Chapter 30
Appendix to Chapter 16

Since the very beginning the literature on the new economic geography has made wide use of numerical methods to explore the models. In this appendix we provide two simple examples. In the first example we use numerical methods to draw the phase line which then allows performing the traditional topological analysis of global stability. The second example deals with the relationship between theory and empirics and consists in comparing the (numerical) solution values obtained from the model with what is observed in (or estimated from) the data.

30.1 Numerical Phase Lines

Consider the Core-Periphery model studied in Sect. 16.3. After operating the substitutions discussed below Eqs. (16.4) and (16.5) these equations become

$$w_1 = \frac{(w_1)^{\frac{1}{1-\mu}}\left[\frac{(1-\gamma)}{2} + \lambda_1 \gamma w_1\right]}{\lambda_1 (w_1)^{\frac{1}{1-\mu}} + (1-\lambda_1)\left(\frac{w_2}{\tau}\right)^{\frac{1}{1-\mu}}} + \frac{\left(\frac{w_1}{\tau}\right)^{\frac{1}{1-\mu}}\left[\frac{(1-\gamma)}{2} + (1-\lambda_1)\gamma w_2\right]}{\lambda_1 \left(\frac{w_1}{\tau}\right)^{\frac{1}{1-\mu}} + (1-\lambda_1)(w_2)^{\frac{1}{1-\mu}}} \qquad (30.1)$$

$$w_2 = \frac{\left(\frac{w_2}{\tau}\right)^{\frac{1}{1-\mu}}\left[\frac{(1-\gamma)}{2} + \lambda_1 \gamma w_1\right]}{\lambda_1 (w_1)^{\frac{1}{1-\mu}} + (1-\lambda_1)\left(\frac{w_2}{\tau}\right)^{\frac{1}{1-\mu}}} + \frac{(w_2)^{\frac{1}{1-\mu}}\left[\frac{(1-\gamma)}{2} + (1-\lambda_1)\gamma w_2\right]}{\lambda_1 \left(\frac{w_1}{\tau}\right)^{\frac{1}{1-\mu}} + (1-\lambda_1)(w_2)^{\frac{1}{1-\mu}}}. \qquad (30.2)$$

G. Gandolfo, *International Trade Theory and Policy*, Springer Texts in Business and Economics, DOI 10.1007/978-3-642-37314-5_30,

Table 30.1 Numerical solutions of the core-periphery model

$\mu = 3/2, \quad \tau = 1/10, \quad \gamma = 4/10$		
λ_1	Nominal wages	Real wage differential
$\lambda_1 = 0.1$	$w_1 = 2.341826921, w_2 = 0.8509081199$	$\overset{\bullet}{\lambda}_1 = 0.2777197895$
$\lambda_1 = 0.2$	$w_1 = 1.838861373, w_2 = 0.7902846567$	$\overset{\bullet}{\lambda}_1 = 0.2558537633$
$\lambda_1 = 0.3$	$w_1 = 1.474281847, w_2 = 0.7967363511$	$\overset{\bullet}{\lambda}_1 = 0.1948332076$
$\lambda_1 = 0.4$	$w_1 = 1.200024688, w_2 = 0.8666502081$	$\overset{\bullet}{\lambda}_1 = 0.1049309257$
$\lambda_1 = 0.5$	$w_1 = 1.000000000, w_2 = 1.000000000$	$\overset{\bullet}{\lambda}_1 = 0$

Substituting for the price index into expression (16.7) the law of motion (16.6) becomes

$$\overset{\bullet}{\lambda}_1 = \frac{w_1}{\left[\lambda_1 (w_1)^{\frac{1}{1-\mu}} + (1-\lambda_1)\left(\frac{w_2}{\tau}\right)^{\frac{1}{1-\mu}}\right]^{\gamma(1-\mu)}} - \frac{w_2}{\left[\lambda_1 \left(\frac{w_1}{\tau}\right)^{\frac{1}{1-\mu}} + (1-\lambda_1)(w_2)^{\frac{1}{1-\mu}}\right]^{\gamma(1-\mu)}}. \tag{30.3}$$

