Introduction and Survey

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Chapter 1

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Chapter 1

The name "Physics" comes from the Greek ("$\varphi\upsilon\sigma\iota\kappa\eta$" = nature, creation, origin) which comprises, according to the definition of *Aristotle* (384–322 BC) the theory of the material world in contrast to *metaphysics*, which deals with the world of ideas, and which is treated in the book by Aristotle after (*Greek: meta*) the discussion of physics.

Definition

The modern definition of physics is: Physics is a basic science, which deals with the fundamental building blocks of our world and the mutual interactions between them.

The goal of research in physics is the basic understanding of even complex bodies and their composition of smaller elementary particles with interactions that can be categorized into only four fundamental forces. Complex events observed in our world should be put down to simple laws which allow not only to explain these events quantitatively but also to predict future events if their initial conditions are known.

In other words: Physicists try to find laws and correlations for our world and the complex natural events and to explain all observations by a few fundamental principles.

Note, however, that complex systems that are composed of many components, often show characteristics, which cannot be reduced to the properties of these components. The amalgamation of small particles to larger units brings about new and unforeseen characteristics, which are based on cooperative processes. *The whole is more than the sum of its parts (Heisenberg 1973, Aristotle; metaphysics VII).* Examples are living biological cells, which are composed of lifeless molecules or molecules with certain chemical properties consisting of atoms that do not show these properties of the molecule.

The treatment of such complex systems requires new scientific methods, which have to be developed.

This should remind enthusiastic physicists, that physics alone might not explain everything although it has been very successful to expands the borderline of its realm farther and farther in the course of time.

1.1 The Importance of Experiments

The more astronomically oriented observations of ancient Babylonians brought about a better knowledge of the yearly periods of the star sky. The epicycle model of *Ptolemy* gave a nearly quantitative description of the movements of the planets. However, modern Physics in the present meaning started only much later with *Galileo Galilei* (1564–1642, Fig. 1.1), who performed as the first physicist well planned experiments under defined conditions, which could give quantitative answers to open questions. These experiments can be performed at any time under conditions chosen by the experimentalist independent of external influences. This distinguishes them from the observations of natural phenomena, such as thunderstorms, lightening or volcanism, which cannot be influenced. This freedom of choosing the conditions is the great advantage of experiments, because all perturbing external influences can be partly or even completely eliminated (e. g. air friction in experiments on free falling bodies). This facilitates the analysis of the experimental results considerably.

Experiments are aimed questions to nature, which yield under defined conditions definite answers.

The goal of all experiments is to find reasons and causes for all phenomena observed in nature, to see connections between the manifold of observations and to categorize them under a common law. Even more ambitious is the quantitative prediction of future experimental results, if the initial conditions of the experiments are known.

A physical law connects measurable quantities and concepts. Its clear form is a mathematical equation.

Such mathematical descriptions give a clearer insight into the relations between different physical laws. It can reduce the manifold of experimental findings, which might seem at first glance uncorrelated but turn out to be special cases of the same general law that is valid in all fields of physics.

Examples

1. Based on many careful measurements of planetary orbits by *Tycho de Brahe* (1546–1601), *Johannes Kepler* (1571–1630) could postulate his three famous laws for the quantitative description of distances and movements of the planets. He did not find the cause for these movements, which was discovered only later by Isaac Newton (1642–1727) as the gravitational force between the sun and the planets. However, Newton's gravitation law did not only describe the planetary orbits but all movements of bodies in gravitational fields. The problem to unite the gravitational force with the other forces (electromagnetic, weak and strong force) has not yet been solved, but is the subject of intense current research.
2. The laws of energy and momentum conservation were only found after the analysis of many experiments in different fields. Now they explain and unify many experimental findings. Such a unified summary of different physical laws and principles to a consistent general description is called a *physical theory*. ◀

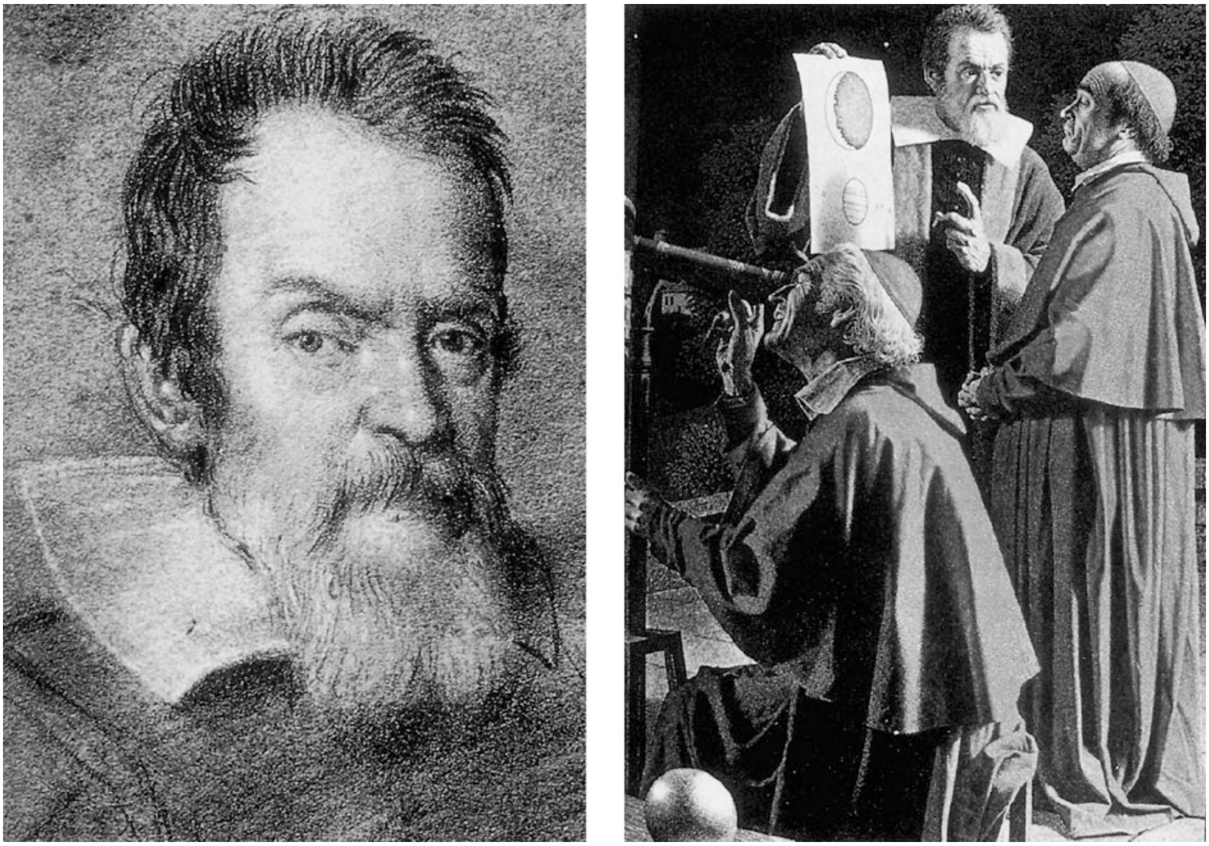

Figure 1.1 *Left:* Galileo Galilei. *Right:* Looking of Cardinales through Galilio's Telescope

Its range of validity and predictive capability is checked by experiments.

Since the formulation of a theory requires a mathematical description, a profound knowledge of basic mathematics is indispensible for every physicist.

1.2 The Concept of Models in Physics

The close relation between theory and experiments is illustrated by the following consideration:

If a free falling body in a vacuum container at the surface of the earth is observed one finds that the fall time over a definite distance is independent of the size or form of the body and also independent of its weight. In contrast to this result is the fall of a body in any fluid, instead of vacuum where the form of the body does play a role because here perturbing influences, such as friction often cannot be neglected. Neglecting these perturbations one can replace the body by the **model of a point mass**. With other words: In these experiments the falling body behaves like a point mass, because its size does not matter. The theory can now give a complete description of the movement of point masses under the influence of gravitational forces and it can predict the results of corresponding future experiments (see Chap. 2).

Now the experimental conditions are changed: For a body falling in water the velocity and fall time do depend on size and weight of the body, because of friction and buoyancy. In this case the model of a point mass is no longer valid and has to be broadened to the **model of spatially extended rigid bodies** (see Chap. 5). This model can predict and quantitatively explain the movements of extended rigid bodies under the influence of external forces.

If we now further extend our experimental condition and let a massive body fall onto a deformable elastic steel plate, our rigid body model is no longer valid but we must include in our model the deformation of the body, This results in the **model of extended deformable bodies**, which describes the interaction and the forces between different parts of the body and explains elasticity and deformation quantitatively (see Chap. 6).

The theory of phenomena in our environment is always the description of a model, which describes the observations. If new phenomena are discovered which are not correctly represented by the model, it has to be broadened and refined or even completely revised.

The details of the model depend on the formulation of the question asked to nature and on the kind of experiments which should be explained. Generally a single experiment tests only certain statements of the model. If such an experiment confirms these statements, we say, that nature behaves in this experiment like the model predicts, i. e. nature gives the same answer to selected experiments as the model.

Since theory can in principle calculate all properties of an accepted model it often gives valuable hints, which experiments could best test the validity of the model.

Such a cooperation and mutual inspiration of theoretical and experimental physics contribute in an outstanding way to the progress in physical knowledge.

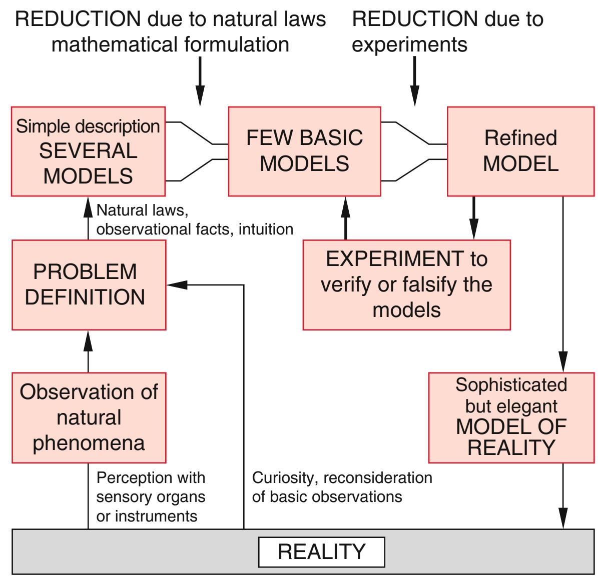

Figure 1.2 Schematic representation of the way, how scientists gain information on nature

An impressive example is the development of *quantum chromodynamics*. This modern theory describes the substructure of particles, which had been regarded as elementary, such as protons, neutrons and mesons, but are really composed of still smaller particles, the **quarks**. Theoretical predictions about the possible masses of unstable particles, composed of these quarks, which appear as resonances in the collision cross sections, allowed the experimentalists to restrict their search which is like the search for a needle in the haystack, to the predicted energy range, which facilitated their efforts considerably.

The model concept for the description of observations in nature is in particular obvious in the world of microphysics (atomic, molecular and nuclear physics), because here the particles cannot be seen with the naked eye and therefore a vivid picture cannot be given. Attempts to transfer vivid models useful in macrophysics to microphysics have often led to misunderstandings and wrong ideas. One example is the particle-wave dualism for the description of microparticles (see Vol. 3).

Figure 1.2 comprises the discussion above. One example shall illustrate the development and refinement of models in physics. The explanation of lightning by Greek philosophers was the god Zeus who flung flashes to the earth while he was in a furious mood. Modern models explain lightning by the separation of positive and negative electrical charges by charged water drops floating in turbulent air, leading to large electric voltages between different clouds or between clouds and earth with resulting strong discharges. This modern model is based on many detailed observations with high speed photographic instruments and on experimental simulations of lightning in high voltage laboratories where discharges can be observed under controlled conditions.

The goal of sciences is the understanding of natural phenomena observed under different conditions and to categorize their explanations under a common law. It is assumed, that the observed reality exists independent of the observer. However, the experiments performed in order to reproduce the observations demand nevertheless characteristic features of the observing subject, such as imagination for the planning of decisive experiments, an open mind for new ideas, etc. Many ideas turn out to be wrong. They can be already excluded by comparison with former experiments. Such ideas which do not contradict already existing knowledge can contribute to a working hypothesis. Even such a hypothesis might be only partly correct and has to be modified by the results of further experiments. If all these results confirm the working hypothesis it can become a **proved theory**, which allows us to summarize many observations to a general law (see Fig. 1.3).

This procedure where a theory is built up from many experimental results is called the **inductive method**.

In theoretical physics often a reverse procedure is chosen. The starting point are fundamental basic equations such as Newton's law of gravitation or the Maxwell equations or symmetry laws. From these general laws the outcome of possible experiments is predicted (**deductive method**).

Both procedures have their justification with advantages and drawbacks. They supplement each other.

An important aspect which one should keep in mind is summarized in the following fundamental statement:

Physics describes objective and as accurate as possible the reality of the material world. For human beings this is, however, only a small section of the world we experience, as a specific example illustrates: From the standpoint of physics a painting can be described, by giving for each point (x, y) the reflectivity $R(\lambda, x, y)$, which depends on the wavelength λ, the spectrum of

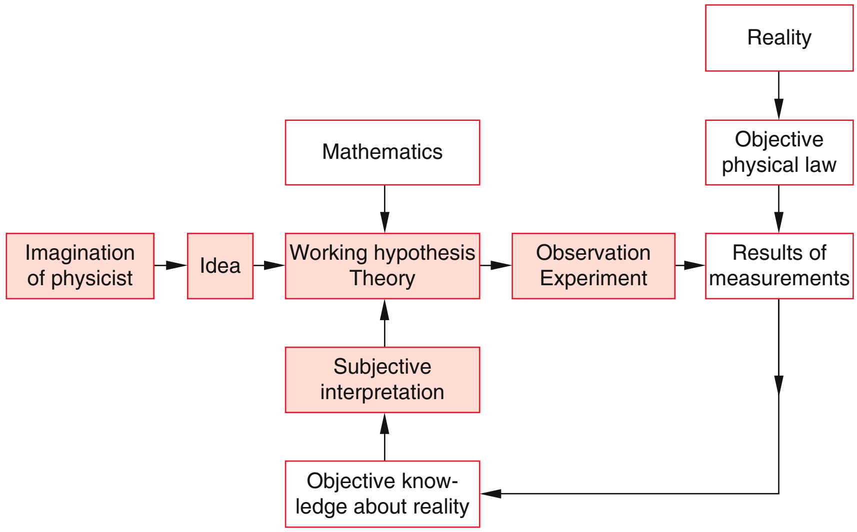

Figure 1.3 Schematic diagram of gaining insight into natural phenomena

the illuminating radiation source and the angles of incidence and observation direction. A computer which is fed with these characteristic input data can reproduce the painting very accurately.

Nevertheless this physical description lacks an essential part of the painting, which is in the mind of the observer. When looking at the painting a human being might remember other similar paintings which he compares with the present painting, even if these other paintings are not present but only in the mind they still change the subjective impression of the observer. The subject of the painting may induce cheerful or sad feelings in the mind of the observer, it may call back remembrances of former events or impressions which are related to this painting. All these different influences will determine the judgement about the painting, which therefore might be different for different observers.

All these aspects are not the realm of physics, because they are subjective, although they are essential for the quality of the painting as judged by human beings and they represent an important part of the "reality" as perceived by us.

These remarks should warn physicists, not to forget that our fascinating science is only competent for the description of the material basis of our world. Although the other nonmaterial realms are based on the material world their description and understanding reaches far beyond physics. The question, how living cells are built from inanimate molecules and how the human mind is related to the structure of the brain are still pending but exciting problems, which might be solved in the future. This is related to the question whether the human brain is more than a highly developed computer, which is the subject of hot discussions between the supporter of artificial intelligence and biologists.

For more detailed discussions of these questions, the reader is referred to the literature [1.1a–1.6].

1.3 Short Historical Review

The historical development of physics can be roughly divided into three periods:

- The natural philosophy in ancient times
- The development of classical physics
- The modern physics.

1.3.1 The Natural Philosophy in Ancient Times

The investigation of natural phenomena and the efforts to explain them by rational arguments started already 4000 years ago. The astronomical observation of the Babylonian and the Egyptian scientists were important for the prediction of annual occurrences, such as the Nile flood or the correct time for sowing. The Greek philosophers produced many ideas for the explanation of the observed natural phenomena. All these ideas were treated within the framework of general philosophy. For example, the textbook on Physics ($\varphi\upsilon\sigma\iota\kappa\eta\ \alpha\kappa\varrho o\alpha\sigma\iota\varsigma$ = lectures on physics) by Aristotle contains mainly philosophical considerations about space and time, movements of bodies and their causes.

Probably the most important achievement of Greek philosophy was the overcoming of the widespread mythology, where the life of mankind was governed by a hierarchy of gods, whose mood was not predictable and everybody had to win the liking of gods by sacrificing precious gifts to them. Most Greek philosophers abandoned the belief, that the world was a playing ground for gods, demons and ghosts who generated thunderstorm, floods, sunshine or disastrous droughts just according to their mood (see Homer's Odyssey).

The Greek philosophers believed that all natural phenomena obeyed eternal unchanging laws which were not always obvious

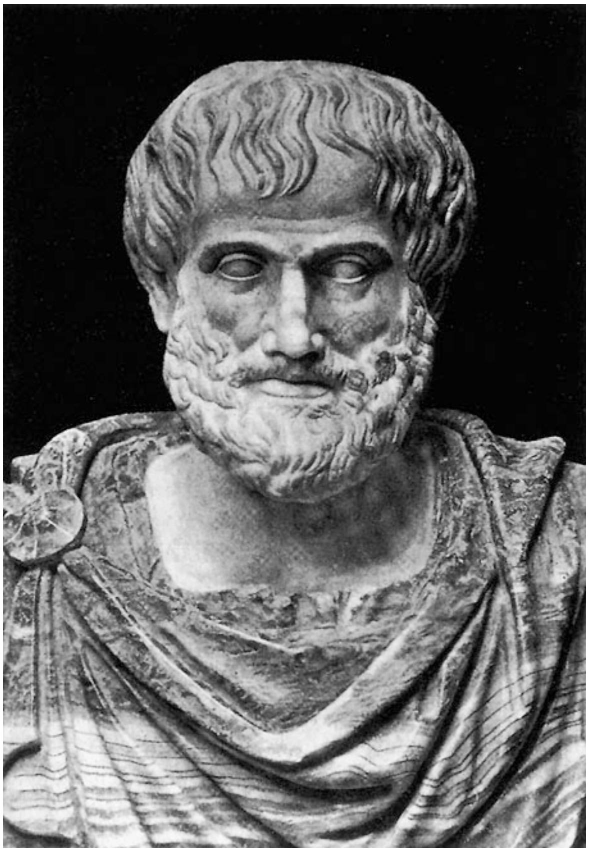

Figure 1.4 Aristotle. With kind permission of the "Deutsches Museum"

because of the complex nature but which were independent of men or gods. This means that it is, at least in principle, possible to find such laws merely by human reason.

