Chapter 24
Appendix to Chapter 10

24.1 Tariffs, Terms of Trade, Domestic Relative Price

If we assume that country 1 imports commodity A and exports commodity B whilst the opposite holds for country 2, international equilibrium is determined in accordance with Eq. (19.27), which we rewrite here

$$E_{2B}(p) + E_{1B}(p) = 0, \tag{24.1}$$

or

$$E_{2B}(p) = -E_{1B}(p), \tag{24.2}$$

that is, the excess demand for commodity B by country 2 (country 2's demand for imports) is equal in absolute value to the excess supply of this commodity by country 1 (country 1's supply of exports).

In the case that a country, say country 2, levies a duty, the domestic relative price of that country—to which its economic agents respond—is no longer p, but $p_d = p(1+d)$. Therefore E_{2B} will be a function of p_d instead of p. Besides, we must introduce the spending of the revenue by the government, which in real terms is dE_{2B}. We assume that the government spends a fraction $0 < \varphi < 1$ of this revenue to purchase commodity B and the remaining fraction $(1-\varphi)$ to purchase commodity A; consequently country 2's total (private + public) demand for imports will be $(1+\varphi d)E_{2B}$.

Thus have the relations

$$\begin{aligned} (1+\varphi d)\, E_{2B}(p_d) + E_{1B}(p) &= 0, \\ p_d - p(1+d) &= 0. \end{aligned} \tag{24.3}$$

Equation (24.3) constitute a set of two implicit functions in three variables (p_d, p, d). Therefore, provided that the Jacobian of these functions with respect

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to p_d and p is different from zero at the equilibrium point, by the implicit function theorem we can express p_d and p as differentiable functions of d in a neighbourhood of the equilibrium point and perform exercises in comparative statics. In particular, we are interested in the effects on p and p_d of the introduction of a tariff and in determining the conditions for the Metzler and Lerner cases to occur.

The Jacobian of (24.3) is

$$J = \begin{vmatrix} (1+\varphi d)\, E'_{2B} & E'_{1B} \\ 1 & -(1+d) \end{vmatrix} = -(1+\varphi d)(1+d)\, E'_{2B} - E'_{1B}, \tag{24.4}$$

which, evaluated at the initial (free trade) equilibrium point (hence $d = 0$), becomes

$$J = -\left(E'_{2B} + E'_{1B}\right). \tag{24.5}$$

If we multiply and divide by E_{2B}/p we get

$$J = -\frac{E_{2B}}{p}\left(E'_{2B}\frac{p}{E_{2B}} + E'_{1B}\frac{p}{E_{2B}}\right), \tag{24.6}$$

and, since $E_{2B} = -E_{1B}$ in the initial equilibrium situation (in which (24.1) holds), we have

$$J = -\frac{E_{2B}}{p}\left(E'_{2B}\frac{p}{E_{2B}} - E'_{1B}\frac{p}{E_{1B}}\right),$$

that is, by using the definitions of the elasticities given in (19.44) and (19.37),

$$J = -\frac{E_{2B}}{p}\left(\xi_2 - \varepsilon_1\right). \tag{24.7}$$

By using the relation (see (19.42)) $\varepsilon_1 = -(1+\xi_1)$, we finally get

$$J = -\frac{E_{2B}}{p}\left(1 + \xi_1 + \xi_2\right). \tag{24.8}$$

If we now apply Samuelson's correspondence principle and assume that the equilibrium is stable on the basis of the dynamic process of adjustment to excess demand, we can use condition (19.49), that is

$$(1 + \xi_1 + \xi_2) < 0, \tag{24.9}$$

and so

$$J > 0.$$

Let us now calculate p_d' and p', the derivatives of p_d and p with respect to d. By totally differentiating system (24.3) with respect to d we get

$$\begin{aligned} \varphi E_{2B} + (1+\varphi d)\, E_{2B}' p_d' + E_{1B}' p' &= 0, \\ p_d' - p'(1+d) - p &= 0, \end{aligned} \tag{24.10}$$

that is, by using the fact that the derivatives are computed at the initial free-trade equilibrium situation ($d = 0$), and rearranging terms,

$$\begin{aligned} E_{2B}' p_d' + E_{1B}' p' &= -\varphi E_{2B}, \\ p_d' - p' &= p. \end{aligned} \tag{24.11}$$

If we solve for p_d' and p' we get

$$\begin{aligned} p_d' &= \frac{\varphi E_{2B} - E_{1B}' p}{J}, \\ p' &= \frac{\varphi E_{2B} + E_{2B}' p}{J}. \end{aligned} \tag{24.12}$$