The system is at rest when real wages equalize. The system is also at rest when the real wage is higher in 1 and $\lambda_1 = 1$ or when the real wage is larger in 2 and $\lambda_1 = 0$; these two cases represent the "core-periphery situations". After assigning values to the three parameters τ, μ, γ and to the state variable λ_1, Eqs. (30.1) and (30.2) can be solved numerically for w_1 and w_2. These numerical solutions may be obtained by use of any mathematical software. Having solved for nominal wages we can compute the real wage differential. Note that the so obtained real wage differential depends on the value assigned to λ_1. Assigning a series of values to λ_1 gives the corresponding series of solution values for the real wage differential. This means that we can associate to any value of λ_1 a corresponding value of $\overset{\bullet}{\lambda}_1$, i.e., we can constructed a (numerically computed) "dotted" phase line. The line is "dotted" in the obvious sense that we can compute only a finite number (not a continuum) of correspondences between λ_1 and $\overset{\bullet}{\lambda}_1$.

As an example we show the solutions obtained by assigning $\mu = 3/2$, $\tau = 1/10$, and $\gamma = 4/10$ for each of the following nine values of λ_1: $\lambda_1 = 0.1,\ 0.2,\ \ldots,\ 0.9$. The solutions are obtained by use of any mathematical software. The solutions are shown in Table 30.1. Given the symmetric structure of the model, the solutions values of nominal wages for $\lambda_1 = 0.6, \ldots, 0.9$ are symmetric around 1 to those for $\lambda_1 = 0.1, \ldots, 0.4$. Likewise, the solution values of the real wage differential for $\lambda_1 = 0.6, \ldots, 0.9$ are symmetric around zero to those for $\lambda_1 = 0.1, \ldots, 0.4$. We recall from Eq. (30.3) that $\overset{\bullet}{\lambda}_1$is equal to the real wage differential. Then, by plotting the values for the real wage differential in the space $\lambda_1, \overset{\bullet}{\lambda}_1$ and then connecting the points we obtain a numerical approximation of the phase line. The simplest way of connecting any two consecutive points is by a straight line (linear interpolation). Of course, a variety of more precise methods exists (see Judd, 1998). Once the numerical approximation of the phase line is drawn we can study the dynamic properties of the model by use of standard topological analysis (see Gandolfo, 2009, sect. 21.3).

30.2 Calibration

Calibration is one way to help assessing the empirical validity of theoretical models. The idea is to find the range of reasonable parameter values and the right modeling structure that replicate the empirical observations. As an example of calibration we use the simple exercise in Brülhart, Carrère, and Trionfetti (2012), from which we draw this subsection. The example shows how the empirical analysis offers guidance for the calibration and, more interestingly, for the right choice of the model.

Their objective is to study how the geographical distribution of economic activity within Austria has adjusted to trade opening with Eastern Europe. The model initially used is a three-region extension of the housing congestion model Sect. 16.4.1. In this extension Austria is composed by two regions between which the economic activity is endogenously allocated. The third region is the Eastern Europe. Trade opening occurs between Austria and Eastern Europe and triggers an endogenous reallocation of economic activity towards the eastern region of Austria since these regions are geographically closer to the new source of demand, Eastern Europe. Interestingly, this model does not replicate the data very well. Specifically the model predicts too high labour mobility with respect to what is observed in the data. Guided by this observation, the model was extended to include a migration inertia modeled along the lines of Tabuchi and Thisse (2002) and Murata (2003). This new model extension replicates the data with much greater precision.

30.2.1 The Model

The world economy consists of two countries and three regions. Regions I and B belong to country A and the third region constitutes a one-region country named R. Regions names are obvious mnemonics for the I(nterior) region and the Eastern B(order) region of A(ustria) and the third region represents the R(est of the world) with which trade opening takes place. There are iceberg trade costs: for a unit of good sent from region i to region j only a fraction $\tau_{ij} \in (0, 1)$ arrives in j. It is assumed that $\tau_{ij} = \tau_{ji}$ $\forall$ i, j and $\tau_{ii} = 1$, $\forall$ i. The geographical structure of the three-region model is represented by the following assumptions on trade costs:

$$\tau_{IR} = \tau_{IB}\tau_{BR},$$

which means that for a variety of the M-good to be transported between I and R it has to transit through B. Thus, the B(order) is nearer to R than the I(nterior) region and according to theory the increase in demand coming from R after trade opening should cause a geographical reallocation of economic activity from I to B. The task of the exercise, however, is not only to verify that the data confirms this theoretical prediction. The task is also and principally to replicate in the model the magnitude of the observed geographical reallocation of economic activity.

