Chapter 3
Formal Methods and the History of Philosophy

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Although not (yet) entirely mainstream, uses of formal methods for the study of the history of philosophy, the history of logic in particular, represent an important trend in recent philosophical historiography. In this chapter, I discuss what can (and cannot) be achieved by the application of formal methods to the history of philosophy, addressing both motivations and potential pitfalls. The first section focuses on methodological aspects, and the second section presents three case studies of historical theories which have been investigated with formal tools: Aristotle's syllogistic, Anselm's ontological argument, and medieval theories of supposition.

3.1 Methodological Considerations

3.1.1 Why (Not) Apply Formal Methods to the History of Philosophy?

Let us begin by discussing motivations and potential objections to the use of formal methods in the study of the history of philosophy. A recurring concern pertains to the risk of *anachronism*: formal methods are for the most part recent inventions, and applying these modern frameworks to theories of the past is bound to bring along a range of presuppositions and assumptions that have no counterpart in the historical framework in question.

However, while this issue may be more acute in the case of formal methods, it in fact pertains to philosophical historiography in general. Indeed, a certain amount of

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anachronism is inherent to any historical analysis, and it is not immediately obvious why the anachronism brought in by formal methods would be substantially different from the anachronism brought in by other recent methodologies and frameworks. Thus, even acknowledging that philosophical theories bear a mark of historicity, formal methods can still be seen as legitimate interpretive tools for historical investigations.

Nonetheless, the risk of excessive anachronism when employing formal methods is real, and perhaps more acute than with other methodologies. Thus, the historian of philosophy who employs formal methods must remain particularly alert so as to minimize or in any case take into account the inevitable traces of anachronism in her investigations. The choice of the formalism to be used must be judicious, as for a given historical analysis some formalisms will bring in a lesser degree of anachronism and inadequacy than others.

This being said, formal methods can in fact be valuable tools in the context of textual exegesis. Much of what the historian of philosophy does consists in working with *texts*, and formalization may help elucidate particularly thorny passages or arguments.[1] (However, it must be stressed that a formalization of a historical theory usually does not consist in taking the very linguistic expression of the theory in the original text as its object.[2]) In other words, formal methods can serve as a hermeneutical tool among others; by engaging in the formalization of a given historical theory, the interpreter may obtain a deeper understanding of the theory, possibly an understanding that other interpretive methods could not provide.

Indeed, formal methods seem particularly well-placed to unveil certain aspects of the target theory. A formalization presupposes a deconstruction of the historical theory so that some of its key elements are isolated from the others, thus outlining its logical scaffolding. Furthermore, formal methods may disclose 'hidden' aspects of a historical theory, which are not visible to the 'naked eye' (to pursue Frege's metaphor of a formalism as a microscope, in the preface of the *Begriffsschrift*).

Hence, provided they are used with caution and that their inherent anachronism is taken into account, formal methods can be irreplaceable items in a historian's toolkit. But their use is only justified if they truly shed new light on the object of analysis; unless new insight is obtained, fancy formalization may simply be overkill.

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[1]More recently, computational methods have been gaining quite a lot of traction for research in history of philosophy, under the umbrella of 'digital humanities'. These are exciting developments that may well change substantially how historians of philosophy approach their topics, but for now they are still at early stages. While these can be broadly understood as formal methods, in this piece I do not discuss them any further for reasons of space.

[2]In fact, I have argued elsewhere ([11], chap. 3) that it is a mistake to think about formalizations in general merely as taking portions of 'natural language' as their starting point and translating them into a formal language.

3.1.2 How (Not) to Apply Formal Methods in History of Philosophy

How does a historian of philosophy who applies formal methods proceed? It cannot be sufficiently emphasized that the formal historian remains above all a *historian*: it is solely on the basis of solid conceptual knowledge of her object of study that she can successfully apply formal methods in her investigations. While formal tools can be instrumental even in the interpretive process of textual analysis, ultimately the formal historian must be thoroughly familiar with the historical framework in question before formalization begins.

