Chapter 22
Appendix to Chapter 6

22.1 The Specific Factors Model

The specific factors model (Jones, 1971; Samuelson, 1971) can be conveniently examined we follow (Jones, 1971) extending to the present case the treatment already introduced in Sect. 19.5 for the traditional case. Equations (19.59) have to be modified to take account of the presence of specific factors.

Let $a_{ij}, i = K^A, K^B, L; j = A, B$, denote the quantity of factor i required to produce a unit of commodity j. Then we have

$$\begin{aligned} a_{K^A A} A &= K^A, \\ a_{K^B B} B &= K^B, \\ a_{LA} A + a_{LB} B &= L, \end{aligned} \tag{22.1}$$

since $a_{K^A B} \equiv 0, a_{K^B A} \equiv 0$ by the specific factors assumption. These equations emphasize the dual relations between factor endowments and commodity outputs, and derive from full factor employment. We also have the dual relations between commodity prices and factor prices, which derive from competitive equilibrium:

$$\begin{aligned} a_{LA} p_L + a_{K^A A} p_{K^A} &= p_A, \\ a_{LB} p_L + a_{K^B B} p_{K^B} &= p_B. \end{aligned} \tag{22.2}$$

Since, in general, the four input coefficients a_{ij} are variable, the above five equations must be supplemented by four equations to determine these coefficients. Such equations derive from the firm's optimization procedure. As is well known, with constant returns to scale, the input coefficients depend solely upon the factor-price ratio. Therefore

$$a_{ij} = a_{ij}\left(\frac{p_L}{p_{K^j}}\right), \quad i = K^A, K^B, L; \quad j = A, B, \tag{22.3}$$

which are the four equations that we need.

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The nine equations (22.1)–(22.3) describe the production side of the model, and make it possible to determine the nine unknowns a_{ij}, A, B, p_L, p_{K^A}, p_{K^B} given the five parameters L, K^A, K^B, p_A, p_B.

Before going on to examine the equations of change, it is possible to simplify the model by substituting from the first two equations of (22.1) into the third one, thus obtaining

$$\frac{a_{LA}}{a_{K^A A}} K^A + \frac{a_{LB}}{a_{K^B B}} K^B = L. \tag{22.4}$$

Since the a_{ij} depend on factor prices, Eqs. (22.2) and (22.4) provide a set of three equations to determine the three factor prices in terms of the parameters. Let us begin with the total differentials of Eqs. (22.2), that are

$$\begin{aligned} p_L \mathrm{d}a_{LA} + a_{LA}\mathrm{d}p_L + p_{K^A}\mathrm{d}a_{K^A A} + a_{K^A A}\mathrm{d}p_{K^A} &= \mathrm{d}p_A, \\ p_L \mathrm{d}a_{LB} + a_{LB}\mathrm{d}p_L + p_{K^B}\mathrm{d}a_{K^B B} + a_{K^B B}\mathrm{d}p_{K^B} &= \mathrm{d}p_B. \end{aligned} \tag{22.5}$$

If we denote relative changes by an asterisk (namely, $a^*_{LA} = \mathrm{d}a_{LA}/a_{LA}$, etc.) we can rewrite Eqs. (22.5), after simple manipulations (these are the same as shown in the Sect. 19.5) in the form

$$\begin{aligned} \theta_{K^A A} p^*_{K^A} + \theta_{LA} p^*_L &= p^*_A, \\ \theta_{K^B B} p^*_{K^B} + \theta_{LB} p^*_L &= p^*_B, \end{aligned} \tag{22.6}$$

where the θ's denote the factor shares in each sector ($\theta_{K^A A} = a_{K^A A} p_{K^A}/p_A$ etc.), which of course add up to 1, and use has been made of the fact that, as shown in the Eqs. (19.63)

$$\begin{aligned} \theta_{K^A A} a^*_{K^A A} + \theta_{LA} a^*_{LA} &= 0, \\ \theta_{K^B B} a^*_{K^B B} + \theta_{LB} a^*_{LB} &= 0. \end{aligned} \tag{22.7}$$

We next consider the total differential of Eq. (22.4), which is

$$\begin{aligned} &\frac{a_{LA}}{a_{K^A A}} \mathrm{d}K^A + \frac{\mathrm{d}a_{LA}}{a_{K^A A}} K^A - \frac{a_{LA}}{a_{K^A A}} \frac{\mathrm{d}a_{K^A A}}{a_{K^A A}} K^A \\ &+ \frac{a_{LB}}{a_{K^B B}} \mathrm{d}K^B + \frac{\mathrm{d}a_{LB}}{a_{K^B B}} K^B - \frac{a_{LB}}{a_{K^B B}} \frac{\mathrm{d}a_{K^B B}}{a_{K^B B}} K^B \\ = \; &\mathrm{d}L, \end{aligned} \tag{22.8}$$

whence

$$\frac{K_A}{L}\left(\frac{a_{LA}}{a_{K^A A}}\mathrm{d}K^A/K^A+\frac{\mathrm{d}a_{LA}}{a_{K^A A}}-\frac{a_{LA}}{a_{K^A A}}\frac{\mathrm{d}a_{K^A A}}{a_{K^A A}}\right)$$
$$+\frac{K^B}{L}\left(\frac{a_{LB}}{a_{K^B B}}\mathrm{d}K^B/K^B+\frac{\mathrm{d}a_{LB}}{a_{K^B B}}-\frac{a_{LB}}{a_{K^B B}}\frac{\mathrm{d}a_{K^B B}}{a_{K^B B}}\right)$$
$$=\mathrm{d}L/L.$$

Further simple manipulations and use of the definitions of starred variables give

$$\begin{aligned}&\lambda_{LA}K^{*A}+\lambda_{LA}a^*_{LA}-\lambda_{LA}a^*_{K^A A}\\&+\lambda_{LB}K^{*B}+\lambda_{LB}a^*_{LB}-\lambda_{LB}a^*_{K^B B}\\&=L^*,\end{aligned}\tag{22.9}$$

where $\lambda_{LA}\equiv a_{LA}A/L,\lambda_{LB}\equiv a_{LB}B/L$ denote the fractions of the labour force used in sector A and B respectively. These fractions must of course add up to one, given the full employment condition.

From the definition of elasticity of substitution between factors (see Sect. 19.5) in the two sectors, σ_A,σ_B, we have

$$\begin{aligned}a^*_{K^A A}-a^*_{LA}&=\sigma_A(p^*_L-p^*_{K^A}),\\a^*_{K^B B}-a^*_{LB}&=\sigma_B(p^*_L-p^*_{KB}),\end{aligned}\tag{22.10}$$

which allows us to rewrite (22.9) in the form

$$\begin{aligned}&\lambda_{LA}\sigma_A p^*_{K^A}+\lambda_{LB}\sigma_B p^*_{K^B}-(\lambda_{LA}\sigma_A+\lambda_{LB}\sigma_B)\,p^*_L\\&=\left(L^*-\lambda_{LA}K^{*A}-\lambda_{LB}K^{*B}\right).\end{aligned}\tag{22.11}$$

The system made up of Eqs. (22.6) and (22.11) gives us the equations of change that allow us to determine the effects on factor returns of changes in commodity prices and factor endowments.

Before solving this system, it is as well to observe that it immediately shows why FPE (factor price equalization) does not hold.

In the standard model, two relationships are given to determine two factor prices once commodity prices are known: these are the last two equations in set (19.65). Hence, since commodity prices are internationally identical, with the assumed identical technologies also factor prices are identical across countries. In the present model, the two equations (22.6) are obviously not sufficient to determine the three factor prices in terms of commodity prices only.

Let us now solve our three-equation system. Its determinant is

$$D\equiv\begin{vmatrix}\theta_{K^A A}&0&\theta_{LA}\\0&\theta_{K^B B}&\theta_{LB}\\\lambda_{LA}\sigma_A&\lambda_{LB}\sigma_B&-(\lambda_{LA}\sigma_A+\lambda_{LB}\sigma_B)\end{vmatrix}$$
$$=-\theta_{K^A A}\{\theta_{K^B B}\,(\lambda_{LA}\sigma_A+\lambda_{LB}\sigma_B)+\theta_{LB}\lambda_{LB}\sigma_B\}-\theta_{LA}\theta_{K^B B}\lambda_{LA}\sigma_A$$

$$= -\lambda_{LA}\sigma_A\theta_{K^AA}\theta_{K^BB}\left(\theta_{K^AA}+\theta_{LA}\right) - \lambda_{LB}\sigma_B\theta_{K^AA}\left(\theta_{K^BB}+\theta_{LB}\right)$$

$$= -\theta_{K^AA}\theta_{K^BB}\left(\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}} + \lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}\right), \tag{22.12}$$

where use has been made of the fact that $\theta_{K^AA} + \theta_{LA} = \theta_{K^BB} + \theta_{LB} = 1$.

Simple calculations (for example by Cramer's rule) yield

$$p^*_{K^A} = \frac{1}{\Delta}\left\{\left[\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}} + \frac{1}{\theta_{K^AA}}\lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}\right]p^*_A - \frac{\theta_{LA}}{\theta_{K^AA}}\lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}p^*_B + \frac{\theta_{LA}}{\theta_{K^AA}}\left[L^* - \lambda_{LA}K^{*A} - \lambda_{LB}K^{*B}\right]\right\}, \tag{22.13}$$

$$p^*_{K^B} = \frac{1}{\Delta}\left\{\left[\lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}} + \frac{1}{\theta_{K^BB}}\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}}\right]p^*_B - \frac{\theta_{LB}}{\theta_{K^BB}}\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}}p^*_A + \frac{\theta_{LB}}{\theta_{K^BB}}\left[L^* - \lambda_{LA}K^{*A} - \lambda_{LB}K^{*B}\right]\right\}, \tag{22.14}$$

$$p^*_L = \frac{1}{\Delta}\left\{\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}}p^*_A + \lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}p^*_B + \left[\lambda_{LA}K^{*A} + \lambda_{LB}K^{*B} - L^*\right]\right\}, \tag{22.15}$$

$$p^*_{K^A} - p^*_{K^B} = \frac{1}{\Delta}\left\{\left[\frac{1}{\theta_{K^BB}}\lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}} + \frac{1}{\theta_{K^AA}}\lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}\right]\left(p^*_A - p^*_B\right) + \frac{1}{\theta_{K^AA}\theta_{K^BB}}\left(\theta_{LA} - \theta_{LB}\right)\left[\lambda_{LA}K^{*A} + \lambda_{LB}K^{*B} - L^*\right]\right\}, \tag{22.16}$$

where

$$\Delta \equiv -\frac{D}{\theta_{K^AA}\theta_{K^AA}} = \lambda_{LA}\frac{\sigma_A}{\theta_{K^AA}} + \lambda_{LB}\frac{\sigma_B}{\theta_{K^BB}}. \tag{22.17}$$

Let us observe that the expression σ_j/θ_{K^jj}, $j = A, B$, which frequently appears in the above formulae, is the elasticity of the marginal product curve of the mobile factor in sector j. Hence Δ is a weighted average of these elasticities.

It is then an easy matter to show that (a form of) the Stolper-Samuelson theorem holds for the specific factors but not for the mobile factor.

Suppose, for example, that commodity A is the numéraire, so that $p^*_A = 0$. A positive (negative) value of p^*_B then means an increase (decrease) in the relative price p_B/p_A. Consider now an increase in the relative price of commodity B at unchanged factor endowments. From Eqs. (22.13) and (22.14) we see that

$p^*_{K^B} > 0$, $p^*_{K^A} < 0$: the unit real reward of the specific factor used in sector B increases while that of the specific factor used in the other sector decreases. From Eq. (22.15) we see that $p^*_L > 0$, hence the wage rate increases in terms of commodity A.