Continuing with the model description, labour is mobile within countries but immobile between countries. Individuals derive utility from consumption of goods and also from the pleasure of residing in a region. The component of utility that is associated with consumption is a Cobb-Douglas defined over M and H with expenditure share on M equal to γ. Demand for any domestic and any imported variety of good M are, respectively, $q_{ii}^{d} = (p_{ii})^{1-\sigma} (P_i)^{\sigma-1} \gamma E_i$ and $q_{ji}^{d} = \left(p_{ji}\right)^{1-\sigma} (P_i)^{\sigma-1} \gamma E_i$ where the first subscript refers to the region where the variety is produced and the second subscript refers to the region where the variety is consumed. Total indirect utility, V_i^k, is given by the sum of the real wage, ω_i, which represents the indirect utility derived from consumption and is common to all individuals in a given region, and utility derived from the idiosyncratic appreciation that each individual k associates with region i, ξ_i^k; that is:

$$V_i^k = \omega_i + \xi_i^k.$$

The letter ξ_i^k denotes a random variable that is identically and independently distributed across individuals according to a double exponential (Gumbel) distribution with zero mean and variance $\pi^2\chi^2/6$ (the letter π here is the trigonometric $\pi \approx 3.14$) where $\chi \geq 0$ is a parameter. For notational convenience let $\bar{A}$ be the set of regions in A, i.e., $\bar{A} = \{I, B\}$. Given this distribution, the probability that an individual will choose to reside in region i of country A is given by the logit formula

$$\Pr{}_i\,(\omega_i, \chi) = \frac{\exp\left(\frac{\omega_i}{\chi}\right)}{\sum\limits_{i\in\bar{A}} \exp\left(\frac{\omega_i}{\chi}\right)}, \tag{30.4}$$

When $\chi \rightarrow 0$, individuals tend to have identical preferences and choose their region of residence solely according to the indirect utility derived from consumption of M and H. This is the preference structure of the model we considered in Sect. 16.4.1. As χ increases, idiosyncratic location preferences become more important and in the extreme case of $\chi \rightarrow \infty$ they are all that matters for workers' location choices.

The stock of H in each region is constant. The relative number of varieties, the price of H, total expenditure, and the real wage are given, respectively, by expressions (16.8)–(16.11) in Sect. 16.4.1. The zero profit condition determines the equilibrium size of firm output $q^* = (F/a)\,(\sigma - 1)$. Product-market equilibrium requires equality of supply and demand for any variety of M produced in each region. For notational convenience let $\bar{W}$ be the set of all regions, i.e., $\bar{W} = \{I, B, R\}$. Using the expressions for optimal prices, the price index for tradeable, total expenditure, the relative number of varieties, and equilibrium output of any variety, the system of goods market equilibrium equations is

$$1 = \sum_{j \in \bar{W}} \frac{\left(\tau_{ij}\right)^{\sigma-1} (w_i)^{-\sigma}}{\sum_{j \in \bar{W}} \lambda_i \left(\tau_{ij}\right)^{\sigma-1} (w_i)^{1-\sigma}} w_j L_j \quad . \qquad i = I, B. \tag{30.5}$$

A spatial equilibrium is defined by the condition that *net* migration flows be zero:

$$L_B \operatorname{Pr}_I - L_I \operatorname{Pr}_B = 0. \tag{30.6}$$

The first summand in Eq. (30.6) is the migration flow from region B to region I, and the second summand is the migration flow from region I to region B. The spatial equilibrium requires them to be equal. Lastly,

$$\sum_{i \in \bar{A}} L_i = L_A, \tag{30.7}$$

where L_A is the exogenously given population in A.

We choose w_R as numéraire and set $w_R = 1$. The equilibrium is characterized by a vector of wages $\left[w_I^*, w_B^*\right]$ and labour allocations $\left[L_I^*, L_B^*\right]$ that satisfies the system of four independent equations composed by the product-market equilibrium equations (30.5), the spatial equilibrium equation (30.6) and the labour market equilibrium equation (30.7). Equilibrium values of all other endogenous variables can be computed from the equilibrium values of wages and labour allocations.