Next, an important step is the choice of an adequate formalism. The first uses of formal methods to study the history of philosophy, in the second half of the twentieth century, tended to adopt uncritically the 'standard' logical systems of the time, in particular classical predicate logic. But as we will see with the case studies below, uncritically adopting an inadequate framework is likely to lead to poor results. An inadequate formalism will bring along unwarranted assumptions and presuppositions, and/or fail to capture some key components of the historical theory if they have no counterpart in the formalism.

The point is not that there will be at most one adequate formal framework for each historical theory; there may well be different, equally adequate frameworks, or frameworks adequate for different aspects of the theory. In other words, conceptual as well as semi-pragmatic considerations will play a role, but some formalisms are hopelessly unsuitable for a given historical theory. The choice of a formalism is already an *interpretive* choice; there is no such thing as a theoretically neutral formalization.

A formalization is always a process of abstraction, but one which promises to offer further insight precisely because it separates what is crucial from what is secondary in a given theory (relative to a given purpose), allowing for a more uniform analysis. In any formalization, some elements of the target phenomenon are represented by certain features of the model – what Shapiro[3] refers to as the *representors* – while other features of the model are *artifacts* (again in Shapiro's terminology), introduced for convenience. So a good formalization is not one where every aspect of what is being formalized is represented, but rather one where the tradeoff between simplification and accuracy of representation is favorable.

In particular, the chosen formalism must have the right level of *granularity* with respect both to the target historical theory and the purpose of the formalization: it must abstract the right amount of information away – not too much, not too little. The formalization is too coarse if it fails to capture important aspects of the historical theory, and it is too fine-grained if it projects distinctions and concepts into the theory that are not there to start with (naturally, it can be both too coarse and too fine-grained).

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[3]Shapiro [30].

Note that these general considerations must be viewed as no more than schematic guidelines for what is to count as an adequate formalization. Actual criteria must be discussed on a case-by-case basis, as will be illustrated by the case studies below.

3.1.3   Interpreting the Results

Assume that the historian has undertaken a formalization of an episode in the history of philosophy, and is now looking at the end-product. What does the formalization mean? Has it succeeded in outlining aspects of the historical theory that alternative methodologies had failed to identify?

There is a sense in which the goal of a formalization (of a historical theory or otherwise) is precisely to reveal novel, hidden aspects of its object of study. In some sense, the goal is to obtain a situation of *mismatch* between one's initial beliefs about a given historical theory and the results of the formalization.[4] But if a formalization makes a prediction that is not explicitly to be found in the informal theory being formalized (or vice-versa) – i.e. if there is a mismatch between formalization and what is formalized – then this may mean two things: either the formalization is not sufficiently faithful to the informal theory – in which case it is a 'bad' formalization; or the formalization in fact 'sees' something in the original theory that was not immediately apparent – in which case it is a 'good' formalization in that it is illuminating.

If, however, the historian's prior views on the historical theory and the results of the formalization match completely, then on the one hand one may say that the formalization is entirely accurate and adequate, but on the other hand one may also say that it is uninformative in that it produced no new insights. So there is a sense in which precisely the cases of mismatch are the interesting ones; when mismatch occurs, further analysis is required in order to establish whether it is indeed a novel result revealed by the formalization or rather a sign that it is inadequate.[5]

Again, there is no one-fits-all answer here; in each case, further analysis is required to establish whether a mismatch between initial expectations and the results of the formalization signals inadequacy, or alternatively, novelty and informativeness. This may also be done with a critical stance, i.e. the formalization may be able to outline shortcomings and flaws in the historical theory itself (e.g. the potential invalidity of Anselm's ontological argument). But often, what may appear to be a shortcoming in the historical theory is, on further scrutiny, an unwarranted projection of presuppositions (e.g. some of Łukasiewicz's criticisms of Aristotelian syllogistic). Thus, although a certain amount of critical stance is to be commended, the principle of charity remains an important guideline for the formal historian of philosophy.

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[4]See [13].

[5]For an example of formal analysis actually revealing something new about a historical theory, see [8] on Bradwardine's solution to the Liar paradox.

