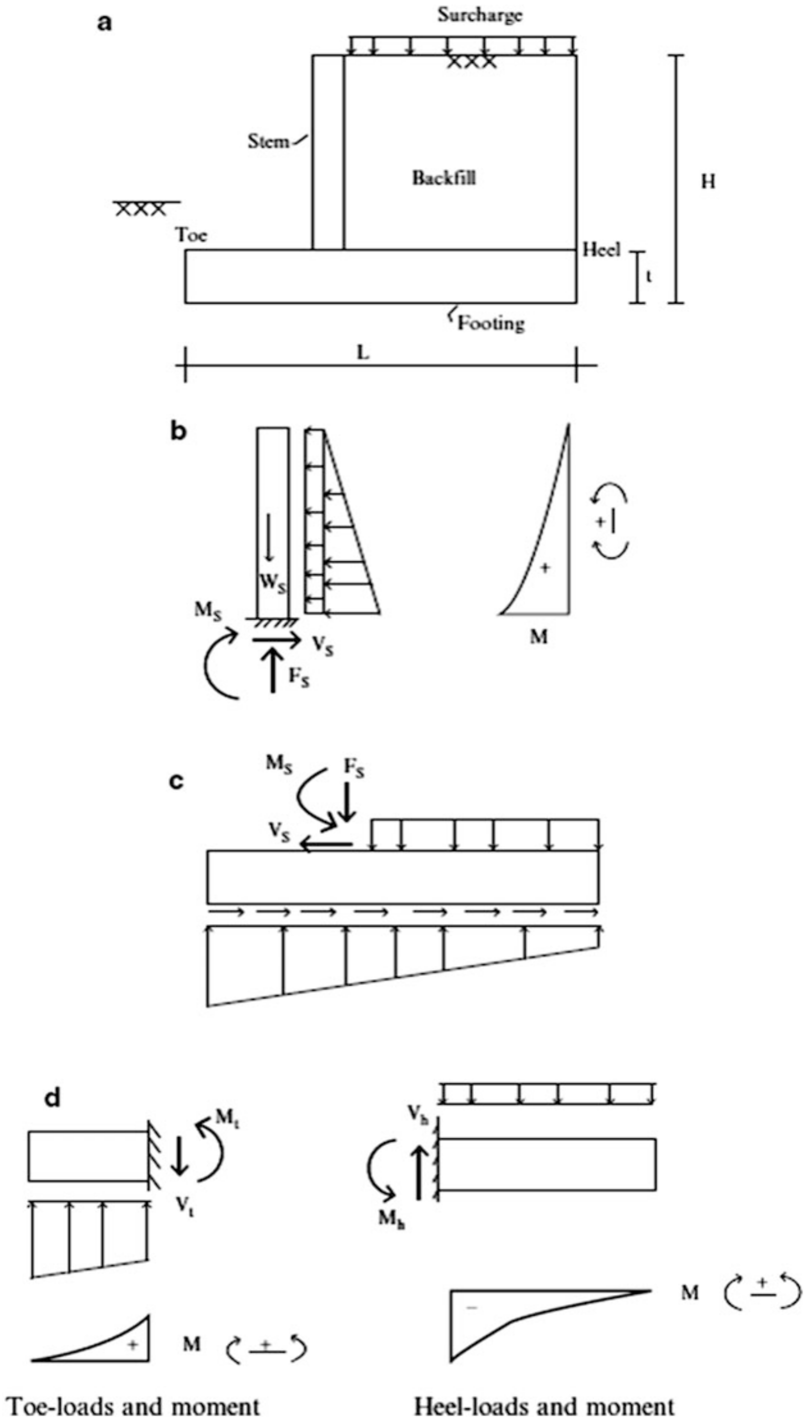


Fig. 8.15 Loadings and response pattern for cantilever retaining wall structure. (**a**) Cantilever retaining wall components. (**b**) Stem—loads and bending moment. (**c**) Footing—loads. (**d**) Components of footing

bending moment distributions are also plotted in Fig. 8.15d. Note that for this type of structure, the bending moment distribution in the footing has both positive and negative regions. *The critical region for design is the stem–footing junction*.