Let us now take commodity B as the numéraire ($p^*_B = 0$), whereby an increase in the relative price of B means $p^*_A < 0$. From Eqs. (22.13) and (22.14) we again see that $p^*_{K^B} > 0$, $p^*_{K^A} < 0$: the result as regards specific factor rewards is independent of the choice of the numéraire. However, from Eq. (22.15) we see that $p^*_L < 0$, namely the wage rate *decreases* in terms of commodity B. Hence the real wage rate may move in either direction, depending on the composition of the expenditure of wage earners.

The effects of changes in factor endowments on factor prices are unambiguous: a change in any factor endowment causes the return to the mobile factor to change in a direction opposite to the returns to both specific factors. For example, an increase in the labour force has a positive effect on both $p^*_{K^A}$, $p^*_{K^B}$ and a negative effect on p^*_L.

To obtain the comparative statics results concerning outputs, we totally differentiate Eqs. (22.1), thus obtaining

$$\begin{aligned} A\mathrm{d}a_{K^A A} + a_{K^A A}\mathrm{d}A &= \mathrm{d}K^A, \\ B\mathrm{d}a_{K^B B} + a_{K^B B}\mathrm{d}B &= \mathrm{d}K^B, \\ A\mathrm{d}a_{LA} + a_{LA}\mathrm{d}A + B\mathrm{d}a_{LB} + a_{LB}\mathrm{d}B &= \mathrm{d}L. \end{aligned} \tag{22.18}$$

Simple manipulations (see Sect. 19.5) yield

$$\begin{aligned} \lambda_{K^A A}A^* &= K^{*A} - \lambda_{K^A A}a^*_{K^A A}, \\ \lambda_{K^B B}B^* &= K^{*B} - \lambda_{K^B B}a^*_{K^B B}, \\ \lambda_{LA}A^* + \lambda_{LB}B^* &= L^* - (\lambda_{LA}a^*_{LA} + \lambda_{LB}a^*_{LB}). \end{aligned} \tag{22.19}$$

It is possible to express the proportional changes in input coefficients in terms of the proportional changes in factor prices using Eqs. (22.7) and (22.10), whence

$$\left.\begin{aligned} a^*_{Lj} &= -\theta_{K^j j}\sigma_j(p^*_L - p^*_{K^j}), \\ a^*_{K^j j} &= \theta_{Lj}\sigma_j(p^*_L - p^*_{K^j}). \end{aligned}\right\} \; j = A, B. \tag{22.20}$$

Substitution of (22.20) into (22.19) yields

$$\begin{aligned} \lambda_{K^A A}A^* &= K^{*A} - \lambda_{K^A A}\theta_{LA}\sigma_A(p^*_L - p^*_{K^A}), \\ \lambda_{K^B B}B^* &= K^{*B} - \lambda_{K^B B}\theta_{LB}\sigma_B(p^*_L - p^*_{K^B}), \\ \lambda_{LA}A^* + \lambda_{LB}B^* &= L^* + \lambda_{LA}\theta_{K^A A}\sigma_A(p^*_L - p^*_{K^A}) + \lambda_{LB}\theta_{K^B B}\sigma_B(p^*_L - p^*_{K^B}). \end{aligned} \tag{22.21}$$

Differently from the standard 2×2 model, factor prices are not constant in the face of constant commodity prices. In fact, letting $p^*_A = p^*_B = 0$, from Eqs. (22.13) to (22.15) we have

$$
\begin{aligned}
p^*_{K^A} &= \frac{1}{\Delta}\frac{\theta_{LA}}{\theta_{K^A A}}[L^* - \lambda_{LA}K^{*A} - \lambda_{LB}K^{*B}],\\
p^*_{K^B} &= \frac{1}{\Delta}\frac{\theta_{LB}}{\theta_{K^B B}}[L^* - \lambda_{LA}K^{*A} - \lambda_{LB}K^{*B}],\\
p^*_L &= \frac{1}{\Delta}[\lambda_{LA}K^{*A} + \lambda_{LB}K^{*B} - L^*],
\end{aligned}
\tag{22.22}
$$

whence

$$
\begin{aligned}
K^{*B} = L^* = 0 \text{ and } K^*_A > 0 &\Longrightarrow p^*_{K^A} < 0, p^*_{K^B} < 0, p^*_L > 0,\\
K^{*A} = L^* = 0 \text{ and } K^*_B > 0 &\Longrightarrow p^*_{K^A} < 0, p^*_{K^B} < 0, p^*_L > 0,\\
K^{*A} = K^*_B = 0 \text{ and } L^* > 0 &\Longrightarrow p^*_{K^A} > 0, p^*_{K^B} > 0, p^*_L < 0.
\end{aligned}
\tag{22.23}
$$

Equations (22.21) and (22.23) in turn imply

$$
\begin{aligned}
K^{*B} = L^* = 0 \text{ and } K^*_A > 0 &\Longrightarrow A^* > 0, B^* < 0,\\
K^{*A} = L^* = 0 \text{ and } K^*_B > 0 &\Longrightarrow A^* < 0, B^* > 0,\\
K^{*A} = K^*_B = 0 \text{ and } L^* > 0 &\Longrightarrow A^* > 0, B^* > 0,
\end{aligned}
\tag{22.24}
$$

which show that (a form of) the Rybczynski theorem is only valid for specific factors (an increase in a specific factor causes an output increase in the corresponding sector and an output decrease in the other sector) but not for the mobile factor (an increase in the labour force brings about an output increase in both sectors).

22.2 The Cost of Transport

As we pointed out in the text, the rigorous treatment of the cost of transport requires a model which maintains the two-country assumption but with at least four variables present: the two transport services in addition to the two commodities. This takes us at once to a general equilibrium model of the type mentioned in Sect. 3.7. It is of course necessary to add the equations which establish equilibrium between demand and supply on the market for transport services and also the relations stating that exports of a given commodity by a given country occur only if the price in the importing country is equal to that in the exporting country plus the cost of transport. By applying to the resultant model the methods used in mathematical economics to demonstrate the existence of general economic equilibrium in a closed economy, one can see that in effect an equilibrium does exist. The extension of the model of general world equilibrium to a model with more than two countries does not present any further difficulties.

The price to be paid for this generality is, as we have already seen in Sect. 3.7, the loss of the explicative and interpretative power of the model, which does not allow us to establish empirically significant propositions regarding the structure of international trade or the other problems that the pure theory of international trade deals with. For a demonstration of the existence of equilibrium, see Hadley and Kemp (1966) . For further considerations regarding the cost of transport, see Casas (1983) and Casas and Choi (1985b) .

The role of transport cost in bringing about a core-periphery pattern will be examined in Sect. 31.2.

22.3 Intermediate Goods

22.3.1 Final Goods as Inputs

Let us first look at the case in which each product existing in the economy can be used as both an intermediate and final good. For simplicity, we assume that each good enters as an intermediate good only in the production of the other good and let A_B and B_A be respectively the quantity of A used as an intermediate good in the production of B and the quantity of B used in the production of A; with A and B we shall now indicate the *net* quantities of the two goods. We thus have the relationships

$$\begin{aligned} A &= F_A(K_A, L_A, B_A) - A_B, \\ B &= F_B(K_B, L_B, A_B) - B_A, \end{aligned} \tag{22.25}$$

where F_A and F_B are first-degree homogeneous production functions. Samuelson's theorem states that Eqs. (22.25) can be transformed into the net production functions

$$\begin{aligned} A &= N_A\left(K_A^c, L_A^c\right), \\ B &= N_B\left(K_B^c, L_B^c\right), \end{aligned} \tag{22.26}$$

where K_A^c, L_A^c denote the total quantities of capital and labour (directly and indirectly) required in the production of A as final good, and similarly for K_B^c, L_B^c. On the basis of Eqs. (22.26), each sector may be considered as an *integrated industry*, which produces internally all the intermediate goods (which are not observed from the outside) which are needed to produce the final good. Equations (22.26) are derived from a process of efficient allocation of resources, which consists in maximizing the quantity of the final good that can be obtained with any given combination of total use (direct and indirect) of capital and labour.[1]

Let us consider one of the two integrated industries, for example, that of commodity A (the same argument applies to B). From the point of view of the

[1] Remember that, in general, a production function gives the *maximum* quantity of output for any given combination of inputs. This maximum, in the case of ordinary production functions, such as Eqs. (22.25), is set for us by the state of technology, while in the case we are examining, in which we are trying to cause the intermediate goods to disappear, it is necessary to solve a further problem, that of the efficient allocation of resources.

integrated industry, the other commodity serves solely as an intermediate good, with a production function $B_A = F_B(\ldots)$, so that it is as if we placed $B = 0$ in the second equation of (22.25). The production function of A can therefore be rewritten as

$$A = F_A\left[K_A, L_A, F_B\left(K_A^c - K_A, L_A^c - L_A, A_B\right)\right] - A_{B,} \tag{22.27}$$

since, given the assumptions, made, $K_A^c = K_A + K_B, \quad L_A^c = L_A + L_B$. It is thus a question of maximizing A in (22.27), given K_A^c, L_A^c. The first-order conditions are

$$\begin{aligned} \frac{\partial A}{\partial K_A} &= \frac{\partial F_A}{\partial K_A} - \frac{\partial F_A}{\partial B_A}\frac{\partial F_B}{\partial K_A} &= 0, \\ \frac{\partial A}{\partial L_A} &= \frac{\partial F_A}{\partial L_A} - \frac{\partial F_A}{\partial B_A}\frac{\partial F_B}{\partial L_A} &= 0, \\ \frac{\partial A}{\partial A_B} &= \frac{\partial F_A}{\partial B_A}\frac{\partial F_B}{\partial A_B} - 1 &= 0. \end{aligned} \tag{22.28}$$

The interpretation is very simple: the first two conditions tell us that the marginal productivity (in terms of A) of each primary factor must be the same whether the factor is used directly or indirectly in the production of A (by producing B, which is used as an intermediate good in the production of A). The third condition tells us that the marginal productivity of A in terms of itself (that is, when A is used as an intermediate good to produce B which is used as an intermediate good to produce A) must be equal to one.

The integrated industry is completely described by (22.27) and (22.28). On the basis of the theory of comparative statics, it is possible—provided that the second order conditions for a maximum have been satisfied—to use Eqs. (22.28) to express K_A, L_A, A_B as differentiable functions of the two parameters K_A^c, L_A^c. By substituting these functions in (22.27), we can see that A is ultimately expressed as a function only of K_A^c, L_A^c, that is, $A = N_A(K_A^c, L_A^c)$,[2] which is in fact the first of Eqs. (22.26). The second of Eqs. (22.26) can be obtained in the same way.

22.3.2 Pure Intermediate Goods

Let us now examine the model with a "pure" intermediate good. The first point to be considered is that the classification of goods on the basis of the apparent factor intensity can be different from the classification of goods on the basis of the total factor intensity. If we indicate the pure intermediate good by Z, we get the following

[2] Still making use of the method of comparative statics, it is possible to obtain explicit expressions for the partial derivatives of the N_A function and to show that it is homogeneous of the first degree. See, for example, Chacholiades (1978, pp. 231–232).

equations, which express the full employment of the primary factors and of the intermediate good:

$$\begin{aligned} a_{KA}A + a_{KB}B + a_{KZ}Z &= K, \\ a_{LA}A + a_{LB}B + a_{LZ}Z &= L, \\ a_{ZA}A + a_{ZB}B &= Z, \end{aligned} \tag{22.29}$$

where $a_{KA} = K_A/A$ etc., are the apparent technical coefficients. By substituting from the third equation into the previous ones, we get

$$\begin{aligned} a^c_{KA}A + a^c_{KB}B &= K, \\ a^c_{LA}A + a^c_{LB}B &= L, \end{aligned} \tag{22.30}$$

where

$$\begin{aligned} a^c_{KA} &= a_{KA} + a_{KZ}a_{ZA},\ a^c_{KB} = a_{KB} + a_{KZ}a_{ZB}, \\ a^c_{LA} &= a_{LA} + a_{LZ}a_{ZA},\ \ a^c_{LB} = a_{LB} + a_{LZ}a_{ZB}, \end{aligned} \tag{22.31}$$

are the total technical coefficients.

