



Together with the debt capital, another important part of the financing of a firm is the equity capital, a source of financing that is based on cash inflows provided by the ownership of the company or by retention of profits.

When the company cannot have access to debt, as a cheaper source of capital, or in general the management is not keen to increase the leverage of the capital structure, equity becomes the primary source of funding for every business.

The analysis of equity capital starts from the understanding of how equity markets work and how the equity capital is priced, in order to make a first distinction with the debt capital.

The valuation of equity capital is one of the most challenging tasks in corporate finance and involves different types of models, according to what is the information available at the time of valuation.

Like in the case of debt, equity capital can take many forms, from the simplest common stocks to more structured instruments like preferred stocks and more. The differences among those are important to be understood, to take advantage of them.

The case of common stock valuation is a foundation element of correct company valuation and equity pricing, and that makes the chapter of major importance for understanding the following parts of the book.

After studying this chapter, you will be able to answer the following questions, among others:

- What is market microstructure, and how does it affect the price formation process?
- How can the price formation model be described mathematically?
- What are the differences between different types of liquidity measurable on the market?
- What are the main models for common equity pricing? In what way do they differ from each other?

- What are the other sources of equity, and how do they differ from common stocks?

The first section of the chapter is dedicated to the market prices, analyzing the price formation process, and describing liquidity. The second section deals with the valuation of equity capital by introducing the dividend discount model and the free cash flow model. The last section focuses on the non-common sources of equity, including preferred stocks, warrants, and convertibles.

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## 6.1 The Stock Market

### Learning Outcomes

- Define market microstructure.
- Describe the price formation process in financial markets.
- Explain the difference between funding liquidity and market liquidity.

#### 6.1.1 Market Microstructure

In finance, the branch analyzing price formation mechanisms and market kernels is called market microstructure. As a field, it focuses on how the working process of a financial market affects the level of prices.

The analysis of market features, and their impact on the various market variables, like prices and volumes of transactions, is the core aim of the field and allows for a better understanding of how markets process the available information.

Standard transactions on regulated markets involve the transfer of an asset from one investor to another. The operation involves trading costs having an impact on the final price of the asset (Ball et al. 2011).

One of the main contributions of market microstructure is to establish a connection between transaction costs associated with trading and the bid-ask spread on the market, which itself depends on the asset liquidity.

Depending on how the transactions take place, it is possible to identify several types of market structure. Sometimes different types are mixed, and the differences depend on how transactions take place. A first important distinction is between auction markets and dealer markets.

Pure auction markets are those where the investors trade directly with each other without the intervention of dealers. In a call auction market, on the other hand, trading happens at specific times, when the securities are called for trading. In such a market, investors place orders, specifying desired price and quantity of the target security, and trading happens at specific times following specific rules.

Another type of auction market is the continuous auction market. As opposed to call auction, such a market is characterized by investors trading on orders placed at an earlier time by other investors.

The second type of market, the dealer market, is made of major dealers placing bid and offers to be answered by investors. It is therefore a market characterized by market makers, setting the initial prices that will adapt to the price formation process.

Besides the distinction among markets, it is also possible to distinguish between different types of orders to be input in the market, depending on the mechanism of order execution and satisfaction (Almgren and Chriss 2000).

A limit order sets minimum and maximum price for the trade to take place. A limit order to buy sets a maximum price, above which the investor will not accept to buy. A limit order to sell, on the other hand, sets a minimum price, below which the investor is not willing to sell the security.

The best limit order to buy results in being the lowest higher price in a centralized continuous auction market, while the best limit order to sell is the higher price. Both establish the market level, and the quantities at those prices represent the depth of the market.

**Example 6.1** Consider an investor who wants to sell 18,500 shares of some stock A at no less than €30 dollars per share, but only if the whole quantity can be traded at once, in the current session. Such an investor should then issue an all-or-nothing, day order to sell the shares.

A particular type of order is the stop-loss, which can be placed to buy or sell when the asset price reaches a predetermined threshold. In this way the trader experiencing a bullish trend will manage to limit the losses.

The technique is to usually set the stop-loss price at a level right below the price paid for the asset. Such an order gives the advantage that monitoring of the asset price is not needed, once the limit has been set. It is therefore an efficient way to trade in case of long absence from the market, at the desired price (Handaa et al. 2003).

The disadvantage is that sometimes the stop-loss price could be activated by a sudden and temporary shock on the asset price, which creates a short-term fluctuation not reflected by the real trend of price.

**Example 6.2** An investor can buy a stock for €40.00 and immediately after that enter a stop-loss order for €38.50, so that the stock will be sold at market price in case it falls below the stop-loss price. This limits the loss on the position to

$$40.00 - 38.50 = €1.50$$

with no need of further action after the stop order has been sent.

The stop-loss order automatically turns into a market order when the threshold is reached. Technically the final selling price could be different from the stop price, especially in fast-moving markets, due to the sudden changes.

Regarding the orders they place, it is also possible to set distinctions among investors. There are active traders, generally placing market orders and interested in continuous trading. Passive traders, on the other hand, normally issue limit orders, to

make the best out of limited transactions, therefore earning higher margins from active traders.

A popular type of trading is buying on margin, which involves borrowing cash from some broker and using it to buy assets. The strategy clearly leverages the position of the investor, increasing the risk, but allows to trade on assets that would not be affordable otherwise (Bangia et al. 1998).

When opening a margin account, an investor is required to place a minimum initial investment, as minimum margin. The account becomes operational, and the investor can borrow a certain percentage (initial margin) of the purchase price of a stock.

The percentage of stock price that can be borrowed is usually 50%, with maturity chosen by the investor. When the position is closed, and the stock is sold on the market, the proceedings go to the broker up to full payment of the loan.

The marginable securities on the account are collateral of the loan, and some interest must be paid to the broker. Interests are accrued on the part of the loan which is not reimbursed, and increase as debt increases.

**Example 6.3** An investor deposits an amount of €1000 in a 40% margin account, so that an investment of €1800 can be afforded. If the investor buys €500 value of stocks, the amount left on the margin account is €1300. The investor has used only a fraction of the equity in the account, meaning that the loan is not active yet, so no interest is due. When securities are bought for more than the equity of €1000, the loan position starts, and interests are accrued on it.

A margin agreement also includes maintenance margin, to be paid on top of the initial margin, as the equity required on the margin account after each trade. If the equity in the margin account falls below the maintenance margin, a margin call is issued, so that the investor must close the position in the stock or add more cash to the account.

**Example 6.4** Referring to Example 6.3, assume the full €1800 available to the investor is used to buy stocks, so that the whole loan is used. If the market value of the securities goes to €1300, the value of the equity falls from the original €500 to

$$1300 - 1000 = €300$$

Assuming a maintenance margin is 25%, the amount of required equity to be maintained is given by

$$1000 \times 0.25 = €250$$

Since the equity is higher than this, the situation is fine. If the maintenance margin was 40% instead, the amount of maintained equity would be

$$1000 \times 0.40 = €400$$

which is higher than the €300 equity. In this case, a margin call will be issued by the broker, and the investor will have to restore an appropriate level of equity.

The structure of the market is such as to accommodate the needs of the many different types of investors. From the small investor to the big player, the market must specify the order in which transactions are executed.

To do so, some markets, for example, adopt price criteria, giving priority to orders with the best price and secondary priority to the order posted first at a given price. Some other markets modify secondary priority rules to accommodate large transactions.

**Example 6.5** The current trading price of a futures contract is bid at €134.10 and asks €134.35 cents. There is a trader A who is the bidder with time precedence at that price. In order to buy at €134.10, a trader B must wait until investor A trades, or he can gain precedence by improving the bid to €134.15. Trader B would then have price priority over his bid and time precedence over all subsequent bids at 134.15. If trader A then wants to reclaim his precedence, he would have to bid higher at €134.20. Time precedence encourages traders to jump over each other's prices with improved prices.

The whole trading process can be divided in four steps, from information processing to clearing. In the first step, the information about past and current prices is made available, followed by brokers taking orders from investors and forwarding them to the market.

The third step is the actual execution of the orders through kernel processing, while the last step is the clearing of orders which takes place comparing the transaction orders of buyers and sellers in order to clear them.

The whole process results in a bid-ask spread on the market, representing the difference between asking price and offer price. It shows the trading prices available to investors, and it is a reliable proxy of the liquidity of the market.

When dealers place bid or ask prices, they give an option to the market investors to trade at this actual bid and ask, before it changes according to new information hitting the market.

**Example 6.6** Consider an actively traded asset A and a rarely traded asset B. The closing price of the most liquid asset A is more likely to reflect new information hitting the market, rather than the price of asset B. The returns of asset B will reflect the new information by showing autocorrelation, due to the fact that asset B has a zero return in periods of non-trading, reverting to the cumulative mean return when trading is active. Thus, negative autocorrelation arises due to the mean reversion.

Market structure is affected and determined by several important factors, mainly regarding liquidity. When a market is liquid, the elasticity is such that there are no large changes in price due to shifts in demand. When a market is less liquid instead, the bid-ask spread is much more sensitive to shifts in demand.