Example

A solar eclipse is no longer described by a monster that engulfs the sun, but by the temporarily blocking of the sunlight by the moon. This changes the solar eclipse from an accidental event to a predictable occurrence. ◀

Famous representatives of Greek philosophy were *Thales from Milet* (624–546 BC), who discovered magnetism and frictional electricity, but could not correctly explain his findings. *Empedocles* (495–435 BC) assumed that fire, water, air and soil formed the four basic elements, which can mix, divide and build compositions from which all other material is composed. The mathematical aspect of natural phenomena was introduced by *Pythagoras* (572–492 BC) and his scholars who assumed that numbers and mathematical relations between these numbers reflect the reality. They made acoustic experiments with striking chords of different lengths and measured the resulting tones. However, they erroneously generalized their results to other fields such as the movement of the planets.

Anaxagoras (499–428 BC) was the first to postulated that the world consists of many infinitely small different particles. The force which keeps them together is the Nus (= world spirit). *Leucippus* (489–428 BC) and his student *Democritus* (455–370 BC) followed these ideas and refined this hypothesis. *Democritus* assumed that the world consists of atoms ($\alpha\tau o\mu o\varsigma$ = indivisble), very small indivisible identical particles, which move forever in an infinite empty space. The different forms of matter differ only by the number and arrangement of atoms of which they are composed. This hypothesis comes close to our present understanding of the atomic composition of the different elements in the periodic table (see Sect. 1.4).

The doctrine of the "atomists" was declined by *Plato* (427–347 BC) and *Aristotle* (Fig. 1.4) since it contradicted their view of a continuous world. Since these two philosophers had such a great reputation the atomistic theory was forgotten for nearly 2000 years.

Aristotle (384–322 BC) (Fig. 1.4) regarded nature as the forever moving and developing universe, where at the beginning a "divine mover" was assumed who started the whole world. The planets move apparently without obvious mover and therefore Aristotle assumed that they do not consist of the four earthly elements fire, air, water and soil but of a fifth "divine element" which he called "Ether". This ether should be massless and elastic and should penetrate the whole world, including rigid bodies.

Archimedes (287–212 BC) studied in Alexandria, the centre of science at that time. Later he moved to Syracuse on Sicily. He was the greatest mathematician, physicist and technical expert of his time. He succeeded to calculate the area and the perimeter of a circle, the surfaces of spheres, cones and cylinders and he solved third order equations. As a physicist he determined the centre of mass for bodies of different shape, he found the lever principle, calculated the buoyancy of bodies in water (Archimedes' principle), he built a planetarium and measured star positions and proved the curvature of the sea surface. He was famous for his technical achievements. He invented and constructed about 40 different machines, such as the worm gear drive, catapults, hydraulic levers for lifting ships and many machines used for warfare.

In spite of great success in many fields the Greek philosophers could not reach natural science in the present sense, because they did not accept the experiment as the touchstone for every theory. They believed that an initial observation was sufficient and that all subsequent conclusions and knowledge could be achieved by pure thinking without further confirming or disproving experiments.

This rather speculative procedure has influenced, due to the great impact of Aristotle's generally accepted teaching, many generations of philosophers for more than 1500 years. Even when *Galilei Galileo* observed through his telescope the four moons of Jupiter, most philosophers and high members of the church did not believe him, because his observation contradicted the theory of Aristotle, who taught that the planets were fixed on crystal spheres moving with the planet around the earth. If moons circled around Jupiter they had to penetrate these crystal spheres and would smash them. Therefore, the moons should be impossible. Even when Galilei offered to the sceptics to look through the telescope (Fig. 1.1b) many of them refused and said: "Why should we look and be deceived by optical illusions when we are sure about Aristotle's statements".

Although some inconsistencies in Aristotle's teaching had been found before, Galilee was the first to disprove by his observations and experiments the whole theory of the shining example of Greek philosophy, in particular when he also advertised the new astronomy of *Copernicus*, which brought him many enemies and even a trial before the catholic court.

1.3.2 The Development of Classical Physics

One may call Galileo the first physicist in the present meaning. He tried as the first scientist to prove or disprove physical theories by specific well-planned experiments. Famous examples are his experiments on the movement of a body with constant acceleration under the influence of gravity. He also considered how large the accuracy of his experimental results must be in order to decide between two different versions for the description of such movements. He therefore did not choose the free fall (it is often erroneously reported, that he observed bodies falling from the Leaning tower in Pisa). This could never reach the required accuracy with the clocks available at that time. He chose instead the sliding of a body on an inclined plane with an angle α against the horizontal. Here only the fraction $g \cdot \sin\alpha$ acts on the body and thus the acceleration is much smaller.

His astronomical observations (phases of Venus, Moons of Jupiter) with a self-made telescope (after he had learned about its invention by the optician *Hans Lipershey* (1570–1619) in Holland) helped the Copernican model of the planets circling around the sun instead of the earth, finally to become generally accepted (in spite of severe discrepancies with the dogmatic of the church and heavy oppression by the church council).

The introduction of mathematical equations to physical problems, which comprises several different observations into a common law, was impressively demonstrated by *Isaac Newton* (Fig. 1.5). In his centennial book "*Philosophiae Naturalis Principia Mathematica*" he summarizes all observations and the knowledge of his time about mechanics (including celestial mechanics = astronomy) by reducing them to a few basic principles (principle of inertia, actio = reactio, the force on a body equals the time derivative of his momentum and the gravitational law).

Supported by progress of mathematics in the 17th century (analytical geometry, infinitesimal calculus, differential equations) the mathematical description of physical observations becomes more and more common. Physics emancipates from Philosophy and develops its own framework using mathematical language for the clear formulation of physical laws. For example classical mechanics experiences its complete and elegant mathematical form by *J. L. de-Lagrange* (1736–1813) and *W. R. Hamilton* (1805–1865) who reduced all laws for the movement of bodies under arbitrary forces to a few basic equations.

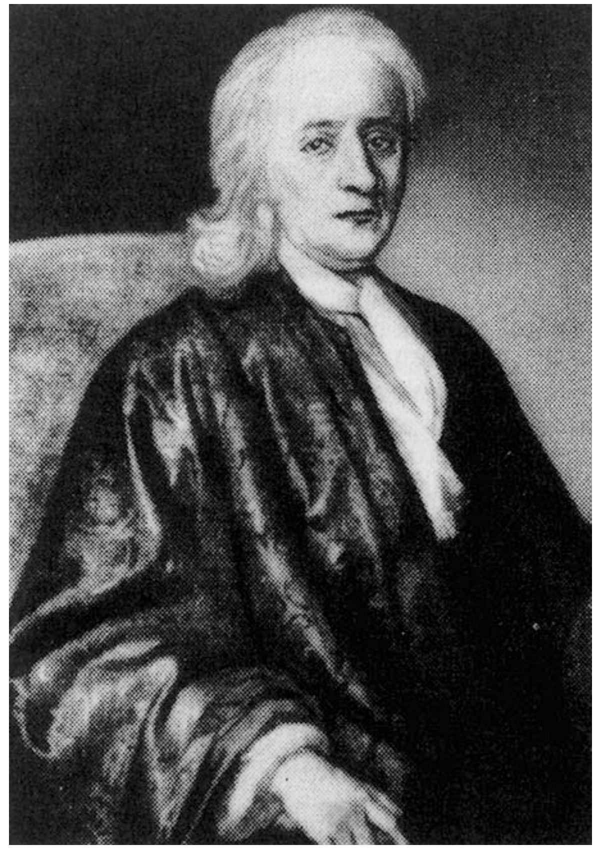

Figure 1.5 Sir Isaac Newton. With kind permission of the "Deutsches Museum München"

Contrary to mechanics which had developed already in the 18th century to a closed complete theory the knowledge about the structure of matter was very sketchy and confused. Simultaneously different hypotheses were emphasized: One taken form the ancient Greek philosophy, where fire, water, air and soil were assumed as the basic elements, or from the alchemists who favoured mercury, sulphur and salt as basic building blocks of matter.

Robert Boyle (1627–1591) realized after detailed experiments that simple basic elements must exist, from which all materials can be composed, which however, cannot be further divided. These elements should be separated by chemical analysis from their composition. Boyle was able to prove that the former assumption of elements was wrong. He could, however, not yet find the real elements.

A major breakthrough in the understanding of matter was achieved by the first critically evaluated quantitative experiments investigating the mass changes involved in combustion processes, published in 1772 by *A. L. de Lavoisier* (1743–1794). These experiments laid the foundations of our present ideas about the structure of matter. Lavoisier and *John Dalton* (1766–1844) recognised metals as elements and postulated like Boyle that all substances were composed of atoms. The atoms were now, however, not just simple non-divisible particles, but had specific characteristics which determined the properties of the composed substance. *Karl Wilhelm Scheele* (1724–1786) found that air consisted of nitrogen and oxygen.

Antoine-Laurent Lavoisier furthermore found that the mass of a substance increased when it was burnt, if all products of the combustion process were collected. He recognized that this mass increase was caused by oxygen which combined with the substance during the burning process. He formulated the law of mass conservation for all chemical processes. Two elements can combine in different mass ratios to form different chemical products where the relative mass ratios always are small integer numbers.

The British Chemist *John Dalton* was able to explain this law based on the atom hypothesis.

Examples

1. For the molecules carbon monoxide and carbon dioxide the mass ratio of oxygen combining with the same amount of carbon is 1 : 2 because in CO one oxygen atom and in CO_2 two oxygen atoms combine with one carbon atom.
2. For the gases N_2O (Di-Nitrogen oxide), NO (nitrogen mono oxide), N_2O_3 (nitrogen trioxide), and NO_2) nitrogen dioxide) oxygen combines with the same mass of nitrogen each time in the ratio 1 : 2 : 3 : 4. ◀

Dalton also recognized that the relative atomic weights constitute a characteristic property of chemical elements. The further development of these ideas lead to the periodic system of elements by *Julius Lothar Meyer* (1830–1895) and *Dimitri Mendelejew* (1834–1907), who arranged all known elements in a table in such a way that the elements in the same column showed similar chemical properties, such as the alkali atoms in the first column or the noble gases in the last column.

Why these elements had similar chemical properties was recognized only much later after the development of quantum theory.

The idea of atoms was supported by *Amedeo Avogadro* (1776–1856), who proposed in 1811 that equal volumes of different gases at equal temperature and pressure contain an equal number of elementary particles.

A convincing experimental indication of the existence of atoms was provided by the *Brownian motion*, where the random movements of small particles in gases or liquids could be directly viewed under a microscope. This was later quantitatively explained by Einstein, who showed that this movement was induced by collisions of the particles with atoms or molecules.

Although the atomic hypothesis scored indisputable successes and was accepted as a working hypothesis by most chemists and physicists, the existence of atoms as real entities was a matter of discussion among many serious scientists until the end of the 19th century. The reason was the fact that one cannot see atoms but had only indirect clues, derived from the macroscopic behaviour of matter in chemical reactions. Nowadays the improvement of experimental techniques allows one to see images of single atoms and the theoretical basis of atomic theory leaves no doubt about the real existence of atoms and molecules.

The theory of heat began to become a quantitative science after thermometers for the measurement of temperatures had been developed (air-thermoscope by Galilei, alcohol thermometer 1641 in Florence, mercury thermometer 1640 in Rome). The Swedish physicist *Anders Celsius* (1701–1744) introduced the division into 100 equal intervals between melting point (0 °C) and boiling point (100 °C) of water at normal pressure. *Lord Kelvin* (1824–1907) postulated the absolute temperature, based on gas thermometers and the general gas law. On this scale the zero point $T = 0\,\mathrm{K} = -273.15\,°\mathrm{C}$ is the lowest temperature which can be closely approached but never reached (see Chap. 10).

Denis Papin (1647–1712) investigated the process of boiling and condensation of water vapour (Papin's steam pressure pot). He built the first steam engine, which *James Watt* (1736–1819) later improved to reliable technical performance. The terms ***amount of heat*** and ***heat capacity*** were introduced by the English physicist and chemist *Joseph Black* (1728–1799). He discovered that during the melting process heat was absorbed which was released again during solidification.

The more precise formulation of the theory of heat was essentially marked by establishing general laws. *Robert Mayer* (1814–1878) postulated the first law of the theory of heat, which states that for all processes the total amount of energy is conserved. *Nicolas Carnot* (1796–1832) started 1831 after some initial errors a fresh successful attempt to describe the conversion of heat into mechanical energy (Carnot's cycle process). This was later more precisely formulated by *Rudolf Clausius* (1822–1888) in the **second law of heat theory**.

A real understanding of heat was achieved, when the kinetic gas theory was formulated. Here the connection between heat properties and mechanical energy was for the first time clearly formulated. Since the dynamical properties of molecules moving around in a gas were related to the temperature of a gas, the heat theory was now called **thermodynamics**, which was formulated by several scientists (Clausius, Avogadro, Boltzmann) (see Fig. 1.6). They proved under the assumption that gases consist of many essentially free atoms or molecules, which move randomly around and collide with each other, that the heat energy of a gas is equivalent to the kinetic energy of these particles. The Austrian physicist *Joseph Loschmidt* (1821–1895) found that under normal pressure the gas contains the enormous number of about $3 \cdot 10^{19}$ atoms per cm^3.

Optics is one of the oldest branches of physics which was already studied more than 2000 years ago where the focussing of light by concave mirrors was used to ignite a fire. However, only in the 17th century optical instruments and their imaging properties were studied systematically. A milestone was the fabrication of lenses and the invention of telescopes. *Willibrord Snellius* (1580–1626) formulated his law of refraction (see Vol. 2, Chap. 9), Newton found the separation of different colours when white sun light passed through a prism. The explanation of the properties of light was the subject of hot discussions. While Newton believed that light consisted of small particles (in our present model these are the *photons*) the experiments on interference and diffraction of light by *Grimaldi*

Figure 1.6 Ludwig Boltzmann. With kind permission from Dr. W. Stiller Leipzig

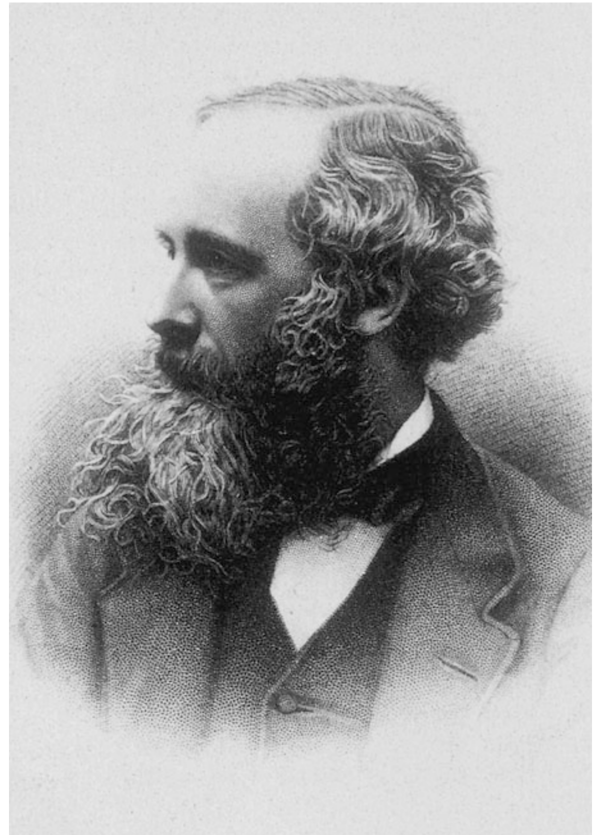

Figure 1.7 James Clerk Maxwell. With kind permission from the American Institute of Physics, Emilio Segre Visual hives, College Park MD

(1618–1663), *Christiaan Huygens* (1629–1695), *Thomas Young* (1773–1829) and *Augustin Fresnel* (1788–1827) decided the dispute in favour of the wave theory of light. *Melloni* showed 1834 that the laws for visible light could be extended into the infrared region and *Max Felix Laue* (1879–1960) and *William Bragg* (1862–1942) demonstrated the wave character of X-rays, which had been discovered by *Conrad Roentgen* (1845–1923), by their famous experiments on X-ray diffraction in crystals.

The velocity of light was first estimated by *Ole Rømer* (1644–1710) by astronomical observations of the appearance time of Jupiter moons and later more precisely determined by Huygens. With measurements on earth *Jean Foucault* (1819–1868) and *Armand Fizeau* (1819–1896) could obtain a rather accurate value for the velocity of light.

William Gilbert (1544–1603) was called "the father of electricity". He investigated the magnetic field of permanent magnets and measured the magnetic field of the earth with the help of magnetic needles. He made extensive experiments on friction electricity and divided the different materials into electrical and non-electrical substances. He built the first electroscop and measured the forces between charged particles. *Stephen Gray* (1670–1736) discovered the electrical conductivity of different materials and made detailed experiments on electric induction. He made electricity very popular by spectacular demonstrations.

Charles Augustin Coulomb (1736–1806) built the first electrometer, constructed the Coulomb torsion balance and formulated the famous Coulomb law for the forces between charged particles. *Benjamin Franklin* (1706–1790) recognized that lightening is not a fire but an electrical discharge and constructed the first lightning conductor. *Luigi Galvani* (1737–1798) discovered the stimulation of nerves by electrical currents (frog's leg experiments); and the contact voltage between different conductors, which lead to the construction of batteries (Galvanic element). *Allessandro Volta* (1745–1827) continued the experiments of Galvani and he categorized the different metals in an electrochemical series.

Hans *Christean Oersted* (1777–1851) discovered the magnetic field of an electric current. *Andre Marie Ampere* (1775–1836) coined the terms "***electrical current***" and ***electrical voltage***. By many detailed experiments, he established modern electrodynamics.

Michael Faraday (1791–1867) performed basic experiments on the relations between electric currents and magnetic fields (Faraday's induction law). He prepared the foundations for the development of alternating currents and their applications.

James Clerk Maxwell (1831–1879) (Fig. 1.7) summarized all known results of former experiments by a few basic equations (Maxwell's equations) and gave them a general mathematical formulation, which represents the basis for electrodynamics and optics. Their solutions are electro-magnetic waves, which found a brilliant confirmation by the experiments of *Heinrich Hertz* (1857–1894), who showed that these waves were transversal and propagate in space with the velocity of light.

1.3.3 Modern Physics

At the end of the 19th century, all problems in physics seemed to be solved and many physicists believed, that a closed theory describing all known facts could be realized in the near future.

This optimistic opinion changed, however, in a dramatic way, induced by the following experimental findings.