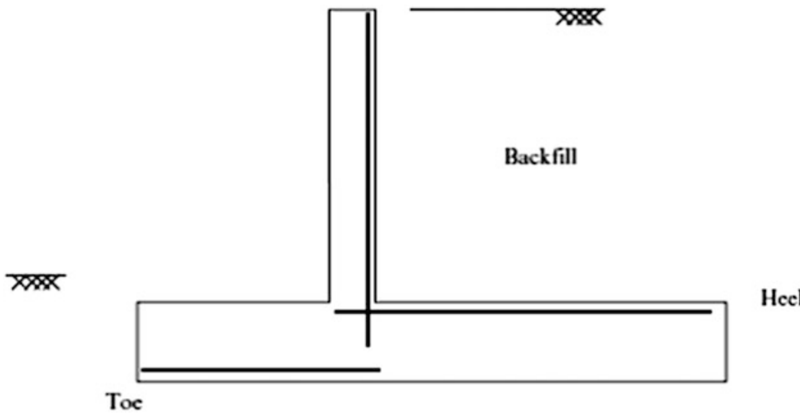


Fig. 8.16 Typical bending steel reinforcement patterns

Retaining wall structures are constructed using reinforced concrete. The thickness of the footing sections is governed by the shear capacity. The location and magnitude of the bending steel reinforcement is dictated by the sense of the bending moment distribution (i.e., positive or negative). Noting that the function of the reinforcement is to provide the tensile force required by the moment, the moment diagrams shown in Fig. 8.15d require the reinforcement patterns defined in Fig. 8.16. The actual size/number of the rebars depends on the magnitude of the moment and the particular design code used to dimension the member.

Example 8.6

Given: The structure shown in Fig. E8.6a.

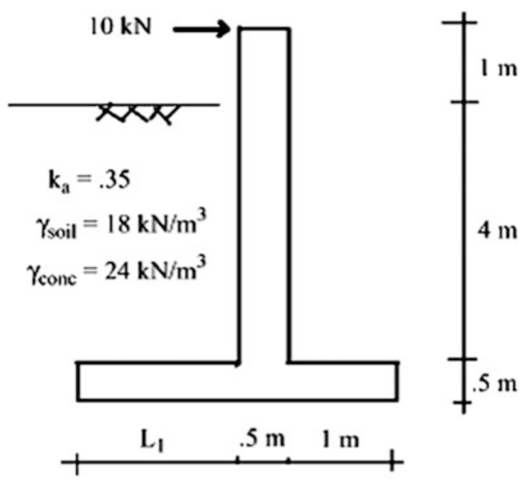


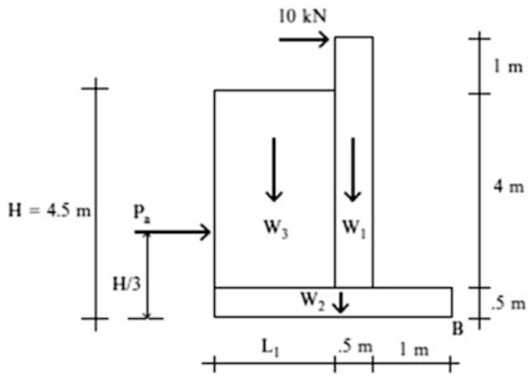
Fig. E8.6a

Determine:

(a) The required L_1 such that the factor of safety with respect to overturning is equal to 2.

(b) The tension areas in the stem, toe, and heel and show the reinforcing pattern.

Solution:



$$P_a = \frac{1}{2} k_a \gamma_s H^2 = \frac{1}{2}(0.35)(18)(4.5)^2 = 63.8\,\text{kN}$$

$$W_1 = (0.5)(5)(24) = 60\,\text{kN}$$
$$W_2 = (0.5)(1.5 + L_1)(24)$$
$$W_3 = (4)(L_1)(18)$$

$$M_{B_{\text{overturning}}} = 63.8\left(\frac{4.5}{3}\right) + 10(5.5) = 150.7$$

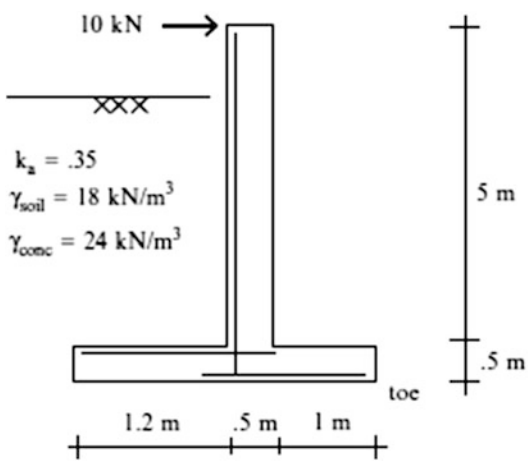
$$M_{B_{\text{resisting}}} = W_1(1.25) + W_2\left(\frac{L_1 + 1.5}{2}\right) + W_3\left(\frac{L_1}{2} + 1.5\right)$$