Apparent and total factorial intensities are then[3]

$$\begin{aligned} &\varrho_A = \frac{a_{KA}}{a_{LA}}, \quad \varrho_B = \frac{a_{KB}}{a_{LB}}, \quad \varrho_Z = \frac{a_{KZ}}{a_{LZ}}, \\ &\varrho^c_A = \frac{a^c_{KA}}{a^c_{LA}} = \frac{a_{KA} + a_{KZ}a_{ZA}}{a_{LA} + a_{LZ}a_{ZA}}, \quad \varrho^c_B = \frac{a^c_{KB}}{a^c_{LB}} = \frac{a_{KB} + a_{KZ}a_{ZB}}{a_{LB} + a_{LZ}a_{ZB}}. \end{aligned} \tag{22.32}$$

By introducing the quantities

$$\gamma_A = \frac{a_{LZ}a_{ZA}}{a_{LA} + a_{LZ}a_{ZA}}, \quad \gamma_B = \frac{a_{LZ}a_{ZB}}{a_{LB} + a_{LZ}a_{ZB}}, \tag{22.33}$$

it is possible to express the total factor intensities of A and B as weighted averages of the respective apparent intensities and of the factor intensity of Z, that is

$$\begin{aligned} \varrho^c_A &= (1 - \gamma_A)\,\varrho_A + \gamma_A\varrho_Z, \\ \varrho^c_B &= (1 - \gamma_B)\,\varrho_B + \gamma_B\varrho_Z, \end{aligned} \tag{22.34}$$

as can be ascertained by direct substitution. Given the properties of the average, ϱ^c_A will be included between ϱ_A and ϱ_Z, and ϱ^c_B between ϱ_B and ϱ_Z. Thus, if ϱ_Z is included between ϱ_A and ϱ_B, the classification based on total intensities coincides with that based on apparent intensities. In fact, if $\varrho_A > \varrho_Z > \varrho_B$ then, as ϱ^c_A is

[3]As the intermediate good is produced exclusively with primary factors, it shows no distinction between apparent and total coefficients or between apparent and total factor intensities.

included between ϱ_A and ϱ_Z while ϱ_B^c is included between ϱ_Z and ϱ_B, ϱ_A is also greater than ϱ_B^c; likewise if $\varrho_A < \varrho_Z < \varrho_B$.

On the other hand, whenever ϱ_Z is not included between ϱ_A and ϱ_B it is possible for the classification based on total intensity to be different from that based on apparent intensity, giving rise to the problems mentioned in the text.[4]

We now pass to the demonstration of the theorem stated in Sect. 6.4, according to which, if it is assumed that one of the three goods is non-traded and the apparent capital intensity of this good is intermediate between the apparent intensities of the two traded goods, then the Heckscher-Ohlin theorem is valid. For this purpose we use the dual approach (see Sect. 19.5) extended to our case. As the majority of empirical studies regarding intermediate goods take as reference Leontief's input-output model, in which the input coefficients of intermediate goods are assumed to be constant, we too adopt this simplification. The coefficients a_{ZA}, a_{ZB}, are therefore assumed to be constant. The price equations are

$$\begin{aligned} a_{LA}p_L + a_{KA}p_K + a_{ZA}p_Z &= p_A, \\ a_{LB}p_L + a_{KB}p_K + a_{ZB}p_Z &= p_B, \\ a_{LZ}p_L + a_{KZ}p_K \qquad\qquad &= p_Z, \end{aligned} \tag{22.35}$$

from which, calculating the total differentials, assuming A as numéraire (whence $\mathrm{d}p_A = 0$) and rearranging terms, we have

$$\begin{aligned} a_{LA}\mathrm{d}p_L + a_{KA}\mathrm{d}p_K + a_{ZA}\mathrm{d}p_Z &= -\left(p_L\mathrm{d}a_{LA} + p_K\mathrm{d}a_{KA}\right), \\ a_{LB}\mathrm{d}p_L + a_{KB}\mathrm{d}p_K + a_{ZB}\mathrm{d}p_Z &= \mathrm{d}p_B - \left(p_L\mathrm{d}a_{LB} + p_K\mathrm{d}a_{KB}\right), \\ a_{LZ}\mathrm{d}p_L + a_{KZ}\mathrm{d}p_K - \mathrm{d}p_Z \qquad &= -\left(p_L\mathrm{d}a_{LZ} + p_K\mathrm{d}a_{KZ}\right). \end{aligned} \tag{22.36}$$

As the minimum cost conditions imply that $p_L\mathrm{d}a_{Li} + p_K\mathrm{d}a_{Ki} = 0, i = A, B, Z$, the terms in brackets on the right-hand side of (22.36) disappear. If we now solve this system, we obtain

$$\begin{aligned} \mathrm{d}p_L &= \frac{a_{KA} + a_{ZA}a_{KZ}}{D}\mathrm{d}p_B, \\ \mathrm{d}p_K &= \frac{-\left(a_{LA} + a_{ZA}a_{LZ}\right)}{D}\mathrm{d}p_B, \\ \mathrm{d}p_Z &= \frac{a_{LA} + a_{LZ}\left(\varrho_A - \varrho_Z\right)}{D}\mathrm{d}p_B, \end{aligned} \tag{22.37}$$

[4]For the two classifications to coincide even in this case, it is necessary for the final commodity, with a capital/labour ratio between the capital/labour ratio of the intermediate good and the capital/labour ratio of the other final good, to have an intensity of use of the intermediate good equal to or greater than that of the other final good. This can be demonstrated by starting from Eqs. (22.32) and afterwards examining the appropriate inequalities.

It is as well at this point to note that, in the model previously examined (A and B are used both as final and intermediate goods) the two classifications necessarily coincide: see Vanek (1963).

where

$$D \equiv a_{LA}a_{LB}(\varrho_A - \varrho_B) + a_{ZA}a_{LB}a_{LZ}(\varrho_Z - \varrho_B) + a_{ZB}a_{LA}a_{LZ}(\varrho_A - \varrho_Z). \tag{22.38}$$

Let us now assume that country 1 is relatively capital abundant in economic terms (see Sect. 4.2), that is, $q_1 > q_2$, where $q = p_L/p_K$. We then begin to consider the case in which the intermediate good is non-traded. As

$$\frac{\mathrm{d}q}{\mathrm{d}p_B} = \left(p_K \frac{\mathrm{d}p_L}{\mathrm{d}p_B} - p_L \frac{\mathrm{d}p_K}{\mathrm{d}p_B}\right) / p_K^2,$$

given Eqs. (22.37) we have

$$\frac{\mathrm{d}q}{\mathrm{d}p_B} = \frac{1}{p_K^2 D},$$

and therefore, assuming that D will be different from zero

$$\frac{\mathrm{d}p_B}{\mathrm{d}q} = p_K^2 D. \tag{22.39}$$

We must remember that, having used A as numéraire ($p_A = 1$), p_B is in effect the relative price of the final goods. Equation (22.39) therefore expresses the relationship between the relative price of the final goods and the relative price of the factors, which must be single-valued for the Heckscher-Ohlin theorem to be valid. In fact, it is necessary that a different relative price of goods in autarky corresponds uniquely to a different relative factor endowment (in economic terms). If, for example, $D > 0$, we have $\mathrm{d}p_B/\mathrm{d}q > 0$ and, with $q_1 > q_2$, this means that $(p_B)_1 > (p_B)_2$ in autarky, so that on opening international trade (which determines a single common price lying between the two autarkic prices) country 2 will export B and country 1 will export A. Does this conform to the Heckscher-Ohlin theorem? The answer is yes, provided that ϱ_Z is included between ϱ_A and ϱ_B. In fact, with $\varrho_A > \varrho_Z > \varrho_B$ we have $D > 0$ and country 1 in fact exports the capital intensive good. Similarly, with $\varrho_B > \varrho_Z > \varrho_A$ we have $D < 0$, and given (22.39), it follows that $(p_B)_1 < (p_B)_2$, so that country 1 will export B, which is now the capital-intensive one. Thus, in any case in which the capital intensity of the intermediate good (which, as we have assumed, is non-traded) is included between those of the two traded goods, it is true that country 1, with a relatively high capital endowment (in economic terms) will export the more capital-intensive good in conformity with the Heckscher-Ohlin theorem.

Let us now examine the case in which the non-traded good is one of the two final goods, for example, A. It is now necessary to find an expression which will give us the derivative of the relative price p_Z/p_B with respect to q and establish the conditions under which it has a unique sign in relation to the factor intensities. Since

$$\frac{d(p_Z/p_B)}{dq} = \frac{1}{p_B^2}\left(p_B\frac{dp_Z}{dq} - p_Z\frac{dp_B}{dq}\right), \tag{22.40}$$

the procedure consists in calculating dp_Z/dq, as dp_B/dq is already known from (22.39). This calculation can be carried out if one notes that $dp_Z/dq = (dp_Z/dp_B)(dp_B/dq)$ and if one uses Eq. (22.37) to determine dp_Z/dp_B. We refer the reader to Batra and Casas (1973, p. 307) for the details, and we shall limit our observations to the fact that (22.40) will certainly have a unique sign, if the factor intensity of the non-traded good A is intermediate between that of B and that of Z. More precisely, we have

$$\frac{d(p_Z/p_B)}{dq} > 0 \quad \text{if} \quad \varrho_B > \varrho_A > \varrho_Z, \tag{22.41}$$

and therefore, given that $q_1 > q_2$ we get $(p_Z/p_B)_1 > (p_Z/p_B)_2$, so that country 2 will export Z and country 1 will export B (which is more capital-intensive than Z), in conformity with the Heckscher-Ohlin theorem.

Similarly, it can be seen that, as

$$\frac{d(p_Z/p_B)}{dq} < 0 \quad \text{if} \quad \varrho_B < \varrho_A < \varrho_Z, \tag{22.42}$$

the assumption $q_1 > q_2$ implies $(p_Z/p_B)_1 < (p_Z/p_B)_2$, so that country 1 will export Z (which is now the most capital-intensive).

This completes the demonstration of the theorem given in the text. For other approaches to trade in intermediate goods, see, e.g., Sanyal and Jones (1982) and Sarkar (1985).