- The Michelson experiment (see Sect. 3.4) showed without doubt, that the velocity of light is constant, independent of the direction or the velocity of the observer. This result was in sharp contrast to former concepts and induced Albert Einstein (Fig. 1.8) to formulate his theory of special relativity (see Sect. 3.6).
- Experimentally found deviations from the theoretically expected spectral intensity distribution of the thermal radiation of hot bodies, as calculated by *Stephan Boltzmann* and *Wilhelm Wien*, could not be explained by classical physics. This discrepancy led *Max Planck* (1858–1947) (Fig. 1.9) to the conclusion of quantized energy of radiation fields. This bold assumption, which could perfectly reproduce the experimental results, represented the beginning of quantum theory that was later on imbedded in a concise mathematical framework by *Erwin Schrödinger* (1887–1961) and *Werner Heisenberg* (1901–1976) (see Vol. 3). The concept of energy quanta was further supported experimentally by the photoelectric effect, which was quantitatively explained by Einstein, who received the Nobel Prize for his theory of the photo-effect (not for his theory of relativity!).
- New experimental techniques allowed investigating the structure of atoms and molecules. The light emitted from atoms or molecules could be sent through a spectrograph and showed discrete lines, indicating that it has been emitted from discrete energy levels. Through the development of spectral analysis by *Gustav Robert Kirchhoff* (1824–1887) and *Robert Bunsen* (1811–1899) it was found that atoms of a specific element emitted spectral lines with wavelengths characteristic for this element. The results could not be explained by classical physics but needed quantum theory for their interpretation. Today the physics of atomic electron shells and their energy levels can be completely described by a closed theory called **quantum-electrodynamics**.

Figure 1.8 Albert Einstein. With kind permission of the "Deutsches Museum München"

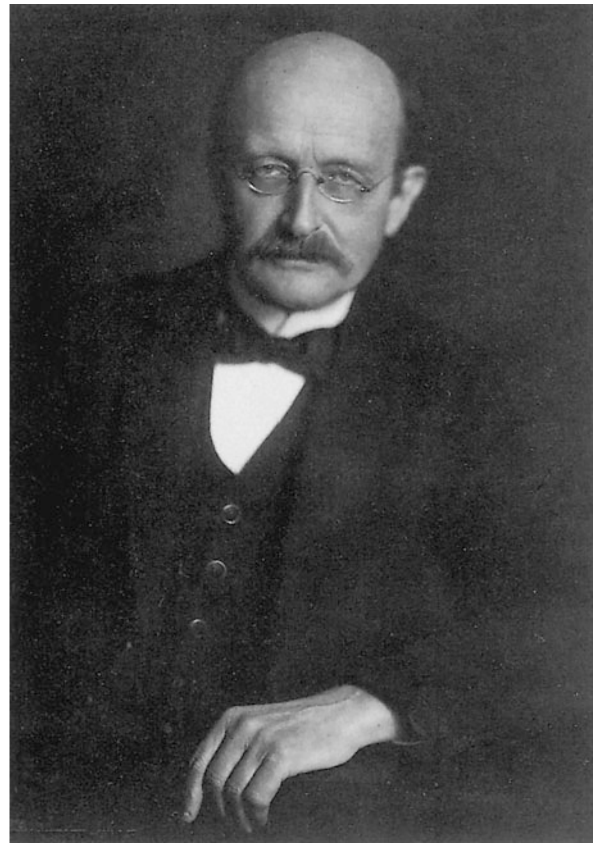

Figure 1.9 Max Planck. With kind permission of the "Deutsches Museum München"

This illustrates that always in the history of natural sciences new experimental results forced physicists to revise former concepts and to formulate new theories which, however, should include proved earlier results. In most cases the old theories were not completely abandoned but their validity range was restricted and more precisely characterized. For example the classical physics is perfectly correct for the description of the motion of macroscopic bodies or for many applications in daily life, while for the description of the micro-world of atoms and molecules it may completely fail and quantum theory is necessary.

The properties of atomic nuclei could be only investigated after appropriate detectors had been developed. Nuclear physics is therefore a rather new field where most of the results were obtained in the 20th century. The substructure of atomic nuclei and the physics of elementary particles could start after particle accelerators could be operated and many results in this field have been achieved only recently.

This short historical review should illustrate that many concepts which today are taken for granted, are not as old and have been accepted only after erroneous ideas and a long way of successive corrections, guided by new experiments. It is worthwhile for every physicist to look into some original papers and follow the gradual improvements of concepts and representation of results.

More extensive literature about the historical development of physics and about bibliographies of physicists can be found in the references [1.6–1.14c].

1.4 The Present Conception of Our World

As the result of all experimental and theoretical investigations our present model of the material world has been established (Fig. 1.10). In this introduction, we will give only a short summary. The subject will be discussed more thoroughly in Vol. 3 and 4 of this textbook series.

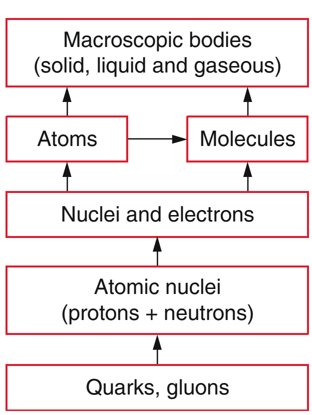

Figure 1.10 Build up of our material world (H. J. Jodl [1.14b])

Elementary Particles

The entire material world known up to now is composed of only a few different particles. The three most important are the electron (e^-), the proton (p^+) and the neutron (n). All other elementary particles (muons, π-Mesons, Kaons, Λ-particles etc.) exist after their production only a very short time (10^{-6}–10^{-15} s). They convert either spontaneously or by collisions into other particles which finally decay into p^+, e^-, neutrinos or photons $h\cdot\nu$. Although neutrinos are stable particles they show such a small interaction with matter that they are difficult to detect and they therefore play no role in daily life.

Recent experiments and theoretical consideration have shown, that the particles p^+, n, mesons and hyperons, which had been regarded as elementary, show a substructure (see Vol. 4), According to our present understanding they consists of smaller particles, called "quarks", which occur in 6 different species.

All building blocks of matter can be divided into two groups:

1. the quarks, which build up the heavy particles (*baryons*), such as proton, neutron, mesons and hyperons
2. the light particles (*leptons*) electron, myon and neutrino.

Each of these two groups consists of three families of elementary particles, which are listed in Tab. 1.1. For each of these particles there exists an anti-particle with equal mass but opposite charge. For instance the anti-particle of the electron e^- is the positron e^+, the proton p^+ has as anti-particle the anti-proton p^- and the anti-neutron has the same mass and the charge zero as the neutron.

According to present theories the interaction between the particles can be described by the exchange of "interaction particles", which are called the quanta of the interaction field. For example the quanta of the electromagnetic field, which determine the interaction between charged particles are the ***photons*** $h\cdot\nu$.

The quanta of the strong interaction between nucleons are called ***gluons***. The ***gravitons*** are the quanta of the gravitational field. Our present knowledge is that there exist only four different kinds of interaction, which are summarized in Tab. 1.2.

An essential goal of present research is to reduce the four types of interaction to one common force (grand unification). The reduction of the manifold of different particles to two groups of elementary particles was in a certain sense successful, because the classification into two groups with three families in each group gives a rather simple arrangement. However, the number of 24 different particles together with their antiparticles is still large and adding the 15 interaction quanta the total number of elementary particles is 39. Whether the "grand unification" will allow a further reduction or a simpler ordering scheme is still an open question.

This field of research is very interesting because it ventures to the limit where matter and energy might become indistinguishable. It is also closely related to processes occurring at the very beginning of our universe where elementary particles and their interaction played a major role in the extremely hot fireball during the first seconds of the big bang.

Table 1.1 The three families of Leptons and Quarks

Leptons				Quarks			
Name	Symbol	Mass MeV/c^2	Charge	Name	Symbol	Mass MeV/c^2	Charge
Electron	e^-	0.51	−1	Up	u	≈ 300	2/3
Electron neutrino	ν_e	$< 10^{-5}$	0	Down	d	≈ 306	−1/3
Myon	μ^-	−105.66	−1	Charm	c	≈ 1200	2/3
Myon neutrino	ν_μ	$< 10^{-4}$	0	Strange	s	≈ 450	−1/3
Tau-lepton	τ	1840	−1	Top	t	$1.7 \cdot 10^5$	2/3
Tau-neutrino	ν_τ	$< 10^{-4}$	0	Bottom	b	≈ 4300	−1/3

Table 1.2 The four types of interaction between particles (known up to now) and their field quanta. There are 8 gluons, 2 charged (W^+ and W^-) W bosons, 1 neutral boson (Z^0) and probably only 1 graviton with spin $I = 2$

Interaction	Field quantum	Rest mass MeV/c^2
Strong interaction	Gluons	0
El. magn. interaction	Photons	0
Weak interaction	W bosons	81,000
	Z bosons	91,010
Gravitational interaction	Gravitons	0

Atomic Nuclei

Protons and Neutrons can combine to larger systems, the atomic nuclei. The smallest nucleus is the proton as the nucleus of the hydrogen atom. The largest naturally existing nucleus is that of the uranium atom with 92 protons and 146 neutrons. Its diameter is about 10^{-14} m. Besides the nuclei found in nature there are many artificially produced nuclei, which are however, generally not stable but decay into other stable nuclei. Nearly every atom has many isotopes with nuclei differing in the number of neutrons. Meanwhile there is a wealth of information about the strong attractive forces, which keep the protons and neutrons together in spite of the repulse electrostatic force between the positively charged protons.

Atoms

Atomic nuclei together with electrons can form stable atoms, where for neutral atoms the number of electrons equals the number of protons. The smallest atom is the hydrogen atom, which consists of one proton and one electron. The diameter of atoms ranges from $5 \cdot 10^{-11}$ m to $5 \cdot 10^{-10}$ m and is about 10,000 times larger than that of the nuclei, although the mass of the nuclei is about 2000 times larger than that of the electrons. The electrons form a cloud of negative charge around the nucleus. The electro-magnetic interaction between electrons and protons has been investigated in detail and there is a closed theory, called **quantum electrodynamics**, which describes all observed phenomena of atomic physics very well.

The chemical properties of the different atoms are completely determined by the structure of the atomic electron shell. This is illustrated by the periodic system of the elements (Mendelejew 1869, Meyer 1870), where the elements are arranged in rows and columns and ordered according to the number of electrons of the atoms (see Vol. 3). With each new row a new electron shell starts. In each column the number of electrons in the outer shell (valence electrons) is equal and the chemical properties of the elements in the same column are similar. A real understanding of the periodic table could only be reached 60 years later after the quantum theory of atomic structure had been developed.

Molecules

Two or more atoms can combine to form a molecule, where the atoms are held together by electro-magnetic forces. The magnitude of the binding energy depends mainly on the electron density between the nuclei. Biological molecules such as proteins or DNA-molecules may consist of several thousand atoms and have diameters up to 0.1 µm, which is about 1000 times larger than the hydrogen atom. Molecules form the basis of all chemical and biological substances. The properties of these substances depend on the kind and structure of the molecules, such as the geometrical arrangement of the atoms forming the molecule.

Macroscopic Structures, Liquids and Solid Substances

Under appropriate conditions many equal or different atoms can form large macroscopic bodies which can contain a huge number of atoms. Depending on temperature they can exist in the solid or liquid phase. The interaction between the atoms is in principle known (el. magn. forces) but difficult to calculate because of the enormous number of participating atoms ($10^{22}/\text{cm}^3$). Most theoretical treatments therefore use statistical methods. Up to now many characteristics of macroscopic bodies can be calculated and understood from their atomic structure but a general exact theory of liquids and solids, which can explain also finer details, is still not available. Therefore approximations are used where each approximate model can describe special features quite well but others less satisfactorily. Examples are the band structure model, which can explain the electrical conductivity but not as well the elastic properties.

Structure and Dynamics of Our Universe

In our universe all of the constituents discussed so far are present.

- *Free elementary particles* (p^+, n, e^-, photons $h\nu$, also short lived mesons in the cosmic radiation, in the atmosphere of stars and in hot interstellar clouds, in the hot fireball during some minutes after the big bang, of our universe).

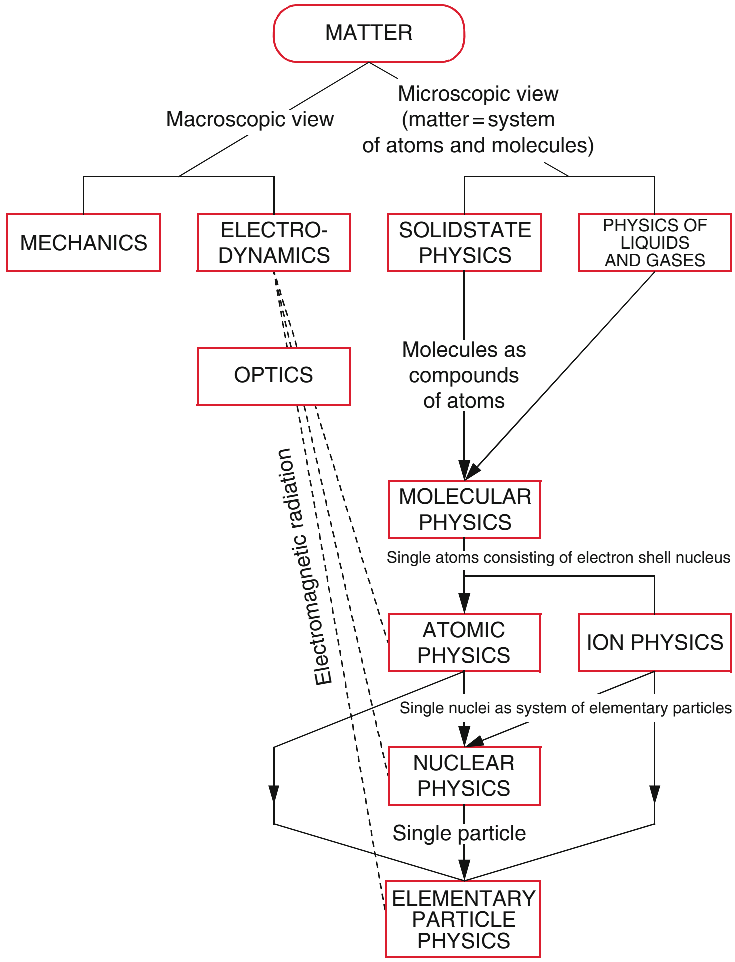

Figure 1.11 Family tree of physics (with kind permission of Dr. H. J. Jodl) [1.14b]

- *Atomic nuclei* in the inner part of stars, in neutron stars and in hot gas clouds.
- *Atoms* in atmospheres of planets and stars and in the interstellar medium.
- *Molecules* in molecular clouds, in comet tails, in interstellar space, in the atmospheres of cold stars and of planets.
- *Solid and liquid macroscopic bodies* (in planets and moons, in meteorites).

For the understanding of the origin and the development of our universe the interactions between these particles have to be known. Although in the early stage of the universe and later on in the interior of stars all four kinds of interaction played a role, gravitation is by far the most important force between celestial bodies such as stars, planets and moons.

Systematic Hierarchy of Physics

The systematic building up principle from small to larger entities discussed so far would suggest to start studies of physics with elementary particles and then proceed gradually to larger systems. However, since the theoretical treatment of elementary particles and nuclear physics is rather difficult, it is advisable from the didactical point of view to go the opposite way, We therefore start with classical physics of macroscopic bodies and proceed then to smaller structures like atoms, molecules, nuclei and elementary particles (see Fig. 1.11). The Physics courses therefore start with classical mechanics and thermodynamics (Vol. 1), continue with electrodynamics and optics (Vol. 2) and then with a basic knowledge of quantum mechanics treat the physics of atoms, molecules, solid and liquid states (Vol. 3) to arrive finally at nuclear physics, elementary particle physics and astrophysics (Vol. 4).

There exist a large number of good books on the subjects treated in this section [1.14b–1.19], which discuss in more detail the questions raised here. In order to gain a deeper understanding of how all this knowledge has been achieved, a more thorough study of basic physics, its fundamental laws and the experimental techniques, which test the developed theories, is necessary. The present textbook will help students with such studies.

Chapter 1

1.5 Relations Between Physics and Other Sciences

Since physics deals with the basic elements of our material world it represents in principle the foundations of every natural science. However, until a few decades ago the scientific methods in chemistry, biology and medicine were more empirically oriented. Because of the complex nature of the objects studied in these sciences it was not possible to start the investigations "ab initio" in order to understand the atomic structure of large complex molecules and biological cells to say nothing of the human body and its complex reactions as the research object in medicine. Therefore, in former years a more phenomenological method was preferred.

With refined experimental techniques developed in recent years (electron microscopy, (Fig. 1.12), tunnel microscopy, x-ray structural analysis, neutron diffraction, nuclear magnetic resonance tomography and laser spectroscopy) in many cases it became possible to uncover the atomic structure even of complex molecules such as the DNA (Fig. 1.13). Here physics was helpful in a twofold way: First of all physicists developed, often in cooperation with engineers, the experimental equipment and secondly it provided the theoretical understanding for the atomic basis of the research objects. Therefore the differences in the research methods become less and less important and the cooperation between researchers of different fields is rapidly increasing, indicated by the growing number of interdisciplinary research projects. For example the essential question of the relation between molecular structure and chemical binding is attacked in common efforts by experimental chemists, theoretical quantum chemists and physicists. Overstated one may say that chemistry is applied quantum theory and therefore a branch of physics.

Due to the complex diagnostic techniques in medicine the cooperation between physicists and medical doctors has enormously increased as will be outlined in the next section.

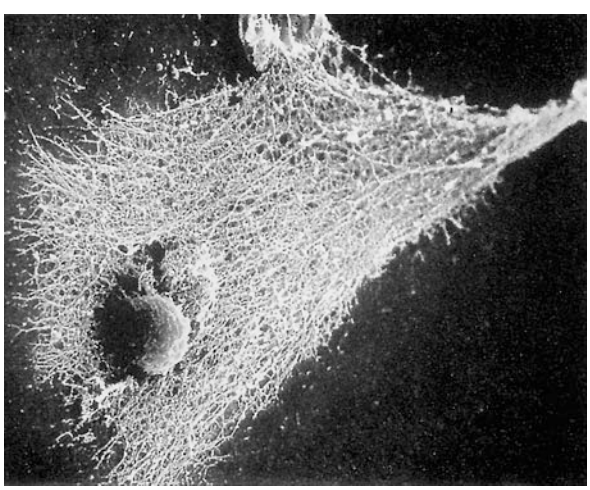

Figure 1.12 Scavenger Cells visualized with an electron microscope

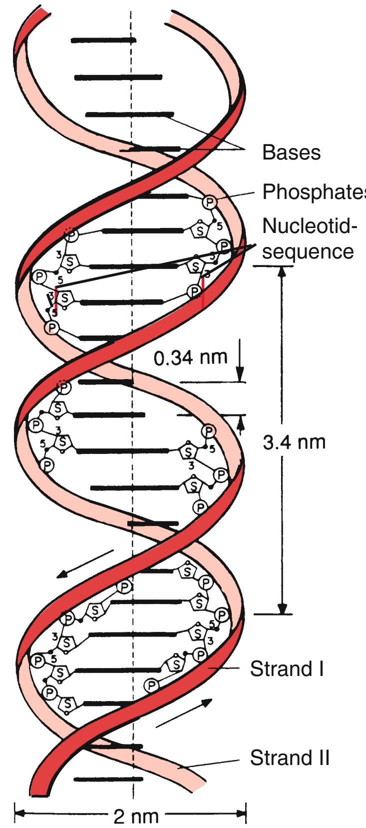

Figure 1.13 Double Helix of DNA (deoxyribonucleic acid)

1.5.1 Biophysics and Medical Physics

Meanwhile biophysics has developed to an independent branch of physics. Some of the many research projects are the physical processes in living cells, e. g. the energy balance during cell processes, the ion transport through cell membranes, the penetration of bacteria and viruses into cells, the different steps of photosynthesis or the visual process. The very sensitive detection techniques for the detection of single molecules, developed in physics laboratories, allow the tracing of single laser excited molecules on their way from outside a cell through membrane channels into the cell interior. In particular the realization of ultra short laser pulses down to below a femtosecond (10^{-15} s) opens for the first time the possibility to view ultrafast processes such as molecular isomerisation.