By replacing J with expression (24.8) we get

$$\begin{aligned} p_d' &= \frac{p}{E_{2B}} \frac{\varphi E_{2B} - E_{1B}' p}{-(1+\xi_1+\xi_2)} = p\frac{\varphi - E_{1B}' \frac{p}{E_{2B}}}{-(1+\xi_1+\xi_2)} \\ &= p\frac{\varphi + E_{1B}' \frac{p}{E_{1B}}}{-(1+\xi_1+\xi_2)} = p\frac{\varphi + \varepsilon_1}{-(1+\xi_1+\xi_2)}. \end{aligned} \tag{24.13}$$

Similarly we obtain

$$p' = p\frac{\varphi + \xi_2}{-(1+\xi_1+\xi_2)}. \tag{24.14}$$

Given condition (24.9), the sign of p_d' and p' depends only on the numerator of the relevant fraction.

It should be remembered that *Metzler's case* (Metzler, 1949) occurs when, as a consequence of the imposition of a tariff by country 2 on its imports of B, this country's domestic relative price (p_B/p_A) decreases, instead of increasing, with respect to that (equal to the terms of trade) existing in the initial free trade situation. Formally, this amounts to $p_d' < 0$, that is, $\varphi + \varepsilon_1 < 0$. Since $\varepsilon_1 = -(1+\xi_1)$ from (19.42), we have

$$\varphi - \xi_1 - 1 < 0, \tag{24.15}$$

that is,

$$\varphi - \xi_1 < 1. \tag{24.16}$$

In the normal case (non-inferior goods etc.) the elasticity $\xi_1$is negative, so that the condition for Metzler's case to occur is that the sum of the fraction φ and the absolute value of the elasticity of the rest-of-the-world's demand for imports should be smaller than one. This is equivalent to saying that the rest-of-the-world's import demand must be sufficiently rigid. If, on the contrary, we have an abnormal case (for example, commodity A is an inferior good for country 1), the elasticity ξ_1 is positive and (24.16) is satisfied for any non negative φ. This is the case illustrated graphically in Fig. 10.5.

As regards *Lerner's case* (Lerner, 1936), this occurs when, after the imposition of tariff, the terms of trade are higher, instead of being lower, than in the initial free trade situation. In formal terms this means $p' > 0$, that is, given (24.14), $\varphi + \xi_2 > 0$ or

$$-\xi_2 < \varphi. \tag{24.17}$$

As before, two cases must be distinguished. In the normal case the elasticity ξ_2 is negative, so that the condition for Lerner's case to occur is that the tariff-imposing country's demand for imports is sufficiently rigid, with an elasticity in absolute value smaller than the fraction φ. On the contrary, in abnormal cases (for example, when commodity B is an inferior good for country 2), the elasticity ξ_2 is positive and (24.17) is verified for any non-negative φ. This is the case illustrated graphically in Fig. 10.6.

24.2 Cartels

Let q_i be the quantity produced by the i–th country participating in the cartel, and $C_i(q_i)$ the corresponding total cost. The whole output $q = \sum_{i=1}^{n} q_i$, is sold by the cartel as a monopolist. If we denote total revenue by $R = p \cdot q$, where p is related to q through the demand function, the problem is to maximize the profit function

$$\pi = R(q) - \sum_{i=1}^{n} C_i(q_i) = R\left(\sum_{i=1}^{n} q_i\right) - \sum_{i=1}^{n} C_i(q_i). \tag{24.18}$$

The first order conditions are

$$\frac{\partial \pi}{\partial q_i} = R' - C_i' = 0, \tag{24.19}$$

that is,

$$R' - C_1' = C_2' = \ldots = C_n'. \tag{24.20}$$

Marginal cost in each country must equal the marginal revenue of the output as a whole.

The second order conditions require that the leading principal minors of the Hessian

$$\begin{bmatrix} R'' - C_1'' & R'' & R'' & \ldots & R'' \\ R'' & R'' - C_2'' & R'' & \ldots & R'' \\ \ldots.. & \ldots.. & \ldots.. & \ldots.. & \ldots.. \\ R'' & R'' & R'' & \ldots & R'' - C_n'' \end{bmatrix} \tag{24.21}$$

alternate in sign, beginning with minus. In the normal case, $R'' < 0$ and $C_i'' > 0$, so that the second order conditions are satisfied.