22.4 Elastic Supply of Factors

We propose to examine formally the behaviour of the offer curve of a country with the endogenous variation of the supply of labour, with the aim of ascertaining the conditions under which this curve will have an anomalous shape. This means ascertaining the conditions under which the country increases its demand for imports when their price is greater. Let us assume that the country concerned imports B.[5] The demand for imports will be given by the difference between domestic demand and domestic production of the commodity in question:

$$E_B = B^D(I_A, p) - B(p, L), \tag{22.43}$$

[5] In the text we assumed that A is the imported commodity, but this has no effect on the conclusions.

where the meaning of the symbols is as given in the Chap. 19: the one thing to note here is that since the supply of labour is variable, the quantity of goods produced is also a function of employment L, in addition to being a function of the relative price $p = p_B/p_A$. We shall now calculate the total derivative of E_B with respect to p, bearing in mind that

$$I_A = A(p, L) + pB(p, L), \tag{22.44}$$

and that employment L is also a function of p through the labour market. Thus we have

$$\begin{aligned}\frac{\mathrm{d}E_B}{\mathrm{d}p} &= \frac{\partial B^D}{\partial I_A}\frac{\mathrm{d}I_A}{\mathrm{d}p} + \frac{\partial B^D}{\partial p} - \frac{\partial B}{\partial p} - \frac{\partial B}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}p} \\ &= \frac{\partial B^D}{\partial I_A}\left(\frac{\partial I_A}{\partial p} + \frac{\partial I_A}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}p}\right) + \frac{\partial B^D}{\partial p} - \frac{\partial B}{\partial p} - \frac{\partial B}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}p}.\end{aligned} \tag{22.45}$$

We now recall that $\partial I_A/\partial p = B$ (see Eq. (19.22)) and that (see Eq. (27.36)) $\partial I_A/\partial L = g_A - \varrho_A g'_A > 0$; defining the marginal propensity to import A as $\mu \equiv p(\partial B^D/\partial I_A)$, we get

$$\frac{\mathrm{d}E_B}{\mathrm{d}p} = \frac{\partial B^D}{\partial p} + \frac{\mu}{p}\left[B + \left(g_A - \varrho_A g'_A\right)\frac{\mathrm{d}L}{\mathrm{d}p}\right] - \left(\frac{\partial B}{\partial p} + \frac{\partial B}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}p}\right). \tag{22.46}$$

If we note that, on the basis of (27.33), $\partial B/\partial L = -\varrho_A g_B/\left(\varrho_B - \varrho_A\right)$ and rearrange the terms, we get

$$\frac{\mathrm{d}E_B}{\mathrm{d}p} = \left(\frac{\partial B^D}{\partial p} + \frac{\mu}{p}B - \frac{\partial B}{\partial p}\right) + \left[\left(g_A - \varrho_A g'_A\right)\frac{\mu}{p} + \frac{\varrho_A g_B}{\varrho_B - \varrho_A}\right]\frac{\mathrm{d}L}{\mathrm{d}p}. \tag{22.47}$$

As, on the basis of (19.17), $g_A - \varrho_A g'_A = p\left(\varrho_B - \varrho_B g'_B\right)$, we can rewrite the expression in square brackets appropriately and we finally get

$$\begin{aligned}\frac{\mathrm{d}E_B}{\mathrm{d}p} &= \left(\frac{\partial B^D}{\partial p} + \frac{\mu}{p}B - \frac{\partial B}{\partial p}\right) \\ &\quad + \left[\left(g_B - \varrho_B - \varrho_B g'_B\right)\left(\mu - \frac{\varrho_A}{\varrho_A - \varrho_B}\frac{g_B}{g_B - \varrho_B g'_B}\right)\right]\frac{\mathrm{d}L}{\mathrm{d}p}.\end{aligned} \tag{22.48}$$

In the case where the supply of labour is constant, $\mathrm{d}L/\mathrm{d}p = 0$ and the derivative $\mathrm{d}E_B/\mathrm{d}p$ will be given by the first expression in parentheses, which we shall assume to be negative, given the assumption that the basic offer curve is normal.

In the case where the supply of labour is endogenously variable, $\mathrm{d}L/\mathrm{d}p \neq 0$ and the expression in square brackets also comes into play. Let us suppose that B is the labour-intensive commodity, so that $\varrho_A > \varrho_B$. We thus get $\varrho_A/\left(\varrho_A - \varrho_B\right) > 1$; and

$g_B/\left(g_B - \varrho_B g_B'\right)$ is also a magnitude greater than one. Under normal conditions $0 < \mu < 1$, and therefore the expression in square brackets is negative. It can at once be seen that, if $\mathrm{d}L/\mathrm{d}p$ is negative, it is possible that $\mathrm{d}E_B/\mathrm{d}p > 0$, i.e., that the country under consideration demands more imports when their price is higher.

The economic meaning of $\mathrm{d}L/\mathrm{d}p < 0$ has already be clarified in Sect. 6.5: when p increases, the real reward of labour grows (Stolper-Samuelson theorem) and, as long as it lies along the backward-bending branch of the labour supply curve, the supply decreases. The reader can obtain further information, for example, in Kemp (1969b, chap. 5); an alternative approach to the one followed here will be found in Laffer and Miles (1982, chap. 8) .

The fact that $\mathrm{d}L/\mathrm{d}p < 0$ can lead to anomalous (or, as some would say, perverse) quantity-price relations is therefore a theoretically admissible possibility; however, some theorists argue against the probability of this actually occurring (Martin & Neary, 1980) .

22.5 Non-traded Goods

Let us take three goods, A, B, N, of which the first is imported, the second exported and the third non-traded; consequently, excess demand for A is positive, for B negative, and for N zero. The production functions have the usual properties (first-degree homogeneity, etc.), so that

$$A = L_A g_A\left(\varrho_A\right), \quad B = L_B g_B\left(\varrho_B\right), \quad N = L_N g_N\left(\varrho_N\right). \tag{22.49}$$

We also have, in equilibrium—see Eqs. (19.17)—that

$$\begin{aligned} g_A' &= p g_B' = p_n g_N', \\ g_A - \varrho_A g_A' &= p\left(g_B - \varrho_B g_B'\right) = p_n\left(g_N - \varrho_N g_N'\right), \end{aligned} \tag{22.50}$$

where $p = p_B/p_A$ and $p_n = p_N/p_A$ are the relative prices.

Given the existing quantities of factors, the full employment conditions are

$$\sum_i L_i = L, \quad \sum_i K_i = \sum_i \varrho_i L_i = K, \quad i = A, B, N. \tag{22.51}$$

Let us now assume that the prices of the two non-traded goods (or their relative price) are given and let us consider the following equations, which are a sub-set of Eqs. (22.50)

$$\begin{aligned} g_A' - p g_B' &= 0, \\ g_A - \varrho_A g_A' - p\left(g_B - \varrho_B g_B'\right) &= 0. \end{aligned} \tag{22.52}$$

These allow us to express ϱ_A, ϱ_B as single-valued functions of p, as already seen in Eqs. (20.20) and (20.21).

If we now consider the sub-system

$$\begin{aligned} g_A' - p_n g_N' &= 0, \\ \left(g_A - \varrho_A g_A'\right) - p_n\left(g_N - \varrho_N g_N'\right) &= 0, \end{aligned} \tag{22.53}$$

we can solve it—provided there is no factor intensity reversal so that its Jacobian is other than zero—obtaining uniquely ϱ_N and p_n as functions of ϱ_A and therefore of p, which demonstrates that, in the context of the traditional model, the price of the non-traded good is uniquely determined by the terms of trade.

22.5.1 The Behaviour of the Offer Curve

Let us now go on to examine the behaviour of the offer curve in order to ascertain the conditions under which the country under consideration has an increased demand for imports when their price is higher. We therefore have to calculate the derivative

$$\frac{\mathrm{d}E_A}{\mathrm{d}p} = \frac{\mathrm{d}A^D}{\mathrm{d}p} - \frac{\mathrm{d}A}{\mathrm{d}p}.$$

We begin with the observation that the production of A is no longer, as in the two-commodity model, an increasing function of p, because of the fact that, following the variations of p, p_n also varies and therefore shifts of resources occur between sector N and sector A. In order to calculate $\mathrm{d}A/\mathrm{d}p$ it is therefore necessary to take account of all these effects.[6] If we start from the production functions (22.49) we get

$$\frac{\mathrm{d}A}{\mathrm{d}p} = \frac{\mathrm{d}}{\mathrm{d}p}\left[L_A g_A\left(\varrho_A\right)\right] = g_A\frac{\mathrm{d}L_A}{\mathrm{d}p} + L_A g_A'\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}. \tag{22.54}$$

Let us now calculate the derivatives $\mathrm{d}L_A/\mathrm{d}p$ and $\mathrm{d}\varrho_A/\mathrm{d}p$. As we shall see, when calculating $\mathrm{d}L_A/\mathrm{d}p$ we shall also calculate $\mathrm{d}\varrho_A/\mathrm{d}p$.

By differentiating Eqs. (22.51) with respect to p, we get

$$\begin{aligned} \frac{\mathrm{d}L_A}{\mathrm{d}p} + \frac{\mathrm{d}L_B}{\mathrm{d}p} + \frac{\mathrm{d}L_N}{\mathrm{d}p} &= 0, \\ \varrho_A\frac{\mathrm{d}L_A}{\mathrm{d}p} + L_A\frac{\mathrm{d}\varrho_A}{\mathrm{d}p} + \varrho_B\frac{\mathrm{d}L_B}{\mathrm{d}p} + L_B\frac{\mathrm{d}\varrho_B}{\mathrm{d}p} + \varrho_N\frac{\mathrm{d}L_N}{\mathrm{d}p} + L_N\frac{\mathrm{d}\varrho_N}{\mathrm{d}p} &= 0, \end{aligned} \tag{22.55}$$

[6] Since these effects also involve the demand for N—as we shall find from (22.65)—it can be seen at once that it is now no longer possible, as in the traditional model given in Chap. 3 and Appendix, to consider the productive side of the model separately from the demand side.

from which

$$\begin{aligned}\frac{\mathrm{d}L_A}{\mathrm{d}p}+\frac{\mathrm{d}L_B}{\mathrm{d}p} &= -\frac{\mathrm{d}L_N}{\mathrm{d}p},\\ \varrho_A\frac{\mathrm{d}L_A}{\mathrm{d}p}+\varrho_B\frac{\mathrm{d}L_B}{\mathrm{d}p} &= -\varrho_N\frac{\mathrm{d}L_N}{\mathrm{d}p}-\sum_i L_i\frac{\mathrm{d}\varrho_i}{\mathrm{d}p},\end{aligned}\tag{22.56}$$

and therefore

$$\frac{\mathrm{d}L_A}{\mathrm{d}p}=-\frac{\varrho_B-\varrho_N}{\varrho_B-\varrho_A}\frac{\mathrm{d}L_N}{\mathrm{d}p}+\frac{1}{\varrho_B-\varrho_A}\sum_i L_i\frac{\mathrm{d}\varrho_i}{\mathrm{d}p}.\tag{22.57}$$

The value of $\mathrm{d}L_A/\mathrm{d}p$ is therefore dependent on $\mathrm{d}L_N/\mathrm{d}p$ and $\mathrm{d}\varrho_i/\mathrm{d}p$, $i=A,B,N$. To get $\mathrm{d}L_N/\mathrm{d}p$ one has merely to start from the condition of equilibrium in the market of the non-traded good, $L_N g_N(\varrho_N)=N^D$ from which, on the basis of the implicit-function rule,

$$\frac{\mathrm{d}L_N}{\mathrm{d}p}=-\frac{1}{g_N}\left(\frac{\mathrm{d}N^D}{\mathrm{d}p}-L_N g_N'\frac{\mathrm{d}\varrho_N}{\mathrm{d}p}\right).\tag{22.58}$$

As for the derivatives $\mathrm{d}\varrho_i/\mathrm{d}p$, one has simply to start out from the conditions of equilibrium given in Eqs. (22.50), and calculate the total derivative thereof with respect to p; solving the consequent system, we get $\mathrm{d}\varrho_i/\mathrm{d}p$. A simpler alternative is to determine $\mathrm{d}\varrho_A/\mathrm{d}p$ and $\mathrm{d}\varrho_B/\mathrm{d}p$ by differentiating system (22.52) and then to calculate $\mathrm{d}\varrho_N/\mathrm{d}p$ by differentiating system (22.53).