Transparency is another factor related to liquidity, in that it resembles the amount of information held by market agents. If a market can communicate prices and spreads in real time to both buyers and sellers, then it is considered highly transparent.

The features of a microstructure are generally defined by econometric issues, given that the trading kernel most commonly consists of a series of discrete events in continuous time, thus defining a point process.

In a perfect market, the price of an asset would be determined at the equilibrium between demand and supply. The reality is not that straightforward because every market has frictions that must be taken into consideration.

Market price of an asset normally reflects all the expenses involved in its trading, so as to carry all the information available at the time of trading. Consequently, the bid-ask spread reflects all these additions, therefore widening, as they increase.

When natural players are not present in the market, investors are obliged to deal with market makers in order to get a quick transaction. That generates illiquidity on the market.

In fact, the market makers bear the risk of price fluctuations while holding the asset in inventory, so they must be compensated, widening the bid-ask spread and reducing the profitability for the small investors.

The role of information for market liquidity is also crucial, and big players generally tend to benefit from the most complete information about fundamental values on the markets.

Since the price in equilibrium reflects all the available information, the asymmetry in information generates a mismatch in the marginal prices for different types of investors, giving rise to disequilibrium.

### **6.1.2 Price Formation**

In the past years, up till recent ones, financial markets have set new rules for the management of trading orders on the exchange. Thus order-driven markets are the focus of recent attention from practitioners.

The development of electronics limits order book trading platforms in virtually all of the market centers in the world. And that has improved the efficiency of market transactions in terms of speed and effectiveness.

The matching of order on the two sides of the order book is not an easy task, given that in general the high-valuation investors are willing to buy from the low-valuation shareholders, in order to gain from the price difference.

Therefore, low-valuation investors have a problem from not gaining anything from the deal. And on the opposite side, the same concept holds for low-valuation investors trying to buy from low-valuation shareholders.

Among the several models describing price formation in stock markets, a popular one is the Foucault's model. The assumptions underlying the model are continuous trading with a single risky asset, with investors trading one share of it sequentially through market or limit order.

The market is populated by two groups of investors, giving a different value to the asset. One group assigns a high value  $V_h$  while the other assigns a low value  $V_l < V_h$ . High-value and low-value investors populate the market with proportions  $h$  and  $l$ , respectively.

Investors are risk-neutral and maximized their expected utility. For a buy order processed at price  $b$ , it is given by

$$E(u) = \eta(V_i - b), \quad i = h, l$$

where:

$\eta$  is the probability of execution of the order.

Similarly, the expected utility from a sell order at a specified price is

$$E(u) = \eta(b - V_i), \quad i = h, l$$

In the absence of a trade, the utility is normalized to zero.

Private information is scarce and only available to a share  $\delta$  of the investors. This type of information on the asset value is worth to the investors a high value  $H_+$  or a low value  $H_-$  with 50% probability each.

Consider a game of mutual strategies. Traders are given an optimal strategy, given the strategies of other traders. Each trader must choose a type of order (market vs. limit) and (for limit orders) the bid or ask order placement price.

Optimal bid price  $b^*$  and offer price  $a^*$  define the equilibrium in which a counterparty in the next period is induced to trade at these prices via a market order. The expected utility of a limit buy order placed at bid price  $b$  can then be rewritten as

$$E(u) = (1 - k)[(1 - p)(V_h - b) - pH]$$

where:

$p = \frac{\delta}{2}$  is the proportion of informed investors weighted by the 50% probability.

When a buyer is not informed and the utility from the two different trading methods is the same, it makes no difference to make a market order or a limit order, as defined by

$$V_h - a_m = (1 - k)[(1 - p)(V_h - b) - pH]$$

where:

$A_m$  is the market ask price.

The optimal ask price  $A^*$ , sufficient to induce an uninformed coming buyer to trade via a market order, can be then expressed as

$$a^* = V_h - (1 - k)[(1 - p)(V_h - b) - pH]$$

Specular to it, it is possible to write the optimal bid price  $B^*$  that will induce an uninformed coming seller to trade via market order, by the formula

$$b^* = V_l + k[(1 - p)(a - V_l) - pH]$$

There exist parameter values for  $V_h$ ,  $V_l$ ,  $H$ ,  $\delta$ , and  $k$  for which equilibrium bid and ask prices are given by

$$\begin{aligned} a^* &= \lambda V_l + (1 - \lambda)(V_h - qH) \\ b^* &= \mu V_h + (1 - \mu)(V_l + qH) \end{aligned}$$

where

$$\begin{aligned} q &= \frac{p}{1 - p} \\ \lambda &= \frac{1 - k(1 - p)}{1 - k(1 - k)(1 - p)^2} \\ \mu &= \frac{1 - (1 - k)(1 - p)}{1 - k(1 - k)(1 - p)^2} \end{aligned}$$

### 6.1.3 Funding vs. Market Liquidity

An important distinction to be made when talking about liquidity is between funding liquidity and market liquidity. The distinction is important to understand the link between corporations and financial markets at full.

Funding liquidity can be defined as the ability to settle obligations with immediacy, and a financial institution is said to be liquid when it can face obligations in time. Funding liquidity risk is the possibility that an institution will not be able to meet its obligations over a specific time horizon (Brunnermeier and Motohiro 2009).

There is a difference between the point in time when a liquidity event happens and the risk associated with that event. Risk outcome in fact can take infinitely many values according to the distribution of future outcomes.

Consider banks as an example. A bank in fact is considered liquid if at each point in time the outflows of central bank money are smaller than the sum of inflows and stock held by the bank.

The net liquidity demand (NLD) indicator measures the fulfilment of the above conditions, to monitor liquidity, and can be calculated as

$$D_{NL} = C_{OUT} - C_{IN} - M$$

where:

$C_{OUT}$  is the sum of the outflows at a specified time.

$C_{IN}$  is the sum of known inflows.

$M$  is the stock of central bank money.

In order for a bank to be liquid, the money demand must be completely filled, which is allowed by the existence of an interbank market, together with deposits and other liabilities. That means the following inequality must hold

$$D_{NL} \leq w_D L_D + w_{IB} L_{IB} + w_A A + w_{CB} L_{CB}$$

where:

- $L_D$  is the amount borrowed from new depositors.
- $w_D$  is the price of new deposits.
- $L_{IB}$  is the amount borrowed from interbank market.
- $w_{IB}$  is the price of interbank market funds.
- $A$  is the amount of assets sold.
- $w_A$  is the price of assets sold.
- $L_{CB}$  is the amount borrowed from central bank.
- $w_{CB}$  is the price of central bank funds.

As from the model, there are basically two stochastic elements driving the funding liquidity risk. One is the future evolution of the NLD, and the other is the future level of liquidity priced (Drehmann and Nikolaou 2010).

In particular, when NLD is negative, there is an excess of liquidity supply, to be sold out to the market. On the other hand, when NLD is positive, and cannot be funded in any way, the bank becomes illiquid.

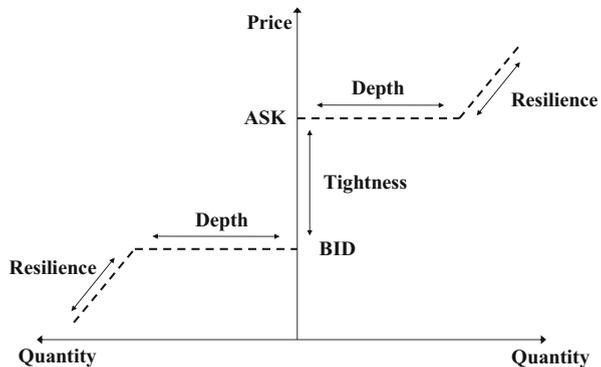
The theory of market microstructure is useful in the analysis of liquidity and illiquidity sources. There are three transactional properties governing the relationship between market microstructure and liquidity, namely, tightness, depth, and resiliency (see Fig. 6.1).

Tightness is the cost of closing a position over a short period of time, and it is defined as the difference between the bid and the ask prices. Depth is the volume of trades that is possible to process without affecting the price of assets. Resiliency is an elasticity measure, defined as the speed at which the price converges to the liquidation value of the underlying commodity.

There are specific values for the properties in case a market is liquid, and that helps in identifying illiquidity issues, when present. In a liquid market, for example, tightness approaches zero, meaning the bid-ask spread is minimized (Ernst et al. 2009).

The depth of a liquid market is small enough to not affect asset prices, and resiliency is high enough to ensure that prices eventually approach the underlying value, so as to avoid arbitrage opportunities.

**Fig. 6.1** The scheme resembles the properties of a market, in terms of liquidity



## 6.2 Common Equity

### Learning Outcomes

- Learn how to use dividends for equity valuation.
- Apply the most common types of dividend discount models.
- Apply the free cash flow models to equity valuation.