In recent years, medical physics has been established at many universities and research institutes. The development of new diagnostic techniques and therapy methods are based on experimental techniques invented and optimized in physics laboratories and on new insights about the interaction between radiation and tissue. Examples of such new methods are ultrasonic diagnostics with improved spatial resolution, nuclear magnetic resonance tomography, thermography or laser-induced cell fluorescence. One specific example is the localization of brain tumours by optical coherence tomography and methods for their operation with laser techniques, which are investigated in cooperation between laser physicists and neurosurgeons. [1.20a–1.23b]

1.5.2 Astrophysics

For ages the closest relation with physics had the astronomy, which tried to determine the positions of stars, the movement of planets and the prediction of eclipses. Modern astronomy goes far beyond this type of problems and looks for information about the composition of stars, conditions for their birth and the different stages of their development. It turns out that nearly all branches of physics are necessary in order to solve these problems. Therefore, this part of modern astronomy is called *astrophysics*. The cooperation with physicists who measure in the laboratory processes relevant for the understanding of star atmospheres and the energy production in the interior of stars has greatly improved our knowledge in astrophysics (see Vol. 4). One of the results is for example, that in the universe the same elements are present as can be found on earth and that the same physical laws are valid as known from experiments on earth. The correct interpretation of many astrophysical observations could only be given, because laboratory experiments had been performed which could give unambiguous decisions between several possible explanations of astrophysical phenomena.

The following facts have contributed essentially to the impressive progress in astronomy.

- The development of new large telescopes in the optical, near infrared and radio region, of satellites and space probes (Fig. 1.14) and sensitive detectors.
- New and deeper knowledge in the fields of atomic, nuclear and elementary particle physics, in plasma physics and magneto-hydrodynamics.
- Faster computers for the calculation of more complex models for the present composition, the birth, evolution and final stages of stars [1.24a–1.24c].

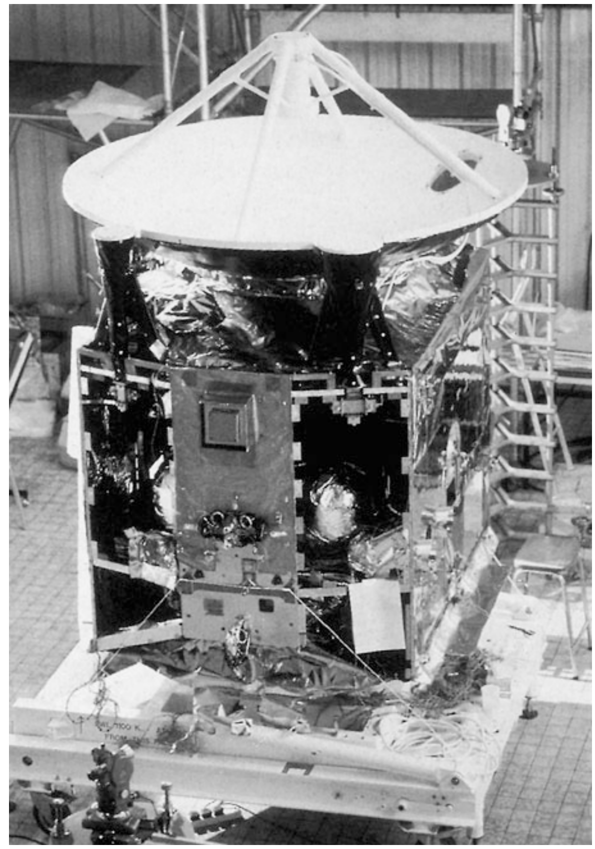

Figure 1.14 Last inspection of the Giotto-space probe before its journey to the comet Halley (with kind permission of the European Space Agency ESA)

1.5.3 Geophysics and Meteorology

Although geophysics and meteorology have developed into autonomous disciplines, they are completely based on fundamental physical laws. In particular, in meteorology it is evident how important fundamental physical processes are, such as the interaction of light with atoms and molecules, collisions between electrons, ions, atoms and molecules or light scattering by aerosols and dust particles. Without the detailed understanding of these and other processes the complex preconditions for the local and global climate could not be calculated within a climate model. However, it turns out, that in spite of the knowledge of these basic processes it is often not possible to give a reliable long term weather forecast, because already tiny changes of the present status of the atmosphere could result in huge changes of its future development. The system shows a chaotic behaviour. This astonishing feature has lead to a new branch of physical and mathematical sciences, called chaos research (see Chap. 12). [1.25–1.30b]

1.5.4 Physics and Technology

The application of physical research has pushed the development of our industrial society in a way, which can hardly be overestimated. Examples are the inventions of the steam engine, the electromotor, research on semiconductors, which form the basis of computers, information technology, such as the telephone and extremely fast optical communication over glass fibres, Lasers and their various applications, precision measuring techniques down into the nanometre range. This connection between applied physics and technology has received new impetus through the urgent problems of energy crisis, lack of raw materials, global warming, which have to be solved within a limited time. Urgent problems are, for example

- the development of new energy sources, such as nuclear fusion, which demands a profound knowledge of plasma physics under extreme conditions,

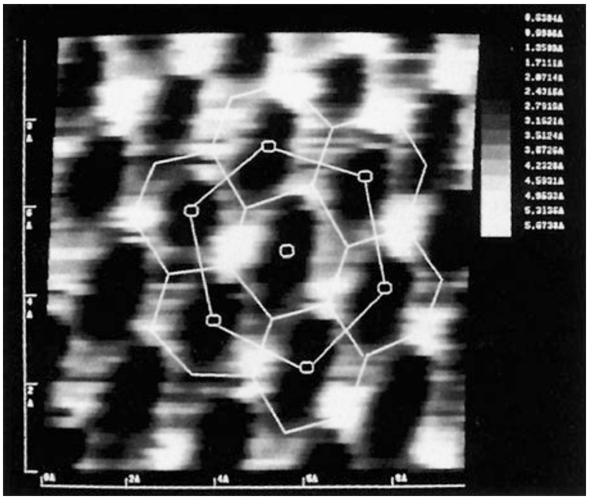

Figure 1.15 Hexagonal structure of a graphite surface, visualized by a tunnel microscope (M. Müller, H. Öchsner, TU Kaiserslautern)

- the optimization of wind converters,
- the development of solar cells with sufficiently high efficiencies,
- increasing the conversion efficiency from heat into electrical energy,
- improving the transport efficiency of energy.

Further examples are the development of reliable electrically driven cars with new designs of batteries, hydrogen technology, magnetically levitated railways (trans-rapid), development of "-clean air cars" etc.

Of particular interest for many branches of industry is the research on new materials such as *met-glasses* (amorphous metals with particular properties such as high tensile strength), compound materials or amorphous semiconductors, which have found meanwhile numerous applications. Surface science (Fig. 1.15) has given the basic understanding for corrosion processes, catalytic effects and the properties of thin films in optics and for the creation of very hard surfaces of tools, which decrease the wear and tear of such tools considerably.

One should keep in mind that for densely populated countries such as Germany, which do not have sufficient raw material at their disposal, technological innovations and inventions of new products as well as progress in environmental protection are essential for a better and safe life in the future. Here physicists encounter great challenges and new ideas and a critical but pragmatic way of thinking are demanded, characteristics, which are trained during the physics education. [1.30a–1.30b]

1.5.5 Physics and Philosophy

Since its beginning in the Greek period, physics always had a close relation to philosophy (see Sect. 1.3). Already for the Greek philosophers recognition in natural sciences gave new directions to the philosophical way of thinking. The essential goal of modern physics is the understanding and the detailed description of our world and the reduction of many observations to a few general laws. The essential point is, that the human consciousness and the attitude against the human surroundings are changed by this new knowledge. The fascinating question, how cognitive faculty is received by communication with other thinking persons and whether the structured mind which allows to process this information to form a unique world view, had been already formed prenatal had been extensively discussed by the great philosopher *Immanuel Kant* (1724–1804) in his famous book "*Kritik der reinen Vernunft*".

Nowadays biophysicists and neurologists try to understand by well aimed experiments the connection between specified parts of the brain and the storage of information which we receive from outside. All these progress in natural sciences has influenced philosophical theories. Although the approach to this subject is often different for philosophers and scientists, an intense discussion between the representatives of the two camps could remove many misunderstandings and could lead to a more extensive view of our world. If such discussions should be fruitful, both sides have to learn more about the way of thinking and arguing of the other side. The study of physics and its way of arguing can shape the way we are looking onto our world and represents an essential part of our culture.

An important aspect of such cooperation is the critical evaluation of ethical questions related to scientific research, which have found more and more concern in our society. Since the developments in physics and their applications, essentially change our daily life, physicists have to think about the consequences of their scientific results. The research itself is unbiased and value-free. Ethical problems arise when the results of basic research are applied in such a way, that society might be damaged by such applications. For instance, the discovery of nuclear fission by Otto Hahn could be used for peaceful applications as well as to build an atomic bomb; lasers can be used for health treatment in medicine or as laser weapons.

People who demand social relevance for every research projects forget that this is a question of possible applications, which can often not be predicted from basic research. There are many examples where basic research was done without any ideas of possible benefit for the public, such as the beginning of solid state physics, low temperature physics, semiconductor research. [1.31–1.35]

1.6 The Basic Units in Physics, Their Standards and Measuring Techniques

Since any objective description of nature demands quantitative relations between measurements of different objects, which can be expressed by numbers, one has to define units for the results of measurements. This means that every numerical result of a measurement must be expressed in multiples of such units. One needs a scale that can be compared with the measured quantity.

To measure always means to compare two quantities!

There are several possibilities for choosing units. For the length unit for instance one may use units which are given by nature such as foot or the distance between two atoms in a crystal; for the time unit the time interval between two successive heart beats, or the time between two culminations of the sun. A better choice of physical units is to use arbitrary but suitable units, which are conveniently adapted to daily life. Such units have to be defined by standards with which they can be always compared (calibration).

Every standard has to meet the following demands:

- It must be possible to compare with sufficient accuracy the quantity in question with the standard.
- The standard must be reproducible with the demanded accuracy.
- The production and the safekeeping of the standard and the comparison with measurable elements must be possible with justifiable expenditure.

According to these demands ulna, foot or heartbeat period are not good standards, because they are dependent on the person who measures them. They may change with time and are not general constants.

The **quality** of a measurement is judged according to the following aspects:

- How reliable is the measurement?
  Here the experimental apparatus plays an important role, the interpretation of the experimental results by the observer; his ability and experience (see for instance temperature estimations guided by our senses (Chap. 10 or "optical illusions" Vol. 2)).
- How accurate is the measurement, i.e. how large is the maximum possible error of the result?
- Are measurements performed under different experimental conditions reproducible?

Of course, each physical quantity cannot be measured more accurately than the accuracy of the normal's measurement. Therefore such a normal should be chosen which is so accurately defined that it does not represent a limitation for the accuracy of the measurement. For many measurements, a stopwatch or a micrometre-screw might not be accurately enough and should not be used as normal.

The question is now how many basic units are necessary to describe all physical quantities. Since all physical processes go off in space and time one certainly needs basic units for length and time. We will see that all physical quantities can be derived from three basic units for length, time and mass. One would therefore need in principle only these three basic units. It turns out, however, that it is useful to add four more basic units for the temperature, the mole fraction of material, for the strength of an electric current and the luminous intensity of radiation sources, because many derived units can be simpler expressed when these four additional units are included [1.37–1.39].

In the following we will discuss the different basic units and also give a short outline of the historical development of this units and their increasing accuracy. This shall illustrate how new measurement techniques have improved the quality of a measurement and asked for new and better standards that could meet the demands for higher accuracy and reproducibility.

1.6.1 Length Units

As length unit the metre (m) was chosen in 1875 which was originally meant as the 1/10,000,000 fraction of the equator quadrant (¼ of the earth circumference). The prototype as the primary standard was kept in Paris. In order to maintain this normal as reproducible as possible, it was realized by the distance between two markers on a platinum-iridium rod with a low thermal expansion coefficient. The rod was kept in a box at 0 °C. More precise later measurements of the earth circumference showed that the metre deviated from the original definition by about 0.02%. The comparison of length standards with this prototype was only possible with a relative uncertainty of 10^{-6}. This means that it is only possible to detect a deviation of larger than 1/1000 mm. This does not meet modern requirements of accuracy.

Therefore in 1960 a new length standard was defined by the wavelength λ of the orange fluorescence line of a discharge lamp filled with the krypton isotope 86 (Fig. 1.16), where the conditions in the krypton lamp (pressure, discharge current and temperature) were fixed. The metre was defined as $1{,}650{,}763.73 \cdot \lambda$. The wavelength λ can be measured with an uncertainty of 10^{-8}, which is 100 times more accurate than the comparison with the original metre standard in Paris.

With increasing accuracy of measurements this standard was again abandoned and a new standard was chosen, which was based on a completely new definition. Since time can be measured much more accurate than length, the length standard was

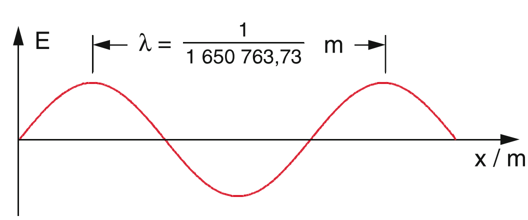

Figure 1.16 The old definition of the length unit, based on the wavelength of a Krypton line (valid from 1960–1983)

Table 1.3 Range of actual lengths in our world

Object	Dimension/m
Radius of the electron	$\leq 10^{-18}$
Radius of the proton	10^{-15}
Distance between atoms in solids	10^{-10}
Thickness of the skin of a soap bubble	10^{-7}
Mean distance between air molecules at 10^5 Pa	10^{-6}
Radius of the earth	$6 \cdot 10^6$
Distance earth–moon	$4 \cdot 10^8$
Distance earth–sun	$1.5 \cdot 10^{11}$
Diameter of the solar system	10^{14}
Distance to the nearest star	$4 \cdot 10^{16}$
Diameter of our galaxy	$3 \cdot 10^{20}$
Extension of the universe	$3 \cdot 10^{25}$

related to time measurements via the velocity c of light. The weighted average of the most precise measurement of the speed of light in vacuum is now defined as

$$c = 299{,}792{,}458\,\text{m/s}\ .$$

This means that the speed of light is no longer a result of new measurements but ***is defined*** as a fixed value.

Definition

The length unit 1 m is now fixed by the following definition:

One metre is the length of the path that is travelled by light in vacuum during the time interval $1/299{,}792{,}485$ s.

From the relation $c = \nu \cdot \lambda$ between speed of light c, frequency ν and wavelength λ of an electro-magnetic wave the wavelength λ of any spectral line can now be determined from the frequency ν (which can be measured with a much higher accuracy than wavelengths) and the defined speed of light (see Sect. 1.6.2 and 1.6.4).

The order of magnitude of length-scales in physics covers the enormous range from 10^{-18} m for the size of elementary particles to 10^{+25} m for the radius of the present universe (Tab. 1.3). It is therefore appropriate to give metre scales in powers of ten. For specific powers a shorthand notation is used, e. g. 10^{-6} m = 1 micrometer (μm); 10^3 m = 1 kilometer (km). These shorthand notations are listed in Tab. 1.4.

In astronomy, the distances are very large. Therefore, appropriate units are used. The astronomical unit AU is the mean distance between earth and sun. The new and more exact definition, adopted 1976 by the International Astronomical Union is the following:

Table 1.4 Labels for different orders of magnitude of length units

1 attometer	= 1 am	$= 10^{-18}$ m
1 femtometer	= 1 fm	$= 10^{-15}$ m
1 picometer	= 1 pm	$= 10^{-12}$ m
1 nanometer	= 1 nm	$= 10^{-9}$ m
1 micrometer	= 1 μm	$= 10^{-6}$ m
1 millimeter	= 1 mm	$= 10^{-3}$ m
1 centimeter	= 1 cm	$= 10^{-2}$ m
1 dezimeter	= 1 dm	$= 10^{-1}$ m
1 kilometer	= 1 km	$= 10^{3}$ m
Often used units in		
– atomic and nuclear physics		
1 fermi = 1 femtometer		$= 10^{-15}$ m
1 X-unit	= 1 XU	$= 1.00202 \cdot 10^{-13}$ m
1 Ångström	= 1 Å	$= 10^{-10}$ m
– astronomy:		
1 astronomical unit		= 1 AU
≈ mean distance earth–sun		$\approx 1.496 \cdot 10^{11}$ m
1 light year	= 1 ly	$= 9.5 \cdot 10^{15}$ m
1 parsec	= 1 pc	$= 3 \cdot 10^{16}$ m $= 3.2$ ly

Definition

1 AU is the distance to the centre of the sun, which a hypothetical body with negligible mass would have, if it moves on a circle around the sun in 365.256 8983 days.

One light-year (1 ly) is the distance, which light travels in 1 year. An object has a distance of one parsec (1 pc) if the astronomical unit seen from this object appears under an angle of one second of arc (1″) (Fig. 1.17). The distance d of a star, where this angle is α is $d = 1\,\text{AU}/\tan\alpha$. With $\tan 1'' = 4.85 \cdot 10^{-6}$ we obtain

$$1\,\text{pc} = 2.06 \cdot 10^5\,\text{AU} = 3.2\,\text{ly}\ .$$

Note: In some countries other non-metric length units are in use: 1 inch = 2.54 cm = 0.0245 m and 1 yard (1 yd) = 0.9144 m, 1 mile (1 mi) = 1609.344 m.

However, **in this textbook only SI units are used**.