The second method gives us

$$\begin{aligned}&g_A''\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}-g_B'-pg_B''\frac{\mathrm{d}\varrho_B}{\mathrm{d}p}=0,\\ &g_A'\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}-\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}g_A'-\varrho_A g_A''\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}-\left(g_B-\varrho_B g_B'\right)\\ &\quad -p\left(g_B'\frac{\mathrm{d}\varrho_B}{\mathrm{d}p}-g_B'\frac{\mathrm{d}\varrho_B}{\mathrm{d}p}-\varrho_B g_B''\frac{\mathrm{d}\varrho_B}{\mathrm{d}p}\right)=0,\end{aligned}\tag{22.59}$$

and, if we simplify and rearrange the terms, we get

$$\begin{aligned}g_A''\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}-pg_B''\frac{\mathrm{d}\varrho_B}{\mathrm{d}p} &= g_B',\\ -\varrho_A g_A''\frac{\mathrm{d}\varrho_A}{\mathrm{d}p}+p\varrho_B g_B''\frac{\mathrm{d}\varrho_B}{\mathrm{d}p} &= g_B-\varrho_B g_B'.\end{aligned}\tag{22.60}$$

By solving, we obtain

$$\begin{aligned}\frac{d\varrho_A}{dp} &= \frac{g_B}{g_A''(\varrho_B-\varrho_A)},\\ \frac{d\varrho_B}{dp} &= \frac{g_A}{p^2 g_B''(\varrho_B-\varrho_A)}.\end{aligned} \tag{22.61}$$

Similarly, if we differentiate system (22.53) with respect to p, simplifying and rearranging the terms gives us

$$\begin{aligned} g_A''\frac{d\varrho_A}{dp} - p_n g_N''\frac{d\varrho_N}{dp} &= g_N'\frac{dp_n}{dp},\\ -\varrho_A g_A''\frac{d\varrho_A}{dp} + p_n\varrho_N g_N''\frac{d\varrho_N}{dp} &= \left(g_N - \varrho_N g_N'\right)\frac{dp_n}{dp},\end{aligned} \tag{22.62}$$

from which, by solving,

$$\begin{aligned}\frac{d\varrho_A}{dp} &= \frac{g_N}{g_A''(\varrho_N-\varrho_A)}\frac{dp_n}{dp},\\ \frac{d\varrho_B}{dp} &= \frac{g_N}{p_n^2 g_N''(\varrho_N-\varrho_A)}\frac{dp_n}{dp},\end{aligned} \tag{22.63}$$

where the first of Eqs. (22.61) and the first of Eqs. (22.63) must naturally coincide, a fact that enables us to determine the derivative of p_n with respect to p:

$$\frac{dp_n}{dp} = \frac{(\varrho_N-\varrho_A)\,g_B}{(\varrho_B-\varrho_A)\,g_N}. \tag{22.64}$$

We now have all the elements necessary to determine dA/dp, by substituting in Eq. (22.54) the results obtained by means of Eqs. (22.57), (22.58), (22.61) and (22.63). We thus get

$$\frac{dA}{dp} = -\frac{g_A}{g_N}\frac{\varrho_B-\varrho_N}{\varrho_B-\varrho_A}\frac{dN^D}{dp} + H, \tag{22.65}$$

where

$$H \equiv \frac{pL_N g_A g_B^2}{p_n^3 g_N g_N''(\varrho_B-\varrho_A)^2} + \frac{pL_A g_B^2}{g_A''(\varrho_B-\varrho_A)^2} + \frac{L_B g_A^2}{p^2 g_B^2(\varrho_B-\varrho_A)^2} < 0. \tag{22.66}$$

Term H tends therefore to make dA/dp take on the right sign for the normality of the offer curve. However, we also have to take into account the first terms on the right-hand side of Eq. (22.65), which may very well be positive and of a higher absolute value than H, so that $dA/dp > 0$ (the economic meaning of this apparently anomalous sign has been clarified in Sect. 6.6). Even without determining the sign of dA^D/dp (which can, in turn, be anomalous: the reader can consult Komiya, 1967; Kemp, 1969b, chap. 6), this is sufficient to establish the possibility that $dE_A/dp < 0$

that is, $dE_A/d\,(1/p) > 0$; this result means that the demand for imports can rise with the rise in the price of imports $p_A/p_B = 1/p$.

22.6 Specific Factors and De-industrialization

Following Corden and Neary (1982) , we shall analyse the problem by means of the dual approach (see Sect. 19.5), appropriately extended to the case of three goods and modified so as to take into account the presence of specific factors (see also Jones, 1971). Bearing in mind that labour is the only mobile factor between sectors and is fully employed, we get the equation

$$a_{LA}A + a_{LB}B + a_{LN}N = L, \tag{22.67}$$

where a_{Li}, $i = A, B, N$, are the technical coefficients. The demand for the non-traded good N (the market for which is constantly in equilibrium) is a function of real national income y and of the price p_N; for simplicity, we shall neglect the effects of the prices of the other goods, and of income distribution.[7] Using the asterisk to indicate proportional variations, we have

$$N^{D^*} = -\varepsilon_N p_N^* + \eta y^*, \tag{22.68}$$

where ε_N and η are the price elasticity and the income elasticity of demand respectively.

In this model the only source of increase in real income is technical progress in the extractive sector which generates the boom, so that

$$y^* = \theta_A \pi, \tag{22.69}$$

where θ_A is the share of the extractive sector in national income and π is the Hicksian measure of technical progress. By substituting (22.69) in (22.68) we have

$$N^{D^*} = -\varepsilon_N p_N^* + \eta\theta_A\pi. \tag{22.70}$$

If we indicate the specific capital of each sector with K_i, it is necessary to add the full employment conditions of each specific factor, that is

$$a_{KA}A = K_A, \quad a_{KB}B = K_B, \quad a_{KN}N = K_N. \tag{22.71}$$

[7] For the complications introduced by the effects that a changed income distribution at a constant price of N has on spending on N see Corden (1984a, fn. 5 on p. 361).

If we differentiate Eq. (22.67) and transform the result into proportional variations, by following the procedure illustrated in Sect. 19.5 (bearing in mind that now L is constant), we have

$$\lambda_{LA}\left(A^* + a_{LA}^*\right) + \lambda_{LB}\left(B^* + a_{LB}^*\right) + \lambda_{LN}\left(N^* + a_{LN}^*\right) = 0, \tag{22.72}$$

where $\lambda_{LA} = a_{Li}A/L$, etc., denote the fractions of the total labour force employed in the various sectors. Following the usual procedure, from Eqs. (22.71) we get

$$a_{KA}^* + A^* = 0, \quad a_{KB}^* + B^* = 0, \quad a_{KN}^* + N^* = 0, \tag{22.73}$$

and by substituting in Eqs. (22.72), we obtain

$$\lambda_{LA}\left(a_{LA}^* - a_{KA}^*\right) + \lambda_{LB}\left(a_{LB}^* - a_{KB}^*\right) + \lambda_{LN}\left(a_{LN}^* - a_{KN}^*\right) = 0. \tag{22.74}$$

From Eqs. (19.63) we have

$$a_{Li}^* - a_{Ki}^* = -\sigma_i\left(p_L^* - p_{Ki}^*\right), \quad i = A, B, N, \tag{22.75}$$

where σ_i is the elasticity of substitution in sector i. As labour is mobile, p_L^* is equal throughout, while the p_{Ki}^* are specific for each sector. From the equality between price and unit cost—see Eqs. (19.59) and (19.62)—account being taken of the technical progress factor and using B as numéraire, we have

$$\begin{aligned} p_A^* &= \theta_{LA}p_L^* + \theta_{KA}p_{KA}^* - \pi, \\ 0 &= \theta_{LB}p_L^* + \theta_{KB}p_{KB}^*, \\ p_N^* &= \theta_{LN}p_L^* + \theta_{KN}p_{KN}^*, \end{aligned} \tag{22.76}$$

where $\theta_{LA} = a_{LA}p_L/p_A$ etc. is the share of labour in the value of output in sector A and so on. By substituting Eqs. (22.75) and (22.76) in (22.74), assuming that $p_A^* = 0$ as the price of good A is given by the international market and simplifying, we have

$$p_L^* = \xi_A\pi + \xi_N p_N^*, \tag{22.77}$$

where $0 < \xi_i < 1$ is the proportional contribution of sector i to Δ, the elasticity with respect to wages of the aggregate demand for labour:

$$\begin{aligned} \xi_i &\equiv \frac{1}{\Delta}\lambda_{Li}\frac{\sigma_i}{\theta_{Ki}}, i = A, B, N, \\ \Delta &\equiv \lambda_{LA}\frac{\sigma_A}{\theta_{KA}} + \lambda_{LB}\frac{\sigma_B}{\theta_{KB}} + \lambda_{LN}\frac{\sigma_N}{\theta_{KN}}. \end{aligned} \tag{22.78}$$

Turning now to the market for N, supply depends solely on the real wage which entrepreneurs have to meet in this sector. In fact, as K_N is assumed fully employed and immobile, the quantity of N produced will depend on the quantity of labour utilized, which in turn is a function of the real wage,[8] following the optimization principle, according to which the entrepreneur equates the marginal productivity of labour to the real wage. Thus, if, as usual, we consider the proportional variations, we get

$$N^* = \Phi_N \left(p_N^* - p_L^* \right), \tag{22.79}$$

where $\Phi_N \equiv \sigma_N \theta_{LN} / \theta_{KN}$ is the price elasticity of supply.

By equating demand (22.68) and supply (22.79), we obtain

$$\left(\Phi_N + \varepsilon_N \right) p_N^* = \Phi_N p_L^* + \eta \theta_A \pi. \tag{22.80}$$

We can now solve the system made up of Eqs. (22.77) and (22.80) for the unknowns p_N^* and p_L^*, obtaining

$$\begin{aligned} H p_N^* &= \left(\eta \theta_N + \Phi_N \xi_A \right) \pi > 0, \\ H p_L^* &= \left[\eta \xi_N \theta_A + \left(\Phi_N + \varepsilon_N \right) \xi_A \right] \pi > 0, \end{aligned} \tag{22.81}$$

where

$$H \equiv \Phi_N \left(1 - \xi_N \right) + \varepsilon_N > 0. \tag{22.82}$$

22.6.1 Effects on Prices, Outputs and Factor Rewards

Relations (22.81) confirm what was said in Sect. 6.7, namely, *that both the relative price of N and the real wage increase*.

In order to see how the production of N varies it is sufficient to substitute p_N^* and p_L^* from (22.81) into (22.79), thus obtaining

$$N^* = \left(\Phi_N / H \right) \left[\eta \theta_A \left(1 - \xi_N \right) - \xi_A \varepsilon_N \right]. \tag{22.83}$$

As can be seen, N can be either positive or negative (i.e., *the production of the non-traded good may either increase or decrease*); with regard to the argument in the text, note that η determines the magnitude of the *spending effect* (which causes the production of N to increase), while ξ_A determines the magnitude of the *resource movement effect* (which causes the production of N to decrease).

[8] It is as well to point out that we use "real wage" in the sense of wage expressed in terms of the product; the real wage expressed in terms of wage-earners' purchasing power will be examined later.

Since, from Eqs. (22.81), the real wage in sector B increases (remember that we have taken B as numéraire, so that p_L^* is expressed in terms of that commodity), employment, and therefore output, in this sector necessarily decrease (*de-industrialization*).

We come now to factor rewards. The real wage, measured in terms of workers' purchasing power, may vary in any direction according to the direction in which p_N varies (remember that p_A and p_B are assumed constant). If we indicate with α_N the share of wages used by workers to buy N, the variation in the real wage from the point of view of the workers will be

$$p_L^* - \alpha_N p_N^* = \frac{1}{H} \{\eta\theta_A (\xi_N - \alpha_N) + \xi_A [\Phi_N (1 - \alpha_N) + \varepsilon_N]\} \pi, \tag{22.84}$$

which may also be negative if $p_N^* > 0$ and if α_N is sufficiently large.

