Chapter 23
Appendix to Chapter 9

23.1 A Neo-Heckscher-Ohlin Model

We examine Falvey's model (Falvey, 1981). For the reader's convenience we report here Eqs. (9.1) from the text:

$$p_1(\alpha) = W_1 + \alpha R_1,$$
$$p_2(\alpha) = W_2 + \alpha R_2,$$

where α is a continuous index over the interval $\underline{\alpha}$, $\overline{\alpha}$; the units are chosen such that the production of one unit of α requires the input of α units of capital and one unit of labour.

The solution for α_0, the marginal quality such that $p_1(\alpha) = p_2(\alpha)$, is

$$\alpha_0 = \frac{W_1 - W_2}{R_2 - R_1}, \qquad (23.1)$$

which is clearly positive, since we have assumed that $W_1 > W_2$ and $R_1 < R_2$. For any other quality we have $p_1 \neq p_2$, and precisely

$$p_1(\alpha) - p_2(\alpha) = (W_1 - W_2) + \alpha(R_1 - R_2),$$

from which, using the fact that Eq. (23.1) yields $R_2 - R_1 = (W_1 - W_2)/\alpha_0$,

$$p_1(\alpha) - p_2(\alpha) = (W_1 - W_2)(\alpha_0 - \alpha)/\alpha_0. \qquad (23.2)$$

It can readily be seen from (23.2) that $p_1(\alpha) \lesseqgtr p_2(\alpha)$ according as $\alpha \gtreqless \alpha_0$; this means that the home country produces the qualities higher than the marginal quality α_0 at lower unit costs than the rest of the world and vice versa. From this result, one can anticipate that under free trade and with no transport costs the home country will

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export the qualities higher than α_0 and import the qualities lower than α_0. This intra-industry trade will follow the lines of the Heckscher-Ohlin proposition, as shown in the text, Sect. 9.1.

Let us now explicitly consider the demand side. The demand for each quality is assumed to depend only on the relative prices of qualities; since we are in a partial equilibrium context, consumers' income as well as the prices of the products of other industries can be taken as given and hence can be ignored. Since perfect competition obtains in the industry, prices will equal unit production costs and so, as the wage rate is given, will depend only on the rate of profit. Thus we can write the demands for quality α as

$$\begin{aligned} D_1 &= D_1(R_1, R_2; \alpha), \\ D_2 &= D_2(R_1, R_2; \alpha). \end{aligned}$$

We must now determine the equilibrium rates of return on capital, R_{1E} and R_{2E}, which are the rates that bring into equality the demand for capital and the (given) supply of it. Let $\underline{\alpha}$ and $\overline{\alpha}$ be the indices of the lowest and highest quality respectively, and K_1, K_2 the industry's stock of capital in the two countries. Since α also measures the capital input, and given the results on the pattern of trade, we have

$$D_{1K}(R_{1E}, R_{2E}) \equiv \int_{\alpha_0}^{\overline{\alpha}} \alpha[D_1(R_{1E}, R_{2E}; \alpha) + D_2(R_{1E}, R_{2E}; \alpha)]\mathrm{d}\alpha = K_1, \tag{23.3}$$

because all of the world demand (domestic plus foreign) for the qualities higher than α_0 will be met by the home country's output. Similarly,

$$D_{2K}(R_{1E}, R_{2E}) \equiv \int_{\underline{\alpha}}^{\alpha_0} \alpha[D_1(R_{1E}, R_{2E}; \alpha) + D_2(R_{1E}, R_{2E}; \alpha)]\mathrm{d}\alpha = K_2, \tag{23.4}$$

since all of the world demand for the qualities below α_0 will be met by the rest-of-the-world's output. Note that in (23.3) and (23.4), α_0 is a function of $(R_{1E} - R_{2E})$ through (23.1).

We observe that in (23.3) an increase in R_1 reduces the home country's excess demand for capital for two reasons. First, this increase raises the prices of domestically produced qualities relative to foreign-produced ones and so—assuming that demand functions are normal—induces a substitution of the latter for the former. Second, the increase reduces the range of qualities where the home country has a cost advantage over the rest of the world.