### 6.2.1 Dividend Discount Models

The cost of equity is essentially represented by the returns demanded by the investors in order to invest in the company. It is the reward granted in exchange to the capital inflows provided by the buyers of corporate shares.

It is therefore an important part of the corporate capital structure, and the estimation of the cost of equity represents a major stake of corporate finance literature. The cost of equity can be estimated using models based on either dividend payments or the cash flows.

The first method is the dividend growth model that measures the stock price and capital to be raised by using the information about the dividends paid out to shareholders by the company.

The model can then be used to estimate the cost of equity, and this model can take into account the dividend growth rate. The equation sheet for the Paper F9 exam will give the following formula:

$$P_0 = \frac{D_0(1 + g)}{r_e - g} \quad (6.1)$$

where:

- $D_0$  is the current dividend.
- $g$  is the expected dividend future growth rate.
- $r_e$  is the cost of equity.

As can be observed, the equation aims to predict the ex dividend market price of the share by processing the dividend today, the expected future dividend growth, and the appropriate cost of equity.

The numerator indicates the expected dividend in one period, due to the growth  $g$ . It is possible to rework Eq. (6.1) in order to isolate the cost of equity. The result is the formula

$$r_e = \frac{D_0(1 + g)}{P_0} + g$$

Since all the terms on the right-hand side are known at time zero, for a listed company, the calculation of the cost of equity is straightforward. When other

information is not available, the future dividend growth is assumed to continue at the recent historical rate.

**Example 6.7** A company is about to pay a dividend of €2.50 on a stock whose market price is €27 (cum dividend). The historical dividend growth rate, which is expected to continue in the future, is 3%. The estimated cost of capital is

$$r_e = \frac{2.50(1 + 0.03)}{27.00} + 0.03 = 0.1254 = 12.54\%$$

To improve the accuracy of the measurement, one must however use the ex dividend price of the stock that can be calculated as the cum dividend market price less the impending dividend.

**Example 6.8** The exdividend market price in the previous example is

$$P_{0,Ex} = 27.00 - 2.50 = €24.50$$

The cost of equity is therefore given by

$$r_e = \frac{2.50(1 + 0.03)}{24.50} + 0.03 = 0.1351 = 13.51\%$$

Stock valuation is based on the general assumption that if a market is populated by rational investors, the value of the stock today represents the present value of all future cash flows to be accrued in the future.

By recalling the time value of money, one can discount the future cash flows to present value and obtain the price today of an asset. Such a price is the intrinsic value of the stock because it is the value of the stock that is perceived based on all available information.

The buyer of a stock usually expects to get two types of cash flows from it. First of all, there is a dividend paid out during the holding period; plus there will be an expected price to be cashed when the stock is sold back on the market. Since this expected price is itself determined by future dividends, the value of a stock is the present value of dividends through infinity:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + r_e)^t}$$

where:

$D_t$  is the expected dividends per share.

$r_e$  is the cost of equity.

The model is based on two main inputs, namely, the expected dividends and the cost of equity. The expected dividends can be obtained by making some assumptions about expected future growth rates in earnings and payout ratios.

The discount rate depends on the riskiness of the asset and can be measured with the already seen models of asset pricing like CAPM or arbitrage and multifactor

models. The model is flexible enough to allow for time-varying discount rates, where the time variation is caused by expected changes in interest rates or risk across time.

The issue with projecting future dividends is complicated because it is not possible to make such a projection through infinity. Therefore, several versions of the dividend discount model are available, making different assumptions about future growth.

The simplest model is designed to value stocks in an environment of stable growth, with the firm paying out as much of dividends as it can afford. When the firm is in such a steady state, the Gordon growth model is the good choice for valuation.

The model can be used to value a firm that is in “steady state” with dividends growing at a rate that can be sustained forever and relates the value of a stock to its expected dividends in the next time period, the cost of equity, and the expected growth rate in dividends:

$$P_0 = \frac{D_1}{r_e - g}$$

where:

$D_1$  is the dividend expected in the next period.

It is worth understanding what is meant for stable growth rate, and there are two insights worth keeping in mind when estimating a “stable” growth rate. When considering the growth rate in the firm’s dividends, one can expect them to last forever.

As a consequence, all performance financial measures are expected to grow at the same rate. If a company is growing stably at some rate, while the dividends grow at a higher rate, the dividends will exceed earnings over time. If the opposite holds, and the earnings grow at a higher rate than dividends, the latter will then converge toward zero, which is different from a steady state.

It follows that for a firm to grow at a steady state, analysts should be able to substitute in the expected growth rate in earnings and get precisely the same result, if the firm is truly in a steady state.

There is also an issue related to how to judge whether a growth rate can be considered reasonable for a stable growth. The rule of thumb is that the growth rate has to be less than or equal to the growth rate of the economy in which the firm operates.

The analysts are generally not in agreement on the rate to be adopted, but the common agreement in the academia and research is that there are three reasons why a firm is in stable growth.

First of all, there is usually uncertainty about expected inflation, which affects the real growth in the economy. Different analysts may have different expectations about inflation, which in the long term may project a nominal growth rate in the economy that is higher.

The growth rate of the company can be much less than the growth rate of the overall economy, and in some cases the firms become smaller over time, in proportion to the economy.

Finally, if a firm is likely to maintain a few years of “above-stable” growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate, to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited.

When the growth of the company is not assumed to be stable, but it varies over time, often from initial high growth to a final stable growth, the analyst will be better served using a two-stage or a three-stage model.

It is very difficult in reality for a firm to meet the requirement of a constant growth rate. Earnings are in fact usually quite volatile, but the model can still be applied when there is an average over cyclical periods.

Thus, a cyclical firm that can be expected to have year-to-year swings in growth rates, but has an average growth rate that which is defined, can be valued using the Gordon growth model, without a significant loss of generality.

An intuition that strengthens this point is that dividends are usually smoothed overtime, therefore not linked to the cyclicity of earnings. Moreover, the mathematical effects of using an average growth rate rather than a constant growth rate are small.

By assuming that dividends are constant forever, the stock valuation is given by the present value of dividends per share in perpetuity:

$$P_0 = \frac{D_1^*}{r_e}$$

where:

$D_1^*$  is the constant dividend per share expected from the next period.

The rate of discount (return) is proportional to the time value of the money tied up in the investment and also reflects the risk associated with the uncertainty about the amount of future cash flows.

**Example 6.9** Assume a stock paying a current dividend of €2.00 per share and a required rate of return of 10%. The value of a share of stock is therefore

$$P_0 = \frac{2.00}{0.10} = €20.00$$

Therefore, by paying €20.00 per share and assuming the dividends remain constant at €2.00 per share, the investor will earn a 10% return per year on the investment every year.

By assuming that the dividend grows at a constant rate, one can decompose the  $D_1$  into the previous dividend multiplied by the growth rate  $g$ . The present value of the common stock is the present value of all future dividends. The specific case of dividends growing at the constant rate  $g$  is commonly referred to as the dividend valuation model (DVM):

$$P_0 = \frac{D_0(1+g)}{r_e - g} = \frac{D_1}{r_e - g}$$

This model is also referred to as the Gordon model, and it is one of a general class of models referred to as the dividend discount model (DDM).

**Example 6.10** A stock pays a current dividend of €7.00, and the dividends per share are expected to grow at a rate of 3% per year. If the required rate of return (cost of equity) is 12% in perpetuity, the value of the share is

$$P_0 = \frac{7.00(1 + 0.03)}{0.12 - 0.03} = \text{€}80.11$$

The case is different when the growth is expected to change over time, representing the common scenario that most firms encounter in real life. Companies in fact experience life cycles up and down turns with initial rapid growth when they start, a decreased growth in the intermediate phase of operations, and a situation of declining growth in their final stage. Further, companies may experience changes in their growth due to acquisitions and divestitures.

A modification of the dividend growth model allows for a two-stage growth representation, with an initial phase of unstable growth and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term.

The standard setting of the model usually accounts for an initial stage of higher growth and a second stage of much lower stable growth, but it can be adapted to cases where the firm is expected to post low or even negative growth rates for a few years and then revert back to stable growth.

The model is built on two stages of growth, with initial growth lasting for a set period of  $T$  years and the stable growth phase starting immediately afterward. So there are two growth rates, one for the first phase and one for the stable infinite growth. The value of the stock according to the model is given by

$$\begin{aligned} P_0 &= \sum_{t=1}^T \frac{D_t}{1 + r_e^*} + \frac{P_T}{(1 + r_e^*)^T} \\ &= \sum_{t=1}^T \frac{D_t}{1 + r_e^*} + \left[ \frac{1}{(1 + r_e^*)^T} \times \frac{D_{T+1}}{(r_e - g)} \right] \end{aligned} \quad (6.2)$$

where:

$r_e^*$  is the cost of equity in the high growth stage.

$r_e$  is the cost of equity in the stable growth stage.