In order to determine the variations in the rewards of the specific factors, all that is needed is to combine Eqs. (22.81) with Eqs. (22.76), by which we obtain

$$\begin{aligned} \theta_{KA} H p_{KA}^* &= [-\eta\xi_N \theta_{LA}\theta_A + \Phi_N (1 - \theta_{LA}\xi_A - \xi_N) + \varepsilon_N (1 - \theta_{LA}\xi_A)] \pi, \\ \theta_{KB} H p_{KB}^* &= -\theta_{LB} [\eta\xi_N \theta_A + \xi_A (\Phi_N + \varepsilon_N)] \pi < 0, \\ \theta_{KN} H p_{KN}^* &= [\eta (1 - \theta_{LN}\xi_N) \theta_A + \xi_A (\theta_{KN}\Phi_N - \theta_{LN}\varepsilon_N)] \pi. \end{aligned} \tag{22.85}$$

As can be seen, only the sign of p_{KB}^* is certain, that is, we are able to establish a priori that the reward for specific capital in sector B decreases, but we can say nothing a priori about the direction in which the reward for specific capital will vary in the other two sectors.

22.7 International Factor Mobility

The role of factor mobility in the Heckscher-Ohlin model was examined for the first time by Mundell (1957b: see Mundell, 1968, chap. 6), whose contribution has been set out in the text. Subsequently a line of research was developed (Jones, 1967; Kemp, 1969b, etc.) which dealt with the optimum tax to be imposed on movements of capital and the problem of what the tax should be if at the same time an optimum tariff is also levied on imports (see Sect. 11.1).

A third line of research (Bhagwati, 1973; Markusen & Melvin, 1979, etc.) looked into the effects on the welfare of the host country of a foreign capital inflow, followed by repatriation of profits. This literature aims to throw light on the age-old debate on the question of whether an inflow of capital is indeed a propitious event and thus to be encouraged, or whether it is damaging. It is necessary to note that in this type of analysis a continuous and potentially unlimited inflow is not considered (as in that case Mundell's results are valid), but a once-and-for-all inflow. The ownership of capital remains abroad and profits are repatriated.

The result of this analysis is that the capital inflow may in general have any effect on the welfare of the host country, as the welfare may either increase or decrease. It is fairly easy to demonstrate this result through our previous findings (see in particular Sect. 21.3) and the results of Sect. 27.2 below. In fact, a once-and-for-all capital inflow can be treated—leaving aside for the moment the question of repatriation of profits—as an exogenous increase in the existing stock of capital. The effects of this increase are well known (Rybczynski's theorem) and it is also known that under certain conditions there can be a decrease in welfare (the so-called immiserizing growth case: see Sect. 27.2). Furthermore, account must be taken of the decrease in welfare due to the fact that the profits accruing to foreign capital are to be deducted from national income, because they are repatriated. In other words, the final effect is given by the algebraic sum of two effects:

(a) The loss (or gain) that comes from the increase in capital stock;
(b) The loss that derives from the repatriation of profits on foreign capital.

Effect (a) is the one we shall meet in the analysis in Sect. 27.2, and it is clear that the addition of effect (b), which is certain to be negative, can cause the situation following the capital inflow to worsen in comparison to the initial one, not only when there is immiserizing growth (in which case effect (b) does no more than strengthen effect (a)), but also when (a) would in itself be positive.

By adopting the same criterion of comparison as in Sect. 27.2 (which allows us to avoid the problems inherent in social indifference curves) and taking up Eq. (27.25) below, we see that there will be an improvement or worsening according to whether

$$\frac{\partial I_A}{\partial \gamma} + \frac{\partial E_A/\partial \gamma}{1+\xi_1+\xi_2} \gtrless 0, \tag{22.86}$$

where, for brevity, we have omitted the subscript 1. By substituting the value of $\partial E_A/\partial \gamma$ from (27.19),[9] we have

$$\frac{\partial I_A}{\partial \gamma} + \left(\frac{\partial A^D}{\partial I_A}\frac{\partial I_A}{\partial \gamma} - \frac{\partial A}{\partial \gamma}\right)/1+\xi_1+\xi_2 \gtrless 0, \tag{22.87}$$

that is, by identifying factor γ with capital and rearranging the terms

$$\frac{\partial I_A}{\partial K}\left(1 + \frac{\partial A^D/\partial I_A}{1+\xi_1+\xi_2}\right) - \frac{\partial A/\partial K}{1+\xi_1+\xi_2} \gtrless 0. \tag{22.88}$$

It is now necessary to calculate $\partial I_A/\partial K$, taking account of effect (b). We get

[9]Equation (27.19) has been used rather than (27.20), because, as will be seen, $\partial I_A/\partial \gamma = 0$ and thus the passage from the first to the second expression is not valid in this case.

$$\frac{\partial I_A}{\partial K} = \frac{\partial}{\partial K}(A + pB) - \left(g'_A k_A + p g'_B k_B\right), \tag{22.89}$$

where the second expression in parentheses in the right-hand side is the variation in income due to the repatriation of profits: k_A and k_B are the fractions of the capital inflow that are utilized in the two sectors and g'_A, pg'_B are the respective marginal productivities. As—see (19.17)—in equilibrium $g'_A = pg'_B$ and as $k_A + k_B = 1$ by definition, we have

$$\frac{\partial I_A}{\partial K} = \left(\frac{\partial A}{\partial K} + p\frac{\partial B}{\partial K}\right) - g'_A = 0, \tag{22.90}$$

since the expression in parentheses is equal to g'_A, on the basis of Eqs. (27.42) and (27.43). *The repatriation of profits thus entirely absorbs the increase in national income consisting of the additional output made possible by the capital inflow.* This is obvious if one thinks that the increase in output is given by the capital increase (inflow) times its marginal productivity (which in equilibrium is levelled in all sectors); by rewarding foreign capital on the basis of its marginal productivity the balance is zero.

Therefore Eq. (22.88), account being taken of (27.41), becomes

$$-\frac{\partial A/\partial K}{1 + \xi_1 + \xi_2} = \frac{g_A}{(\varrho_B - \varrho_A)(1 + \xi_1 + \xi_2)} \gtrless 0. \tag{22.91}$$

If we assume that A is the imported commodity, and bear in mind that $(1 + \xi_1 + \xi_2) < 0$ for stability, there will be an improvement or a worsening according to whether $\varrho_A \gtrless \varrho_B$ that is, according to whether the imported commodity is more or less capital-intensive than the exported one. This in turn is the same as saying that there will be an improvement or a worsening according to whether the terms of trade are better or worse: in fact, if we consider Eqs. (27.17) and (27.19) and bear in mind that $\partial I_A / \partial K = 0$, we have

$$\frac{\mathrm{d}p}{\mathrm{d}K} = -\frac{\partial A/\partial K}{E_{2B}(1 + \xi_1 + \xi_2)}, \tag{22.92}$$

which—as its denominator is negative—has a sign that coincides with that of (22.91).

This result must not be taken as to be in conflict with that in Sect. 27.2, where it will be demonstrated that the worsening in the terms of trade is only a necessary, not a sufficient, condition for immiserizing growth.

In fact, this result is true when only effect (a) is considered; by introducing effect (b) it can be seen that, *as national income has remained unvaried at the level prior to the foreign capital inflow*, the worsening in the terms of trade is evidently a necessary *and* sufficient condition to produce a worsening in the situation.

On international factor movements in general, see Various Authors (1983), Jones and Dei (1983) , Ruffin (1984), and Wong (1995).

22.7.1 The Theorems of International Trade Theory Under Factor Mobility

A fourth line of research (Ethier & Svensson, 1986; Wong, 1995, chap. 4) has examined the validity of the four core theorems of international trade theory (Heckscher-Ohlin, factor price equalization, Rybczynski, Stolper and Samuelson) in the presence of factor mobility. The general result (Ethier & Svensson, 1986) is that appropriate versions of these theorems still hold provided that the number of commodities and mobile factors is at least as large as the total number of factors. This shows that the theorems are sensitive to the total number of markets (and not to the number of commodities) relative to the number of factors.

We shall illustrate this result by a two-country, two-commodity, three-factor (one which is mobile) model due to Wong (1995, chap. 4, sect. 4.1), on which the following treatment is based.

The basic assumption is that, in addition to capital (K) and labour (L), there is a third primary factor, land (D). Capital is the internationally mobile factor, while labour and land are immobile. The production function in sector $i = A, B$ is

$$Q_i = F_i(K_i, L_i, D_i), \tag{22.93}$$

with the usual properties (first-degree homogeneity, etc.).

The representative firm's optimization problem is to choose the inputs (and hence the output) so as to maximize profit for any given set of prices, namely

$$\max_{K_i, L_i, D_i} \{p_i F_i(K_i, L_i, D_i) - p_K K_i - p_L L_i - p_D D_i\}. \tag{22.94}$$

This maximization can also be carried out in two stages: in the first, the firm maximizes the objective function with respect to K_i taking L_i, D_i as given; in the second stage the result of the first stage is plugged in the objective function, which is maximized with respect to L_i, D_i. Thus we have

$$\max_{L_i, D_i} \left\{ \max_{K_i} [p_i F_i(K_i, L_i, D_i) - p_K K_i] - p_L L_i - p_D D_i \right\}. \tag{22.95}$$

Let us now define for each sector the function

$$H_i(L_i, D_i, r_i) \equiv \max_{K_i} [F_i(K_i, L_i, D_i) - r_i K_i], \tag{22.96}$$

where $r_i \equiv p_K / p_i$ is the real rental rate in terms of commodity i. The solution to this maximization problem is given by

$$\frac{\partial F_i(K_i, L_i, D_i)}{\partial K_i} - r_i = 0. \tag{22.97}$$

Since the conditions of the implicit function theorem are satisfied (we have $\partial^2 F_i/\partial K_i^2 \neq 0$, in particular $\partial^2 F_i/\partial K_i^2 < 0$ by the assumption of decreasing marginal productivity), Eq. (22.97) can be solved for the optimal value of K_i in terms of the parameters, say

$$K_i = G_i(L_i, D_i, r_i).$$

The function G_i is a continuously differentiable function of its arguments by the implicit function theorem.