In the case of a quasi-monopolistic cartel, the demand for the cartel's output is, by definition, equal to the difference between total world demand for the commodity, D, and the supply of independent producers, S, that is, for any given price p,

$$D_c(p) = D(p) - S(p), \tag{24.22}$$

so that

$$\frac{\mathrm{d}D_c}{\mathrm{d}p} = \frac{\mathrm{d}D}{\mathrm{d}p} - \frac{\mathrm{d}S}{\mathrm{d}p}. \tag{24.23}$$

By simple manipulations, we get

$$-\frac{D_c}{p}\left(-\frac{p}{D_c}\frac{\mathrm{d}D_c}{\mathrm{d}p}\right) = -\frac{D}{p}\left(-\frac{p}{D}\frac{\mathrm{d}D}{\mathrm{d}p}\right) - \frac{S}{p}\left(\frac{p}{S}\frac{\mathrm{d}S}{\mathrm{d}p}\right). \tag{24.24}$$

We now define the various elasticities

$$\eta_c \equiv -\frac{p}{D_c}\frac{\mathrm{d}D_c}{\mathrm{d}p}, \quad \eta_w \equiv -\frac{p}{D}\frac{\mathrm{d}D}{\mathrm{d}p}, \quad \eta_s \equiv \frac{p}{S}\frac{\mathrm{d}S}{\mathrm{d}p}, \tag{24.25}$$

so that (24.24) becomes

$$-\frac{D_c}{p}\eta_c = -\frac{D}{p}\eta_w - \frac{S}{p}\eta_s, \tag{24.26}$$

whence

$$\eta_c = \frac{D\eta_w + S\eta_s}{D_c} = \frac{\eta_w + (S/D)\,\eta_s}{D_c/D}. \tag{24.27}$$

The fraction D_c/D is the cartel's share in total world consumption, that we denote by k; given Eq. (24.22) we have $S/D = 1-k$. Therefore the final formula is

$$\eta_c = \frac{\eta_w + (1-k)\,\eta_s}{k}. \tag{24.28}$$

24.3 The Effective Rate of Protection

In the general case of n intermediate goods, the pre-tariff value added in sector j is

$$v_j = p_j - \sum_{i=1}^{n} p_i q_{ij} = p_j \left(1 - \sum_{i=1}^{n} a_{ij}\right), \tag{24.29}$$

where $a_{ij} = p_i q_{ij}/p_j$.

After the introduction of a tariff schedule we have

$$v'_j = \left(1 + d_j\right) p_j - \sum_{i=1}^{n} (1 + d_i)\, p_i q_{ij} = p_j \left[\left(1 + d_j\right) - \sum_{i=1}^{n} (1 + d_i)\, a_{ij}\right], \tag{24.30}$$

so that the effective rate of protection turns out to be

$$g_j = \frac{v'_j - v_j}{v_j} = \frac{d_j - \sum_{i=1}^{n} a_{ij} d_i}{1 - \sum_{i=1}^{n} a_{ij}} = d_j + \frac{\left(d_j - \overline{d}_i \sum_{i=1}^{n} a_{ij}\right)}{1 - \sum_{i=1}^{n} a_{ij}}, \tag{24.31}$$

where $\overline{d}_i = \sum_{i=1}^{n} a_{ij} d_i / \sum_{i=1}^{n} a_{ij}$ is a weighted average of the nominal tariff rates. It immediately follows from (24.31) that the same conclusions reached in the text in the case of a single intermediate good hold in the general case as well, if we consider the average rate $\overline{d}_i$ in the place of d_i.

This analysis, it should be noted, is based on the simplifying assumptions of fixed input coefficients of intermediate goods which are all traded. For a more general analysis which relaxes these assumptions see, for example, Various Authors (1973), Yabuuchi and Tanaka (1981, and references therein).

A second observation concerns the definition itself of effective rate of protection. The one used in the text and here is that originally suggested by Corden (1966), who subsequently (Corden, 1969) suggested an alternative definition, namely the proportionate change (due to the tariff structure) in the "price of value added". In general the two definitions give different results, but in the case of separable production functions with fixed input coefficients of intermediate goods they coincide (see, for example, Bhagwati and Srinivasan in Various Authors, 1973).

24.4 Imperfect Competition and Trade Policy

24.4.1 A Tariff Under Vertical Product Differentiation

We consider the effects on the returns to capital and on the range of intra-industry trade of a tariff imposed by country 1 in the context of the model examined in Sect. 23.1.