If the initial growth is stable, and the dividend payout ratio is stable too, over the  $n$  years of extraordinary growth, Eq. (6.2) can be simplified as

$$P_0 = \frac{D_0(1 + g^*) \left[ 1 - \frac{(1 + g^*)^T}{(1 + r_e^*)^T} \right]}{(r_e^* - g^*)} + \frac{D_{T+1}}{(r_e - g)(1 + r_e)^T}$$

where:

$g^*$  is the growth in the high growth stage.

**Example 6.11** A share of common stock currently pays €1.70 per share and is expected to grow at a rate of 5% per year for 2 years and afterward at a rate of 3% per year. The required rate of return is 7%. The stock price can be calculated as

$$P_0 = \left[ \frac{1.70(1.05)}{1 + 0.07} + \frac{1.70(1.05)^2}{(1 + 0.07)^2} \right] + \frac{P_2}{(1 + 0.07)^2}$$

The term  $P_2$  is calculated as

$$P_2 = \frac{1.70(1.05)^2(1.03)}{0.07 - 0.03} = €48.26$$

so that

$$P_0 = \frac{1.785}{1.07} + \frac{1.874}{1.145} + \frac{48.26}{1.145} = 1.668 + 1.637 + 42.148 = €45.453$$

The result is a typical two-stage model, which looks similar to the standard dividend valuation model. But in this case, the DVM is used to determine the price beyond which there is constant growth. The dividends in the first growth phase are instead discounted using basic cash flow discounting.

A further step in the analysis is to introduce an extra stage and account for a three-stage dividend growth. It allows for an initial period of high growth, a transitional period where growth declines, and a final stable growth phase. It is the most general of the models because it does not impose any restrictions on the payout ratio.

The first stage of the model is of stable growth, but higher than classical stable growth, followed by a decline in growth in the second stage and a new stable growth at a lower level, in the last stage.

The value of the stock is given by discounting the expected dividends during the high growth and the transitional periods, plus the terminal price at the start of the final stable growth phase.

$$P_0 = \sum_{t=1}^{T_1} \frac{\text{EPS}_0(1 + g_{\text{HG}})^t \theta_{\text{HG}}}{(1 + r_{\text{HG}})^t} + \sum_{t=1}^{T_2} \frac{\text{DPS}_0}{(1 + r_{\text{TR}})^t} + \frac{\text{EPS}_{T_2}(1 + g_{\text{SG}})^t \theta_{\text{SG}}}{(r_{\text{SG}} - g_{\text{SG}})(1 + r_f)^t}$$

$$P_0 = \sum_{t=1}^{T_1} \frac{\text{EPS}_0 \times (1 + g_{\text{HG}})^t \times \theta_{\text{HG}}}{(1 + r_{\text{HG}})^t} + \sum_{t=T_1+1}^{T_2} \frac{\text{DPS}_t}{(1 + r_{\text{TR}})^t} + \frac{\text{EPS}_{T_2} \times (1 + g_{\text{SG}})^t \times \theta_{\text{SG}}}{(r_{\text{SG}} - g_{\text{SG}})(1 + r_f)^t}$$

where:

$\text{EPS}_0$  are the earnings per share in year  $t$ .

$\text{DPS}_t$  are the dividends per share in year  $t$ .

$g_{\text{HG}}$  is the growth rate in high growth phase, lasting  $T_1$  years.

$g_{\text{SG}}$  is the growth rate in stable phase.

$\theta_{HG}$  is the payout ratio in high growth phase.

$\theta_{SG}$  is the payout ratio in stable growth phase.

$r_{HG}$  is the cost of equity in high growth phase.

$r_{TR}$  is the cost of equity in transition phase.

$r_{SG}$  is the cost of equity in stable growth phase.

The model is more flexible than the other dividend discount models, but it requires more inputs to run. The advantages from the additional flexibility must be compared to the disadvantages of having higher noise in the estimation due to possible errors in the inputs.

It is quite common to find companies that experience an actual three-stage growth process during their life. Most companies go through a development stage with high growth, a maturing phase with moderate growth, and a declining phase with little, no, or negative growth.

This is exactly the pattern to be described by a three-stage dividend model. The practical application of the model is more cumbersome than classical DVM, and it runs through six steps, as illustrated in the following example.

Consider the valuation of a stock that has a current dividend of €3.00 per share. Dividends are expected to grow at a rate of 12% for the next 5 years. Following that, the dividends are expected to grow at a rate of 8% for 5 years.

After 10 years, the dividends are expected to grow at a rate of 3% per year, forever. If the required rate of return is 15%, it is possible to calculate the value of the stock by breaking the calculation in to six steps:

First of all, the dividends for the years from 1 to 11 must be calculated as

Year	Dividend growth rate	Dividend
1	12%	3.360
2	12%	3.763
3	12%	4.215
4	12%	4.721
5	12%	5.287
6	8%	5.710
7	8%	6.167
8	8%	6.660
9	8%	7.193
10	8%	7.768
11	3%	8.001

Year	Dividend	Present value
1	3.360	2922
2	3.763	2845
3	4.215	2771
4	4.721	2699
5	5.287	2629
6	5.710	2469
7	6.167	2318

(continued)

8	6.660	2177
9	7.193	2045
10	7.768	1920
11	8.001	1720

Present value of dividends after year 10 can be calculated as the present value of a growing perpetuity:

$$PV_{\infty} = \frac{8.001}{0.15 - 0.03} = \text{€}66.67$$

The discounted value at time zero of the growing perpetuity is then given by

$$PV_0 = \frac{66.675}{(1 + 0.15)^{10}} = \text{€}16.48$$

The sum of the present value of the dividends in the first 10 years is given by

$$PV_{10} = \sum_{t=1}^{10} \frac{DIV_t}{(1 + 0.15)^t} = \text{€}24.79$$

The value of the stock at present is given by the sum of the present value of the growing perpetuity and the present value of the dividends in the first 10 years, as for

$$PV_{\infty} = 16.48 + 24.79 = \text{€}41.27$$

It is possible to compare the model to the CAPM in order to grasp the differences among them. The dividend growth model gives a measure of cost of equity through the analysis of empirical data publicly available for most companies.

The calculation comes straight through an algorithm that involves measuring the dividends, estimating the dividend growth, copying the market value of the shares, and using the amounts in the equation to estimate the cost of equity.

The model is however limited in that it does not give any information about why different shares have different costs of equity. This is due to the fact that dividend growth models ignore the risk aspect of valuation.

That model simply measures what's there without offering an explanation. Note particularly that a business cannot alter its cost of equity by changing its dividends. The equation

$$r_e = \frac{D_0(1 + g)}{P_0} + g$$

might suggest that the rate of return would be lowered if the company reduced its dividends or the growth rate.

The reality is different, and normally a dividend cut or growth decrease would result in a lower market value of the company. The value would decrease until the level corresponding to the point where investors obtain the required return.

The CAPM is a more complete model, making a step further by introducing systematic (market) risk in the equation for valuation. Other returns in the economy as well as the relationship among the various risks translate into beta and asset return.

Another important feature of CAPM is to offer several ways to measure the inputs, as the risk-free rate, the market return, and the beta. They can be estimated from empirical data or are normally available as public information.

## 6.2.2 Discounted Free Cash Flow Models

The models of free cash flows aim at establishing how much cash the firm can return to the shareholders through its operations. It is therefore a direct derivation from the net income, which is the first entry to be considered.

Recall that net income is the accounting measure of the stockholders' earnings during the period. The task is to convert it to a cash flow by subtracting out a firm's reinvestment needs.

The net income must be further netted of some types of expenses, in order to get to the free cash flows. For example, one must subtract any capital expenditures, defined broadly to include acquisitions, since they represent cash outflows.

On the other hand, depreciation and amortization must be added back to the net income, because they do not represent cash flows. Net expenditure is then the difference between capital expenditures and depreciation.

It is usually a function of the growth characteristics of the firm. High-growth firms tend to have high net capital expenditures relative to earnings, whereas low-growth firms may have low, and sometimes even negative, net capital expenditures.

When working capital increases, the firm's cash flows are reduced, and vice versa, with lower working capital translating into higher cash flows available to equity investors. Companies experiencing a fast growth are subject to large increases in working capital, especially in industries characterized by high levels of working capital, like retailing. In order to capture cash flow effects of working capital, one must focus on the noncash working capital.

The last point in determining cash flow is debt. Equity holders are typically subject to changes in the level of debt that impact on their cash flows. Repaying the principal on existing debt represents a cash outflow; but the debt repayment may be fully or partially financed by the issue of new debt, which is a cash inflow.

By accounting for the cash flow effects of net capital expenditures, changes in working capital and net changes in debt, it is possible to define the net cash flows after all netting, named as free cash flow to equity (FCFE), as from the formula

$$\text{FCFE} = \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ - (\Delta \text{ in non-cash working capital}) + (\text{New debt issued} - \text{Debt repayments})$$

This remaining net cash flow is then available to be repaid to shareholders in the form of dividends or become retained earnings that constitute additional equity for the company.

