Chapter 21
Appendix to Chapter 5

21.1 The Factor-Price-Equalization Theorem

A proof of this theorem in its general version can easily be given by using the dual relations due to Jones (1965) and illustrated in Sect. 19.5. If we consider the last two equations of set (19.65) and solve for p_L^*, p_K^*, we get

$$p_L^* = \frac{\theta_{KB} p_A^* - \theta_{KA} p_B^*}{\theta_{LA}\theta_{KB} - \theta_{LB}\theta_{KA}}, \quad p_K^* = \frac{\theta_{LA} p_B^* - \theta_{LB} p_A^*}{\theta_{LA}\theta_{KB} - \theta_{LB}\theta_{KA}}. \tag{21.1}$$

This shows that the prices of factors depend solely on the prices of commodities: as the latter are internationally identical, so also are the former.

Note that when there is complete specialization these relations would not exist: in fact, with complete specialization, either $\theta_{LA} = \theta_{KA} = 0$ (complete specialization in commodity B) or $\theta_{LB} = \theta_{KB} = 0$ (complete specialization in commodity A).

21.2 The Stolper-Samuelson Theorem

The same dual relations given in Eqs. (21.1) allow us to give a simple proof of this theorem (Stolper and Samuelson, 1941) in its general version.

We can assume, without loss of generality, that commodity A is the numéraire, so that $p_A^* = 0$. A positive (negative) value of p_B^* therefore means an increase (decrease) in the relative price p_B/p_A and, likewise, a positive (negative) value of p_L^* means an increase (decrease) in the unit real reward (i.e., in terms of the numéraire) of labour.

Let us now assume, for example, that commodity B is labour-intensive and that the relative price of this commodity increases. Given the definitions of the θ's (see Sect. 19.5), the greater relative labour intensity of B amounts to the inequality $\theta_{LB}/\theta_{KB} > \theta_{LA}/\theta_{KA}$ and, therefore, the denominator of the fractions in (21.1) is

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negative. As we have assumed $p_B^* > 0$, it follows that $p_L^* > 0$, $p_K^* < 0$. The increase in the unit real reward of the factor used intensively in the industry producing the commodity with the relative price increase is thus proved.

The *magnification effect* is also easily proved. Assuming $p_A^* = 0$, from Eqs. (21.1) we have

$$p_L^* = \frac{\theta_{KA}}{\theta_{LB}\theta_{KA} - \theta_{LA}\theta_{KB}} p_B^*.$$

Now, $p_L^* > p_B^*$ when

$$\frac{\theta_{KA}}{\theta_{LB}\theta_{KA} - \theta_{LA}\theta_{KB}} > 1,$$

which is certainly true. In fact, $\theta_{LB}\theta_{KA} - \theta_{LA}\theta_{KB} < \theta_{KA}$ because this is equivalent to $\theta_{KA}(\theta_{LB} - 1) - \theta_{LA}\theta_{KB} < 0$, which follows from the fact that $\theta_{LB} < 1$.

21.3 The Rybczynski Theorem

A simple proof of the Rybczynski theorem (Rybczynski, 1955) can be given by way of the dual relations illustrated in Sect. 19.5.

From the first two equations in (19.65), we can express A^* and B^* in terms of L^*; since $K^* = 0$ and $p_L^* = p_K^* = 0$ by assumption, we obtain

$$A^* = \frac{\lambda_{KB}L^*}{\lambda_{LA}\lambda_{KB} - \lambda_{KA}\lambda_{LB}}, \quad B^* = \frac{-\lambda_{KA}L^*}{\lambda_{LA}\lambda_{KB} - \lambda_{KA}\lambda_{LB}}. \tag{21.2}$$

If commodity A is labour intensive, the denominator of these expressions is positive and so $A^* > 0$, $B^* < 0$, which proves the theorem.

Besides, given the assumptions, the expression $\lambda_{KB}/(\lambda_{LA}\lambda_{KB} - \lambda_{KA}\lambda_{LB})$ is greater than one, so that $A^* > L^*$ (the *magnification effect*).

This is another example of the fact that the dual approach in various cases enables us to give simple proofs of the fundamental theorems of the pure theory of international trade.

Also note that, if we compare this proof with that of the Stolper-Samuelson theorem given in the previous section, we see that changes in outputs are related to changes in factor endowments through the λ coefficients in the same way as the θ coefficients link factor price changes to commodity price changes. This *duality* between the Rybczynski and Stolper-Samuelson theorems is a basic feature of the general equilibrium model.

To be more precise, the effect of an increase in the endowment of a factor on the output of a commodity (at unchanged prices of factors and goods) is *exactly the same* as the effect of an increase in the price of that commodity (*ceteris paribus*) on that factor's reward. The relations stating the equality of these effects are also called the *reciprocity relations.*

This can be easily checked by using the dual approach. If we compare the results given in Eqs. (21.2) with those of Eqs. (21.1), and substitute the definitions of the λ's and θ's (given in Sect. 19.5) in these expressions, we immediately find the result stated. Alternatively, we could solve the first two equations in (19.61) for $\mathrm{d}A/\mathrm{d}L$, $\mathrm{d}B/\mathrm{d}L$, etc., and the third and fourth for $\mathrm{d}p_L/\mathrm{d}p_A$, $\mathrm{d}p_L/\mathrm{d}p_B$, etc., and find that the resulting expressions are respectively equal.

References

Jones, R. W. (1965). The structure of simple general equilibrium models.
Rybczynski, T. M. (1955). Factor endowment and relative commodity prices.
Stolper, W. F., & Samuelson, P. A. (1941) Protection and real wage.