As regards the returns to capital, the increase in the tariff-inclusive prices of the qualities imported by country 1 will give rise to a range of qualities that country 1 can now produce at a lower cost than the cum-tariff import price instead of importing them as before. Country 1's consumers switch from imports to domestic production of these qualities, hence the demand for domestic capital grows and that for foreign capital decreases. The impact effect is a tendency for the domestic return to capital to increase and for the foreign return to capital to decrease, but the final effect is less clear-cut. Formally, if we introduce the tariff rate d in the demands for capital D_{1K}, D_{2K} as a shift parameter and differentiate the excess demand functions totally, we get

$$E^1_{R_1}\mathrm{d}R_1 + E^1_{R_2}\mathrm{d}R_2 + E^1_d\mathrm{d}d = 0, \tag{24.32}$$

$$E^2_{R_1}\mathrm{d}R_1 + E^2_{R_2}\mathrm{d}R_2 + E^2_d\mathrm{d}d = 0, \tag{24.33}$$

where $E^1_d > 0$, $E^2_d < 0$ according to the impact effect. Also note that $E^1_d + E^2_d < 0$, because at the world level there is a net decrease in the demand for capital since overall prices are higher. Hence by solving this system for $\mathrm{d}R_1$, $\mathrm{d}R_2$ we obtain the final effect

$$\mathrm{d}R_1 = -\frac{E^1_d E^2_{R_2} - E^2_d E^1_{R_2}}{\Delta}\mathrm{d}d, \tag{24.34}$$

$$\mathrm{d}R_2 = -\frac{E^2_d E^1_{R_1} - E^1_d E^2_{R_1}}{\Delta}\mathrm{d}d. \tag{24.35}$$

Given our assumptions we have $\left|E^1_{R_1}\right| > \left|E^2_{R_1}\right|$, $\left|E^2_{R_2}\right| > \left|E^1_{R_2}\right|$ and $\left|E^1_d\right| < \left|E^2_d\right|$, so that from (24.35) we have $\mathrm{d}R_2/\mathrm{d}d < 0$; but the sign of $\mathrm{d}R_1/\mathrm{d}d$ remains ambiguous.

24.4.1.1 Tariffs and Intra-industry Trade

Let us now consider the effects on intra-industry trade. We have stated above that the tariff imposed by country 1 will give rise to a range of qualities that country 1 can now produce at a lower cost than the cum-tariff import price instead of importing them as it did in the pre-tariff situation. Country 2 will of course go on producing these qualities for its internal consumption. More precisely, we must now distinguish two marginal qualities, $\alpha_2^d < \alpha_1^d$, with country 2 being the sole producer in the range $(\underline{\alpha},\alpha_2^d)$, country 1 only producing in the range $(\alpha_1^d, \overline{\alpha})$,and both countries producing (but neither trading) in the range (α_2^d, α_1^d). To determine these marginal qualities we first observe that country 1, account being taken of the tariff, will import a quality α, be indifferent between importing it or producing it domestically, produce it domestically, according as $p_1(\alpha) \gtreqless (1+d)p_2(\alpha)$; similarly country 2 will import a quality α, be indifferent between importing it or producing it domestically, produce it domestically, according as $p_2(\alpha) \gtreqless p_1(\alpha)$. Hence the two marginal qualities are defined by

$$p_1(\alpha_2^d) = (1+d)p_2(\alpha_2^d), \tag{24.36}$$

$$p_1(\alpha_1^d) = p_2(\alpha_1^d). \tag{24.37}$$

If we take account of Eq. (9.1), from (24.36) we get

$$W_1 + \alpha_2^d R_1 = (1+d)(W_2 + \alpha_2^d R_2),$$

whence

$$\alpha_2^d = \frac{W_1 - (1+d)W_2}{(1+d)R_2 - R_1}, \tag{24.38}$$

and from (24.37) we get

$$\alpha_1^d = \frac{W_1 - W_2}{R_2 - R_1}, \tag{24.39}$$

where R_1, R_2 are the cum-tariff rental rates.

It is easy to check that $p_1(\alpha) \gtreqless (1+d)p_2(\alpha)$ according as $\alpha \lesseqgtr \alpha_2^d$, and that $p_2(\alpha) \gtreqless p_1(\alpha)$ according as $\alpha \gtreqless \alpha_1^d$. Hence country 1 will import the qualities lower than α_2^d, and country 2 will import the qualities higher than α_1^d. When $d = 0$, it is clear that $\alpha_2^d = \alpha_1^d = \alpha_0$, and we are back in the initial free trade situation. To complete our demonstration we must show that $\alpha_2^d < \alpha_1^d$. This follows from the fact that the fraction in (24.38) has both a greater denominator and a smaller numerator than the fraction in (24.39). Hence in the range (α_2^d, α_1^d) both countries will produce but neither will trade. It is also easy to see that α_2^d is a decreasing function of d, hence the range of non-traded qualities is an increasing function of the tariff rate.