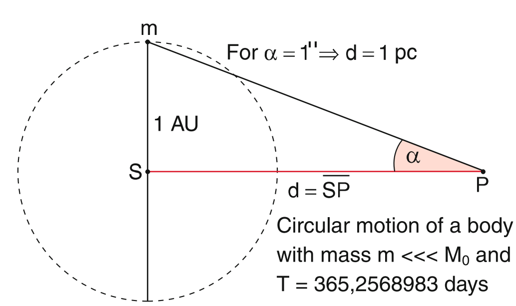

Figure 1.17 Definition of the astronomical units 1 AU and 1 pc

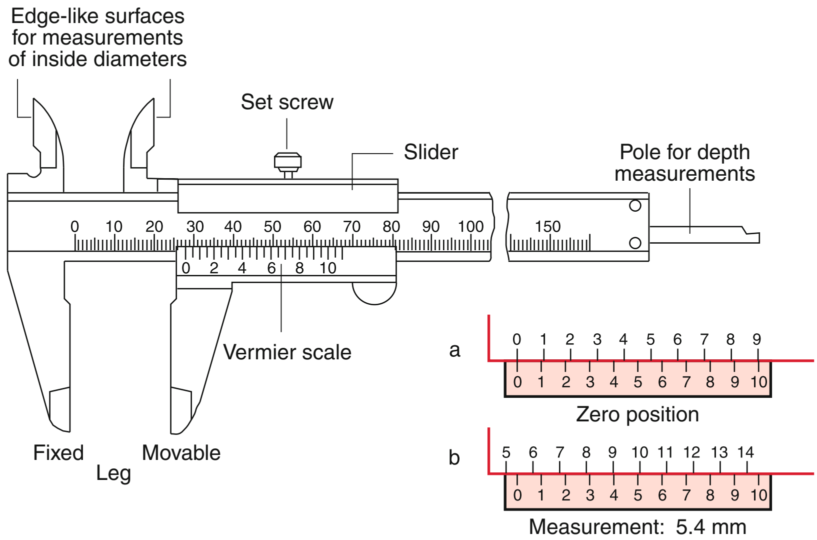

Figure 1.18 Caliper gauge with vernier scale

1.6.2 Measuring Techniques for Lengths

For measuring of lengths in daily life secondary standards are used which are not as accurate as the primary standards but are more readily usable. The accuracy of such standards is adapted to the application for which they are constructed. One simple example is the sliding vernier (Fig. 1.18). Its accuracy is based on the nonius principle. The upper scale is divided into millimetres, the lower scale has 10 scale divisions for 9 mm, which means that every division is 9/10 mm. For the situation in Fig. 1.18b the division mark 9 mm on the upper scale coincides with the division mark 4 on the lower scale. The distance D between the two fold limbs is then

$$D = (9 - 4 \cdot 9/10)\,\text{mm} = 5.4\,\text{mm}\,.$$

The uncertainty of the measurement is about 0.1 mm.

Higher accuracies can be reached with a micrometer screw (Fig. 1.19) where a full turn of the micrometer drum corresponds to a translation of 1 mm. If the scale on the drum is divided into 100 divisions each division mark corresponds to 0.01 mm. The shackle is thermally isolated in order to minimize thermal expansion, With differential micrometer screws, which have two coaxial drums turning into opposite directions, where one drum produces a translation of 1 mm per turn, the other of -0.9 mm in the backward direction, one full turn corresponds now to 0.1 mm. This allows an accuracy of $0.001\,\text{mm} = 1\,\mu\text{m}$. This is about the accuracy limit of mechanical devices.

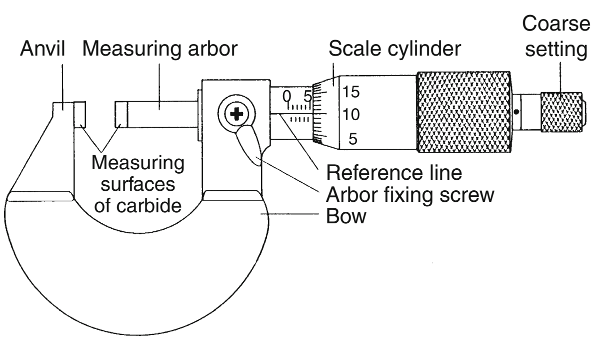

Figure 1.19 Micrometer caliper

More accurate length measurements are based on optical techniques. For distances below 1 m interferometric methods are preferable (see Vol. 2) where lasers (see Vol. 3) are used as light sources. Here distances are compared to the wavelength of the light source. Modern interferometers reach accuracies of $\lambda/100$. With a wavelength of $\lambda = 500\,\text{nm}$ an accuracy of $5\,\text{nm} = 5 \cdot 10^{-9}$ m can be achieved.

Larger distances can be measured via the travel time of a light pulse. For instance the distance of the retro-reflector which the astronauts have positioned on the moon, can be measured within a few cm using laser pulses with 10^{-12} s pulse width (LIDAR technique see Fig. 1.20). Measuring this distance from different locations on earth at different times even allows to detect continental drifts of the earth crust plates [1.41–1.42].

For the exact location of planes, ships or land vehicles the global positioning system GPS has been developed. Its principle is illustrated by Fig. 1.21.

The navigator, who wants to determine his position, measures simultaneously the phases of radio signals emitted from at least four different satellites. The radio signals on frequencies at 1575 MHz and 1227 MHz are modulated. This allows to determine unambiguously the distances d_i from the receiver to the satellites S_i from the measured phase differences ϕ_i. From these four distances d_i the position (x, y, z) of the receiver can be determined with an uncertainty of only a few cm if relativistic effects (see Sect. 3.6) are taken into account! In order to achieve this accuracy, the frequencies of the radio signals must be kept stable within 10^{-10}. This can be realized with atomic clocks which

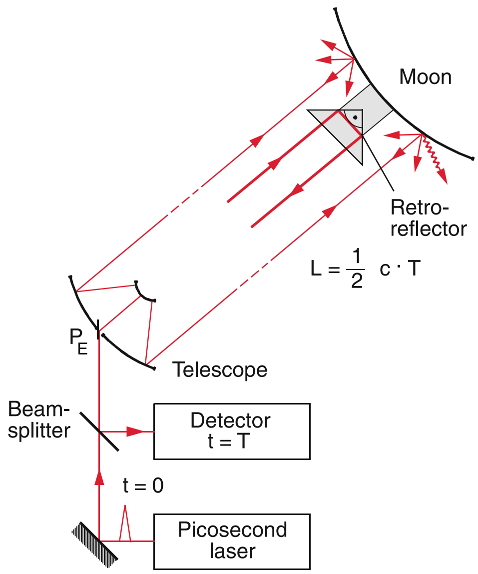

Figure 1.20 Measurement of the distance Earth–Moon with the LIDAR-technique

reach a relative stability $\Delta\nu/\nu = 10^{-14}$. The exact position of the satellites is fixed by radio signals from several stations and receivers at selected precisely known locations on earth. The European Space Agency has launched several satellites for the realization of a new GPS System called Galileo with predicted higher accuracy.

Also a more precise value of the astronomical unit 1 AU can be obtained by measuring the travel time of short light pulses. A radar pulse is sent from the earth to Venus where it is reflected. The time delay between sending and receiving time is measured for a time of closest approach of Venus to Earth. which gives a precise value of the distance between Earth and Venus. From the angle between the radii Earth–Sun and Earth–Venus at the time of the measurement the distance Earth–Sun can be obtained by trigonometric relation in the triangle Earth–Venus–Sun and using Kepler's 3rd law (see Sect. 2.9).

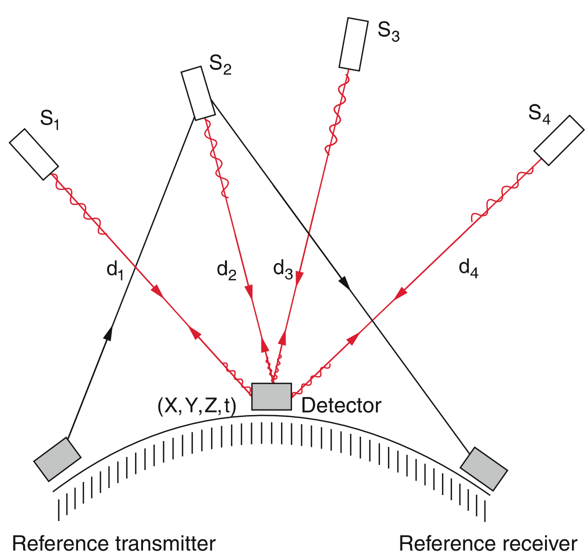

Figure 1.21 Principle of the Global Positioning System GPS

As the result of many different measurements, which became more and more accurate, the Astronomical Union has recommended in 2012 to take the average of these measurements as the **definition** of the Astronomical Unit:

$$1\,\text{AU} \stackrel{\text{def}}{=} 149{,}597{,}870{,}700\,\text{m}\,.$$

1.6.3 Time-Units

The unit of time is the second (1 s). Its initial definition was

$$1\,\text{s} = 1/(60 \cdot 60 \cdot 24)\,\text{d} = (1/86{,}400)\ \text{of a solar day}\,,$$

where a ***solar day*** is defined as the time between two lower culminations of the sun i.e. between two successive midnights.

When the earth rotates around its axis with the angular velocity ω one sun day is $d = (2\pi + \alpha)/\omega$, where the additional angle α is due to the revolution of the earth around the sun. On the other hand a **sidereal day** (= time between two culminations of a star) is $d = 2\pi/\omega$ and therefore shorter by 1/365 d (Fig. 1.22a). 365.25 solar days correspond to 366.25 sidereal days.

Later it was found that the period of a solar day showed periodic and erratic changes, which can amount up to 30 s per day. (Fig. 1.22b) These changes are caused by the following effects:

- A yearly period due to the non-uniform movement of the earth on an ellipse around the sun (Fig. 1.23 and Sect. 2.9). The velocity v_2 around the perihelion (minimum distance between earth and sun) is larger than v_1 around the aphelion (maximum distance). Since the revolution of the earth around the sun and the rotation of the earth around its axis have the same rotation sense, a solar day is longer around the perihelion than around the aphelion.
- A half-year period due to the inclination of the earth axis against the ecliptic (the plane of the earth's movement around the sun), which causes a variation of the sun culmination at a point P on earth (Fig. 1.24).

In order to eliminate the effect of such changes on the definition of the second, a fictive "mean sun" is defined which (seen by an observer on earth) moves with uniform velocity (= yearly average) along the earth equator. The time between two successive culmination points of this fictive sun defines the mean solar day $\langle d\rangle$. This gives the definition of the **mean solar second**

$$1\,\text{s} = (1/86{,}400)\langle d\rangle\,.$$

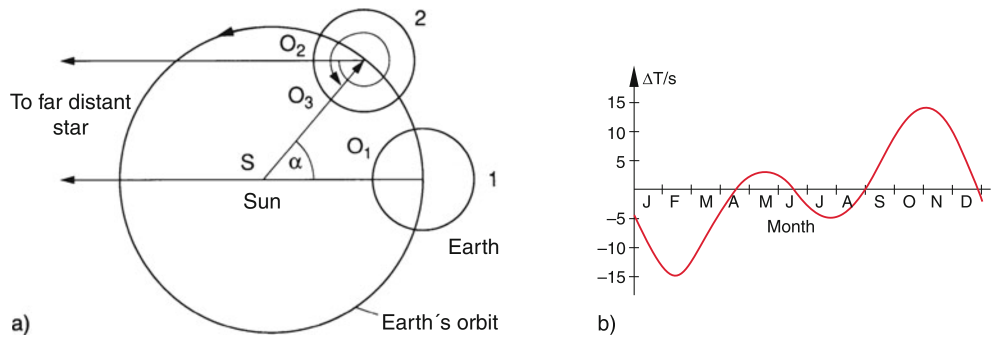

Figure 1.22 **a** Difference between solar day and sidereal day, **b** Difference between the true and the mean solar time

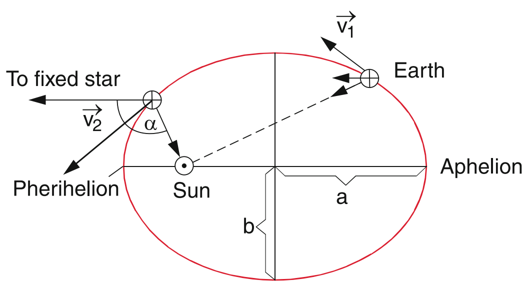

Figure 1.23 Changing velocity of the earth during one revolution on its elliptical path around the sun

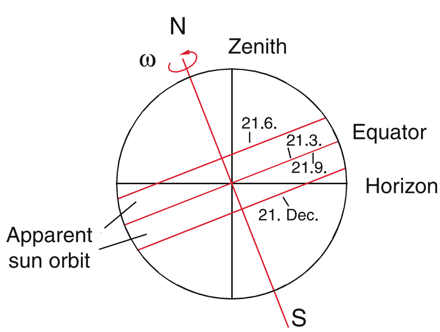

Figure 1.24 Variation of the point of culmination of the sun with a half-year period, due to the inclination of the earth axis

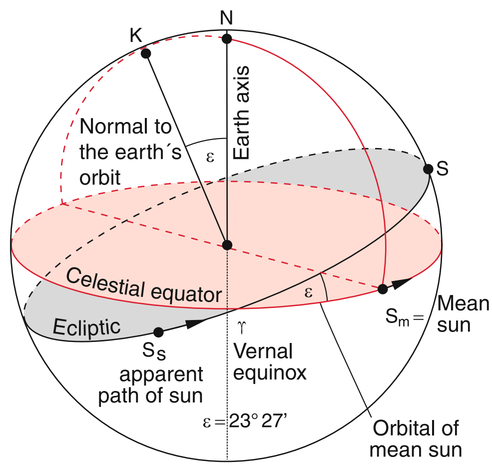

Figure 1.25 Definition of the tropical year

With the development of modern precise quartz clocks it was found that even this mean solar day showed periodic and irregular variations due to changes of the earth's moment of inertia caused by melting of glaciers at the poles, falling of leaves in autumn, volcano eruptions, earth quakes, and turbulent movements of material in the liquid part of the earth's interior. The deviations from the mean sun day amount up to 10 milliseconds per day and cause a relative deviation of $10^{-2}/85{,}400 \approx 10^{-7}$ per day. Therefore the astronomers no longer use the earth rotation as a clock but rather the time span of the **tropical year**. This is the revolution period of the earth around the sun between two successive spring equinoxes, which are the intersection point of the ecliptic and the equator plane vertical to the earth's axis (Fig. 1.25). This tropical year equals the annual period of the mean sun on its way along the earth's equator.

Since even the tropical year suffers in the course of time small variations, the astronomers introduced 1960 the **ephemeris time**, based on tables which give the calculated positions of sun, moon and planets at a given time [1.24d].

The astronomical definition of the second is now 1 s = period of the tropical year 1900 divided by 31,556,925.9747.

For daily use, quartz clocks are more convenient and therefore more useful secondary time standards. Their essential part is a quartz rod of definitive length, which is excited by an external electric high frequency field to length oscillations (see Vol. 2). If the exciting frequency is tuned to the resonance frequency of the quartz rod, the oscillation amplitude reaches a maximum. By appropriate feedback the system becomes a stable self sustaining oscillator which does not need an external frequency source. The relative frequency deviation of good quartz clocks are $\Delta\nu/\nu \le 10^{-9}$. The second is then counted by the number of oscillation periods per time. Of course, the quartz clocks need a calibration with primary time standards.

The subdivisions of the second and longer time periods are listed in Tab. 1.5.

A better time standard which is still valid up to now is the **caesium atomic clock**. Its principle is illustrated in Fig. 1.26.

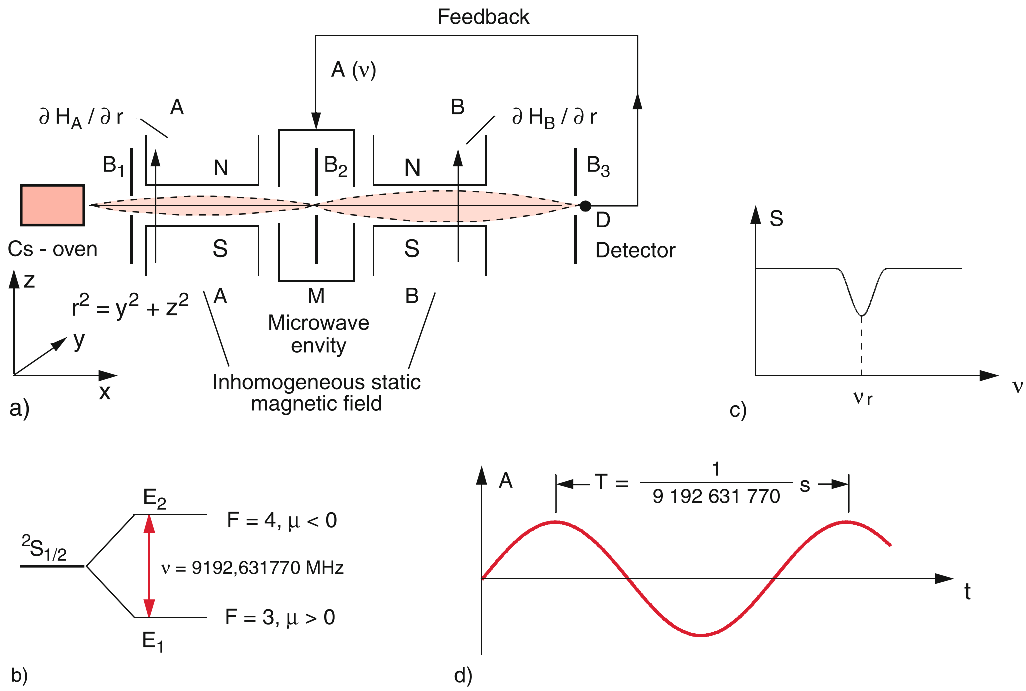

Figure 1.26 Caesium atomic clock. **a** Experimental arrangement; **b** level scheme of the hyperfine-transition; **c** detector signal as a function of the microwave frequency; **d** Definition of the second as a multiple of the oscillation period T

Table 1.5 Labelling of subdivisions of the second or of longer time intervalls

Subdivisions of second		
1 millisecond	= 1 ms	$= 10^{-3}$ s
1 mikrosecond	= 1 μs	$= 10^{-6}$ s
1 nanosecond	= 1 ns	$= 10^{-9}$ s
1 picosecond	= 1 ps	$= 10^{-12}$ s
1 femtosecond	= 1 fs	$= 10^{-15}$ s
1 attosecond	= 1 as	$= 10^{-18}$ s
Larger time units		
1 hour	= 1 h	$= 3.6 \cdot 10^3$ s
1 day	= 1 d	$= 8.64 \cdot 10^4$ s
1 year	= 1 a	$= 3.15 \cdot 10^7$ s

Cs-atoms evaporate through a hole in an oven into a vacuum tank. Several apertures collimate the evaporating atoms and form a collimated atomic beam which passes through a microwave resonator M placed between two six pole magnets A and B. They act on atoms with a magnetic moment like an optical lens and focus the atomic beam onto the detector D where the focusing characteristics depend on the hyperfine structure level of the atoms. If the resonator is excited on the frequency $\nu = (E_2 - E_1)/h$ which corresponds to the transition between the two hyperfine levels $F = 3 \rightarrow F = 4$ in the $S_{1/2}$ electronic ground state of Cs (Fig. 1.26b) (see Vol. 3), the atoms can absorb the microwave radiation and are transferred from the $F = 3$ level into the $F = 4$ level. In this level they have a different magnetic moment and are therefore defocused in the magnetic field B. They cannot reach the detector D and the measured signal decreases (Fig. 1.26c). When the microwave frequency ν is tuned over the resonance at $\nu = 9{,}192{,}631{,}770\,\mathrm{s}^{-1}$ a dip in the signal S(ν) appears which is transferred by a feedback circuit to the microwave generator and keeps its frequency exactly on resonance. The frequency stability of the microwave generator is now determined by the atomic transition frequency and serves as a very stable clock, called **atomic clock**. The achieved frequency stability of modern versions of the Cs-clock is $\Delta\nu/\nu = 10^{-15}$.

> The new definition of the second, which is still valid today, is: 1 s is the time interval of 9,192,631,770.0 oscillation periods of the Cs clock.

Table 1.6 gives a survey about the time scales of some natural phenomena, which extend from 10^{-23} to 10^{+18} s.