Conversely, an increase in R_2 causes the excess demand for capital in the home country to increase. Therefore, if we denote this excess demand by $E^1(R_1, R_2) = D_{1K} - K_1$, the partial derivatives will be $E^1_{R_1} < 0$, $E^1_{R_2} > 0$.
Similar considerations applied to $E^2(R_1, R_2)$ give $E^2_{R_1} > 0$, $E^2_{R_2} < 0$.

Let us now consider the stability of equilibrium. This requires that any change which raises (reduces) the price of the qualities produced in a country, with other prices constant, brings about a decrease (increase) in the overall demand for capital. This implies that $E^1_{R_1} + E^2_{R_1} < 0$, $E^1_{R_2} + E^2_{R_2} < 0$; these inequalities will be used in the following comparative statics analysis (this use is an application of Samuelson's correspondence principle).

We now examine the effects of an increase in the home country's wage rate on the free trade equilibrium values of R_1 and R_2. Since the wage rate is given, it can be introduced as a shift parameter in the excess demand functions for capital defined above. We can then calculate the total differentials of these excess demand functions and obtain the system

$$E^1_{R_1}\mathrm{d}R_1 + E^1_{R_2}\mathrm{d}R_2 + E^1_{W_1}\mathrm{d}W_1 = 0, \tag{23.5}$$

$$E^2_{R_1}\mathrm{d}R_1 + E^2_{R_2}\mathrm{d}R_2 + E^2_{W_1}\mathrm{d}W_1 = 0, \tag{23.6}$$

which has the solution

$$\mathrm{d}R_1 = -\frac{E^1_{W_1}E^2_{R_2} - E^2_{W_1}E^1_{R_2}}{\Delta}\mathrm{d}W_1, \tag{23.7}$$

$$\mathrm{d}R_2 = -\frac{E^2_{W_1}E^1_{R_1} - E^1_{W_1}E^2_{R_1}}{\Delta}\mathrm{d}W_1, \tag{23.8}$$

where $\Delta \equiv E^1_{R_1}E^2_{R_2} - E^2_{R_1}E^1_{R_2}$ is positive given the stability condition discussed above. If we extend stability considerations to the effects of a change in wages, we can assume that $E^1_{W_1} + E^2_{W_1} < 0$, with $E^1_{W_1} < 0$ and $E^2_{W_1} > 0$.

From all these stability considerations it follows that $\left|E^2_{R_2}\right| > \left|E^1_{R_2}\right|$ and $\left|E^1_{W_1}\right| > \left|E^2_{W_1}\right|$, so that from (23.7) we have $\mathrm{d}R_1/\mathrm{d}W_1 < 0$; but the sign of $\mathrm{d}R_2/\mathrm{d}W_1$ remains ambiguous since $\left|E^1_{R_1}\right| > \left|E^2_{R_1}\right|$.

The economic interpretation of these results is the following. At the initial rates of return to capital, the increase in W_1 causes an increase in the domestically produced qualities and so a decrease in the range of qualities in which the home country has a cost advantage (as can be seen from (23.1), an increase in W_1 raises α_0 at unchanged R_1, R_2, W_2). Since foreign prices are unchanged, in world demand there will be a substitution in favour of foreign-produced qualities, and so an excess supply of capital in the home country industry. This excess reduces the rate of return to the domestic industry's capital, which tends to offset the initial effect of the higher wage on costs. In the new equilibrium, R_1 will therefore be lower, while the final position of R_2 is ambiguous (since it increases initially, because of the excess demand for it due to the excess demand for foreign-produced qualities, and then decreases).