24.4.2 Monopolistic Competition and Welfare-Improving Tariff

Let us examine commercial policy in the context of the model studied in Sect. 23.2, in particular the effects of the imposition of a tariff. A surprising result (Helpman & Krugman, 1989; Venables, 1987) is that the imposition of a tariff seems to cause a decrease in the consumer price index of differentiated goods in the tariff-imposing country, which will then be unambiguously better off.

To show this, let us assume that country 1 imposes a tariff at the rate d on imports of the differentiated good, but not on imports of the homogeneous good. The domestic price of the imported goods will rise to $(1+d)p$, hence the market-clearing condition for country 2's firm becomes

$$x_2 = (1+\phi)^{1-\sigma}(1+d)^{-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\epsilon} + B_2 p^{-\sigma} P_2^{\sigma-\epsilon}, \tag{24.40}$$

where $x_2 = x$ as before. To ascertain the effects of the imposition of a tariff on the transformed consumer-price indices for the differentiated goods we compute the differentials of Eqs. (23.24) and (24.40) with respect to d. These are

$$\begin{aligned} &B_1 p^{-\sigma} \mathrm{d}(P_1^{\sigma-\epsilon}) + (1+\phi)^{1-\sigma} B_2 p^{-\sigma} \mathrm{d}(P_2^{\sigma-\epsilon}) = 0, \\ &[(1+\phi)^{1-\sigma}(1+d)^{-\sigma} B_1 p^{-\sigma}]\mathrm{d}(P_1^{\sigma-\epsilon}) + B_2 p^{-\sigma} \mathrm{d}(P_2^{\sigma-\epsilon}) \\ &= [(1+\phi)^{1-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\epsilon}]\sigma(1+d)^{-\sigma-1}\mathrm{d}d, \end{aligned} \tag{24.41}$$

from which

$$\begin{aligned} \frac{\mathrm{d}P_1^{\sigma-\epsilon}}{\mathrm{d}d} &= -\{[(1+\phi)^{1-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\epsilon}]\sigma(1+d)^{-\sigma-1}\}(1+\phi)^{1-\sigma} B_2 p^{-\sigma}/\Delta, \\ \frac{\mathrm{d}P_2^{\sigma-\epsilon}}{\mathrm{d}d} &= \{B_1 p^{-\sigma}[(1+\phi)^{1-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\epsilon}]\sigma(1+d)^{-\sigma-1}\}/\Delta, \end{aligned} \tag{24.42}$$

where

$$\begin{aligned} \Delta &\equiv B_1 p^{-\sigma} B_2 p^{-\sigma} - [(1+\phi)^{1-\sigma}(1+d)^{-\sigma} B_1 p^{-\sigma}][(1+\phi)^{1-\sigma} B_2 p^{-\sigma}] \\ &= B_1 B_2 p^{-2\sigma}[1-(1+\phi)^{2(1-\sigma)}(1+d)^{-\sigma}] \end{aligned}$$

is positive, because both $(1+\phi)^{2(1-\sigma)}$ and $(1+d)^{-\sigma}$ are smaller than one, given the definition of σ. The numerator of $\mathrm{d}(P_1^{\sigma-\epsilon})/\mathrm{d}d$ is clearly negative while the numerator of $\mathrm{d}(P_2^{\sigma-\epsilon})/\mathrm{d}d$ is positive. These signs remain valid when the derivatives are evaluated at the pre-tariff point ($d = 0$).

Hence the imposition of a tariff causes a decrease in the (transformed) price index of differentiated goods in the tariff-imposing country and an increase in the other country's index. Is this enough to say (as in Flam & Helpman, 1987; Helpman &

Krugman, 1989; Venables, 1987, chap. 7) that a tariff is beneficial? Not at all. What we have shown is that the transformed price indices vary in the directions indicated. What we need to know is how the price indices themselves vary. This depends on the sign of $\sigma - \epsilon$. If this is positive, then the price indices will vary in the same direction as the transformed indices, and the result of the welfare-improving effect of a tariff holds. But this is no longer true when $\sigma - \epsilon$ is negative: in this case the actual price index of the tariff-imposing country will increase, leading to the standard result of a welfare loss. Hence, as noted by Helpman (1990, chap. 4), all depends on the magnitude of the elasticity of substitution in the consumer's subutility function relative to the magnitude of the price-elasticity of aggregate demand.

Be it as it may, the economic reason behind the result that a tariff may improve welfare is the "home market effect": the protected home market becomes a preferential place where to produce to supply goods also to the foreign market. The gains, when the domestic price index falls, derive from the fact that domestic consumers obtain a greater number of cheaper domestic goods and a smaller number of more expensive foreign goods at an overall cost, as measured by the price index, which is lower than in the pre-tariff situation.