Since the production function $F_i(K_i, L_i, D_i)$ is first-degree homogeneous, the function $G_i(L_i, D_i, r_i)$ is homogeneous of the first degree with respect to L_i, D_i when given r_i. It follows that the function $H_i(L_i, D_i, r_i)$ is also homogeneous of the first degree with respect to L_i, D_i when given r_i. Besides, the envelope theorem (see, for example, Mas-Colell, Whinston, & Green, 1995, pp. 964–966) shows that the partial derivatives of $H_i(L_i, D_i, r_i)$ with respect to L_i, D_i are equal to the corresponding derivatives of $F_i(K_i, L_i, D_i)$, namely

$$\frac{\partial H_i}{\partial L_i} = \frac{\partial F_i}{\partial L_i}, \quad \frac{\partial H_i}{\partial D_i} = \frac{\partial F_i}{\partial D_i}. \tag{22.98}$$

Finally, the (strict) concavity of F_i implies that H_i is (strictly) concave with respect to L_i, D_i when given r_i. In fact, consider the Hessian matrix of F_i

$$M_{F_i} = \begin{bmatrix} \frac{\partial^2 F_i}{\partial K_i^2} & \frac{\partial^2 F_i}{\partial K_i \partial L_i} & \frac{\partial^2 F_i}{\partial K_i \partial D_i} \\ \frac{\partial^2 F_i}{\partial L_i \partial K_i} & \frac{\partial^2 F_i}{\partial L_i^2} & \frac{\partial^2 F_i}{\partial L_i \partial D_i} \\ \frac{\partial^2 F_i}{\partial D_i \partial K_i} & \frac{\partial^2 F_i}{\partial D_i \partial L_i} & \frac{\partial^2 F_i}{\partial D_i^2} \end{bmatrix},$$

which is negative definite when F_i is (strictly) concave. The Hessian matrix of H_i is

$$M_{H_i} = \begin{bmatrix} \frac{\partial^2 H_i}{\partial L_i^2} & \frac{\partial^2 H_i}{\partial L_i \partial D_i} \\ \frac{\partial^2 H_i}{\partial D_i \partial L_i} & \frac{\partial^2 H_i}{\partial D_i^2} \end{bmatrix}.$$

From (22.98) it follows that, in the neighbourhood of the optimum point, M_{H_i} coincides with the south-east leading principal submatrix of M_{F_i} (the matrix obtained by deleting the first row and column of M_{F_i}). Hence if M_{F_i} satisfies the conditions for negative definiteness (the principal minors alternate in sign, beginning with minus), M_{H_i} satisfies them as well.

From all this it follows that $H_i(L_i, D_i, r_i)$ behaves like a production function in the two factors L_i, D_i.

Consider now the firm's optimization problem, that—by Eqs. (22.95) and (22.96)—can be written as

$$\max_{L_i, D_i} \{p_i H_i(L_i, D_i, r_i) - p_L L_i - p_D D_i\}. \tag{22.99}$$

As shown above, we can take $H_i(L_i, D_i, r_i)$ as a production function, so that we can use (22.99) to define a framework similar to the standard two-factor, two-sector framework. This stratagem greatly simplifies the analysis.

22.7.1.1 The Heckscher-Ohlin Theorem

We make the usual assumptions (identical technologies, homothetic preferences etcetera: see Chap. 4). The functions $H_i(L_i, D_i, r_i)$ are internationally equal since the only possible element of difference (r_i) is equalized by the international mobility of goods and capital. Thus we can concentrate on labour and land and their (relative) abundance. In exactly the same manner as in Sect. 4.2, we can show that, at the same commodity-price ratio, a country abundant in a factor has a production bias in favour of the commodity which uses that factor more intensively, and hence that it will export that commodity given the internationally identical and homothetic structure of demand.

22.7.1.2 Factor Price Equalization

Rental rates are equalized by free trade and free capital mobility. Then we can use the traditional arguments (Sect. 4.3) on the functions $H_i(L_i, D_i, r_i)$ to show that with internationally identical commodity prices and rental rates, the prices of labour and land are also equalized.

22.7.1.3 The Rybczynski Theorem

Consider a closed economy, and suppose that p_K, p_A, p_B are given, hence r_i is also given. We know that, given r_i, the functions $H_i(L_i, D_i, r_i)$ behave like ordinary production functions in the arguments (factors) L_i, D_i. Without loss of generality we can assume that commodity A is labour intensive, with a higher labour/land ratio than B. Then we can apply the traditional arguments (see Sect. 5.4) to show that the increase in the quantity of a factor (say, labour) causes an increase in the output of the commodity intensive in that factor (A) and a decrease in the output of the other commodity, at unchanged commodity and factor prices (i.e., given also p_L, p_D).

This proves the Rybczynski theorem.

22.7.1.4 The Stolper-Samuelson Theorem

Let $a_{ij}, i = K, L, D; j = A, B$ the (optimal) input coefficients, namely the amount of factor i required to produce one unit of commodity j when costs are minimized. Then we have

$$\begin{aligned} p_A &= a_{KA} p_K + a_{LA} p_L + a_{DA} p_D, \\ p_B &= a_{KB} p_K + a_{LB} p_L + a_{DB} p_D. \end{aligned} \tag{22.100}$$

If we differentiate both sides (keeping p_K constant) and consider proportional changes (denoted by an asterisk) we obtain, by the same procedure followed in Sect. 19.5,

$$\begin{aligned} p_A^* &= \theta_{LA} p_L^* + \theta_{DA} p_D^* + \theta_{KA} a_{KA}^* + \theta_{LA} a_{LA}^* + \theta_{DA} a_{DA}^*, \\ p_B^* &= \theta_{LB} p_L^* + \theta_{DB} p_D^* + \theta_{KB} a_{KB}^* + \theta_{LB} a_{LB}^* + \theta_{DB} a_{DB}^*, \end{aligned} \tag{22.101}$$

where θ_{ij} is the share of factor i in sector j ($\theta_{LA} = a_{LA} p_L / p_A$, etc.). Cost minimization (see Sect. 19.5) implies

$$\theta_{KA} a_{KA}^* + \theta_{LA} a_{LA}^* + \theta_{DA} a_{DA}^* = 0,$$

$$\theta_{KB} a_{KB}^* + \theta_{LB} a_{LB}^* + \theta_{DB} a_{DB}^* = 0,$$

hence Eqs. (22.101) reduce to

$$\begin{aligned} p_A^* &= \theta_{LA} p_L^* + \theta_{DA} p_D^*, \\ p_B^* &= \theta_{LB} p_L^* + \theta_{DB} p_D^*. \end{aligned} \tag{22.102}$$

These equations can be solved for p_L^*, p_D^* in terms of p_A^*, p_B^*, thus obtaining

$$\begin{aligned} p_L^* &= \frac{\theta_{DB} p_A^* - \theta_{DA} p_B^*}{\theta_{LA}\theta_{DB} - \theta_{LB}\theta_{DA}}, \\ p_D^* &= \frac{\theta_{LA} p_B^* - \theta_{LB} p_A^*}{\theta_{LA}\theta_{DB} - \theta_{LB}\theta_{DA}}. \end{aligned} \tag{22.103}$$

Let us assume, for example, that commodity B is labour intensive. Given the definition of the θ's, this implies $\theta_{LB}/\theta_{DB} > \theta_{LA}/\theta_{DA}$ and hence that the denominator of the fractions in Eqs. (22.103) is negative. Without loss of generality we can assume that commodity A is the numéraire, $p_A = 1$, hence $p_A^* = 0$. Thus a positive (negative) value of p_B^* means an increase (decrease) in the relative price p_B/p_A, and a positive (negative) value of p_L^* means an increase in the real reward of labour.

If we then let $p_B^* > 0$, we see from Eqs. (22.103) that $p_L^* > 0$, $p_D^* < 0$. Thus an increase of the relative price of a commodity causes an increase in the real reward of the factor intensively used in the production of this commodity. This proves the Stolper-Samuelson theorem.

Let us now again differentiate Eqs. (22.100), this time keeping commodity prices constant but letting p_K vary. We obtain, using the cost minimization conditions,

$$\begin{aligned} 0 &= \theta_{KA} p_K^* + \theta_{LA} p_L^* + \theta_{DA} p_D^*, \\ 0 &= \theta_{KB} p_K^* + \theta_{LB} p_L^* + \theta_{DB} p_D^*. \end{aligned} \tag{22.104}$$

These equations show that, if the reward to capital increases, the price of at least one immobile factor must decrease. To obtain more definite results we can solve Eqs. (22.104) for p_L^*, p_D^* in terms of p_K^*, whence

$$\begin{aligned} p_L^* &= \frac{\theta_{KB}\theta_{DA} - \theta_{KA}\theta_{DB}}{\theta_{LA}\theta_{DB} - \theta_{LB}\theta_{DA}} p_K^*, \\ p_D^* &= \frac{\theta_{KA}\theta_{LB} - \theta_{KB}\theta_{LA}}{\theta_{LA}\theta_{DB} - \theta_{LB}\theta_{DA}} p_K^*. \end{aligned} \tag{22.105}$$

Let us keep for the moment to the assumption that commodity B is labour intensive (hence A is land intensive), which means that $D_A/L_A > D_B/L_B$, or $D_A/D_B > L_A/L_B$. The denominator of the fractions in Eqs. (22.105) is negative. Then as a result of an increase in p_K the price of labour increases when $\theta_{KB}\theta_{DA} - \theta_{KA}\theta_{DB} < 0$, or, using the definitions of the θ's and a's, when $K_A/K_B > D_A/D_B$. Since we have assumed $D_A/D_B > L_A/L_B$, the condition for p_L^* to be positive when p_K^* is positive becomes

$$K_A/K_B > D_A/D_B > L_A/L_B. \tag{22.106}$$

When commodity B is land intensive, the denominator of the fractions in Eqs. (22.105) is positive, and the condition becomes

$$L_A/L_B > D_A/D_B > K_A/K_B. \tag{22.107}$$

In both cases D_A/D_B is included between L_A/L_B and K_A/K_B, and land is called a middle factor by Wong (1995, p. 143). Obviously, when $p_L^* > 0$, then $p_D^* < 0$.

Similarly it can be shown than, when labour is the middle factor, then $p_K^* > 0$ gives rise to $p_D^* > 0$, $p_L^* < 0$.

Finally, if capital is the middle factor, then Eqs. (22.105) imply that an increase in p_K causes a decrease in both p_L and p_D.

All these results can be summarized by saying that, when capital is not the middle factor, and increase in its reward causes a decrease in the reward of the middle factor and an increase in the reward of the other immobile factor. When capital is the middle factor, an increase in its reward causes a decrease in the rewards of both immobile factors.

22.7.2 Factor Mobility in the Specific Factors Model

The effects on factor rewards and outputs of an inflow of labour or of a specific capital have already been determined in the treatment of this model in Sect. 22.1, so that we only reproduce them here:

(I) Effects on factor rewards:

$$\begin{aligned} K^{*B} &= L^* = 0 \text{ and } K_A^* > 0 \Longrightarrow p_{K^A}^* < 0, p_{K^B}^* < 0, p_L^* > 0, \\ K^{*A} &= L^* = 0 \text{ and } K_B^* > 0 \Longrightarrow p_{K^A}^* < 0, p_{K^B}^* < 0, p_L^* > 0, \\ K^{*A} &= K_B^* = 0 \text{ and } L^* > 0 \Longrightarrow p_{K^A}^* > 0, p_{K^B}^* > 0, p_L^* < 0. \end{aligned}$$

(II) Effects on outputs:

$$\begin{aligned} K^{*B} &= L^* = 0 \text{ and } K_A^* > 0 \Longrightarrow A^* > 0, B^* < 0, \\ K^{*A} &= L^* = 0 \text{ and } K_B^* > 0 \Longrightarrow A^* < 0, B^* > 0, \\ K^{*A} &= K_B^* = 0 \text{ and } L^* > 0 \Longrightarrow A^* > 0, B^* > 0. \end{aligned}$$

22.8 Uncertainty and International Trade

Here, following Dumas (1980), we shall examine the case of generalized uncertainty, in which the production function of a generic good Y takes the form

$$Y_s = F_s(K, L), \tag{22.108}$$

where the subscript s indicates the states of nature. Thanks to first-degree homogeneity, we can write

$$y_s = g_s(\varrho), \quad y_s \equiv Y_s/L, \quad \varrho \equiv K/L. \tag{22.109}$$

Let us assume that the factors are rewarded at the beginning of the period and let us introduce Arrow-Debreu uncertainty, where we indicate with Φ_s the price of elementary or pure securities. As it would not be possible to show here the basis of these theories of uncertainty, the reader is referred to Arrow (1964), Debreu (1959), and Hirshleifer (1970). We only recall that an "elementary security" of index s is a security with a price equal to one, if the state of nature s occurs, equal to zero otherwise. As $p_s g_s'(\varrho)$ is the value of the marginal product of capital if the state of nature s occurs, and as only one of these states of nature will occur, then $\sum_s \Phi_s p_s g_s'(\varrho)$, given the definition of Φ_s, is the value of the marginal product of capital which is actually found.