In order to simplify the calculation, one can assume that net capital expenditures and working capital are financed by a mix of equity and debt capital. The proportion of net capital expenditures and working capital raised in the form of debt can be indicated as  $d$ , and the effect on cash flows to equity of these items can be represented as

$$\begin{aligned} \text{FCFE} = & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ & - (\Delta \text{ in non-cash working capital}) + (\text{New debt issued} - \text{Debt repayments}) \end{aligned}$$

The equity cash flows associated with capital expenditure needs ( $\text{FCFE}_{\text{CAP}}$ ) can be defined as

$$\text{FCFE}_{\text{CAP}} = -(\text{Capital Expenditures} - \text{Depreciation})(1 - d)$$

and the equity cash flows associated with working capital needs ( $\text{FCFE}_{\text{WC}}$ ) can be defined as

$$\text{FCFE}_{\text{WC}} = -(\Delta \text{ in non-cash working capital})(1 - d)$$

Following the above formulation, it is possible to express the cash flow to equity, net of capital expenditure, and working capital, for a constant debt-to-equity ratio, as

$$\begin{aligned} \text{FCFE} = & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation})(1 - d) \\ & - (\Delta \text{ in non-cash working capital})(1 - d) \end{aligned}$$

The net debt payment does not appear in the equation anymore, given that the new debt issue is financing the former debt, in order to keep the debt-to-equity ratio constant, as per assumption of the model.

It is particularly useful to assume that a specified proportion of net capital expenditures and working capital needs will be financed with debt if the target or optimal debt ratio of the firm is used to forecast the free cash flow to equity that will be available in future periods. For past periods, an alternative approach is to use the firm's average debt-to-equity ratio over the period to arrive at approximate free cash flows to equity.

The traditional formulation of the cash flow calculation assumes there are no preferred dividends paid. The model values common equity, so that the formulas need to be modified in order to account for the existence of preferred stock and dividends.

The preferred dividends must be subtracted from the net income as well, in order to arrive at the FCFE, according to the modified formula

$$\begin{aligned} \text{FCFE} = & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ & - (\Delta \text{ in non-cash working capital}) - (\text{Pref. dividend} + \text{New pref. stock issued}) \\ & + (\text{New debt issued} - \text{Debt repayments}) \end{aligned}$$

which, in the short form, becomes

$$\text{FCFE} = \text{Net Income} - \text{Pref. dividends} - (\text{Capital Expenditures} - \text{Depreciation})(1 - d) - (\Delta \text{ in non-cash working capital})(1 - d)$$

The non-equity financial ratio ( $d$ ) would then have to include the expected financing from new preferred stock issues.

The FCFE model is not that dissimilar from the dividend discount model, if one considers that, in all above forms, it represents a model where we discount potential dividends rather than actual dividends.

The various versions of the model based on cash flow are therefore all variants of the dividend discount model, with a major change given by the fact that the dividends are replaced by the free cash flows to equity.

Such a replacement goes beyond changing one type of cash flow with another, given that it entails the assumption that the FCFE will be paid out to stockholders. Two logical consequences follow.

The first consequence is that there will be no future cash buildup in the firm, since the cash that is available after debt payments and reinvestment needs is paid out to stockholders each period.

Another consequence is that the part of growth in income due to current assets will be excluded from the expected growth in FCFE, leaving only the contribution of the income from operating assets.

A further complication of the model is the inclusion of stock buybacks into dividends to be discounted. It is useful to understand how the FCFE model compares to the modified dividend model in this case.

Stock buybacks can be seen as accumulated cash due to the strategy of not paying out the FCFE as dividends for some time. Thus, FCFE represent a smoothed out measure of what companies can return to their stockholders over time in the form of dividends and stock buybacks.

The growth of cash flows can be estimated by using the same approach used for the dividends, given they both represent cash flows to equity investors. The expected growth rate can then be expressed as

$$E(g) = R_R \times r_e$$

where:

$R_R$  is the retention ratio.

The amount of cash flow that is not paid out as dividends is reinvested into the firm as retained earnings. This is not consistent with the assumption that free cash flows to equity are paid out to stockholders which underlies FCFE models.

In order to make the model more realistic, it is therefore necessary to replace the retention ratio with the equity reinvestment rate. This is a measure of the share of net income to be reinvested into the company. The model can be then formulated as

$$R_E = 1 - \frac{\text{Net capital expenditure} + \Delta \text{ in working capital} - \text{Net debt issues}}{\text{Net income}}$$

Please note that in the FCFE model, there is no extra cash left in the firm after reinvestment. It is therefore not appropriate to use a return on equity that includes interest income from current assets.

The same argument works for the book value of equity, which includes the value of cash and other marketable securities. The best solution is to construct a modified version of the return on equity that measures the noncash aspects.

$$\text{Non-cash ROE} = 1 - \frac{\text{Net income} - \text{After tax income from cash \& marketable securities}}{\text{Book value of equity} - \text{Cash and marketable securities}}$$

The product of the equity reinvestment rate and the modified ROE will yield the expected growth rate in FCFE as

$$\text{Expected Growth in FCFE} = \text{Equity reinvestment rate} \times \text{Non-cash ROE}$$

There are several versions available of the FCFE model, depending on the type of company to value. For example, for a company that is growing at a stable rate, the best model to use is the constant growth FCFE model.

The model assumes a value of the equity that is a function of expected FCFE in the next period, the stable growth rate and the required rate of return.

$$P_0 = \frac{\text{FCFE}_1}{r_e - g}$$

where:

$\text{FCFE}_1$  is the free cash flows to equity in period 1.

The underlying assumption of the constant growth FCFE model is similar to those of the Gordon growth model. A reasonable growth rate must be used in the model, in relation to the nominal growth rate in the economy.

In order to represent a realistic proxy, the chosen long-term growth must not exceed the growth rate of the economy in which the firm operates by more than one or two percent. Additional assumptions relate to other features possessed by the firm that are shared by stable firms in general.

For example, the risk of the company is average, with capital expenditures not too large, relative to depreciation. In terms of asset pricing, this means that if CAPM is used, for example, the beta of the equity should not be significantly different from one.

The reinvestment rate for a stable growth firm can be estimated in two ways. A first approach is to use the typical reinvestment rates for firms in the industry of operation. Alternatively, the relationship between growth and fundamentals can be used to estimate the required reinvestment.

The expected growth in net income can be written as

$$\text{Expected growth rate in net income} = \text{Equity Reinvestment Rate} \\ \times \text{Return on equity}$$

which turns in an estimation of the equity reinvestment rate as

$$\text{Equity Reinvestment Rate} = \frac{\text{Expected growth rate in net income}}{\text{Return on equity}}$$

To illustrate, a firm with a stable growth rate of 4% and a return on equity of 12% would need to reinvest about a third of its net income back into net capital expenditures and working capital needs. Put another way, the free cash flows to equity should be two thirds of net income.

### 6.2.3 Relative Valuation Models

Relative valuation models assess the value of the assets of a company based on comparable assets in the industry. So, for example, the valuation of assets of an IT company will be run based on the values observed in the IT industry.

After the reference asset values are identified, relative valuation moves on in scaling them into a standard variable that can generate comparable prices. This standardization makes sense when the assets to be compared differ significantly from each other.

An example of how the standardization differs from case to case is corporate stocks. It has been observed that higher growth companies should trade at higher multiples than low growth companies.

As from Larsen et al. (2011), relative valuation (as opposed as the FCF and DDM models) does not estimate the intrinsic value of the assets based on the cash flows they generate. It rather estimates the value of an asset by considering how much the market is currently willing to pay for it.

The convergence or divergence of the result of relative valuation with the result of intrinsic valuation is therefore determined by how correctly the market is currently pricing the target asset.

There are several multiples that can be used for relative valuation, which are summarized in Table 6.1, and the following part of this section analyzes some of them.

For example, one can value the asset as a multiple of the earnings generated by it. This is the case when stock prices are calculated as a multiple of the EPS of the company.

**Table 6.1** Most commonly used multiples

Earnings multiples	Price-over-earnings (PE) ratio
	Value over EBIT
	Value over EBITDA
	Value over cash flow
Book value multiples	Market value over book value of equity
	Market value over book value of assets
	Market value over replacement cost
Revenues multiples	Market price over sales per share
	Market Value over Sales

Current EPS allow to estimate the price-over-earnings (PE) ratio, so that a current PE can be used to estimate a projected PE, which is the basis for the valuation of the stock.

Another commonly used earning multiple is EBITDA, and it is rather obvious that from the point of view of the buyer, lower multiples are preferable in that they lead to a lower valuation. Anyway all multiples are sensitive to the growth potential of the company.

Another class of multiples relates to the book value of the assets. One should recall that the accounting value of a business may differ significantly from its market value, thus generating confusion among the analysts.

Ratios like the price-to-book value of the equity of a firm are often used by the investors to realize how the market is overvaluing or undervaluing a business. These measures can vary across industries.

Some analysts like to use the ratio of the value of the company over the replacement value of the assets as multiple, instead of the price-to-book ratio. This ratio is commonly known as Tobin's Q.