24.4.3 Strategic Trade Policy Under Oligopoly with Homogeneous Good

24.4.3.1 Tariffs

In the context of the model treated in Sect. 23.3.1 there is a particularly convenient way of dealing with tariffs, namely to assume that the tariff is levied in terms of the commodity being exported to the tariff-imposing country. This means that, if a quantity x is being exported from, say, country 2 to country 1, a quantity $(1 - d)x$ will actually reach country 1's market, where d is the tariff rate imposed by country 1. Hence $(1 - d)$ can be treated exactly like g, the transport-cost parameter. An increase (decrease) in g can now be taken as a decrease (increase) in the tariff rate. We already know that a decrease in g causes a decrease in q_{21}^E and an increase in q_{11}^E. The overall quantity is

$$q_{21}^E + q_{11}^E = \frac{2a - c - c/g}{3b}, \tag{24.43}$$

which clearly varies in the same direction as g. Hence, the size of the market decreases as the tariff rate increases. Given the market's downward-sloping demand curve, the price will increase.

As regards the generalisation to the free entry case, we must carefully distinguish two cases. The first is when the number of firms in each economy is arbitrarily fixed or, more precisely, taken as exogenously given and unchanged by trade. This case is not very interesting; besides, the results are ambiguous. The interesting case arises when the number of firms is endogenously determined. Venables (1985) studied the effects of tariffs in such a case, and proved that the imposition of a tariff unambiguously raises welfare in the tariff-imposing country and reduces welfare in the other country. For details see Venables (1985, sect. 6) and Helpman and Krugman (1989, sect. 7.5).

24.4.3.2 Subsidies

For this purpose we consider the case in which the two firms only produce for export and compete in a third market (Brander & Spencer, 1985) . Let

$$p = p(q), \quad q = q_1 + q_2, \quad p' \equiv \mathrm{d}p/\mathrm{d}q < 0 \tag{24.44}$$

be the third market's demand function, where q_i is the quantity offered by firm i. Without loss of generality we can take country 1 as the home country, that subsidizes the domestic firm.

The domestic firm maximizes profit π_1 given by

$$\pi_1 = q_1 p(q) - c_1(q_1) + sq_1, \tag{24.45}$$

where $c_1(q_1)$ is the cost function and s the per-unit subsidy. Since the firm behaves like a Cournot duopolist, the first- and second-order conditions are

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &\equiv \pi_1' = q_1 p' + p - c_1' + s = 0, \\ \frac{\partial^2 \pi_1}{\partial q_1^2} &\equiv \pi_1'' = 2p' + q_1 p'' - c_1'' < 0. \end{aligned} \tag{24.46}$$

For country 2's firm (that receives no subsidy) we have the profit function and the optimum conditions:

$$\begin{aligned} \pi_2 &= q_2 p(q) - c_2(q_2), \\ \frac{\partial \pi_2}{\partial q_2} &\equiv \pi_2' = q_2 p' + p - c_2', \\ \frac{\partial^2 \pi_2}{\partial q_2^2} &\equiv \pi_2'' = 2p' + q_2 p'' - c_2'' < 0. \end{aligned} \tag{24.47}$$

The first-order conditions, as usual in Cournot models, define the reaction functions implicitly. Brander and Spencer also introduce the additional conditions

$$\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} \equiv \frac{\partial \pi_1'}{\partial q_2} = p' + q_1 p'' < 0; \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} \equiv \frac{\partial \pi_2'}{\partial q_1} = p' + q_2 p'' < 0, \tag{24.48}$$

$$\frac{\partial^2 \pi_1}{\partial q_1^2} < \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}; \quad \frac{\partial^2 \pi_2}{\partial q_2^2} < \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}. \tag{24.49}$$

Conditions (24.48) state that each firm's marginal revenue declines with an increase in the quantity produced by the other firm. Conditions (24.49) mean that the own effects of output on marginal profit are greater (in absolute value) than the cross effects. Note that conditions (24.49)—given conditions (24.48) and the second-order conditions—are always satisfied if marginal cost is nondecreasing.