23.2 A Model of Monopolistic Competition

23.2.1 Love for Variety and Demand

This model is based on the contributions by Krugman and Helpman (Helpman, 1990; Helpman & Krugman, 1985, 1989; Krugman, 1979, 1980, 1990). It starts from the S-D-S (Spence-Dixit-Stiglitz) approach to consumer preferences and demand, according to which consumers love variety and so their utility increases as the number of goods consumed increases, other things being equal (Dixit & Stiglitz, 1977; Spence, 1976). This means that the consumer will be better off by consuming a greater number of goods at the given prices and income. A simple way of modelling this (Dixit & Stiglitz, 1977) is to assume that the representative consumer has a utility function of the type

$$u = \left(\sum_{i=1}^{n} D_i^{\alpha} \right)^{1/\alpha}, \quad 0 < \alpha < 1, \tag{23.9}$$

where D_i is the quantity consumed of good i, and n the number of goods. This functional form, also used in production theory, is of the well-known constant elasticity of substitution (CES) type, which is homogeneous of degree one in the quantities and has the convenient property that the elasticity of substitution between any two goods is constant, $\sigma = 1/(1-\alpha) > 1$.

The consumer maximises u subject to the budget constraint $I = \sum_{i=1}^{n} D_i p_i$, where I is the consumer's money income. It is a well-known result (Varian, 1992, p. 112) that in the case of a CES utility function the demand functions deriving from the consumer's maximization process have the form

$$D_i = \frac{p_i^{-\sigma}}{\sum_i^n p_i^{1-\sigma}} I. \tag{23.10}$$

To show that utility increases with the number of goods consumed, let us assume that all goods have the same price p. From (23.10) it follows that the optimal quantities of each good will be equal, hence the consumer's income will be divided equally among all available commodities, $D_i = I/np$. By substituting these into the utility function (23.9) we obtain the optimal utility $\overline{u}$

$$\overline{u} = n^{(1-\alpha)/\alpha} \left(\frac{I}{p} \right),$$

which clearly increases as n increases.

The n goods can be taken as the n varieties of a horizontally differentiated product. In the case of m differentiated products, each of which has several varieties, say v_k, the situation is much more complicated. The number of goods will be $n = \sum_{k=1}^{m} v_k$. A convenient way of simplifying the problem is to assume that the overall utility function has the separability property, namely that the subutility deriving from the consumption of the different varieties of a product is independent of the quantities of the varieties of other products being consumed. It follows that the overall utility function can be written as

$$U = U[u_1(.), u_2(.), \ldots, u_m(.)], \tag{23.11}$$

where $u_1(.)$ is the subutility function whose arguments are the different varieties of product 1, and so on. Note that the presence of homogeneous products is easily accommodated, for in the case of a homogeneous product there will only be one variety of it, hence the subutility function relating to it will have one argument only. Thus, if k is a generic good, we have $u_k = u_k(D_k)$ when k is homogeneous, while if k is a differentiated good we have $u_k = u_k(D_{k1}, D_{k2}, \ldots., D_{kv_k,})$, where $D_{k\omega}$ (for $\omega = 1, 2, \ldots, v_k$) is the quantity of variety ω that is being consumed and v_k is the number of varieties of good k. The subutility function of any product is assumed to be of the CES type discussed above, which clearly reduces to $u_k = D_k$ in the case of a homogeneous product.

It is well known (Green, 1964, chap. 4) that in the case of homogeneous functional separability the solution of the consumer's maximization problem can be carried out as a two-stage maximization procedure. For each group, namely for each subset consisting of all the varieties of each differentiated good, we define a price index $P_k = f_k(p_{k1}, p_{k2}, \ldots, p_{kn_k})$ as a function of the prices of members of the group, and a quantity index $D_k = g_k(D_{k1}, D_{k2}, \ldots., D_{kn_k,})$ as a function of their quantities; both functions must be homogeneous of degree one in their respective arguments for homogeneous separability to obtain. Then the two-stage budgeting procedure is carried out as follows.

First, the optimal distribution of the consumer's given income among the groups is determined by reference to the price and quantity indices alone, namely the subutility functions u_k in U are replaced with the quantity indices D_k, and the utility function $U = U(D_1, D_2, \ldots., D_m)$ is maximised with respect to the D_k's subject to $\sum_{k=1}^{m} P_k D_k = I$. This determines the expenditure $I_k = P_k D_k$ on each group.

