Chapter 25
Appendix to Chapter 11

25.1 The Optimum Tariff

If we denote by v the social welfare function having the quantities demanded (consumed) of the two commodities as arguments, we have, for country 2,

$$v = v\left(A_2^D, B_2^D\right) = v\left(A_2 + E_{2A}, B_2 + E_{2B}\right), \tag{25.1}$$

as $E_{2A} = A_2^D - A_2$ etc. (see Sect. 19.3). We have to maximize (25.1) under the constraints of country 1's offer curve and of the relations linking the variables of the model of general international equilibrium. Instead of using Lagrange multipliers, it is simpler here to introduce the constraints directly into the maximand. For this purpose, it should be remembered that $A_2 = \psi(B_2)$ through country 2's transformation curve, that $E_{2B} = -E_{1B}$, that $E_{2A} = -E_{1A} = pE_{1B}$ (see in particular Eqs. (19.25) and (19.27)). We thus have to maximize

$$v = \left[\psi(B_2) + pE_{1B}(p), B_2 - E_{1B}(p)\right], \tag{25.2}$$

with respect to its arguments, which are now B_2 and p. We obtain the first-order conditions (for brevity, we ignore the second order ones)

$$\begin{aligned} \frac{\partial v}{\partial B_2} &= v_A\psi' + v_B = 0, \\ \frac{\partial v}{\partial p} &= v_A\left(E_{1B} + E'_{1B}p\right) - v_B E'_{1B} = 0. \end{aligned} \tag{25.3}$$

From the first, we get

$$v_B/v_A = -\psi', \tag{25.4}$$

G. Gandolfo, *International Trade Theory and Policy*, Springer Texts in Business and Economics, DOI 10.1007/978-3-642-37314-5_25,

and from the second, with simple manipulations,

$$E_{1B}\left[v_A\left(1+E'_{1B}\frac{p}{E_{1B}}\right)-v_B\frac{1}{p}E'_{1B}\frac{p}{E_{1B}}\right]=0, \tag{25.5}$$

whence, given the definition of ε_1 in (19.37) and rearranging terms,

$$\frac{v_B}{v_A}=p\frac{1+\varepsilon_1}{\varepsilon_1}. \tag{25.6}$$

From (25.4) and (25.6) we obtain

$$-\psi'=p\frac{1+\varepsilon_1}{\varepsilon_1}. \tag{25.7}$$

Since (see Sect. 19.1) in equilibrium the marginal rate of transformation equals country 2's domestic relative price, which in turn equals the terms of trade plus tariff, we have

$$p\,(1+d)=p\frac{1+\varepsilon_1}{\varepsilon_1}, \tag{25.8}$$

whence

$$d=\frac{1}{\varepsilon_1}. \tag{25.9}$$

Equation (25.9) states that the optimum tariff for country 2 equals the reciprocal of the elasticity of *country* 1*'s* supply of exports. By using relation (19.41), we can also write

$$d=\frac{1-e_1}{e_1}=\frac{1}{e_1}-1, \tag{25.10}$$

that is, the optimum tariff for country 2 equals the reciprocal of the elasticity of *country* 1*'s* offer curve reduced by one.

In some treatments (see, for example, Johnson, 1950, p. 58 of the 1958 reprint) one finds the following formula for country 2's optimum tariff:

d = elasticity of country 1's offer curve reduced by one

but this depends on the different definition of the elasticity of an offer curve.

25.2 The Theory of Second Best

A Pareto-optimum can always be considered as the solution of a constrained maximum problem. Following Lipsey and Lancaster (1956) consider the following problem

$$\max F\left(x_1, x_2, \ldots, x_n\right),$$

$$\text{subject to } \psi\left(x_1, x_2, \ldots, x_n\right) = 0, \tag{25.11}$$

where, for simplicity, the constraint has been written as an equality. The solution, if we assume that it is found at an interior point, will be characterized by the conditions obtained by maximizing the Lagrangian

$$L = F\left(x_1, x_2, \ldots, x_n\right) - \lambda\psi\left(x_1, x_2, \ldots, x_n\right),$$

where λ is a Lagrange multiplier. The Paretian conditions are given by the n first-order conditions

$$F_i - \lambda\psi_i = 0, \quad i = 1, 2, \ldots, n, \tag{25.12}$$

where the subscript i denotes the partial derivative with respect to the i–th variables. These conditions can also be written as

$$\frac{F_1}{\psi_1} = \frac{F_2}{\psi_2} = \ldots = \frac{F_n}{\psi_n}. \tag{25.13}$$

Let us now assume that an additional constraint (a distortion) prevents the fulfilment of one of these conditions, for example the first one, so that

$$\frac{F_1}{\psi_1} \neq \frac{F_2}{\psi_2},$$

that is

$$\frac{F_1}{\psi_1} = k\frac{F_2}{\psi_2}, \quad k \neq 1, \tag{25.14}$$

whence

$$\frac{F_1}{F_2} = k\frac{\psi_1}{\psi_2}. \tag{25.15}$$

It is not necessary for k to be constant, but, for simplicity, we shall assume it is. The presence of the additional constraint (25.15) requires the reformulation of the optimum problem in the form

$$\max F\left(x_1, x_2, \ldots, x_n\right)$$

subject to

$$\begin{aligned} \psi\left(x_1, x_2, \ldots, x_n\right) &= 0, \\ \frac{F_1}{F_2} = k\frac{\psi_1}{\psi_2} \quad &= 0. \end{aligned} \tag{25.16}$$

If we maximize the Lagrangian

$$L' = F\left(x_1, x_2, \ldots, x_n\right) - \lambda'\psi\left(x_1, x_2, \ldots, x_n\right) - \mu\left(\frac{F_1}{F_2} - k\frac{\psi_1}{\psi_2}\right),$$

we obtain the new optimum conditions

$$F_i - \lambda'\psi - \mu\left(\frac{F_2F_{1i} - F_1F_{2i}}{F_2^2} - k\frac{\psi_2\psi_{1i} - \psi_1\psi_{2i}}{\psi_2^2}\right) = 0, i = 1, 2, \ldots, n. \tag{25.17}$$

We can now ask whether the conditions for the second best optimum, namely (25.17), are the same as those for the first best Pareto optimum for $i = 2, \ldots, n$, that is, whether in a situation in which one of the Pareto-optimum conditions cannot be fulfilled, the second best solution is obtained by fulfilling the remaining Pareto-optimum conditions. By comparing (25.17) with (25.12), we see that the answer is affirmative if, and only if,

(a) $\mu = 0$, or
(b) $\mu \neq 0$, but the expression in parentheses in (25.17) is zero for all i.

Case (a) Must be excluded, as it can be seen from (25.17) that for $i = 1, 2$ this would imply $F_1/\psi_1 = F_2/\psi_2$, which is excluded by (25.14).

We are left with case (b), which cannot be excluded *a priori*, but nothing can be said about the expression under consideration, which in general may be positive, nil, or negative and, besides, may take on different values for different i's. It follows that, in general, the conditions for the second best optimum, given the additional constraint (25.14), will be different from the corresponding conditions for the Pareto-optimum. This implies that, in the presence of such an additional constraint, the application of those of the Paretian conditions which can still be fulfilled will not, in general, bring about the (second) best solution in the assumed circumstances. Naturally we cannot exclude the possibility that in certain cases (for example in the case of separable functions) this application may bring about the second best solution, but it should be stressed that this is not a generally valid prescription.

References

Johnson, H. G. (1950). Optimum welfare and maximum revenue tariffs.
Lipsey, R. G., & Lancaster, K. (1956). The general theory of second best.