The third class of multiples is related to the revenue variables and departs from the accounting choices. The ratios of the corporate value to the generated revenues can be used as a multiple in relative valuation.

One of these ratios is the price over sales ratio that divides the market value of the corporate equity by the revenues. Another one takes into account the overall enterprise value of the firm, represented by the value of the operating assets of the firm, dividing it by the sales.

Revenue multiples are very useful in that they allow to compare companies across different markets. As other multiples, they vary across industries in that they heavily depend on the profit margins normally realized in each industry.

Some multiples are specific to some sector and do not make sense when used in valuating firms in other sectors. These multiples rely on very specific entries to calculate ratios based on firm-specific value or cost measures.

In the Internet sector, it is common, for example, to use the webpage hits generated by the websites of the company, and the corporate value per hit becomes an interesting and specific multiple to be used in that sector.

Sector-specific multiples can be misleading in that they rely on the value that is associated to the variable considered (in the above example the webpage hit). If there is no sense of what is a high, a medium, or a low price for a hit, then the resulting valuation may be hard to interpret and compare to some benchmark value.

In order to be sure the method in use is well defined and the measurements are uniform across the companies in the same sector, one must analyze several aspects of the multiple.

For example, it is very important to analyze the multiple with cross section in order to determine its distribution across companies in the same sector and also across different sectors in the same market.

Moreover, the sensitivity of the multiple to changes in its determinants must be analyzed, and the selection of the right comparable firms is at the basis of the correct analysis of the similarities and differences existing across firms in the industry of reference.

The correct definition of multiples is very important, and in most cases it differs from one analyst to another. Whichever definition is used, a correct relative valuation is possible when the definition of the multiples used in the analysis is consistent over the different companies analyzed and over time.

Consistency of definition of the multiples also includes the way the ratios are calculated. Some of them in fact are controversial in that the numerator and denominator not always represent the same type of financial.

In the price over EBITDA indicator, for example, the numerator is clearly a measure related to the company's equity, while the denominator includes the part of revenues that belong to other stakeholders, other than equity holders.

However, most experts agree that these inconsistencies are easily solved when the ratios used are the same for all the companies compared in the analysis, independently of being a mix of equity and firm measures.

An aspect that is very important is the time span used in the calculation of the various elements involved in the analysis. All the variables involved in the ratio and multiples calculation should be calculated or averaged over the same time period. This ensures consistency in the valuation, especially when the uniform time span is applied to all the companies included in the analysis.

In terms of consistency, it also helps to have a descriptive analysis of the multiple of reference, in order to be able to judge whether a value is high, medium, or low or to establish a benchmark value for the market or industry.

This can be achieved by analyzing the distributional characteristics of the multiple, especially across the market. While in fact it is very common for analysts who are expert of some specific sector to have the sense of the ranking of companies in that sector, in terms of multiple, it is not equally straightforward to get the sense of this ranking in the overall market.

Standard statistics like average, standard deviation, median, maximum, and minimum, help the analyst to build a map of the situation in terms of some multiple, in some market. All supported by a scientific approach.

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## 6.3 Special Issuances

### Learning Outcomes

- Understand the features of preferred stocks.
- Learn what warrants are.
- Learn what convertibles are.

#### 6.3.1 Preferred Stocks

Other than common stocks, a company can also issue preferred stocks to be offered to investors. It is a type of stock with preference over common stocks in dividend payments and firm liquidation.

The particular features preferred stocks make them a very special type of hybrid security, halfway between stocks and bonds. Stock-like features include taxability of dividends, no specific maturity, and the fact that failure to pay dividends does not constitute bankruptcy.

In terms of bond features, preferred stocks come with the promise of a fixed dividend, like for the coupon of a bond. Moreover, they have no voting rights attached, and the liquidation value attached to them is usually pre-stated.

Other bond features are the sensitivity to interest rate change and the seniority for repayment in case of liquidation of the company, just after the bond holders. Moreover, most preferred dividends are cumulative.

Another important feature of preferred stocks is that companies owning preferred stocks can deduct 70% of the preferred dividend from their taxable income. As a consequence, dividend yields are often at or below yields on bonds of similar risk.

The market for preferred stocks has been dominated by corporations and institutional investors for a long time. But lately, individual investors also started finding it attractive and decided to go for the regular income preferred stocks can give.

All of the above mentioned features make preferred stocks a peculiar type of financial security and must be analyzed one by one in order to grasp the sense of having such a type of hybrid equity in the capital structure.

The seniority of preferred stock is an important feature. Dividends to be paid to them are senior to the dividends received by common stocks. However, as a form of equity capital, the owners of preferred stocks are junior compared to bondholders.

In case of cumulative preferred stocks, the dividends accrue in case a payment is missed, and no dividend is paid to common shareholders until the total of the dividends to the preferred stocks is paid out. One should keep in mind that unlike common stockholders, preferred shareholders do not participate in the potential for increased dividends.

Preferred stocks are an attractive investment to those investors who want to improve their yields and current income. The yields of preferred stocks in fact are normally higher than those on bonds from the same issuer.

The volatility of price is strongly related to their bond-like features, and preferred stocks are subject to interest rate risk, even if not to the same extent as bonds. In general, prices will rise as interest rates fall, and prices will move lower as rates rise. Also market factors and industry situation contribute to the change in price.

Attached to preferred stocks, there is also a credit rating, similarly to what happens with bonds. The rating measures the likelihood of the company to pay dividends regularly and in time. Generally, an issuer's preferred stock ratings are two notches below an issuer's senior debt, which is reflective of the junior claim of the preferred.

An advantage of preferred stocks is that they normally come at a price which is lower than the standard price, therefore making it possible for the investor to build up a diversified portfolio with a limited investment.

Given the lower par value, preferred stocks are appealing to the individual investors, as mentioned above. This enhances liquidity, given that as opposed to other fixed-income instruments, preferred stocks are listed on all major stock

exchanges. The benefits in terms of liquidity are huge, and also the visibility and strong marketability of the shares are significantly improved.

Given the lower purchasing price, compared to standard stocks, preferred stocks offer a high opportunity for capital appreciation. The potential capital gains implied by the investment are therefore interesting.

When preferreds trade at a discount to par, investors can invest in these securities relatively inexpensively and earn a favorable yield on the dividends received, and should the outlook for the issuer become more favorable or interest rates fall, they can earn a capital gain on the shares.

The historical reason for companies to invest in preferred stocks has been the very favorable tax treatment of the dividends. In the United States, for example, domestic companies are allowed a tax relief on the 70% of the dividend amount. Therefore, the corporate taxation only hits a small part of the dividends from preferred stocks, and the procedure is called dividend received reduction (RDR).

Securities must be held by the company for at least 46 days in order to qualify for tax relief. Moreover, the issuing company must be a domestic one, and preferred stocks must be straight, meaning they are not a fixed-rate capital security structure.

Several structures are possible for preferred stocks. Traditional preferred stocks are known as straight and represent the equity of a publicly held domestic corporation, while sharing features with bonds.

The issuer of straight preferred stocks pays quarterly dividends set at a fixed amount, similar to the coupon payment on a bond. With the attraction of well-recognized issuers of preferred stocks that are typically listed on markets like the American NYSE, individual investors also benefit from higher yields than fixed-rate corporate bonds.

Preferred stocks enjoy a higher stability than common stocks given the fact that they enjoy the high liquidity of the regulated stock exchange market, with the comfort of the fixed income stream attached.

An innovative type of preferred stocks available to investors is the fixed-rate capital securities (FRCS). They have similar characteristics to preferred stock while allowing the issuing company to deduct the interest.

FRCS are available in two different types. The Junior Subordinated Debentures (JSD) are the simplest form of fixed-rate capital securities. The issuing company in fact directly issues the securities without the need to set up a separate company for it.

The second type of FCRD is called Trust Originated Securities (TOS), representing preferred stocks issued by a separate trust. The proceeds from the sale of TOS are then invested by the trust in junior subordinated securities issued by the parent company.

Most FRCS carry a dividend deferral option, which allows the issuing company to defer the payment of dividends for a term of up to twenty quarters, equivalent to 5 years. In case of such a type of issuance, the company is not allowed to declare any common dividends until accumulated dividends on FRCS have been paid in full.

Foreign preferred stocks (FPS) are issued in one country in a foreign currency. In the case of the American market, they are called Yankee preferred stocks and are issued by foreign companies in the United States.

FPS normally carry a higher yield than domestic issuances, given that the former do not have access to the DRD system, and have a lower reputation compared to domestic preferred stocks. It must be taken into account that in some cases, the dividend for some FPS may be subject to a withholding tax imposed by the issuer's country.

One may consider the example of the United Kingdom, where the regulators impose a 15% withholding tax on US shareholders. Investors can then reclaim this tax provided that the shares are not held in a tax-deferred account.