$$\text{F.S.}_{\text{overturning}} = 2 = \frac{M_{B_{\text{resisting}}}}{M_{B_{\text{overturning}}}} = \frac{W_1(1.25) + W_2\left(\frac{L_1 + 1.5}{2}\right) + W_3\left(\frac{L_1}{2} + 1.5\right)}{150.7}$$

$$\Downarrow$$

$$L_{1\,\text{required}} = 1.2\,\text{m}$$

The figure below shows the reinforcing pattern required for the tension areas.



8.6 Summary

8.6.1 Objectives of the Chapter

- To introduce the topic of vertical retaining wall structures used for embankments, abutments, and underground structures.
- To present a theory for establishing the lateral loading exerted by soil backfill on vertical walls.
- To develop a methodology for evaluating the stability of cantilever retaining walls when subjected to lateral loading due to backfill and surcharges.

8.6.2 Key Concepts and Facts

- The Rankine theory predicts a linear distribution of soil pressure which acts normal to a vertical face and increases with depth. The resultant force is given by

$$P_a = \frac{1}{2}\gamma H^2 k_a$$

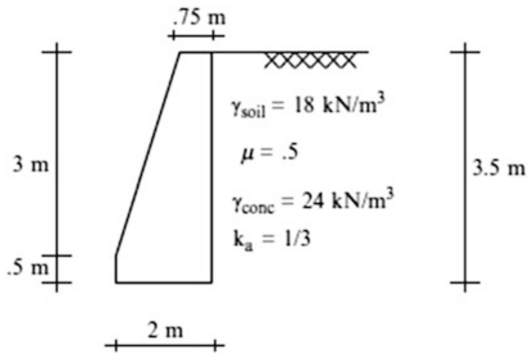
  where H is the vertical wall height, γ is the soil density, and k_a is a dimensionless coefficient that depends on the soil type and nature of the relative motion between the wall and the backfill. For active conditions,

$$k_a = \frac{1 - \sin\varphi}{1 + \sin\varphi}$$

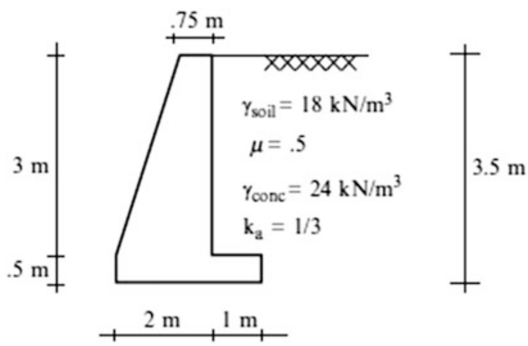
  where φ is the soil friction angle, typically $\approx 30°$.
- Stability is addressed from two perspectives: Sliding and overturning. The factor of safety with respect to sliding is defined as the ratio of the peak available horizontal friction force to the actual friction force. The factor of safety with respect to overturning about the toe is taken as the ratio of the restoring moment to the unbalancing moment.
- One selects the dimensions of the footing, such that these factors of safety are greater than one and the resultant force due to the structural weight and the soil loads acts within the middle third of the footing width.

8.7 Problems

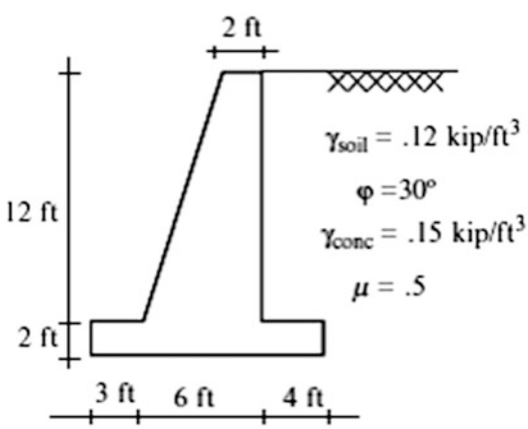
Problem 8.1 For the concrete retaining wall shown, determine the factors of safety against sliding and overturning and the base pressure distribution. Use the Rankine theory for soil pressure computations.