Competitive equilibrium implies

$$\begin{aligned} p_K &= \sum_s \Phi_s p_s g_s'(\varrho), \\ p_L &= \sum_s \Phi_s p_s g_s(\varrho) - \varrho \sum_s \Phi_s p_s g_s'(\varrho), \end{aligned} \tag{22.110}$$

where p_s is the price of the commodity in each state of nature.

Let us assume that two commodities, A and B, are produced and let us consider the *present market value* of each product in each sector

$$\begin{aligned} V_A &= \sum_s \Phi_s p_{sA} L_A g_{sA}(\varrho_A) = L \sum_s \Phi_s p_{sA} l_A g_{sA}(\varrho_A), \\ V_B &= \sum_s \Phi_s p_{sB} L_B g_{sB}(\varrho_B) = L \sum_s \Phi_s p_{sB} l_B g_{sB}(\varrho_B), \end{aligned} \tag{22.111}$$

where $l_A = L_A/L$, $l_B = L_B/L$ are the fractions of the total labour force employed in the two sectors. As there is full employment of labour, $l_A + l_B = 1$: then, considering the condition of full employment of the capital stock and denoting the given total capital/labour ratio with $\bar{\varrho}$, we get

$$l_A \varrho_A + l_B \varrho_B = \bar{\varrho}, \tag{22.112}$$

which, together with the condition of full employment of labour, makes it possible to obtain

$$l_A = \frac{\bar{\varrho} - \varrho_B}{\varrho_A - \varrho_B}, l_B = \frac{\varrho_A - \bar{\varrho}}{\varrho_A - \varrho_B}. \tag{22.113}$$

If we consider the ratio between the present market values of the future outputs, $v = V_A/V_B$, given Eqs. (22.111) and (22.113), we have

$$v = \frac{\sum_s \Phi_s p_{sA} g_{sA}(\varrho_A)}{\sum_s \Phi_s p_{sB} g_{sB}(\varrho_B)} \frac{\bar{\varrho} - \varrho_B}{\varrho_A - \bar{\varrho}}. \tag{22.114}$$

Let us now assume that there are two countries and that commodities are freely traded in all states of nature in both countries, so that p_{sA} and p_{sB} are the same everywhere. Let us also assume that the pure security markets are unified at world level, so that the Φ_s are equal in the two countries. The production functions are internationally identical and there is no factor-intensity reversal.

Without any loss of generality we can assume that A is the capital intensive commodity, so that $\varrho_A > \bar{\varrho} > \varrho_B$. It can then be seen at once from (22.114) that the country in which $\bar{\varrho}$ is higher will have a higher v, that is, a relatively greater V_A. This shows that *the capital-abundant country produces a relatively greater present market value of the capital-intensive commodity*, and vice versa for the labour-abundant country. Obviously, this proposition is the extension to the case of uncertainty (with present market value in the place of certain quantity) of the proposition at the basis of the Heckscher-Ohlin theorem (see Sect. 4.2).

If we now assume, as in the Heckscher-Ohlin theory, identical demand structures in the two countries (no element of uncertainty being introduced on the demand side), it immediately follows that *each country has a positive present value of exports of the commodity which makes relatively intensive use of the relatively plentiful factor.* This extends the Heckscher-Ohlin theorem to the case of uncertainty.[10]

Assuming absence of complete specialization, it is possible to demonstrate the factor-price equalization: given Eqs. (22.110), inside each country we shall have

$$\begin{aligned} p_K &= \sum_s \Phi_s p_{sA} g'_{sA}(\varrho_A) = \sum_s \Phi_s p_{sB} g'_{sB}(\varrho_B), \\ p_L &= \sum_s \Phi_s p_{sA} g_{sA}(\varrho_A) - \varrho_A \sum_s \Phi_s p_{sA} g'_{sA}(\varrho_A) \\ &= \sum_s \Phi_s p_{sB} g_{sB}(\varrho_B) - \varrho_B \sum_s \Phi_s p_{sB} g'_{sB}(\varrho_B), \end{aligned} \tag{22.115}$$

from which

$$\begin{aligned} &\sum_s \Phi_s p_{sA} g'_{sA}(\varrho_A) - \sum_s \Phi_s p_{sB} g'_{sB}(\varrho_B) = 0, \\ &\left[\sum_s \Phi_s p_{sA} g_{sA}(\varrho_A) - \varrho_A \sum_s \Phi_s p_{sA} g'_{sA}(\varrho_A)\right] \\ &-\left[\sum_s \Phi_s p_{sB} g_{sB}(\varrho_B) - \varrho_B \sum_s \Phi_s p_{sB} g'_{sB}(\varrho_B)\right] = 0, \end{aligned} \tag{22.116}$$

which is a system of two implicit functions. On the basis of the implicit-function theorem, if the Jacobian with respect to ϱ_A, ϱ_B is different from zero at the equilibrium point, it is possible to express ϱ_A and ϱ_B as single-valued differentiable functions of the other $3s$ variables (Φ_s, p_{sA}, p_{sB}).

The Jacobian is

$$\begin{vmatrix} \sum_s \Phi_s p_{sA} g''_{sA}(\varrho_A) & -\sum_s \Phi_s p_{sB} g''_{sB}(\varrho_B) \\ -\varrho_A \sum_s \Phi_s p_{sA} g''_{sA}(\varrho_A) & -\varrho_B \sum_s \Phi_s p_{sB} g''_{sB}(\varrho_B) \end{vmatrix}$$

$$= \left[\sum_s \Phi_s p_{sA} g''_{sA}(\varrho_A)\right]\left[\sum_s \Phi_s p_{sB} g''_{sB}(\varrho_B)\right](\varrho_B - \varrho_A), \tag{22.117}$$

which is different from zero because, given the assumption of absence of factor-intensity reversals, there will always be $\varrho_A > \varrho_B$ or $\varrho_B > \varrho_A$,

[10]It is as well to observe that the extension of this theorem from the deterministic case to one with uncertainty is valid only if the *physical* definition of relative abundance is used, whereas if the definition in terms of relative factor prices is used, then such an extension is no longer valid.

As we have assumed that the production functions are internationally identical and the variables Φ_s, p_{sA}, p_{sB} likewise, the values of ϱ_A and ϱ_B derived from Eqs. (22.116) will be identical in both countries so that, by substituting in Eqs. (22.115), we get the same factor prices in both countries.

For a demonstration of the validity of the other traditional theorems (Stolper-Samuelson, Rybczynski) we refer the reader to Dumas (1980). See also Helpman and Razin (1978) , Eaton (1979), Pomery (1979, 1984), Anderson (1981), Grossman and Razin (1985), and Grinols (1985).

22.9 Smuggling

Let us take as example the case in which the real costs of smuggling are made up exclusively of a loss of part of the commodity smuggled. We start from the following model (Bhagwati & Srinivasan, 1974)

$$\begin{aligned} C_A &= A + m_{Ag} + m_{As}, \\ C_B &= f\,(A) - p m_{Ag} - p_s m_{As,} \\ U_A &= p_h U_{B,} \\ -f'(A) &= p_{h.} \end{aligned} \tag{22.118}$$

The first equation defines the domestic consumption of the imported commodity (we assume that it is A), given by domestic output plus imports, distinguished in legal imports m_{Ag} and illegal ones m_{As}. The second equation defines the domestic consumption of commodity B, equal to domestic production less exports. Domestic production of B is connected to that of A by way of the transformation curve $B = f(A)$. Exports of commodity B are equal, in equilibrium, to the values of the corresponding imports of A in the two branches of trade (legal trade and smuggling), where p and p_s are the international relative price of A[11] for legal trade and the relative price of the same commodity illegally traded ($p_s > p$).

Given a social welfare function $U = U(A, B)$, with positive partial derivatives U_A, U_B, the optimum condition is given by the equality between the marginal rate of substitution (U_A/U_B) and the domestic relative price p_h, hence the third equation. The fourth and last expresses the fact that, on the basis of the efficiency conditions (see Sect. 19.1), the marginal rate of transformation is equal to the domestic relative price p_h.

Given that the domestic (relative) price charged by the smugglers (henceforth "domestic illegal price" for brevity) is less than the legal (relative) domestic price (which is equal to the international price plus tariff), legal trade will disappear, so that $p_h = p_s, m_{Ag} = 0$. We propose to calculate the direction in which social

[11]To symplify analysis we use the relative price of commodity A instead of that of B as we did in Sect. 6.10.

welfare moves with the variation in the price of the domestic illegal price $p_s = p_h$, in the interval $p \leq p_s \leq p(1+d)$, where d is the tariff rate, assuming that p is constant. From the social welfare function, we get

$$\frac{\mathrm{d}U}{\mathrm{dp}_h} = U_A \frac{\mathrm{d}C_A}{\mathrm{d}p_h} + U_B \frac{\mathrm{d}C_B}{\mathrm{d}p_h} = U_B \left(\frac{U_A}{U_B} \frac{\mathrm{d}C_A}{\mathrm{d}p_h} + \frac{\mathrm{d}C_B}{\mathrm{d}p_h} \right)$$
$$= U_B \left(p_h \frac{\mathrm{d}C_A}{\mathrm{d}p_h} + \frac{\mathrm{d}C_B}{\mathrm{d}p_h} \right), \tag{22.119}$$

given the third equation of (22.118). The last expression in parentheses is formally identical to the following

$$\frac{\mathrm{d}}{\mathrm{d}p_h} \left(p_h C_A + C_B \right) - C_A.$$

Remembering that $m_{Ag} = 0$, $p_h = p_s$, it follows from the first two equations in (22.118) that

$$p_h C_A + C_B = p_h A + f(A),$$

and therefore

$$\frac{\mathrm{d}}{\mathrm{d}p_h} \left(p_h C_A + C_B \right) = A + p_h \frac{\mathrm{d}A}{\mathrm{d}p_h} + f'(A) \frac{\mathrm{d}A}{\mathrm{d}p_h} = A,$$

given the fourth of Eqs. (22.118). So, by substituting in (22.119), we have

$$\frac{\mathrm{d}U}{\mathrm{d}p_h} = U_B \left(A - C_A \right) = -U_B m_{As} < 0, \tag{22.120}$$

given the first of (22.118) and the fact that $m_{Ag} = 0$. It follows from (22.120) that social welfare is a monotonically decreasing function of the domestic illegal price. There will obviously be maximum welfare at the lower bound of the interval, that is, when $p_s = p$ (the free trade price), while there will be minimum welfare at the upper bound of the interval, that is when $p_s = p(1+d)$. Now, as we have seen in the text, this minimum is inferior to that which the society would have if there were no smuggling and the legal domestic price were equal to $p(1+d)$. We can therefore establish that

$$U^f > U^d > U^s_{\min},$$

where U^f = welfare in the case of free trade, U^d = welfare in the case of a tariff and legal trade, U^s_{min} = welfare in the case of smuggling with a relative price equal to that of legal trade with tariff. The U^s welfare that the society enjoys in the case of smuggling will therefore be included between U^f and U^s_{min} and, given the monotonic relationship between welfare and the domestic illegal price, it is demonstrated that U^s can be less or greater than U^d, according to the value assumed by $p_s = p_h$.

The economic theory of smuggling can be put in the general framework of the theory of DUP (Directly UnProductive) activities, for which see Bhagwati and Srinivasan (1983, chap. 30, and references therein). For a crime-theoretical approach see Martin and Panagariya (1984).

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