The new definition of the second shows that the time measurement is put down to frequency measurements. The frequency of any oscillating system is the number of oscillation periods per second. Its metric unit is [$1\,\mathrm{s}^{-1}$] or [1 hertz = 1 Hz]. Larger units are

- 1 kilohertz = 1 kHz = $10^3\,\mathrm{s}^{-1}$,
- 1 Megahertz = 1 MHz = $10^6\,\mathrm{s}^{-1}$,
- 1 Gigahertz = 1 GHz = $10^9\,\mathrm{s}^{-1}$,
- 1 Terahertz = 1 THz = $10^{12}\,\mathrm{s}^{-1}$.

Smaller units are

- 1 Millihertz = 1 mHz = $10^{-3}\,\mathrm{s}^{-1}$,
- 1 Microhertz = 1 μHz = $10^{-6}\,\mathrm{s}^{-1}$.

Table 1.6 Time scales occuring in natural phenomena

Natural phenomenon	Period/s
Transit time of light over the diameter of an atomic nucleus	10^{-23}
Revolution period of electron in the hydrogen atom	10^{-15}
Transit time of electrons in old tv-tubes	10^{-7}
Oszillation period of tuning fork	$2.5 \cdot 10^{-3}$
Time for light propagation sun–earth	$5 \cdot 10^{2}$
1 day	$8.64 \cdot 10^{4}$
1 year	$3.15 \cdot 10^{7}$
Time since the first appearance of homo sapiens	$2 \cdot 10^{13}$
Rotational period of our galaxy	10^{16}
Age of our earth	$1.6 \cdot 10^{17}$
Age of universe	$5 \cdot 10^{17}$

1.6.4 How to measure Times

For the measurement of times periodic processes are used with periods as stable as possible. The number of periods between two events gives the time interval between these events if the time of the period is known. Devices that measure times are called **clocks**.

Quartz Clocks: Modern precision clocks are quartz clocks with a frequency instability $\Delta\nu/\nu \leq 10^{-9}$. This means that they deviate per day from the exact time by less than 10^{-4} s.

Atomic Clocks: For higher accuracy demands atomic clocks are used, which are available as portable clocks (Rubidium clocks with $\Delta\nu/\nu \leq 10^{-11}$) or as a larger apparatus fixed in the lab e. g. the Cs clock with $\Delta\nu/\nu \leq 10^{-15}$.

As world-standard Cs-clocks are used at several locations (National Institute of Standards and Technology NIST in Boulder, Colorado, Physikalisch-Technische Bundesanstalt PTB in Braunschweig, Germany and the National Physics Laboratory in Teddington, England) which are connected and synchronized by radio signals. Two of such clocks differ in 1000 years by less than 1 millisecond [1.44a–1.44b].

Frequency stabilized Lasers: A helium-Neon laser with a frequency of 10^{14} Hz can be locked to a vibrational transition of the CH_4 molecule and reaches a stability of 0.1 Hz, which means a relative stability $\Delta\nu/\nu \leq 10^{-15}$ comparable to the best atomic clocks [1.45]. With the recently developed optical frequency comb (see Vol. 3) stabilities $\Delta\nu/\nu \leq 10^{-16}$ could be achieved [1.46]. It is therefore expected, that the Cs-standard will soon be replaced by stabilized lasers as frequency and time standards.

The time resolution of the human eye is about 1/20 s. For the time resolution of faster periodic events stroboscopes can be used. These are pulsed light sources with a tuneable repetition frequency. If the periodic events are illuminated by the light source, a steady picture is seen, as soon as the repetition frequency equals the event frequency. If the two frequencies differ the appearance of the event is changing in time the faster the more the two frequencies differ.

Periodic and non-periodic fast events can be observed with high speed cameras, which reach a time resolution down to 10^{-8} s; with special streak cameras even 10^{-12} s can be achieved. Faster events, such as the rearrangement of the atomic electron shell after excitation with fast light pulses or the dissociation of molecules which occur within femtoseconds ($1\,\text{fs} = 10^{-15}$ s) can be time- resolved with special correlation techniques using ultrafast laser pulses with durations down to 10^{-16} s.

1.6.5 Mass Units and Their Measurement

As the third basic unit the mass unit is chosen. The mass of a body has always a fixed value, even if its form and size is altered or when the aggregation state (solid, liquid or gaseous) changes as long as no material is lost during the changes. The mass is the cause of the gravitational force and for the inertia of a body, which means that all bodies on earth have a weight and if they are moving, magnitude and direction of their velocity is not changing as long as no external force acts on the body (see Sect. 2.6).

As mass unit the kilogram is defined as the mass of a platinum-iridium cylinder, which is kept as the primary mass standard in Paris. (Fig. 1.27)

Initially the kilogram should have been the mass of a cubic decimetre of water at 4 °C (at 4 °C water has its maximum density). Later more precise measurements showed, however, that the mass of 1 dm^3 water was smaller by $2.5 \cdot 10^{-5}$ kg = 0.025 g than the primary standard.

In Tab. 1.7 the subunits of the kilogram, which are used today, are listed. For illustration in Tab. 1.8 some examples of masses which exist in nature are presented.

Figure 1.27 Standard kilogram of platin-iridium, kept under vacuum in Paris (https://en.wikipedia.org/wiki/Kilogram#International_prototype_kilogram)

Table 1.7 Subdivisions and multiples of the kilogram

Unit	Denotion	Mass/kg
1 gram	= 1 g	10^{-3}
1 milligram	= 1 mg	10^{-6}
1 microgram	= 1 µg	10^{-9}
1 nanogram	= 1 ng	10^{-12}
1 pikogram	= 1 pg	10^{-15}
1 ton		10^{3}
1 megaton		10^{9}
1 atomic mass unit	= 1 AMU	$1.6605402 \cdot 10^{-27}$

Table 1.8 The masses of particles and bodies found in nature

Body	Mass/kg
Electron	$9.1 \cdot 10^{-31}$
Proton	$1.7 \cdot 10^{-27}$
Uranium nucleus	$4 \cdot 10^{-25}$
Protein molecule	10^{-22}
Bacterium	10^{-11}
Fly	10^{-3}
Man	10^{2}
Earth	$6 \cdot 10^{24}$
Sun	$2 \cdot 10^{30}$
Galaxy	$\sim 10^{42}$

Masses can be measured either by their inertia or they weight, since both properties are proportional to their mass and unambiguously defined (see Sect. 2.6). The inertia of a mass is measured by the oscillation period of a spring pendulum. Here the mass measurement is reduced to a time measurement.

The weight of a mass is determined by comparison with a mass normal on a spring balance or a beam balance and therefore reduced to a length measurement. Today balances are available with a lower detection limit of at least 10^{-10} kg (magnetic balance, electromagnetic balance, quartz fibre microbalance).

Note: In some countries non-metrical units are used: 1 pound = 0.453 kg.

1.6.6 Molar Quantity Unit

As already mentioned in the beginning of this section in addition to the three basic units for length, time and mass four further units (molar quantity, temperature, electric current and luminous intensity of a radiation source) are introduced because of pragmatic reasons. Strictly speaking they are not real basic units because they can be expressed by the three basic units.

Definition

The unit of molar quantity is the mol, which is defined as follows:

1 mol is the amount of a substance that consist of as many particles as the number N of atoms in 0.012 kg of the carbon nuclide ^{12}C.

These particles can be atoms, molecules, ions or electrons. The number N of particles per mol with the numerical value $N = 6.02 \cdot 10^{23}/\text{mol}$, is called **Avogadro's number** (*Amedeo Avogadro* 1776–1856).

Example

1 mol helium has a mass of 0.004 kg, 1 mol copper corresponds to 0.064 kg, one mol hydrogen gas H_2 has the mass $2 \cdot 0.001\,\text{kg} = 0.002\,\text{kg}$. ◄

1.6.7 Temperature Unit

The unit of the temperature is 1 Kelvin (1 K). This unit can be defined by the thermo-dynamic temperature scale and can be reduced to the kinetic energy of the molecules (see Sect. 10.1.4). Because of principal considerations and also measuring techniques, which are explained in Chap. 10, the following definition was chosen:

1 Kelvin is the fraction (1/273.16) of the thermodynamic temperature of the triple point of water.

The triple point is that temperature where all three phases of water (ice, liquid water and water vapour) can simultaneously exist (Fig. 1.28).

There are plans for a new definition of 1 K which is independent on the choice of a special material (here water). It reads:

1 Kelvin is the temperature change which corresponds to a change $\Delta(kT) = 1.3806505 \cdot 10^{-23}$ Joule of the thermal energy kT, where $k = 13{,}806{,}505 \cdot 10^{-23}$ J/K is the Boltzmann constant.

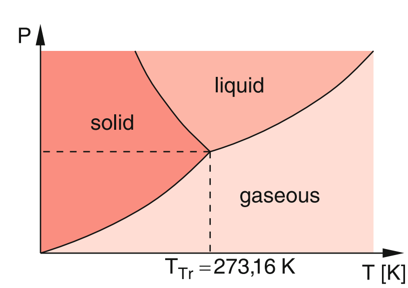

Figure 1.28 Phase diagram and triple point of water

New very accurate measurements of the Boltzmann constant allow a much better definition of the temperature T with an uncertainty of $\Delta T/T \leq 8 \cdot 10^{-6}$.

1.6.8 Unit of the Electric Current

The unit of the electric current is 1 Ampere (1 A) (named after Andre-Marie Ampère 1775–1836). It is defined as follows:

1 Ampere corresponds to a constant electric current through two straight parallel infinitely long wires with a distance of 1 m which experience a mutual force of $2 \cdot 10^{-7}$ Newton per m wire length (Fig. 1.29).

The definition of the electric current unit is therefore based on the measurement of the mechanical quantities length and force (see Vol. 2)

1.6.9 Unit of Luminous Intensity

The luminous intensity of a radiation source is the radiation power emitted into the solid angle 1 Sterad $= 1/(4\pi)$. It could be defined in Watt/Sterad, which gives the radiation power independent of the observing human eye. However, in order to characterize the visual impression of the light intensity of a light source, the spectral characteristics of the radiation must be taken into account, because the sensitivity of the human eye depends on the wavelength. Therefore the definition of the light intensity is adapted to the spectral sensitivity maximum of the eye at a wavelength $\lambda = 555$ nm. The luminosity unit is called 1 **candela** (1 cd).

1 cd is the radiation power of (1/6839)W/Sterad emitted by a source at the frequency 540 THz ($\lambda = 555$ nm) into a selected direction.

Note: 1. The luminous intensity of a source can differ for different directions.
2. The definition of the candela is related to the radiation power in Watt/Sterad, which shows that the candela is not a basic unit.

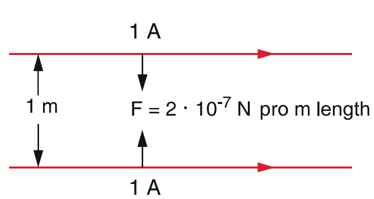

Figure 1.29 Illustration how the unit of the electric current is defined

1.6.10 Unit of Angle

Plane angles are generally measured in degrees of arc. The full angle of a circle is 360°. The subdivisions are minutes of arc ($1° = 60'$) and seconds of arc ($1' = 60'' \rightarrow 1° = 3600''$). Often it is convenient to use dimensionless units by reducing angle measurements to length measurements of the arc length L of a circle, which corresponds to the angle α (Fig. 1.30).

The circular measure (radian) of the angle α is defined as the ratio L/R of circular arc L and radius R of the circle. The unit of this dimensionless quantity is 1 radian (rad) which is realized for $L = R$. Since the total circumference of the circle is $2\pi R$ the angle $\alpha = 360°$ in the unit degrees corresponds to $\alpha = 2\pi$ in the units radian = rad.

The conversion from radians to degrees is

$$1\,\text{rad} = \frac{360°}{2\pi} = 57.296° = 57°17'45''\ .$$

While the plane angle $\alpha = L/R$ cuts the arc with length L out of a circle with radius R the solid angle $\Omega = A/R^2$ is the angel of a cone that cuts the area $A = \Omega R^2$ out of a full sphere with area $4\pi R^2$ and radius R (Fig. 1.31). The dimensionless unit of the solid angle is 1 steradian (1 sr) for which $A = R^2$.

Definition

1 sr is the solid angle of a cone which cuts an area $A = 1\,\text{m}^2$ out of the unit sphere with $R = 1$ m.

Since the total surface of a sphere is $4\pi R^2$ the total solid angle around the centre of the sphere with $A = 4\pi R^2$ is $\Omega = 4\pi$.

The three planes xy, xz, yz through the positive coordinate axis $+x$, $+y$, $+z$ cut a sphere around the origin (0, 0, 0) into 8 octands, The solid angle of one octand is

$$\Omega = \frac{1}{8} \cdot 4\pi = \frac{1}{2}\pi\ sr\ .$$

Note: The numerical values of the units for the basic physical quantities discussed so far have been often adapted by *the International Comission for Weights and Measures* (CIPM for

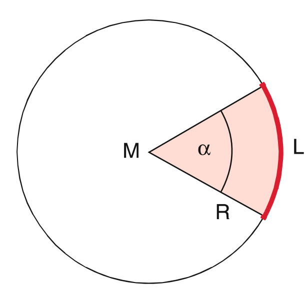

Figure 1.30 To the definition of the radian $\alpha = L/R$

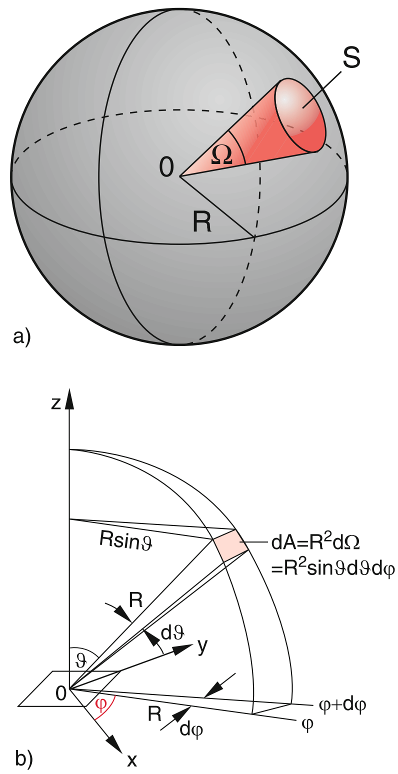

Figure 1.31 **a** To the definition of the solid angle Ω; **b** Illuration of the solid angel element $d\Omega = dA/r^2$

the French *comite international des poids et measures*) in order to take into account the results of new and more accurate measurements. At present, considerations are made to reduce all quantities to combinations of fundamental constants in order to give them more accurate and time independent values. This has been realized up to now only for the length unit which is defined through the fixed speed of light and the frequency of the Cs-clock. This might be soon generalized to all physical units in order to get a system of time-independent values for the units which do not need to be corrected in future times.

One example is the mass unit. There are many efforts in several laboratories to create a better and more accurately defined mass normal. One realistic proposal is a large silicon single crystal in form of a polished sphere, where the atomic distances in the crystal have been precisely measured with X-ray interferometry. This allows the determination of the total number of atoms in the crystal and the mass of the crystal can be related to the mass of a silicon atom and is therefore reduced to atomic mass units and the Avogadro constant [1.48a]. Although it has been shown, that such a mass normal would be more accurate ($\Delta\,m/m \leq 10^{-8}$) and would represent a durable mass standard, it has not yet been internationally acknowledged.

Similar considerations are discussed for the temperature unit 1 K which might be reduced to the Boltzmann constant k (see above).

1.7 Systems of Units

As has been discussed in Sect. 1.6 the three basic quantities and their units in physics are

- length with the unit 1 Meter = 1 m
- time with the unit 1 second = 1 s
- mass with the unit 1 kilogram = 1 kg

with four additional quantities

- molar quantity with the unit 1 mole = 1 mol
- temperature with the unit 1 Kelvin = 1 K
- electric current with the unit 1 Ampere = 1 A
- radiation luminosity with the unit 1 candela = 1 cd

where these four quantities can be reduced in principle to the three basic quantities and are therefore no real basic quantities.

All other quantities in physics can be expressed by these 3 basic quantities with the additional 4 quantities for convenient use. This will be shown for each derived quantity in this textbook when the corresponding quantity is introduced.

Each physical quantity is defined by its unit and its numerical value. For instance the speed of light is $c = 2.9979 \cdot 10^8$ m/s or the earth acceleration $g = 9.81\,\text{m/s}^2$ etc.

> In a physical equation all summands must have the same units.

These units or the products of units are called the dimension of a quantity. The check, whether all summands in a equation have the same dimension is called dimensional analysis. It is a very helpful tool to avoid errors in conversion of different systems of units.

Each physical quantity can be expressed in different units, for example, times in seconds, minutes or hours. The numerical value differs for the different units. For instance the velocity $v = 10$ m/s equals $v = 36$ km/h. In order to avoid such numerical conversions one can use a definite fixed system of units.

If the three basic units are chosen as

- 1 m for the length unit,
- 1 s for the time unit,
- 1 kg for the mass unit.

The system is called the *mks-system*. If the unit Ampere for the electric current is added, the system is called the mksA.-system, often named the **SI-System** after the French nomenclature *System International d'Unites*. It has the very useful advantage that for the conversion from mechanical into electrical and magnetic units all numerical conversion factors have the value 1. All basic units and also the units derived from them are called **SI units**.

In theoretical physics often the *cgs system* is used, where the basic units are 1 cm (instead of 1 m), 1 Gramm (instead of 1 kg) and only the time unit is 1 s as in the SI-system. According to international agreements from 1972 only the SI-system should be used. **In this textbook exclusively SI units are used**.

For a more detailed representation of the subject the reader is referred to the literature [1.37–1.39,1.50].

1.8 Accuracy and Precision; Measurement Uncertainties and Errors

Every measurement has in different ways uncertainties which can be minimized by a reliable measuring equipment and careful observation of the measurement. The most important part in the measuring process is an experienced and critical experimenter, who can judge about the reliability of his results. The final results of an experiment must be given with error limits which show the accuracy of the results. There are two different kinds of possible errors: *Systematic* and *statistical* errors.

1.8.1 Systematic Errors

Most systematic errors are caused by the measuring equipment, as for instance a wrong calibration of an instrument, ignoring of external conditions which can influence the results of the measurement (temperature change for length measurements, lengthening of the string of a threat pendulum by the pendulum weight or air pressure changes for measurements of optical path length). Recognizing such systematic errors and their elimination for precision measurements is often difficult and demands the experience and care of the experimental physicist. Often the influence of systematic errors on the experimental results is underestimated. This is illustrated by Fig. 1.32, which shows the results of measurements of the electron mass during the time from 1950 up to today with the error bars given by the authors. Due to improved experimental techniques the error bars become smaller and smaller in the course of time. The dashed line gives the value that is now accepted. One can clearly see, that all the error bars given by the authors are too small because the systematic error is much larger.

The electron mass can be only determined by a combination of different quantities. For example, from the deflection of electrons in magnetic fields one can only get the ratio e/m of electron charge e and electron mass m. According to the CODATA publication of NIST the value accepted today is $m_e = 9.10938291(40) \cdot 10^{-31}$ kg, where the number in brackets gives the uncertainty of the last two digits.