Given the second-order conditions, inequalities (24.48) imply that the reaction functions are downward sloping. Consider, for example, the domestic firm, whose reaction function is implicitly given by the first-order optimum condition $\partial \pi_1 / \partial q_1 = 0$. By the implicit function theorem we can calculate the slope of the reaction function R_1 as

$$\left(\frac{dq_1}{dq_2}\right)_{R_1} = -\frac{\partial^2 \pi_1 / \partial q_1 \partial q_2}{\partial^2 \pi_1 / \partial q_1^2}, \tag{24.50}$$

which is negative, given $\partial^2 \pi_1 / \partial q_1^2 < 0$, when $\partial^2 \pi_1 / \partial q_1 q_2 < 0$. Similarly we obtain the slope of the reaction function R_2

$$\left(\frac{dq_1}{dq_2}\right)_{R_2} = -\frac{\partial^2 \pi_2 / \partial q_2^2}{\partial^2 \pi_2 / \partial q_2 \partial q_1} < 0. \tag{24.51}$$

Together, conditions (24.48) and (24.49) imply

$$D \equiv \frac{\partial^2 \pi_1}{\partial q_1^2} \frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} > 0. \tag{24.52}$$

Let us now go on to comparative statics. We first calculate the effects of the subsidy on outputs and price. Since, at the equilibrium point, the Jacobian of the system formed by the two first-order conditions is different from zero (this Jacobian is simply D), by the implicit function theorem we can express q_1, q_2 as differentiable functions of the parameter s. Then we can compute the derivatives dq_1/ds, dq_2/ds by differentiating the first order-conditions with respect to s. This gives

$$\frac{\partial^2 \pi_1}{\partial q_1^2} \frac{dq_1}{ds} + \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} \frac{dq_2}{ds} = -1, \tag{24.53}$$

$$\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} \frac{dq_1}{ds} + \frac{\partial^2 \pi_2}{\partial q_2^2} \frac{dq_2}{ds} = 0, \tag{24.54}$$

whence

$$\frac{\mathrm{d}q_1}{\mathrm{d}s} = -\frac{\partial^2 \pi_2}{\partial q_2^2}/D > 0, \tag{24.55}$$

$$\frac{\mathrm{d}q_2}{\mathrm{d}s} = \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}/D < 0, \tag{24.56}$$

$$\frac{\mathrm{d}q_1}{\mathrm{d}s} + \frac{\mathrm{d}q_2}{\mathrm{d}s} = \left(\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} - \frac{\partial^2 \pi_2}{\partial q_2^2}\right)/D > 0, \tag{24.57}$$

where the signs derive from (24.46), (24.48), (24.49), (24.52). This shows that an increase in the subsidy causes a decrease in the foreign firm's output and an increase in the output of the domestic firm, a fairly intuitive result (shown in the text, Fig. 10.12). It also shows that total output $q_1 + q_2$ increases, and hence that price decreases, given the downward-sloping demand function (24.44).

Let us now examine the effects of the subsidy on profits. For the domestic firm we have

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}s} = \frac{\partial \pi_1}{\partial q_1}\frac{\mathrm{d}q_1}{\mathrm{d}s} + \frac{\partial \pi_1}{\partial q_2}\frac{\mathrm{d}q_2}{\mathrm{d}s} + q_1,$$

hence, since $\frac{\partial \pi_1}{\partial q_1} = 0$ by the first-order conditions,

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}s} = \frac{\partial \pi_1}{\partial q_2}\frac{\mathrm{d}q_2}{\mathrm{d}s} + q_1 = q_1 p' \frac{\mathrm{d}q_2}{\mathrm{d}s} + q_1 > 0. \tag{24.58}$$

For the foreign firm we have

$$\frac{\mathrm{d}\pi_2}{\mathrm{d}s} = \frac{\partial \pi_2}{\partial q_1}\frac{\mathrm{d}q_1}{\mathrm{d}s} + \frac{\partial \pi_2}{\partial q_2}\frac{\mathrm{d}q_2}{\mathrm{d}s} = q_2 p' \frac{\mathrm{d}q_1}{\mathrm{d}s} < 0. \tag{24.59}$$

These results show that a subsidy increases domestic profit and lowers foreign profit.

The additional (and less intuitive) effect of the subsidy is to increase domestic surplus (net of the subsidy). Domestic surplus $G(s)$ is defined as the domestic firm's profit (deriving from exports) minus the cost of the subsidy:

$$G(s) = \pi_1 - s q_1 \tag{24.60}$$

hence

$$\begin{aligned} \frac{\mathrm{d}G}{\mathrm{d}s} &= \frac{\mathrm{d}\pi_1}{\mathrm{d}s} - q_1 - s\frac{\mathrm{d}q_1}{\mathrm{d}s} \\ &= q_1 p' \frac{\mathrm{d}q_2}{\mathrm{d}s} - s\frac{\mathrm{d}q_1}{\mathrm{d}s}, \end{aligned} \tag{24.61}$$

where we have used (24.58) to substitute for $\mathrm{d}\pi_1/\mathrm{d}s$. At $s = 0$, $\mathrm{d}G/\mathrm{d}s$ is clearly positive since we have shown above that $\mathrm{d}q_2/\mathrm{d}s < 0$. This shows that a marginal increase in the subsidy (from a zero-subsidy situation) increases domestic welfare.