Taking the natural logarithm of both sides of (30.6) and rearranging gives:

$$\omega_B - \omega_I = \chi \ln \left(\frac{L_B}{L_I}\right). \tag{30.8}$$

We therefore use the two Eqs. (30.5), (30.7), and (30.8) to obtain numerical solutions for w_I, w_B, L_I, and L_B.

30.2.2 Calibration of the Model

The starting point is the empirical observation. Let $\overset{\circ}{L}_i$ be the growth rate of employment between steady states. That is, the difference between the steady state equilibrium value of employment before the shock and the new steady state equilibrium value of employment on which the economy settles after the shock. Let $\overset{\circ}{w}_i$ be the analogous definition referred to wages. The estimated variable is the ratio between the difference in growth rates of employment and the difference in growth rates of wages:

$$\rho \equiv \frac{\overset{\circ}{L}_B - \overset{\circ}{L}_I}{\overset{\circ}{w}_B - \overset{\circ}{w}_I}. \tag{30.9}$$

Let $\hat{\rho}$ denote the estimated value of ρ. The empirical estimations give $\hat{\rho} \approx 3$ (more precisely the hypothesis $\hat{\rho} = 3$ is never rejected). We therefore take $\hat{\rho} = 3$ as the number to be replicated by the model.

To calibrate the model, we need to decide on the values of the following parameters: housing stocks (in each region), H_i, population in A and R, the elasticity of substitution among differentiated goods, $\sigma \equiv \mu / (\mu - 1)$, the expenditure share γ, the location preference parameter χ, and trade costs between regions. While for qualitative results these values can be chosen without any constraint, in the calibration for empirical purposes these values must satisfy two requirements: (1) they must be reasonable and (2) they must be such that the resulting values of the endogenous variables replicate the empirical evidence. The first criterion puts a constraint on the range in which parameter values may lie. Typically it is required that parameter values are in line with measures of them taken form independent sources. For example, in relation to the present calibration, it is reasonable to use the estimated value of σ found in the empirical literature. These estimates have found values that range in the interval from 3 to 6. The measurement for $(1-\gamma)$ are less numerous in the literature but the few existing studies find that the expenditure share on housing is approximately 0.25.[1] We therefore take the range 3–6 as reasonable values that may be assigned to σ and we take 0.25 as a reasonable value for $(1-\gamma)$. Coming to H_i, there is no data that suggest what reasonable values of the housing stocks could be but there is data on population. By inputting into the model the observed regional population before the shock and then solving the model for H_i we obtain consistent values of H_i.[2] These values are then used as exogenous regional stocks of H which remain constant throughout the simulation of trade liberalization. The distribution of the total stock of housing between A and R is instead arbitrarily assigned by choosing $H_R = \frac{H}{3}$ and $L_R = \frac{L}{3}$ and normalizing total stock of housing and labour by setting $H = L = 1$. Hence, A is twice the size of R. This is totally arbitrary but the lack of data does not allow to do any better. Thus, as a robustness check it is necessary to study the sensitivity of the results of different parametrization of H_R and L_R. We anticipate here that

[1] See Brülhart et al. (2012) for references on the empirical studies that have estimated σ and $(1-\gamma)$ as well as for references on the migration inertia reported below.

[2] In our data set region B accounted for 5.1 % of Austrian population prior to liberalization. Their implied housing stock in our calibrations ranges from 6 to 9 % of the total for country A.

Table 30.2 Baseline model: simulated values of ρ

	$\sigma = 3$	$\sigma = 4$	$\sigma = 5$	$\sigma = 6$
$(1-\gamma) = 0.20$	10.33	9.60	9.23	9.00
$(1-\gamma) = 0.25$	7.70	7.16	6.88	6.71
$(1-\gamma) = 0.30$	5.97	5.54	5.33	5.20
$(1-\gamma) = 0.40$	3.82	3.55	3.43	3.33
$(1-\gamma) = 0.50$	2.54	2.36	2.27	2.21

robustness checks have shown that the simulated value of ρ is pretty insensitive to changes in the population and housing distribution between A and R.[3] Coming to the simulation of trade opening the following parameter values for trade costs are chosen to simulate external trade liberalization: $\tau_{IB} = 0.9$ (very low trade costs within A) and $\tau_{BR} = 0.1, 0.2, \ldots, 0.9$ (falling trade costs between A and R). The model is solved for each of the nine levels of τ_{BR} and we compute the relative change in steady state equilibrium nominal wages, $\overset{\circ}{w}_i$, and employment, $\overset{\circ}{L}_i$, for each 0.1 increment of trade cost reduction.[4] It turns out that ρ varies only trivially across pairs of trade costs for which it is calculated. We will therefore report averages of the eight computed ratios.