1.8.2 Statistical Errors, Distribution of Experimental Values, Mean Values

Even if systematic errors have been completely eliminated, different measurements of the same quantity (for instance the falling time of a steel ball from the same heights) do not give the same results. The reasons are inaccurate reading of meters, fluctuations of the measured quantity, noise of the detection system etc. The measured results show a distribution around a mean value. The width of this distribution is a measure of the quality of the results. It is illustrative to plot this distribution of measured values x_i in a histogram (Fig. 1.33), where the area of the rectangles represents the number $n_i \Delta x = \Delta n_i$ of measurements which have given a value within the interval from $x_i - \Delta x/2$ to $x_i - \Delta x/2$.

The mean value $\overline{x}$ of n measurements is chosen in such a way that the sum of the squares of the deviations $(\overline{x} - x_i)$ from the

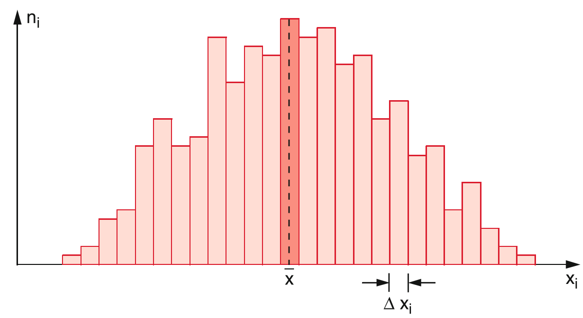

Figure 1.33 Typical histogram of the statistical distribution of measured values x_i around the mean value $\overline{x}$

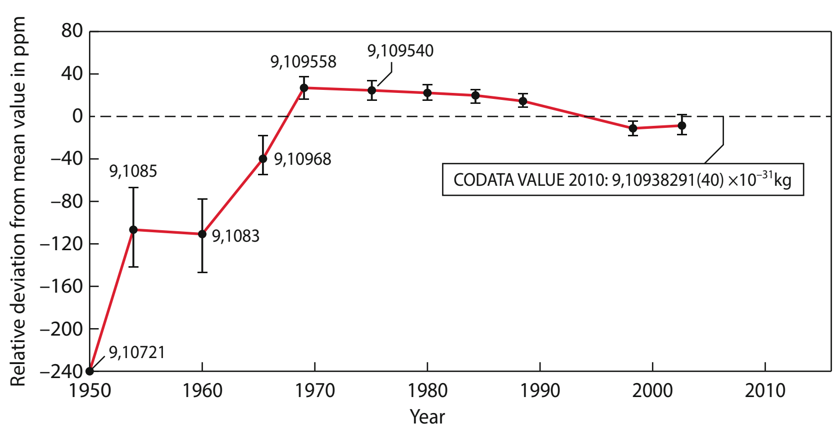

Figure 1.32 Historical values of measurements of the electron mass in units of 10^{-31} kg, demonstrating the underestimation of measuring uncertainties. The relative deviations Δ m/m from the best value accepted today are plotted in units of 10^{-6} (ppm = parts per million)

mean value become a minimum, i.e.

$$S = \sum_{i=1}^{n} (\overline{x} - x_i)^2 = \text{Minimum} . \quad (1.1)$$

For the derivative follows:

$$\frac{dS}{d\overline{x}} = 2 \cdot \sum_{i=1}^{n} (\overline{x} - x_i) = 0 .$$

This gives for the mean value

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i , \quad (1.2)$$

the arithmetic mean value of all measured results. Because $\sum(\overline{x} - x_i) = 0$ the arithmetic mean is at the centre of the symmetric distribution, which means that the sum of the positive deviations equals the sum of the negative ones. Contrary to this symmetric distribution of values with statistical errors the systematic errors cause deviations in one direction.

The question is now how much the mean value deviates from the true, but generally unknown value of the measured quantity. We will now prove, that after elimination of all systematic errors the arithmetic mean converges against the true value x_w with increasing number of measurements. This means:

$$x_\mathrm{w} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i . \quad (1.3)$$

Since it is impossible to perform infinitely many measurements the true value generally remains unknown!

We define the absolute error of the measured value x_i as the difference

$$e_i = x_\mathrm{w} - x_i \quad (1.4)$$

and the absolute error of the mean value as the difference

$$\varepsilon = x_\mathrm{w} - \overline{x} . \quad (1.5)$$

The mean values of these errors are

$$\langle e \rangle = (1/n) \sum e_i ; \quad \langle e^2 \rangle = (1/n) \sum e_i^2 .$$

From (1.2) it follows

$$\varepsilon = x_\mathrm{w} - \overline{x} = \frac{1}{n} \sum_{i=1}^{n} (x_\mathrm{w} - x_i) = \frac{1}{n} \sum_{i=1}^{n} e_i . \quad (1.6)$$

The absolute error ε of the arithmetic mean $\overline{x}$ equals the arithmetic mean $\langle e_1 \rangle = \frac{1}{n} \sum e_i$ of the absolute errors of the individual results x_i.

From (1.6) we obtain by squaring

$$\begin{aligned} \varepsilon^2 &= \frac{1}{n^2} \left(\sum_i e_i \right)^2 = \frac{1}{n^2} \sum_i e_i^2 \\ &\quad + \frac{1}{n^2} \sum_i \sum_{j \neq i} e_i e_j \approx \frac{1}{n^2} \sum_i e_i^2 . \end{aligned} \quad (1.7)$$

The double sum converges for $n \to \infty$ towards zero because for any fixed number j it follows from (1.3)

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} e_i = x_\mathrm{w} - x_\mathrm{w} = 0 .$$

Since for statistical errors the deviations e_i and e_j are uncorrelated.

The quantity

$$\sigma = \sqrt{\langle e^2 \rangle} = \sqrt{\frac{\sum (x_\mathrm{w} - x_i)^2}{n}} \quad (1.8a)$$

is named **standard deviation** or *root mean square deviation*. It equals the square root of the squared arithmetic mean $\langle e^2 \rangle$

$$\langle e^2 \rangle = \frac{1}{n} \sum e_i^2 = \frac{1}{n} \sum_{i=1}^{n} (x_\mathrm{w} - x_i)^2 \quad (1.8b)$$

The smaller quantity

$$\begin{aligned} \sigma_\mathrm{m} &= \sqrt{\varepsilon^2} = \sqrt{\frac{1}{n^2} \sum e_i^2} \\ &= \frac{1}{n} \sqrt{\sum_i (x_\mathrm{w} - x_i)^2} \end{aligned} \quad (1.8c)$$

is the **mean error of the arithmetic mean** $\overline{x}$.

From (1.8a)–(1.8c) we can conclude

$$\sigma_\mathrm{m} = \frac{\sigma}{\sqrt{n}} . \quad (1.9)$$

The mean error of the arithmetic mean equals the mean error of the individual measurements divided by the square root of the total number n of measurements.

In the next section it will be shown that σ approaches a constant value for $n \to \infty$. Equation 1.9 then implies, that $\lim \sigma_\mathrm{m} = 0$, which means that the arithmetic mean $\overline{x}$ approaches the true value x_w for a sufficiently large number n of measurements.

1.8.3 Variance and its Measure

Since for a finite number n of measurements the true value of the measured quantity is generally unknown, also the absolute errors and the mean errors σ and σ_m cannot be directly determined. We will now show how σ and σ_m are related to quantities that can be directly measured.

We introduce instead of the unknown deviations $e_i = x_\mathrm{w} - x_i$ of the measured values from the true value x_w the deviations $v_i = \overline{x} - x_i$ from the mean value, which contrary to e_i are known values.

According to (1.4) and (1.5) we can express the v_i by the quantities e_i and ε.

$$\begin{aligned} v_i &= \overline{x} - x_i \\ &= x_\mathrm{w} - x_i - (x_\mathrm{w} - \overline{x}) \\ &= e_i - \varepsilon \,. \end{aligned} \tag{1.10}$$

The mean square deviation of the measured values x_i from the arithmetic mean $\overline{x}$ can then be written as

$$\begin{aligned} s^2 &= \frac{1}{n}\sum_i v_i^2 = \frac{1}{n}\sum_i (e_i - \varepsilon)^2 \\ &= \frac{1}{n}\left[\sum_i e_i^2 - \left(\frac{2\varepsilon}{n}\sum_i e_i\right) + \varepsilon^2\right] \\ &= \frac{1}{n}\sum_i \left(e_i^2 - \varepsilon^2\right) \,, \end{aligned} \tag{1.11}$$

because according to (1.6) $\varepsilon = (1/n)\sum e_i$. The comparison with (1.8a,b,c) yields the relation

$$s^2 = \frac{1}{n}\sum_i \left(e_i^2 - \varepsilon^2\right) = \sigma^2 - \sigma_\mathrm{m}^2 \,. \tag{1.12}$$

From the equations (1.8b), (1.9) and (1.12) it follows

$$\begin{aligned} s^2 &= \left(\frac{1}{n} - \frac{1}{n^2}\right)\sum_i (x_\mathrm{w} - x_i)^2 \\ &= \frac{n-1}{n^2}\sum_i (x_\mathrm{w} - x_i)^2 \\ &= (n-1)\sigma_\mathrm{m}^2 = \frac{n-1}{n}\sigma^2 \,. \end{aligned}$$

For the standard deviation of the individual results x_i we obtain the mean deviation of the arithmetic mean value

$$\sigma^2 = \frac{n}{n-1}s^2 \to \sigma = \sqrt{\frac{\sum(\overline{x} - x_i)^2}{n-1}} \,, \tag{1.13}$$

which can be obtained from measurements and is therefore a known quantity.

For the mean deviation of the arithmetic mean (also called standard deviation of the arithmetic means) we get

$$\sigma_\mathrm{m}^2 = \frac{1}{n-1}s^2 \to \sigma_\mathrm{m} = \sqrt{\frac{\sum(\overline{x} - x_i)^2}{n(n-1)}} \,. \tag{1.14}$$

Example

For 10 measurements of the period of a pendulum the following values have been obtained:
$T_1 = 1.04\,\mathrm{s}$; $T_2 = 1.01\,\mathrm{s}$; $T_3 = 1.03\,\mathrm{s}$; $T_4 = 0.99\,\mathrm{s}$; $T_5 = 0.98\,\mathrm{s}$; $T_6 = 1.00\,\mathrm{s}$; $T_7 = 1.01\,\mathrm{s}$; $T_8 = 0.97\,\mathrm{s}$; $T_9 = 0.99\,\mathrm{s}$; $T_{10} = 0.98\,\mathrm{s}$.

The arithmetic mean is $\overline{T} = 1.00\,\mathrm{s}$. The deviations $x_i = T_i - \overline{T}$ of the values T_i from the mean $\overline{T}$ are
$x_1 = 0.04\,\mathrm{s}$; $x_2 = 0.01\,\mathrm{s}$; $x_3 = 0.03\,\mathrm{s}$; $x_4 = -0.01\,\mathrm{s}$; $x_5 = -0.02\,\mathrm{s}$; $x_6 = 0.00\,\mathrm{s}$; $x_7 = 0.01\,\mathrm{s}$; $x_8 = -0.03\,\mathrm{s}$; $x_9 = -0.01\,\mathrm{s}$; $x_{10} = -0.02\,\mathrm{s}$. This gives

$$\Sigma(T_i - \langle T\rangle)^2 = \Sigma x_i^2 = 46 \cdot 10^{-4}\,\mathrm{s}^2 \,.$$

The standard deviation is then

$$\sigma = \sqrt{(46 \cdot 10^{-4}/9)} = 2.26 \cdot 10^{-2}\,\mathrm{s}$$

and the standard deviation of the arithmetic mean is

$$\sigma_\mathrm{m} = \sqrt{(46 \cdot 10^{-4}/90)} = 0.715 \cdot 10^{-2}\,\mathrm{s} \,.$$ ◀

1.8.4 Error Distribution Law

In the histogram of Fig. 1.33 the resolution of the different measured values depends on the width Δx_i of the rectangles. All values within the interval Δx_i are not distinguished and regarded to be equal. If Δn_i is the number of measured values within the interval Δx_i and k the total number of intervals Δx_i we can write Eq. 1.2 also as

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{k} \Delta n_i \cdot x_i \quad \text{with} \quad \sum_{i=1}^{k} \Delta n_i = n \,. \tag{1.15}$$

The histogram in Fig. 1.33 can be obtained in a normalized form when we plot the fraction n_i/n ($n_i = \Delta n_i/\Delta x_i$ and $n = \sum \Delta n_i$),

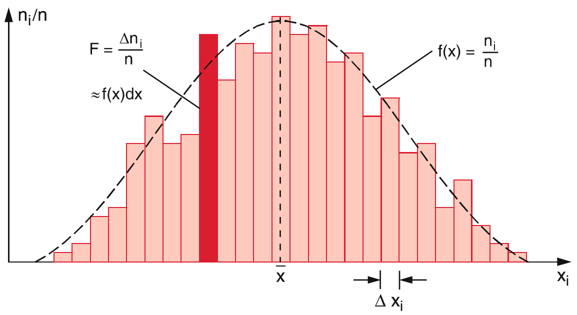

Figure 1.34 Normalized statistical distribution and distribution function of measured data

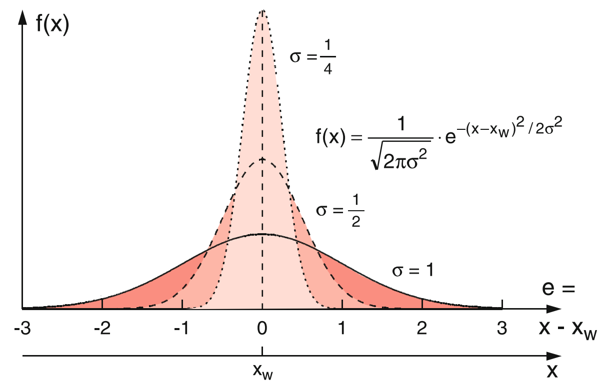

Figure 1.35 Error distribution function (Gaussian distribution) around the true value x_w for different standard deviations σ

which represents the number of measured values within the unit interval $\Delta x_i = 1$ (Fig. 1.34). The heights of the rectangles give these fractions. The quantity $\Delta n_i/n$ can be regarded as the probability that the measured values fall within the interval Δx_i. With increasing number n of measurements we can decrease the width of the intervals Δx_i which means that the total number k of all intervals increases. For $\Delta x_i \to 0$ the number $k \to \infty$ and $\Delta n_i \to 0$ but the fraction $\Delta n_i/\Delta x_i$ approaches a finite value. The sum $n = \sum n_i \Delta x_i$ which represents the total number of measured values, stays of course constant. The discontinuous distribution of the histogram in Fig. 1.34 converges against a continuous function $f(x)$, which is shown in Fig. 1.34 as black dashed curve. The function $f(x)$ is defined as

$$f(x) = (1/n)\lim(\Delta n_i/\Delta x_i) = (1/n) \cdot \mathrm{d}n/\mathrm{d}x \; ; \qquad (1.16a)$$

$f(x)$ is the continuous **distribution function**. The product $f(x) \cdot \mathrm{d}x$ gives the probability to find a measured value in the interval from $x - \mathrm{d}x/2$ to $x + \mathrm{d}x/2$. From (1.16a) and $\sum n_i \Delta x_i$ follows the normalization

$$\int f(x)\mathrm{d}x = \lim\left[(1/n)\sum n_i \Delta x_i\right] = 1 \; . \qquad (1.16b)$$

This means that the probability to find a measured value somewhere within the total x-range must be of course 100% = 1, because it has to be somewhere in this range.

The integral $\int f(x)\mathrm{d}x$ represents the area under the black curve which is normalised to 1 because the ordinate in Fig. 1.34 is given as the normalized quantity n_i/n.

The standard deviation σ is a measure for the width of the distribution $f(x)$. Its square σ^2 gives, as for the discontinuous distribution (1.8b), the mean square deviation of the arithmetic mean from the true value x_w, which determines the centre of the symmetric curve $f(x)$

$$\sigma^2 = \langle e^2 \rangle = \int_{-\infty}^{+\infty} (x_w - x)^2 f(x)\,\mathrm{d}x \; . \qquad (1.17)$$

The quantity σ^2 is named the **variance**.

If only statistical errors contribute, the normalized distribution of the measured values can be described by the normalized Gauss-function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-x_w)^2/2\sigma^2} \; , \qquad (1.18)$$

which has its maximum at $x = x_w$. The inflection points of the curve $f(x)$ are at $x = x_w \pm \sigma$. The full width between the inflection points where $f(x) = f(x_w)/e$ is therefore 2σ. The distribution $f(x)$ is symmetrical around its centre at x_w (Fig. 1.35). For infinitely many measurements the arithmetic mean x becomes x_w.

When the standard deviation has been determined from n measurements, the probability $P(\sigma)$ that further measured values fall within the interval $x = x_w \pm \sigma$ and are therefore within the standard deviation from the true value. It is given by the integral

$$P\left(|x_w - x_i| \le \sigma\right) = \int_{x_w-\sigma}^{x_w+\sigma} f(x)\mathrm{d}x \; . \qquad (1.19)$$

When inserting (1.18) the integral can be solved and yields the numerical values

$$\begin{aligned} P\left(e_i \le \sigma\right) &= 0.683 \quad \text{(68\% confidence range)} \\ P\left(e_i \le 2\sigma\right) &= 0.954 \quad \text{(95\% confidence range)} \\ P\left(e_i \le 3\sigma\right) &= 0.997 \quad \text{(99.7\% confidence range)} \; . \end{aligned}$$

The results of a measurement are correctly given with the 68% confidence range as

$$x_w = \bar{x} \pm \sigma \; . \qquad (1.20)$$

This means that the true value falls with a probability of 68% within the uncertainty range from $\bar{x} - \sigma$ to $\bar{x} + \sigma$ around the arithmetic mean, if all systematic errors has been eliminated. The relative **accuracy** of a measured value x_w is generally given as $\sigma/\bar{x}$.