Second, the expenditure allocated to the various groups is distributed among the members of the group on the basis of their individual prices, namely by carrying out the maximization of each subutility function taking I_k, the expenditure allocated to the group, as given. It is clear that the second stage can also be carried out first, considering I_k as a parameter to be determined in the subsequent optimal-expenditure-allocation stage taken as second. This is the approach chosen by Dixit and Stiglitz (1977) and followed by Helpman and Krugman, but we prefer to follow the traditional sequence for clarity of exposition.

Let us then consider a model in which there are two commodities, one homogeneous and the other differentiated with n varieties. Let Y be the consumption of the homogeneous good, and D_i the consumption of variety i of the differentiated commodity. The subutility functions are of the CES type, hence turn out to be Y for the homogeneous commodity and $\left(\sum_{i=1}^{n} D_i^{\alpha}\right)^{1/\alpha}$ for the differentiated commodity. The overall utility function is assumed to be

$$U = Y + A\theta^{-1} \left[\left(\sum_{i=1}^{n} D_i^{\alpha} \right)^{1/\alpha} \right]^{\theta}, \qquad 0 < \theta < 1, \tag{23.12}$$

where A is a constant.

To carry out the first stage of the maximization process we must preliminarily define a quantity index and a price index for the differentiated commodity. These are

$$D = \left(\sum_{i=1}^{n} D_i^{\alpha} \right)^{1/\alpha}, \qquad P = \left(\sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right)^{(\alpha-1)/\alpha}, \tag{23.13}$$

which clearly satisfy the condition of being homogeneous of degree one. The first stage consists in maximising $U = Y + A\theta^{-1}D^{\theta}$ with respect to Y and D subject to the budget constraint $I = Y + PD$, where the prices are expressed in terms of the homogeneous good. From the first-order conditions we get

$$AD^{\theta-1} = P,$$

hence

$$D = (A^{-1}P)^{-1/(1-\theta)} = BP^{-\epsilon}, \qquad \epsilon = \frac{1}{1-\theta}, \tag{23.14}$$

is the aggregate demand function, with constant price-elasticity ϵ. Having thus determined PD, the budget allocated to the differentiated good, we can go on to the second stage, where we maximise the subutility function $\left(\sum_{i=1}^{n} D_i^{\alpha}\right)^{1/\alpha}$ subject to the budget constraint $PD = \sum_{i=1}^{n} D_i p_i$. The solution is of type (23.10), namely

$$D_i = \frac{p_i^{-\sigma}}{\sum_i^n p_i^{1-\sigma}} PD = \frac{p_i^{-\sigma}}{P^{-1}\sum_i^n p_i^{1-\sigma}} D. \tag{23.15}$$

If we use the definition of P and the fact that $1 - \sigma = -\alpha/(1-\alpha) = \alpha/(\alpha-1)$, we can manipulate the denominator of the last fraction as follows:

$$P^{-1} \sum_{i=1}^{n} p_i^{1-\sigma} = \left(\sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right)^{\frac{-(\alpha-1)}{\alpha}} \left(\sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right) = \left(\sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right)^{\frac{-(\alpha-1)}{\alpha}+1}$$

$$= \left(\sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right)^{1/\alpha}.$$

The last term is clearly $P^{1/(\alpha-1)} = P^{-\sigma}$. Hence the demand for quality i turns out to be

$$D_i = \left(\frac{p_i}{P}\right)^{-\sigma} D, \tag{23.16}$$

which can also be written as

$$D_i = Bp_i^{-\sigma} P^{\sigma-\epsilon} \tag{23.17}$$

since $D = BP^{-\epsilon}$ as shown in (23.14).