3.2   Case Studies

To appreciate the (initially) innovative character of applying modern mathematical logic to the analysis of so-called 'traditional logic', it is important to bear in mind that much (though not all) of modern mathematical logic emerged as a *rejection* of traditional logic. But since then (first half of twentieth century), much has changed, and formal methods have been regularly used for the analysis of philosophical theories of the past. In what follows, I discuss three case studies: Aristotle's syllogistic; Anselm's ontological argument; and medieval theories of supposition. By its very nature, the history of *logic* is particularly amenable to formal analysis, but Anselm's ontological argument illustrates a fruitful application of formal methods outside the history of logic.

3.2.1   Syllogistic

The founder of 'formal history of philosophy' is the Polish logician Jan Łukasiewicz, well known for his work on mathematical logic; the historical theory he set out to formalize was Aristotle's syllogistic. In the *Prior Analytics*, Aristotle presents the logical system which became known as syllogistic, whose language contains only four kinds of sentences (*a* and *b* are arbitrary terms):

> A: All *a* is *b*
> I: Some *a* is *b*
> E: No *a* is *b*
> O: Some *a* is not *b*

Aristotle develops a theory of the pairs of such sentences yielding conclusions that 'follow of necessity' – the famous syllogistic arguments. Of the 256 possible combinations, 24 are said by Aristotle to constitute valid arguments. Łukasiewicz became interested in Aristotle's syllogistic already in the 1920s, but his major work on the topic was published only in 1951: his monograph *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* [20]. Łukasiewicz's account of Aristotelian syllogistic can be thus summarized:

> The logic of Aristotle is a theory of the relations A, E, I, and O (in their mediaeval senses) in the field of universal terms. [...] As a logic of terms, it presupposes a more fundamental logic of propositions, which, however, was unknown to Aristotle and was discovered by the Stoics in the century after him. Aristotle's theory is an axiomatized deductive system, in which the reduction of the other syllogistic moods to those of the first figure is to be understood as the proof of these moods as theorems by means of the axioms of the system. ([23], 134)

Crucially, Łukasiewicz formulates syllogistic as an *axiomatic theory* embedded in a *propositional logic*, thus disregarding its original term-based nature. He arrives at the same results as Aristotle (at least in terms of which arguments are deemed

valid or invalid), but his derivations are nothing like Aristotle's own. In particular, he criticizes Aristotle's *per impossibile* proofs of the syllogisms *Baroco* and *Bocardo* (in the medieval terminology) as incorrect, simply because they are not deemed correct within his axiomatic approach. He himself acknowledges that, taking valid syllogisms to be *rules of inference* rather than axioms, Aristotle's proofs are correct, but rather than viewing this as a sign that his axiomatic interpretation might be inadequate, he prefers to attribute the error to Aristotle.[6] Łukasiewicz's formalization in fact imposes "an order on Aristotle's syllogistic, rather than discovering the order within it" ([33], 192).

In the early 1970s, John Corcoran [5, 6] and Timothy Smiley [31] independently presented alternative formalizations of Aristotle's syllogistic; *contra* Łukasiewicz's axiomatic approach, they emphasized the role of rules of inference in the system. Corcoran, for instance, views syllogistic as a *term-based natural deduction system*. Thus, a valid syllogism such as "All *a* is *b*, all *b* is *c*, thus all *a* is *c*", which is rendered as an axiom by Łukasiewicz (in Polish notation):

$$CKAbcAabAac^7$$

is formalized by Corcoran as a rule of inference:

$$Azy + Axz \models Axy$$

In this way, "Corcoran succeeds, as Lukasiewicz did, in reproducing Aristotle's results, and he succeeds, as Lukasiewicz did not, in reproducing Aristotle's method step by step, so that the annotated deductions of his system D are faithful translations of Aristotle's exposition." ([23], 134) Undoubtedly, Corcoran's formalization (as Smiley's) is a great improvement over Łukasiewicz's from the point of view of historical accuracy.