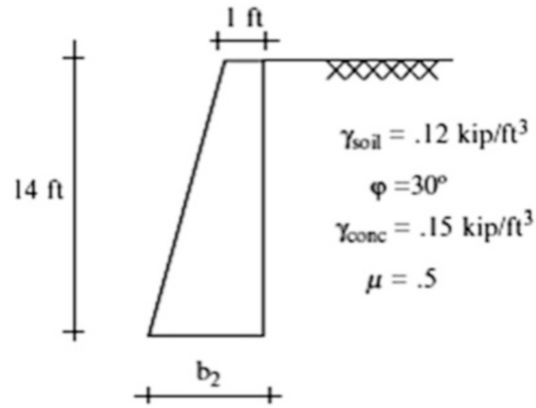
Problem 8.2 For the concrete retaining wall shown, determine the factors of safety against sliding and overturning and the base pressure distribution. Use the Rankine theory for soil pressure computations.



Problem 8.3 For the concrete retaining wall shown, determine the factors of safety against sliding and overturning and the base pressure distribution. Use the Rankine theory for soil pressure computations



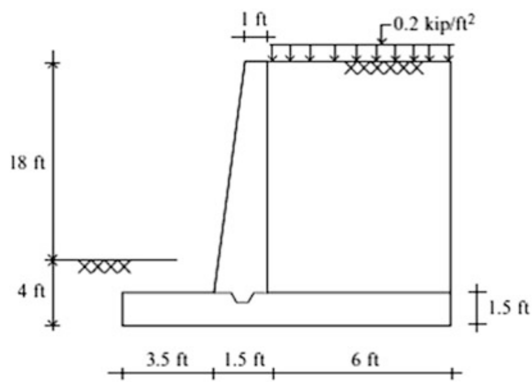
Problem 8.4 For the concrete retaining wall shown, determine the required value for b_2. Take the factors of safety for overturning and sliding to be equal to 1.75 and 1.25, respectively. Use the Rankine theory for soil pressure computations.



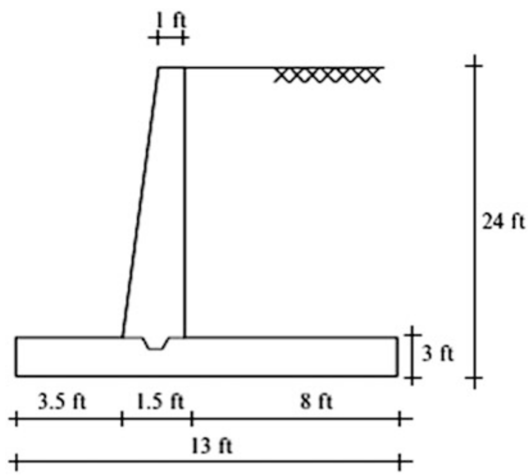
Problem 8.5 For the retaining wall shown, determine

(a) The soil pressure acting on the wall
(b) The factor of safety for overturning
(c) The factor of safety for sliding
(d) The soil pressure distribution under the footing

Assume: $\mu = 0.5$, $\gamma_{soil} = 0.12$ kip/ft^3, $k_a = 1/3$, $\gamma_{concrete} = 0.15$ kip/ft^3, $\mu = 0.5$, and $\Phi = 30°$.



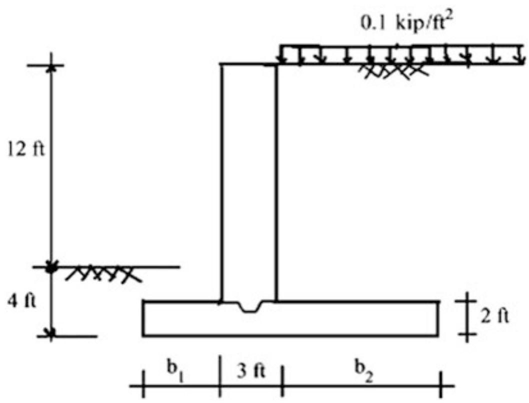
Problem 8.6



(a) Determine the factors of safety against overturning and sliding.
(b) Determine the soil pressure distribution under the footing (q_1, q_2).
(c) Determine the moment distribution in the stem.
(d) Determine the bending moment distribution in the heel.