Cautious researchers extend the uncertainty range to $\pm 3\sigma$ and can than state that their published result lies with the probability of 99.7%, which means nearly with certainty within the given limits around the arithmetic mean. The result is then given as

$$x_w = \bar{x} \pm 3\sigma = \bar{x} \pm 3 \cdot \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \; . \qquad (1.21)$$

Since the arithmetic mean is more accurate than the individual measurements often the uncertainty range is given as the standard deviation σ_m of the arithmetic mean which is smaller than σ. The result is then given as

$$x_w = \bar{x} \pm \sigma_m = \bar{x} \pm \sqrt{\frac{\sum (x_i - \bar{x})^2}{n(n-1)}} \,. \tag{1.22}$$

Example

For our example of the measurements of the periods of a pendulum the result would be given with the 69% confidence range as

$$T_w = \langle T \rangle \pm \sigma = (1.000 \pm 0.025)\,\mathrm{s}$$

and for the 99.7% confidence range as

$$T_w = \langle T \rangle \pm 3\sigma = (1.000 \pm 0.075)\,\mathrm{s}\,.$$

For the standard deviation σ_m of the arithmetic mean one gets

$$T_w = \langle T \rangle \pm \sigma_m = (1.0000 \pm 0.0079)\,\mathrm{s}\,.$$

The relative uncertainty of the true value is then with a probability of 68%

$$\Delta T_w / T_w = 7.9 \cdot 10^{-3} = 0.79\%\,.$$

Remark. For statistical processes where the measured quantity is an integer number $x_i = n_i$ that statistically fluctuates (for instance the number of electrons emitted per sec by a hot cathode, or the number of decaying radioactive nuclei per sec) one obtains instead of the Gaussian function (1.18) a Poisson distribution

$$f(x) = \frac{\bar{x}^x}{x!} e^{-\bar{x}} \qquad x = \text{integer number}\,. \tag{1.23}$$

1.8.5 Error Propagation

If a quantity $y = f(x)$ depends in some way on the measured quantity x, the uncertainty $\mathrm{d}y$ is related to $\mathrm{d}x$ by (Fig. 1.36)

$$\mathrm{d}y = \frac{\mathrm{d}f(x)}{\mathrm{d}x}\,\mathrm{d}x\,. \tag{1.24}$$

When the quantity x has been measured n-times its standard deviation is

$$\sigma_x = \sqrt{\frac{\sum(\bar{x} - x_i)^2}{n-1}}\,,$$

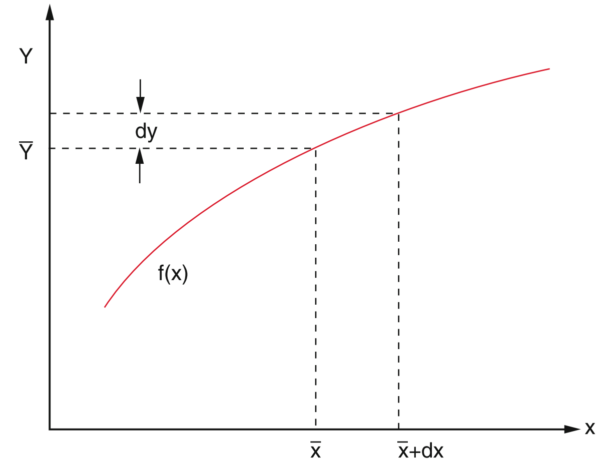

Figure 1.36 Error propagation for a function $y = f(x)$

which results in the standard deviation of the y_i values

$$\sigma_y = \sqrt{\frac{\sum(\bar{y} - y_i)^2}{n-1}} = \sqrt{\frac{\sum\left(f(\bar{x}) - f(x_i)\right)^2}{n-1}} = \left(\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right)_{\bar{x}} \cdot \sigma_x\,. \tag{1.25}$$

Often the value of a quantity, which is not directly accessible to measurements, and its uncertainty should be known. Examples are the density of a body which can be calculated as the ratio of mass and volume of the body, or the acceleration of a moving mass which is determined from measurements of distances and times.

The question is now: What is the accuracy of a quantity $f(x, y)$, if the uncertainties of the measurements of x and y are known.

Assume one has made n measurements of the quantity x from which the uncertainty range of the arithmetic mean is determined as

$$\bar{x} \pm \sigma_x = \bar{x} \pm \sqrt{\frac{\sum v_i^2}{n-1}} \quad \text{with} \quad v_i = x_i - \bar{x}$$

and m measurements of the quantity y with the mean

$$\bar{y} \pm \sigma_y = \bar{y} \pm \sqrt{\frac{\sum u_k^2}{m-1}} \quad \text{with} \quad u_k = y_k - \bar{y}\,,$$

one obtains the quantity

$$\begin{aligned} f_{ik} &= f(x_i, y_k) = f(\bar{x} + v_i, \bar{y} + u_k) \\ &= f(\bar{x}, \bar{y}) + v_i \left(\frac{\partial f(x,y)}{\partial x}\right)_0 \\ &\quad + u_k \left(\frac{\partial f(x,y)}{\partial y}\right)_0 + \ldots \end{aligned} \tag{1.26}$$

by a Taylor expansion, where $(\partial f / \partial x)_0$ is the partial derivative for the values x, y. Often the deviations v_i and u_k are so small that the higher powers in the expansion can be neglected. The mean value of all f_{ik} is then

$$\begin{aligned}\overline{f} &= \frac{1}{n\cdot m}\sum_i\sum_k f_{ik} = \frac{1}{n\cdot m}\sum_{i=1}^{n}\sum_{k=1}^{m}\left[f\left(\overline{x},\overline{y}\right)\right.\\ &\quad\left. + v_i\frac{\partial f}{\partial x}\left(\overline{x},\overline{y}\right) + u_k\frac{\partial f}{\partial y}\left(\overline{x},\overline{y}\right)\right]\\ &= \frac{1}{n\cdot m}\left[n\cdot m\cdot f\left(\overline{x},\overline{y}\right) + m\sum_i v_i\frac{\partial f}{\partial x}\right.\\ &\quad\left. + n\sum_k u_k\frac{\partial f}{\partial y}\right] = f\left(\overline{x},\overline{y}\right) \ ,\end{aligned} \tag{1.27}$$

because $\partial f/\partial x|_{x,y}$ is constant and $\sum v_i = \sum u_i = 0$.

The arithmetic mean $\overline{f}$ of all values f_{ik} equals the value $f(\overline{x},\overline{y})$ of the function $f(x,y)$ for the arithmetic means $\overline{x},\overline{y}$ of the measured values $x_i y_k$.

In books about error calculus [1.53a–1.55] it is shown, that the standard deviation of the derived quantity f is related to the standard deviations σ_x and σ_y of the measured values x_i, y_k by

$$\sigma_f = \sqrt{\sigma_x^2\left(\frac{\partial f}{\partial x}\right)^2 + \sigma_y^2\left(\frac{\partial f}{\partial y}\right)^2} \ . \tag{1.28}$$

The mean uncertainties σ_x and σ_y propagate to the uncertainty σ_f of the derived mean $f(x,y)$. The 68% confidence range of the true value $f_w(x,y) = f(x_w, y_w)$ is then

$$f_w(x,y) = f\left(\overline{x},\overline{y}\right) \pm \sqrt{\sigma_x^2\left(\frac{\partial f}{\partial x}\right)^2 + \sigma_y^2\left(\frac{\partial f}{\partial y}\right)^2} \ . \tag{1.29}$$

With the inequality $\sqrt{a^2+b^2} \le |a| + |b|$ the uncertainty (1.29) can be also written as

$$\Delta f = f_w - f\left(\overline{x},\overline{y}\right) \le \left|\sigma_x\frac{\partial f}{\partial x}\right| + \left|\sigma_y\frac{\partial f}{\partial y}\right| \ . \tag{1.30}$$

Examples

1. The length L is divided into two sections x and y with $L = x + y$ which are separately measured (Fig. 1.37a). The final result of L is then, according to (1.27) and (1.28) with $\partial f/\partial x = \partial f/\partial y = 1$,

$$\overline{L} = \overline{x} + \overline{y} \pm \sqrt{\sigma_x^2 + \sigma_y^2} \ .$$

This means: the mean error of a sum (or a difference) equals the square root of the sum of squared errors of the measured values.

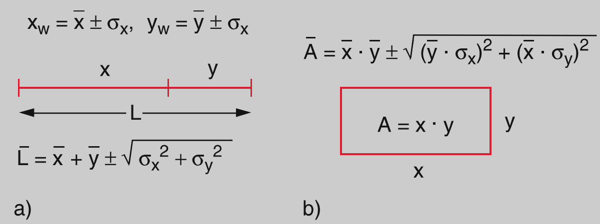

Figure 1.37 **a** Mean error of a length measurement, that consists of two individual measurements x and y; **b** Error propagation for the measurement of an area $x \cdot y$

2. The area $A = x \cdot y$ of a rectangle shall be determined for the measured side lengths x and y. The true values of x and y are

$$\begin{aligned} x_w &= \overline{x} \pm \sigma_x \ , \quad y_w = \overline{y} \pm \sigma_y \ ,\\ \frac{\partial A}{\partial x}\left(\overline{x},\overline{y}\right) &= \overline{y} \ , \quad \frac{\partial A}{\partial y}\left(\overline{x},\overline{y}\right) = \overline{x} \ ,\\ \overline{A} &= \overline{x}\cdot\overline{y} \pm \sigma_{xy}\\ &= \overline{x}\cdot\overline{y} \pm \sqrt{\left(\overline{y}\cdot\sigma_x\right)^2 + \left(\overline{x}\cdot\sigma_y\right)^2} \ .\end{aligned}$$

The relative error of the product $A = x \cdot y$

$$\frac{\sigma_{xy}}{A} = \sqrt{\left(\frac{\sigma_x}{\overline{x}}\right)^2 + \left(\frac{\sigma_y}{\overline{y}}\right)^2}$$

equals the Pythagorean sum of the relative errors of the two factors x and y.

3.

$$\begin{aligned} y &= \ln x \ ; \quad x = \overline{x} \pm \sigma_x \Rightarrow \frac{\partial y}{\partial x} = 1/x\\ \overline{y} &= \ln\overline{x} \pm \sigma_x/\overline{x}\end{aligned}$$

The mean absolute error of the logarithm of a measured value x equals the relative error of x. ◀

1.8.6 Equalization Calculus

Up to now we have discussed the case, where the same quantity has been measured several times and how the arithmetic means of the different measured values and its uncertainty can be obtained. Often the problem arises that a quantity $y(x)$, which depends on another quantity x shall be determined for different values of x and the question is how accurate the function $y(x)$ can be determined if the measured values of x have a given uncertainty.

Example

1. A falling mass passes during the time t the distance $d = \frac{1}{2}g \cdot t^2$ and its velocity $v = g \cdot t$ is measured at different times t_i.

2. The change of the length $\Delta L = L_0 \cdot \alpha \cdot \Delta T$, a long rod with length L and thermal expansion coefficient α experiences for a temperature change ΔT, is measured at different temperatures T. ◀

In our first example distances and velocities are measured at different times. The goal of these measurements is the accurate determination of the earth acceleration g. In the second example length changes and temperatures are measured in order to obtain the thermal expansion coefficient α as a function of temperature T.

The relation between $y(x)$ and x can be linear (e. g. $v = g \cdot t$), but may be also a nonlinear function (e. g. a quadratic or an exponential function). Here wee will restrict the discussion to the simplest case of linear functions, in order to illustrate the application of equalization calculus to practical problems.

This will become clear with the following example.

Example

We consider the linear function

$$y = ax + b$$

and will answer the question, how accurate the constants a and b can be determined when y is calculated for different measured values of x.

Solution

It is often the case that the values x can be measured more accurately than y. For instance for the free fall of a mass the times can be measured with electronic clocks much more accurately than distances or velocities. In such cases the errors of x can be neglected compared to the uncertainties of y. This reduces the problem to the situation depicted in Fig. 1.38. The measured values $y(x)$ are given by points and the standard deviation by the length of the error bars.

The question is now, how it is possible to fit a straight line to the experimental points in such a way that the uncertainties of the constants a and b become a minimum.

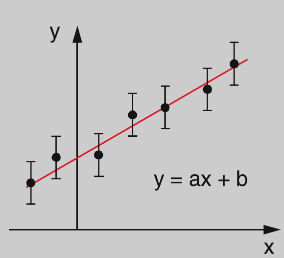

Figure 1.38 Equalization calculus for the function $y = ax + b$, when the values x_i can be measured much more accurate than the values y_i

This is the case if the sum of the squared deviations reaches a minimum.

$$S = \sum (y_i - ax_i - b)^2 \tag{1.31}$$

Differentiating (1.31) gives the two equations. (Note that a and b are here the variables!)

$$\frac{\partial S}{\partial a} = -2 \sum_{i=1}^{n} x_i (y_i - ax_i - b) = 0 \tag{1.32a}$$

$$\frac{\partial S}{\partial b} = -2 \sum_{i=1}^{n} (y_i - ax_i - b) = 0\,. \tag{1.32b}$$

Rearranging yields

$$a \cdot \sum_i x_i^2 + b \cdot \sum_i x_i = \sum_i x_i y_i \tag{1.33a}$$

$$a \cdot \sum_i x_i + b \cdot n = \sum_i y_i\,. \tag{1.33b}$$

The last equation is matched exactly for the point $(\bar{x}, \bar{y})$ with the mean coordinates

$$\bar{x} = (1/n) \sum x_i; \quad \bar{y} = (1/n) \sum y_i\,.$$

Inserting these values into (1.33b) yields after division by the number n the relation

$$a \cdot \bar{x} + b = \bar{y}\,.$$

This proves that the point $(\bar{x}, \bar{y})$ fulfils the equation and is located in Fig. 1.38 exactly on the red straight line.

From (1.33b) one obtains for the slope b of the straight line

$$b = y - ax = (1/n) \sum y_i - (a/n) \sum x_i\,.$$

Inserting this into (1.33a) gives with the abbreviation

$$d = n \cdot \sum x_i^2 - \left(\sum x_i\right)^2\,,$$

the constants a and b as

$$a = \frac{n\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{d}\,, \tag{1.34a}$$

$$b = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{d}\,. \tag{1.34b}$$

The true constants a and b give the true values $y_w(x_i) = ax_i + b$ within the 68% confidence limits $y_i \pm \sigma_y$ around the mean value $\bar{y}$. From (1.18) and (1.19) one obtains the probability $P(y_i)$ to find the measured value y_i

$$P(y_i) \propto \frac{1}{\sigma_y} e^{-(y_i - ax_i - b)^2/2\sigma_y^2}\,. \tag{1.35}$$

The uncertainties of the constants a and b can be obtained according to the error propagation rules. The results are

$$\sigma_a^2 = \frac{n \cdot \sigma_y^2}{d} , \quad \sigma_b^2 = \frac{\sigma_y^2 \sum x_i^2}{d} . \tag{1.36}$$

The full width between the two points $P(y_\mathrm{w})/e$ is $\sigma_y \cdot \sqrt{2}$.

For more information on error analysis and regression fits see [1.53a–1.56]. ◄

Summary

- Physics deals with the basic building blocks of our world, their mutual interactions and the synthesis of material from these basic particles.
- The gain of knowledge is pushed by specific experiments. Their results serve for the development of a general theory of nature and to confirm or contradict existing theories.
- Experimental physics started in the 16th century (e. g. Galilei, Kepler) and led to a more and more refined and extensive theory, which is, however, even today not yet complete and consistent.
- All physical quantities can be reduced to three basic quantities of length, time and mass with the basic units 1 m, 1 s, and 1 kg. For practical reasons four more basic quantities are introduced for molar mass (1 mol), temperature (1 K), electric current (1 A) and the luminous power (1 cd).
- The system of units which uses these basic 3 + 4 units is called SI-system with the units 1 m, 1 s, 1 kg, 1 mol, 1 K, 1 A and 1 cd.
- Every measurement means the comparison of the measured quantity with a normal (standard).
- The *length standard* is the distance which light travels in vacuum within a time interval of (1/299,792,458) s. The *time standard* is the transition frequency between two hyperfine levels in the Cs atom measured with the caesium atomic clock. The present *mass standard* is the mass of the platinum-iridium kilogram, kept in Paris.
- Each measurement has uncertainties. One distinguishes between systematic errors and statistical errors. The mean value of n independent measurements with measured values x_i is chosen as the arithmetic mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i ,$$

which meets the minimum condition

$$\sum_{i=1}^{n} (\bar{x} - x_i)^2 = \text{minimum.}$$

If all systematic errors could be eliminated the distribution of the measured values x show the statistical Gaussian distribution

$$f(x) \propto \mathrm{e}^{-(x-x_\mathrm{w})^2/2\sigma^2} ,$$

about the most probable value, which equals the true value x_w. The half-with of the distribution between the points $f(x_\mathrm{w})/e = f(x_\mathrm{w} \pm \sigma)$ is $\sigma \cdot \sqrt{2}$ Within the range $x = x_\mathrm{w} \pm \sigma$ fall 68% of all measured values. The standard deviation σ of individual measurements is

$$\sigma = \sqrt{\frac{\sum (\bar{x} - x_i)^2}{n-1}} ,$$

the standard deviation of the arithmetic means is

$$\sigma_\mathrm{m} = \sqrt{\frac{\sum (\bar{x} - x_\mathrm{i})^2}{n(n-1)}} .$$

The true value x_w lies with the probability of 68% within the interval $x_\mathrm{w} \pm \sigma$, with a probability of 99.7% in the interval $x_\mathrm{w} \pm 3\sigma$. The Gaussian probability distribution for the measured values x_{oi} has a full width at half maximum of

$$\Delta x_{1/2} = 2\sigma \sqrt{2 \cdot \ln 2} = 2.35\sigma .$$

Problems

1.1 The speed limit on a motorway is 120 km/h. An international commission decides to make a new definition of the hour, such that the period of the earth rotation about its axis is only 16 h. What should be the new speed limit, if the same safety considerations are valid?

1.2 Assume that exact measurements had found that the diameter of the earth decreases slowly. How sure can we be, that this is not just an increase of the length of the meter standard?

1.3 Discuss the following statement: "The main demand for a length standard is that its length fluctuations are smaller than length changes of the distances to be measured".

1.4 Assume that the duration of the mean solar day increases by 10 ms in 100 years due to the deceleration of the earth rotation. a) After which time would the day length be 30 hours? b) How often would it be necessary to add a leap second in order to maintain synchronization with the atomic clock time?

1.5 The distance to the next star (α-Centauri) is $d = 4.3 \cdot 10^{16}$ m. How long is the travelling time of a light pulse from this star to earth? Under which angle appears the distance earth-sun from α-Centauri? If the accuracy of angular measurements is $0.1''$ what is the uncertainty of the distance measurement?

1.6 A length L is seen from a point P which is 1 km (perpendicular to L) away from the centre of L, under an angle of $\alpha = 1°$. How accurate can the length be determined by angle measurements from P if the uncertainty of α is $1'$?

1.7 Why does the deviation of the earth orbit from a circle cause a variation of the solar day during the year? Give some arguments why the length of the mean solar day can change for different years?

1.8 How many hydrogen atoms are included in 1 kg of hydrogen gas?

1.9 How many water molecules H_2O are included in 1 litre water?

1.10 The radius of a uranium nucleus ($A = 238$) is $8.68 \cdot 10^{-15}$ m. What is its mean mass density?

1.11 The fall time of a steel ball over a distance of 1 m is measured 40 times, with an uncertainty of 0.1 s for each measurement. What is the accuracy of the arithmetic mean?

1.12 For which values of x has the error distribution function $\exp[-x^2/2]$ fall to 0.5 and to 0.1 of its maximum value?

1.13 Assume the quantity $x = 1000$ has been measured with a relative uncertainty of 10^{-3} and $y = 30$ with $3 \cdot 10^{-3}$. What is the error of the quantity $A = (x - y^2)$?

1.14 What is the maximum relative error of a good quartz clock with a relative error of 10^{-9} after 1 year? Compare this with an atomic clock ($\Delta\nu/\nu = 10^{-14}$).

1.15 Determine the coefficients a and b of the straight line $y = ax + b$ which gives the minimum squared deviations for the points $(x, y) = (0, 2)$; $(1, 3)$; $(2, 3)$; $(4, 5)$ and $(5, 5)$. How large is the standard deviation of a and b?

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