Alongside a presentation of Aristotle's syllogistic as a natural deduction system, Corcoran also introduces a formal semantics for the system, on the basis of families of non-empty sets. He proves that his deductive system is sound and complete with respect to this semantics, and then goes on to argue that this establishes the adequacy of his deductive system. But why is it that *this* particular semantics should serve as yardstick for the adequacy of the deductive system? Corcoran does not offer much motivation for the choice of this semantics, and indeed other semantics for syllogistic have been proposed in the literature [2].

There is no doubt that formal analysis has greatly improved our understanding of syllogistic as a logical system.[8] But the divergences between Łukasiewicz's formalization and Corcoran's also outline the extent to which conceptual, historical analysis of the texts remains crucial, and illustrate the open-ended nature of formalization in history of philosophy.

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[6]See ([29], 37–39).

[7]Polish notation is based on prefixing operators. 'C' stands for implication and 'K' for conjunction, so this expression roughly means *'Abc & Aab ➔ Aac'*.

[8]See for example [1] for some interesting meta-theoretical results, and [18] for a formal analysis of Buridan's modal syllogism.

3.2.2 Anselm's Ontological Argument

Anselm's so-called ontological argument (most famously presented in chapter II of the *Proslogion*, written c. 1077-78) purports to demonstrate the existence of God on the basis of a seemingly plausible definition of God as 'that than which nothing greater can be thought'. More precisely, it purports to show from this definition alone that a contradiction can be derived from the assumption that God does not exist.[9]

Anselm's argument is one of the most discussed arguments in the history of philosophy, and continues to puzzle commentators. Structurally, it is prima facie a plausible argument, but there is something highly unsettling about deriving such a strong conclusion (God exists) from apparently modest premises, by an apparently valid reasoning. Commentators widely disagree on where the problem lies; as summarized by Uckelman ([35], section 5),

> The verdict on the premises range from "obviously true" to "obviously false", and similarly for the validity of the argument(s). The difficulty of determining the soundness and validity of the argument is also located in different places, with some of the various possibilities put forward including the problem of counterfactual reasoning, the role played by the term 'God', the analysis of definite descriptions, substitution into opaque contexts, the definition of perfection, and the nature of possibility. Others believe that the real error of the proof is still to be found, while some believe that the error is as simple as begging the question or the fallacy of equivocation.

(Uckelman provides extensive references to the different commentators holding these views.) Given this interpretive conundrum, it seems that the application of modern logical apparatuses could be of great use to the interpreter. In effect, an adequate formalization might be able to unveil the logical structure of the argument, making hidden assumptions explicit, and bringing to the fore each of the inferential steps in the argument. However, the different formalizations of Anselm's argument proposed in the literature disagree significantly on how best to interpret and analyze it, which again illustrates the fluidity of formalization in research on the history of philosophy: even a single argument, originally expressed in what amounts to half a page of text, is susceptible to receiving highly diverging formal analyses.

Two notable applications of formal tools to Anselm's argument were proposed by Jacquette [16] and Oppenheimer and Zalta [24, 25]. Jacquette argues that the argument has a strong modal component, more precisely an intensional/epistemic component, introduced by the notion of 'that than which nothing greater *can be thought*'. (In fact, arguably there are two intensional layers: one introduced by 'can be' and the other introduced by 'thought'.) On his reconstruction, the argument commits the fallacy of substitution in opaque contexts, as two co-referential terms

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[9]See [34] for a concise presentation of the argument.

(the definiens and the definiendum in the proposed definition of God) cannot be used interchangeably in opaque contexts. Thus, according to Jacquette, the argument is not valid.

While Jacquette focuses on the intensional/epistemic component of the argument, Oppenheimer and Zalta highlight the fact that the definiens in the proposed definition of God corresponds to a *definite description*.[10] Rather than eliminating the definite description, they maintain that, in an analysis/reconstruction of the argument, the phrase should be explicitly represented as such. For this end, they resort to the framework of free logic, which allows for terms or expressions having no denotation. On their reconstruction, the argument comes out as valid, once the proposed logical behavior of definite descriptions is properly spelled out.