A more sophisticated type of preferred stock is the adjustable rate preferred stock (ARPS), for which the dividends are reset quarterly to reflect market rates. The benchmark for setting the new rate is the higher of the 3-month Treasury bill discount rate, the 10-year constant maturity Treasury rate or the 30-year constant maturity Treasury rate.

For this type of securities, both a minimum and a maximum level are set for the dividend yields, known as collars. Moreover, they qualify for the DRD standard, in case they are owned by corporations.

The ARPS have a lower yield than straight preferred stocks, in that the investors benefit from owning a security that potentially insulates them against adverse changes in the shape of the yield curve, in addition to changes in the absolute levels of interest rates.

**Example 6.12** An investor buys preferred stocks at a price of €25. The stock pays a dividend yield of 7.5%, for a euro amount per share of

$$\text{DIV} = 25 \times 0.075 = \text{€}1.875$$

It is possible to perform standard calculations about preferred stocks. The following are definitions and examples of some common price and yield calculations related to this flexible instrument.

Preferred stocks trade at a clean price, meaning that the price does not account for any interest earned since the last dividend payment. However, theoretically, some interest can be accrued on the stocks from the last dividend to the time of purchase. This represents an implied accrued dividend.

**Example 6.13** Consider a share that paid a dividend of €1.875, 60 days before, is purchased today. The price must then include accrued interest of

$$I_{\text{ACC}} = \frac{60}{360} \times 1.875 = \text{€}0.31$$

The current yield is a measure of the expected annual return on the investment, obtained by dividing the annualized dividend by the current preferred stock price. It gives the sense of the profitability of investing in preferred equity.

**Example 6.14** By inverting the relationship in Example 6.11, the dividend yield can be obtained as

$$r_{\text{DIV}} = \frac{1.875}{25} = 0.075 = 7.5\%$$

Another way to label the clean price, so the price of the preferred share minus the implied accrued dividend, is the stripped price.

**Example 6.15** For the stock in the previous examples, the stripped price is given by

$$P_{\text{STR}} = 25 - 0.31 = \text{€}24.69$$

Starting from the stripped price of the stock, it is possible to calculate a stripped yield, which is given by the ratio of the annualized dividend over the stripped stock price.

**Example 6.16** Shares paying an annualized dividend of €1.875 with a stripped price of €24.69 offer a stripped yield of

$$r_{\text{DIV}} = \frac{1.875}{24.69} = 0.0759 = 7.59\%$$

Sometimes investors are looking for monthly income in their investment. This can be easily done by investing in three different issues of preferred stocks, so that each stock will pay quarterly in different months, to cover the whole calendar.

Many preferreds pay dividends quarterly in one of three cycles: (1) January, April, July, and October; (2) February, May, August, and November; or (3) March, June, September, and December.

It comes naturally to ask why investors like to hold preferred stocks, given that they have a lower yield than bonds, and they are also subordinate to them. The main reason is the tax advantage of DRD to corporations.

Among the several types of preferred stocks available on the market, there are adjustable-rate preferred stocks, with a dividend yield equal to some reference rate plus a spread. The market value of this type of shares is stable, and there are an upper bound and a lower bound to the dividend rate.

Convertible preferred stocks can be converted into certain number of shares of common stock, representing an option granted to the preferred stock holder. Callable preferred stocks on the other hand can be called before the maturity date at a pre-specified price at the option of the issuer and represent an option granted to the issuer.

### 6.3.2 Warrants

Warrants are a type of financial options that give holders the right to buy shares of a company at a fixed price for a given period. Each warrant specifies the number of shares of equity that the holder can buy, the exercise price, and the expiration date.

Even if from the definition they look very similar, the main difference between a warrant and a financial option is that the former has longer maturity periods, with some warrants having an infinite maturity, so that they never expire.

In many cases, warrants are attached to privately placed bonds issued by some company. In this case, the loan agreement will state whether the warrants are detachable from the bond and potentially be sold separately. Normally, the warrant can be detached immediately.

Warrants have been issued infrequently in the last years, and companies issuing them are mostly based in the United States. More recently, governments have purchased warrants from banks as part of the financial rescue plans put in place to combat the global credit crunch in 2008.

A warrant is a security that gives the holder the right to buy an underlying stock of the issuing company at a fixed exercise price, until the expiry date. As one can see, this is similar to the definition of a financial option.

Warrants usually come as an attachment to bonds or preferred stocks of a particular company. It helps the issuer to sell the bond at a lower yield that would have to be offered if nothing were attached to it. Warrants can also be used in private equity deals. Frequently, these warrants are detachable and can be sold independently of the bond or stock.

When the warrants are attached to preferred stocks, it might be the case that they need to detach and sell the warrant before any dividend can be cashed. This is why in most cases the warrants attached to preferred stocks are quickly sold separately.

Warrants have similar characteristics to that of standard equity options. They can in fact be exercised when the holder informs the issuer their intention to purchase the shares underlying the warrant. Right after the bond issuance, the parameters of the warrant, including the strike price, are fixed.

The premium of a warrant is the difference in price between buying the shares through the warrant and buying them directly on the market. The gearing or leverage shows how much exposure on the underlying is given by holding the warrant compared to having bought the shares directly.

The expiration date of a warrant follows the same rules and standard of an option, with the value of the warrant getting lower as the time to maturity reduces. Therefore, the expiry date is the date on which the right to exercise ceases to exist.

Like the options, the warrants can be European style or American style. They are longer-dated options and are normally traded on the over-the-counter markets, rather than on regulated exchanges.

Warrants are issued by private corporations and, when issued by the company itself, are dilutive, because if the warrant is exercised, the company issues new shares of stock, so the number of outstanding shares increases.

Warrants do not carry any voting rights, and the owner receives an existing share at exercise time. In case of employee stock options, new shares are created and issued by the company.

Warrants usually have a longer life than standard options, and their life is measured in years. Upon expiration, the warrants are worthless unless the price of the common stock is greater than the exercise price.

There are traditional warrants that are attached to a bond and simply carry the right to buy shares in the issuing entity. The writer of a traditional warrant is therefore also the issuer of the underlying instrument.

One way to calculate the value of the warrant is to subtract the clean price for the bond from the overall price paid for it with the warrant attached as

$$P_w = B_w - B$$

$$= B_w - \left[ \sum_{t=1}^T \frac{C}{(1+r_r)^t} + \frac{F}{(1+r_r)^T} \right]$$

where:

$B_w$  is the price paid for the bond with warrants.

$C$  is the coupon payment.

$r_r$  is the required rate of return.

Another type of warrant is named naked, and it is issued stand-alone, without being attached to a bond or another instrument. Like traditional warrants, these are traded on the stock exchange.

Warrants are typically issued by banks and securities firms, in which case they are called covered warrants, not involving the firm issuing the outstanding shares. Covered warrants are very popular and generate more volume compared to uncovered ones.

From a financial point of view, they are generally bought by retail investors, rather than investment funds or banks. The latter in fact normally prefer financial options, given that they are keenly priced and trade on a different market.

When warrants are issued by the holders of the underlying asset, they are called third-party warrants. Companies and mutual funds in particular can issue warrants on their assets.

**Example 6.17** A company issues 1,000,000 warrants, giving the right to convert each warrant into a share at €250. Such a warrant is issued by the company. Assume a mutual fund holds 100,000 shares of the company and sells the warrants attached to them. If the stock does not cross €250, the buyer will not exercise the warrant. The seller will therefore keep the warrant premium.

The main advantage of issuing third-party warrants is that they signal the value of the underlying asset by tracking its value. A mutual fund, selling warrants at some exercise price, signals that the value of the fund shares in the short-medium term may tend to that value.

If volumes in such warrants are high, the price discovery process will be that much better, for it would mean that many investors believe the stock will trade at that level in 1 year.

Third-party warrants are long-term call options issued in the form of a covered call or write, given that the issuer of the warrant is the holder of the underlying asset and the option is sold against it.

The name warrant is also used for the issuance of checks by government agencies. The checks are in fact not immediately redeemable due to lack of funds, but they will be in the future, with interest compensation.

In some states, a warrant is a demand draft drawn on a government's treasury to pay its bills. Checks or electronic payments have replaced these warrants, but in Arkansas, some counties and school districts use warrants for nonelectronic payments.

It is possible to express in general terms the profit associated to the exercise of a warrant. The gain from a single warrant can be expressed as

$$V_w = \frac{E + K_w m_w}{m + m_w} - K \quad (6.3)$$

where:

$E$  is the firm's value net of debt.

$K_w$  is the exercise price of the warrant.

$m$  is the number of shares outstanding before the exercise of the warrant.

$m_w$  is the number of new shares from the exercise of the warrant.

The first term on the right-hand side of Eq. (6.3) is the value of a share of equity after the warrant is exercised. The numerator of the left term is the firm's value net of debt after the warrant is exercised.

The sum  $m + m_w$  is the amount of shares outstanding after the exercise of the warrants. By rearranging terms, we can rewrite the equation as

$$V_w = \frac{m_w}{m + m_w} \left( \frac{E}{m} - K \right)$$

The term in the parenthesis on the right-hand side of the equation is the gain on a call option written on the equity of the firm, before the warrants are exercised. It follows that the gain from exercising a warrant is a proportion of the gain from exercising a call in a firm without warrants.