This result has an important implication: if a single firm produces good i, and if this firm is small enough with respect to the economy so that it considers itself as unable to influence D and P, it will perceive itself as facing a downward sloping demand curve with constant elasticity σ. This will indeed be the case in a monopolistically competitive market, with imperfect competition due to economies of scale in the production of the several varieties of the differentiated good: given the large number of symmetric potential products, there is no reason for two firms trying to produce the same good. More precisely, if a firm chose a variety that is already produced by another firm, it would have to share the market for this variety: given the equality of the demand curves for the various goods (varieties) when D and P are taken as given, the profits to be gained are clearly lower than the profits that the incumbent firm could make by choosing another variety as yet unproduced. Hence each good will be produced by a different firm.

23.2.2 The Production Side

Let us now turn to the production side. The homogeneous commodity is produced under constant returns to scale in a perfectly competitive market, while the n varieties of the differentiated good are produced under increasing returns to scale in a monopolistically competitive market. Hence the pricing rule of the representative firm producing the homogeneous commodity is *price = marginal cost*, which in turn equals average cost at the equilibrium point, given the no-profit condition. The representative monopolistically competitive firm will apply the *marginal revenue = marginal cost* pricing rule, with the usual mark-up over price. However, if we assume absent any restriction on entry and exit, monopolistic competition will also reduce profits to zero, hence a selling price equal to average cost in this market as well.

As regards the structure of production, namely the factor inputs, one could consider the traditional two-factor setting (Helpman & Krugman, 1985, chap. 7) , but the essentials of the monopolistic competition approach to international trade can be brought out in a much simpler way if we use the one-factor setting (Helpman & Krugman, 1989; Krugman, 1979, 1980, 1990) .

Let us then assume that there is only one factor, labour (for this reason the model has also been called a "Chamberlinian-Ricardian" model). We first consider the simpler case in which only the differentiated good exists. If $g(x_i)$ is the labour input of the firm producing the quantity x of variety i, we have $g'(x_i) > 0$, but $\mathrm{d}[g(x_i)/x_i]/\mathrm{d}x_i < 0$ due to increasing returns. Marginal cost is $wg'(x_i)$, where w is the given wage rate. From the demand function (23.16) we get the inverse demand function $p_i = (PD^{1/\sigma})D_i^{-1/\sigma}$, hence marginal revenue is $\mathrm{d}(p_i D_i)/\mathrm{d}D_i = \mathrm{d}[(PD^{1/\sigma})D_i^{(\sigma-1)/\sigma}]/\mathrm{d}D_i = [(\sigma-1)/\sigma]p_i$. Thus the pricing rule of the monopolistically competitive firm gives

$$wg'(x_i) = [(\sigma-1)/\sigma]p_i,$$

from which

$$\frac{p_i}{w} = \frac{g'(x_i)\sigma}{\sigma-1}. \tag{23.18}$$

If we assume free entry and exit, we additionally have

$$\frac{p_i}{w} = \frac{g(x_i)}{x_i}. \tag{23.19}$$

These two equations together determine the output and price of the representative firm. Since the demand functions are identical across varieties and the cost functions have also been assumed identical, output per firm and price (relative to the wage rate) turn out to be the same for all varieties produced. It remains to determine the number of varieties produced. This can easily be obtained from the full employment condition and the fact that output per firm is the same and labour input also. Hence $ng(x) = L$, from which

$$n = \frac{L}{g(x)}, \tag{23.20}$$

where x is taken from the previous solution and L is the labour force. We do not know which n goods are produced, but this is unimportant, since all goods are symmetric.

23.2.3 International Trade

If we now consider a world consisting of two such economies, and assume identical tastes (technology needs not be identical, but for simplicity's sake we shall assume that it is), it is easy to see the determinants of international trade. Country 1 will produce n_1 goods and country 2 will produce n_2 different goods. Given the love for variety, each will consume some of the other products, and consumers will be better off since the number of goods increases. Thus there will be mutually beneficial intra-industry trade.