Arguably, each of these two formal analyses of Anselm's argument has illuminated a particular central aspect thereof: the intensional/epistemic component for Jacquette, and the definite description component for Oppenheimer and Zalta. In itself, this is not particularly remarkable; as argued in Sect. 3.1.2, a formalization always entails a decision to focus on certain aspects of its object at the expense of others. Thus, it is perfectly conceivable that there might be more than one adequate formalization for the same object. Nonetheless, the fact that these two analyses disagree on their verdict regarding the validity of Anselm's argument does suggest that they cannot both be equally 'right'. Perhaps a unified analysis taking both elements into account would be required to adjudicate the issue.

In any case, formalizations of Anselm's argument illustrate applications of formal methods in history of philosophy going beyond the history of logic strictly speaking. They also illustrate the fact that formalizations always entail theoretical choices, but suggest as well that, while there is typically room for more than one adequate formalization, at times two formalizations turn out to be true competitors that cannot both be adequate.

3.2.3 Medieval Theories of Supposition

Supposition is a key concept in Latin medieval semantics, but the phrase 'medieval theories of supposition' covers a rather heterogeneous group of theories, ranging from the twelfth to the fifteenth century [9]. The fragments of theories of supposition having attracted the attention of contemporary philosophers and logicians are primarily those (seemingly) related to the modern concept of *quantification*, especially the so-called modes of personal supposition [10]. This is in itself quite revealing: in first instance, modern philosophers were mostly interested in the *similarities*, rather than in the differences, between the historical theories in question and modern frameworks. Indeed, from early on, the 'quantificational fragment' of supposition

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[10]([24], 509).

theories was viewed from the perspective of modern conceptions of quantification: "The theory of supposition is, to a very large extent, one with the modern theory of quantification . . . " ([3], 28).[11]

The different modes of personal supposition offer a semantic account of a wide range of what the medieval authors referred to as *syncategorematic terms* ('every', 'not', 'no', 'some', 'only' etc.).[12] This is spelled out by means of inferential relations between sentences where such terms occur, and sentences of the form 'This *a* is *b*', where '*a*' and '*b*' are terms occurring in the original sentences; the latter, the *categorematic terms*, are those said to have such-and-such supposition. (There are also rules specifying in which syntactic contexts, defined by the syncategorematic terms and word order, a term would have such-and-such supposition)

The main modes of personal supposition can be defined as follows. Let (S) and (Q) stand for any syncategorematic term (or combination thereof), and the general form of a sentence P be '(Q) *a* is (S) *b*'. The generic definitions of the modes of personal supposition in terms of inferential relations are[13]:

– A term *a* has *determinate* supposition in P **if and only if**: A disjunction of sentences of the form 'This *a* is (S) *b*' can be inferred from P, but a conjunction of sentences of the form 'This *a* is (S) *b*' cannot be inferred from P**.**
– A term *a* has *confused and distributive* supposition in P **if and only if**: A conjunction of sentences of the form 'This *a* is (S) *b*' can be inferred from P.
– A term *a* has *merely confused* supposition in P **if and only if**: A sentence with a disjunctive subject term of the form 'This *a*, or that *a* etc . . . is (S) *b*' can be inferred from P, but neither a disjunction nor a conjunction of propositions of the form 'This *a* is (S) *b*' can be inferred from P.

The same applies *mutatis mutandis* to predicate terms, so that P can be fully analyzed in terms of disjunctions and conjunctions of simpler sentences (possibly including disjunctive terms). For example, in 'Every *a* is *b*', '*a*' has confused and distributive supposition and 'b' has merely confused supposition; in 'No *a* is *b*' both terms have confused and distributive supposition; in 'Some *a* is *b*' both terms have determinate supposition; and in 'Some *a* is not *b*' '*a*' has determinate supposition and '*b*' has confused and distributive supposition.