We can therefore value a warrant using the Black-Scholes model, adjusted for the dilution effect:

$$P_w = \frac{c_w}{\left(1 + \frac{m_w}{m}\right)}$$

where

$c_w$  is the value of a call option written on the equity of a firm without warrants.

### 6.3.3 Convertibles

In the case of a warrant attached to a bond, it is always possible to separate the two securities. This is not possible in the case of convertible bonds, which gives the holder the right to exchange it for a given number of shares any time up to and including the maturity date of the bond.

The concepts of conversion ratio, the amount of stocks to be received for every bond converted, and the conversion price to be paid for every stock purchased through conversion of the bond are the key features of a convertible bond.

In fact, both concepts implicitly assume that the bond is selling at par. If the bond is selling at another price, the terms have little meaning. By contrast, conversion ratio can have a meaningful interpretation regardless of the price of the bond.

The value of the convertible bond is determined by three factors, namely, the value of the naked bond, the conversion value, and the value of the embedded option. All of the three components deserve to be analyzed and understood.

The straight (naked) bond value is what the convertible bonds would sell for if they could not be converted into equity. It will depend on the general level of interest rates and on the default risk.

**Example 6.18** Company TXL raises €10,000,000 by issuing 5.25% convertible bonds. Each bond is convertible into four shares, and the share price at the time of bond issuance is €18.75. The conversion price of

$$K = \frac{100}{4} = €25.00$$

is much higher than the actual equity price. Suppose that straight debentures issued by the same company are rated AA, and AA-rated bonds are priced to yield 3.25% per 6 months. The straight bond value of the convertible bonds can be determined by discounting the €2.625 semiannual coupon payment and principal amount at 3.25%:

$$B = \sum_{t=1}^{16} \frac{5.25}{(1 + 0.0325)^t} + \frac{100}{(1 + 0.0325)^{16}} = 61.56 + 63.09 = €124.65$$

The straight bond value of a convertible bond is a minimum value. The price of the convertible could not have gone lower than the straight bond value.

The value of convertible bonds depends on conversion value. Conversion value is what the bonds would be worth if they were immediately converted into equity at current prices.

The conversion value is equal to the share of equity that can be received upon conversion of the bond, multiplied by the current price of the stocks, as determined by the market.

Normally, the value of the convertible bond is higher than the sum of the value of the naked bond and the conversion value. This is due to the fact that the bond does not have to be converted immediately.

The option of waiting for exercising the option embedded in the bond is assumed to have a value, and it raises the value over both the straight bond value and the conversion value.

In case of a low value of the firm, the value of the convertible bond is mostly made by the naked bond, while in case of a high value of the company, most of the value is in the conversion value.

$$B_{CV} = \max(B_C, K_{CV}) + c_{CV}$$

where:

$B_C$  is the straight bond value.

$K_{CV}$  is the conversion value.

$c_{CV}$  is the option value.

Compared to a normal naked bond, a convertible bond pays a lower interest rate, given that the investor will accept a lower interest rate on a convertible because of the potential gain from conversion.

According to the specific situation, issuing straight bonds can be better or worse than issuing convertible bonds. First of all, consider a scenario of an increase of the share price. This is of course beneficial to the company, but it also enhances conversion.

The firm would have benefited even more had it previously issued straight debt instead of a convertible. The gain in terms of the lower interest paid on the convertible bonds is offset by the loss of selling equity at a cheap price to the convertible bond holders.

On the other hand, in case of a fall in price of the share, the company will be upset on one side but will also benefit from having issued convertible bonds compared to straight bonds.

Because conversion does not take place, our comparison of interest rates is all that is needed. It can be concluded that it is not optimal for the firm to issue convertible debt if a future raise in price is expected. The firm is better off having issued convertible debt if the underlying equity subsequently does poorly.

It is not possible in an efficient market to predict the future price of the shares. Therefore, any consideration about the a priori convenience of issuing convertible bonds instead of naked ones is purely speculative.

Another type of dilemma consists in the choice between issuing convertible bonds or equity for financing. Assuming that convertibles are issued, a subsequent raise of the stock price would fully benefit the company.

In case the stock price falls after convertible bonds are issued, the scenario is flipped, and the company suffers from both the loss in equity value and regretting the choice of having issued convertibles instead of equity.

The firm in fact would have benefited by issuing equity above its later market price. That is, the firm would have received more than the subsequent worth of the equity.

The value of the convertible is not severely affected because the straight bond value serves as a floor.

To conclude, the firm is better off having issued convertible debt instead of equity, if the stock price subsequently increases. The firm is worse off having issued convertible debt if the underlying equity subsequently does poorly.

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## 6.4 Summary

Together with debt, equity is the other important source of capital for every company. The stock market resembles lots of investors trading very high volumes of equity shares every day.

The main feature of an equity market is its microstructure, defined by the kernel that processes the orders of the investors translating them into transactions that drive the price of the stocks.

The price formation goes through a process of supply and demand where the information hitting the market at random times plays a crucial role in determining the strategies of the investors.

The availability of the deals and the formation of a fair price are guaranteed by the liquidity of the market. In order for a market to be liquid, there must be sufficient volumes of tradable assets available to the investors to satisfy their demand.

Common stocks are the main junk of equity capital in the structure of a company, and they come with voting rights and other features that result from the ownership of a part of the firm.

Several models are available for common equity valuation including discount dividend models, based on the dividends paid out by the firm, and free cash flow models, based on the cash flows generated by the firm. Also relative valuation models have become of common use among practitioners, in that they are more simple and accounting-based.

On top of the common stocks, there are other forms of equity capital that are relevant to the capital structure of the firm. Preferred stocks, for example, are very popular among investors, in that they guarantee the debt features of a bond, with the advantage of acquiring ownership of the firm.

In the same way, warrants are also very important, as a tool for distributing the chance to buy extra stocks to already existing shareholders and also to give the chance to new investors to enter the capital.

Convertible bonds are a specific type of security with an option embedded. They result in the chance for a debt holder to become a shareholder at some time in the future, so as to change the profile of the investment portfolio.

**Problems**

1. Explain the difference between illiquidity and insolvency. Does the difference matter?
2. Explain why if the government announces it is abolishing insurance on deposits, a typical bank is likely to face liquidity problems.
3. If yield curves, on average, were flat, what would this say about the liquidity premiums in the term structure?
4. Would you expect the bid-ask spread to higher on actively or inactively traded stocks?
5. Discuss the moral hazard aspects created by deposit insurance.
6. Describe and compare the different types of trading orders available in the markets.
7. A bill has a bank discount yield of 6.65% based upon the asked price and 6.75% based upon the bid price. The maturity of the bill (already accounting for skip-day settlement) is 90 days.
  - (a) Find the bid and asked prices of the bill.
  - (b) Calculate the bond equivalent yield of the bill as well as its effective annual yield based upon the asked price. Confirm that these yields exceed the discount yield.
8. The table below provides some price information on Marriott:

Bid price	Ask price
37.55	38.33

You have placed a stop-loss order to sell at €37.80.

- (a) By placing this order, what are you in effect asking your broker to do?
  - (b) Given the market prices, will your order be executed?
9. Consider the following limit order book of a specialist. The last trade in the stock occurred at a price of €45.55.

Limit buy orders		Limit sell orders	
Price	Shares	Price	Shares
35.50	5000	35.75	1000
35.25	6000	35.90	2000
35.00	8000	36.00	5000

- (a) If a market buy order for 3000 shares comes in, at what prices will it be filled?
  - (b) What will happen if a market order to sell 5000 shares comes in?
10. Consider the following limit order book of a specialist. The last trade in the stock occurred at a price of €45.55.

Limit buy orders		Limit sell orders	
Price	Shares	Price	Shares
59.75	4000	55.75	1000
59.50	5000	55.80	2500
59.25	7000	56.00	4500

- (a) If a market buy order for 1000 shares comes in, at what prices will it be filled?
- (b) At what price would the next market buy order be filled?
- (c) You are the specialist: do you wish to increase or decrease your inventory of this stock?
11. You have borrowed €20,000 on margin to buy shares in Disney, which is now selling at €80 per share. Your account starts at the initial margin requirement of 50%. The maintenance margin is 35%. Two days later, the stock price falls to €75 per share.
- (a) Will you receive a margin call?
- (b) How low can the price of Disney shares fall before you receive a margin call?
12. Explain why banks hold more liquid assets than most other business.
13. Explain the difference between illiquidity and insolvency. Does the difference matter?
14. Explain why if the government announces it is abolishing insurance on deposits, a typical bank is likely to face liquidity problems.
15. If yield curves, on average, were flat, what would this say about the liquidity premiums in the term structure?
16. Would you expect the bid-ask spread to higher on actively or inactively traded stocks?
17. Discuss the moral hazard aspects created by deposit insurance.

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