It can also be shown that the optimal subsidy, namely the subsidy that maximizes domestic surplus, is positive. In fact, setting $\mathrm{d}\,G/\mathrm{d}s = 0$ we get

$$s = q_1 p' \frac{\mathrm{d}q_2}{\mathrm{d}s} / \frac{\mathrm{d}q_1}{\mathrm{d}s} > 0. \tag{24.62}$$

Actually, the optimal domestic subsidy moves the domestic firm from a Cournot equilibrium to a Stackelberg equilibrium with the domestic firm as leader. To show this, let us consider what would, in the absence of the subsidy, be the Stackelberg equilibrium with the domestic firm as leader. Without the subsidy, the domestic firm's profit is

$$\pi_1 = q_1 p(q_1 + q_2) - c_1(q_1).$$

The Stackelberg leader (see, for example, Varian, 1992, chap. 16) chooses its optimal quantity taking into account that the follower will react along its Cournot reaction curve. In other words, firm 1 does not take q_2 as given, but knows that $q_2 = f(q_1)$ along firm 2's reaction curve R_2. Firm 2, the follower, continues to behave like a Cournot duopolist.

Thus firm 1's optimum condition is

$$\begin{aligned}\pi_1' &= q_1 p' + q_1 p' \left(\frac{\mathrm{d}q_2}{\mathrm{d}q_1}\right)_{R_2} + p - c_1' \\ &= q_1 p' - q_1 p' \frac{\partial^2 \pi_2/\partial q_2 \partial q_1}{\partial^2 \pi_2/\partial q_2^2} + p - c_1' \\ &= 0,\end{aligned} \tag{24.63}$$

where we have used (24.51).

If we now consider the first-order optimum condition for firm 1 when it behaves like a Cournot duopolist with subsidy—see Eq. (24.46)—and substitute the optimum subsidy as given by (24.62) we get

$$\begin{aligned}\pi_1' &= q_1 p' + p - c_1' + q_1 p' \frac{\mathrm{d}q_2}{\mathrm{d}s} / \frac{\mathrm{d}q_1}{\mathrm{d}s} \\ &= q_1 p' + p - c_1' - q_1 p' \frac{\partial^2 \pi_2/\partial q_2 \partial q_1}{\partial^2 \pi_2/\partial q_2^2} \\ &= 0,\end{aligned} \tag{24.64}$$

where we have used (24.55) and (24.56) to substitute for $\mathrm{d}q_1/\mathrm{d}\,s$ and $\mathrm{d}q_2/\mathrm{d}s$. Conditions (24.63) and (24.64) are identical, which proves the statement.

For further analysis of strategic trade policy in the context of the Brander-Spencer model see, for example, Brainard and Martimort (1997) and Bandyopadhyay (1997). For the case in which firms behave like Bertrand duopolists (i.e., their decisional variable is price rather than quantity) see Neary (1991).

24.4.4 Strategic Trade Policy Under Oligopoly with Differentiated Good

It is easy to see that in the model treated in Sect. 23.5 free trade is not the first-best policy for the foreign country, which can improve its welfare by imposing a tariff on imports of the differentiated commodity. Let us consider a specific tariff d^*. Given the assumptions, the selling price will have to remain the same in both countries as in the free trade situation. Hence, the consumers in the tariff-imposing country will suffer no loss, and the country will have a gain which coincides with $d^*n_2^*$, the fiscal revenue from the tariff. This of course will happen provided that the producing firm finds that after the tariff the alternative of exporting in addition to serving the domestic market remains more profitable than the alternative of serving the domestic market only.

To analyse this point let us observe that the specific tariff can be considered as an additional cost to the producing firm as regards the part of its output exported. Hence its profit will become

$$[\overline{p} - (\theta_2 - \theta_1) - c]n_1 + [\overline{p} - (\theta_2 - \theta_1) - (c + d^*)]n_2^* - K, \tag{24.65}$$

which has to be compared with the profit of serving only the domestic market, $(\overline{p} - c)n_1 - K$. The domestic firm will be indifferent when these two expressions are equal. Thus the optimal specific tariff d_E^*, that is to say the specific tariff that taxes away from the producing firm all profits in excess of profits it makes by selling only to the domestic market, is easily computed by equating the two expressions, from which

$$d_E^* = [\overline{p} - (\theta_2 - \theta_1) - c]\frac{n_1 + n_2^*}{n_2^*} - (\overline{p} - c)\frac{n_1}{n_2^*}. \tag{24.66}$$

If there was trade in the pre-tariff situation, condition (23.60) above had to be satisfied, hence d_E^* is clearly positive.

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