Earlier interpreters noted that, while modern theories of quantification are expressed in the formal language of predicate calculus, medieval theories were expressed in the regimented form of Latin used at the time. But if this is merely a superficial difference in modes of expression – that is, if theories of supposition are indeed "one" with modern theories of quantification – then the translation into the language of predicate calculus should be a straightforward affair. Matthews ([21], 99) was the first to challenge this assumption, noting that "Ockham [and medieval

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[11] See also [22], and [4] for an overview focusing specifically on the scholarship on Ockham.

[12] See [26] for an overview from a contemporary perspective.

[13] See (Ockham [36], chap. 70) and ([17], chaps. 4.3.5 and 4.3.6) for some of the original formulations of these definitions.

authors in general] quantifies over terms whereas modern logicians quantify over variables"; thus, "Ockham and the moderns are not free to agree on the interpretation of any categorical propositions". In a similar vein, Henry [14] suggested that, rather than variable-based theories of quantification, an alternative system, namely Leśniewiski's Ontology, would be the right modern system to formalize medieval theories of supposition. Indeed, Ontology is term-based, and the basic sentential form is the traditional subject-copula-predicate form, thus being closer in spirit to the medieval framework. But it brings along yet other presuppositions alien to the supposition framework, and at any rate it never became widely adopted by historians of philosophy. For the most part, formal treatments of supposition theory continued to rely on standard predicate logic [19, 32], with mixed results.

Another challenge for any formalization of supposition theory with modern predicate logic is the definition of merely confused supposition. As seen above, merely confused supposition relies on term-disjunction: "This *a* or that *a* or that other *a* etc. is *b*". Now, in its standard versions, modern predicate logic does not contain the device of term-disjunction (or of term-conjunction, for that matter). It is not an insurmountable problem, and Priest and Read [28] adapted standard predicate logic so as to accommodate term-disjunction. Nevertheless, the need for such adaptations suggests once again that the equation between medieval theories of modes of supposition and modern standard approaches to quantification is by no means straightforward.

Does this mean that medieval theories of supposition are not amenable to investigations with modern logical tools? This conclusion would be unwarranted. Given the striking similarities between portions of Latin medieval semantics and the modern enterprise of formal semantics, it would seem that formal tools can indeed be fruitfully applied here.[14] Nevertheless, as stressed in section "How (not) to apply formal methods in history of philosophy", formalization requires prior and extensive *conceptual analysis*: one must first grasp the historical theory in its own terms so as to determine which modern formalism, if any, might be adequate for a formalization. With respect to theories of supposition, rather than hastily concluding that they are "one" with modern quantification theory, some of the questions to be asked are: what did theories of supposition represent for the medieval authors themselves? What were the purposes assigned to them by these authors? ([7], chap. 1; [4], 11–15)

There is no doubt that the modes of personal supposition deal with 'quantificational phenomena' broadly construed, but a formalization must also do justice to the profound differences between how medieval authors conceptualized these phenomena and the presuppositions underlying modern systems such as predicate logic. Generally, it would seem that the latter is not a particularly suitable system to formalize the former, especially given the centrality of the concept of variable in

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[14][27] is a particularly ambitious and impressive recent example of applications of modern formal tools borrowed from logic and linguistic to medieval logical theories.

the latter and its complete absence in the former. Indeed, it would seem that tailor-made formalisms are more likely to offer informative analyses of these (and other) medieval theories.

3.3 Conclusion

I have here attempted to offer a nuanced picture of the role of formal methods in the study of the history of philosophy. Views on the matter tend to be extreme, split between those who maintain that the application of formal methods for historical analysis is hopelessly anachronistic and thus unwarranted; and those who deem it entirely unproblematic. I have suggested that formalization can be an illuminating approach for the historian of philosophy, but also that it requires careful reflection and conceptual analysis. I have also suggested that, while generally there is not one unique correct formalization of a historical theory, some formalizations are definitely more adequate than others. Ultimately, a formalization must strive to balance the orthogonal desiderata of faithfulness and informativeness; not an easy task, but one with potentially fruitful results.

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[15]There is not much literature specifically on the application of formal methods for the study of the history of philosophy, but the interested reader can consult in particular [12, 15, 33].

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