Assume: Allowable soil pressure $= 5.0$ ksf, $\gamma_{\text{soil}} = 0.12$ kip/ft^3, $k_a = 1/3$, and $\gamma_{\text{concrete}} = 0.15$ kip/ft^3

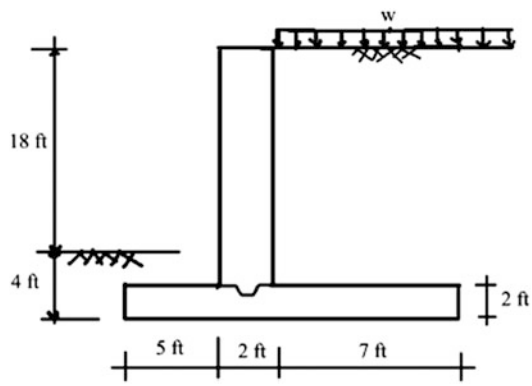
Problem 8.7



Suggest values for b_1 and b_2. Take the safety factors for sliding and overturning to be equal to 2.

Assume: $\gamma_{\text{soil}} = 0.12$ kip/ft^3, $\gamma_{\text{concrete}} = 0.15$ kip/ft^3, $\mu = 0.57$, and $\Phi = 30°$.

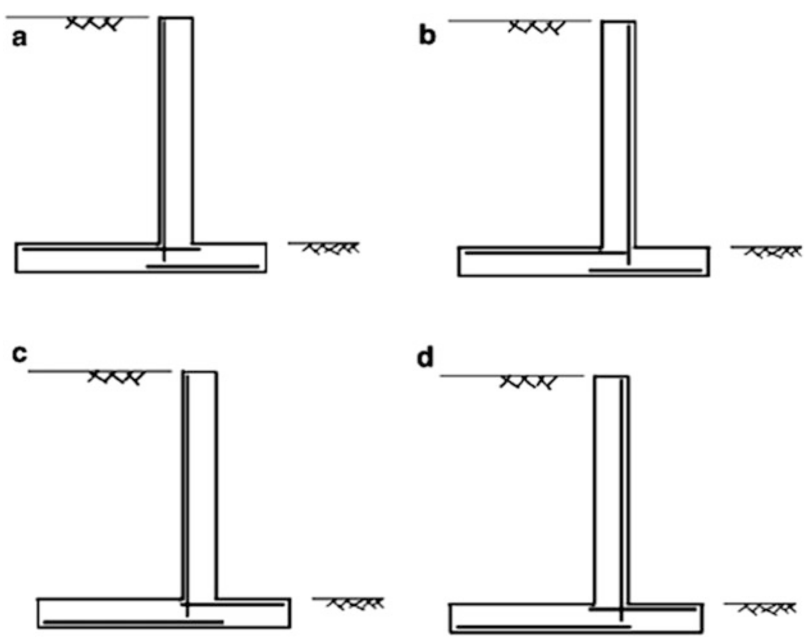
Problem 8.8



Determine the minimum value of w at which soil failure occurs (i.e., the soil pressure exceeds the allowable soil pressure).

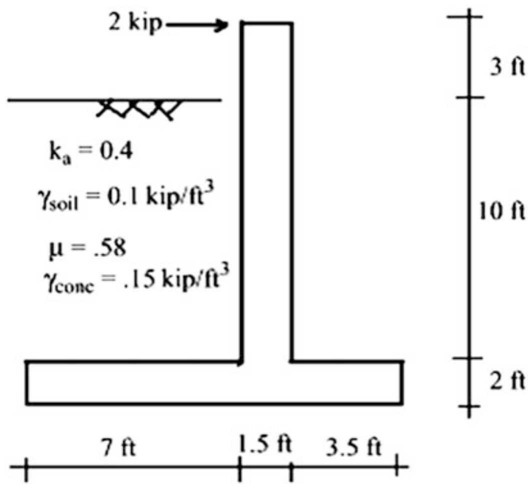
Assume: $q_{\text{allowable}} = 5$ kip/ft^2, $\gamma_{\text{soil}} = 0.12$ kip/ft^3, $\gamma_{\text{concrete}} = 0.15$ kip/ft^3, $\mu = 0.57$, and $\Phi = 30°$.

Problem 8.9 Which of the retaining walls shown below is adequately reinforced for bending?



Problem 8.10

(a) Determine the factor of safety with respect to overturning and sliding.
(b) Identify the tension areas in the stem, toe, and heel and show the reinforcing pattern.
(c) Determine the location of the line of action of the resultant at the base of the footing



References

1. Lambe TW, Whitman RV. Soil mechanics. New York: Wiley; 1969.
2. Terzaghi K, Peck RB. Soil mechanics in engineering practice. New York: Wiley; 1967.
3. Huntington WC. Earth pressures and retaining walls. New York: Wiley; 1957.