One last important matter concerns the stability of the equilibrium. The range of reasonable parameter values turns out to be such that the (only) equilibrium is globally stable for any value of trade costs. This means that the exogenous shock considered in the exercise will move the position of the equilibrium but will not move the economy on a different equilibrium.

Armed with the set of reasonable values to be assigned to the exogenous variables and to parameters, except for χ which we discuss below, and knowing that the equilibrium we shock is stable we can now come to confrontation with data. To show how empirical evidence is a guidance to calibration we solve the model first for $\chi \to 0$. The results are shown in Table 30.2 which reports the simulated values of ρ. It is clear that reasonable parameter values for $(1-\gamma)$ and σ do not easily replicate $\hat{\rho} = 3$. Only expenditure shares on housing well above 0.25 or unreasonably high values of σ yield corresponding simulated values of ρ near 3. The model simulated in the absence of migration inertia therefore is not able to satisfactorily replicate the data. In particular, the simulated values of ρ are too high with respect to $\hat{\rho}$ which, by inspection of expression (30.9), means that the model allows for too

[3] This insensitivity is not surprising. By increasing the size of R, for instance, trade liberalization becomes more important for both I and B, but more so for B. Yet, I is not a measure of the locational attractiveness of B relative to I; rather, it captures whether that increased attractiveness manifests itself more in terms of employment growth or in terms of nominal wage growth. This ratio is largely insensitive to the overall attractiveness of B with respect to I.

[4] $\overset{\circ}{w}_i$ and $\overset{\circ}{L}_i$ are growth rates between steady states. Their empirical counterparts are the average or cumulative growth rates over the entire pre- and post-liberalization subperiods, assuming that these subperiods are sufficiently long to capture the full transition between steady states.

Table 30.3 Extended model: implied immobility for $\rho = 3$

	$\sigma = 3$	$\sigma = 4$	$\sigma = 5$	$\sigma = 6$
$(1-\gamma) = 0.20$	25	29	30	31
	[19]	[22]	[23]	[25]
$(1-\gamma) = 0.25$	24	28	31	32
	[14]	[16]	[17]	[18]
$(1-\gamma) = 0.30$	20	25	27	29
	[10]	[12]	[12]	[12]

much labour interregional mobility with respect to what is implied by the data. Therefore, introducing migration inertia, here in the form of idiosyncratic location preferences, promises to be a suitable way of adapting the model to replicate the empirical evidence. We therefore set $\chi > 0$.

There is no data to offer guidance about plausible values of χ. Still it is necessary to constrain χ in reasonable ranges to make the calibration meaningful. One way of doing so is to take account that when χ is positive the spatial equilibrium gives rise to real wage differences which do not induce migration because they are compensated by location preferences. Therefore, one way of constraining χ in a reasonable range is to find the values of χ such that the resulting equilibrium real wage differences between regions appear to be reasonable. But what are reasonable real wage differences that do not induce migration? There is only few evidence, mainly based on surveys, on this type of migration inertia. One study reports that 34 % of EU15 unemployed and 25 % of Czech unemployed stated in 2002 that they would not move under any circumstances even if a job became available elsewhere. One other study has found that the percentage of Italian unemployed refusing to move out of their town of residence if a job were available elsewhere ranges from 20.7 % (Northern male university graduates) to 61 % (Southern low-education females). These surveys would then serve as measures of reasonable migration inertia.

Solving the model for χ such that $\rho = 3$ gives the results shown in Table 30.3. Each cell reports the implied percentage real-wage difference between regions of A and, in brackets, the implied share of A's population that prefers not to migrate at the prevailing real-wage difference. It is clear that allowing for heterogenous location preferences aligns the simulated value of ρ with the estimated ρ under more reasonable parameter values. For instance, the necessary degree of preference heterogeneity when $\sigma = 4$ and $(1-\mu) = 0.25$ is such that 16 % of the population would not move even if the real wage were 28 % higher in the other region. In light of the available European evidence on the issue, this does not appear to be an excessive dose of assumed intrinsic migration inertia.

References

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