Let us now introduce the homogeneous good into the picture. If we denote by a_{LY} the constant labour-input coefficient in the production of Y, we have $p_Y = wa_{LY}$. Now let us assume that in equilibrium both countries produce some of this good, and that trade in Y can occur costlessly (no transport costs, no tariffs, etc.). Then p_Y must be the same in both countries, and this ties down relative wage rates in the two countries:

$$\frac{w_1}{w_2} = \frac{a_{2LY}}{a_{1LY}}. \tag{23.21}$$

We already know from Eqs. (23.18) and (23.19) the producer price and output of differentiated products in terms of labour and thus also in terms of the homogeneous good. If we denote by x and p the output and the selling price of a generic firm producing a variety of the differentiated commodity, and assume identical technology, x and p will be the same in both countries, and will also be the same across varieties. Let us then consider the varieties which are internationally traded. Clearing of the product market requires output to equal the sum of the two countries' demands, $x = D^1 + D^2$, where D^1, D^2 are given by Eq. (23.17), namely $D^1 = B_1 p^{-\sigma} P_1^{\sigma-\epsilon}$, $D^2 = B_2 p^{-\sigma} P_2^{\sigma-\epsilon}$, where we have omitted the country subscript from p since it is equal in both countries, as we have seen above. We now introduce transport costs of the usual iceberg type, so that for every unit shipped, only $1/(1+\phi)$ units reach the foreign market, where $\phi > 0$. Hence the price to domestic consumers of one unit of an imported good will be $(1+\phi)p$. Taking transport costs into account and letting x_{12} be the quantity produced by country 1 to serve country 2's market we can write the usual *supply* = *demand* condition

$$(1+\phi)^{-1} x_{12} = B_2 [p(1+\phi)]^{-\sigma} P_2^{\sigma-\epsilon},$$

whence

$$x_{12} = (1+\phi)^{1-\sigma} B_2 p^{-\sigma} P_2^{\sigma-\epsilon}. \tag{23.22}$$

As regards the domestic market, we have

$$x_{11} = B_1 p^{-\sigma} P_1^{\sigma-\epsilon}, \tag{23.23}$$

where x_{11} is the quantity produced by country 1's firm to serve the domestic market.

It follows that the overall market-clearing condition for country 1's firm can be written

$$x_1 = B_1 p^{-\sigma} P_1^{\sigma-\epsilon} + (1+\phi)^{1-\sigma} B_2 p^{-\sigma} P_2^{\sigma-\epsilon}. \tag{23.24}$$

We similarly find that the market clearing condition for country 2's firm is

$$x_2 = (1+\phi)^{1-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\epsilon} + B_2 p^{-\sigma} P_2^{\sigma-\epsilon}. \tag{23.25}$$

Since $x_1 = x_2 = x$ as seen above, and p is also given, the system consisting of Eqs. (23.24) and (23.25) determines the price indices P_1, P_2 or, to simplify the solution, $P_1^{\sigma-\epsilon}, P_2^{\sigma-\epsilon}$, that is to say, *transformations* of the consumer price indices for differentiated products in each country. Note that the fact that producer prices and quantities of each variety are given implies that any change in the price indices is brought about by a change in the number of firms active in each country, as can be immediately seen from the definition of the price index given in Eqs. (23.13).

23.2.3.1 A Simple Gravity Equation

The monopolistic competition model gives rise to the gravity equation in a very simple and direct way. A number of slightly different specification of the gravity equation exist in the literature. Here we derive the *odds and friction*[1] specification since it obtains directly from the model described above. In this specification the dependent variable is the ratio of foreign to domestic trade (purchase from abroad divided by purchase from home). Consider for instance country 1 and let n_i be the number of varieties produced in country i. Recall from (23.24) and (23.25) that x_{21} and x_{11} are, respectively, country 1's imports and domestic sales of a any single variety. Therefore country 1's total import of the differentiated good is n_2 times the imports of a single variety, $n_2 x_{21} = n_2 (1+\phi)^{1-\sigma} B_1 p^{-\sigma} P_1^{\sigma-\varepsilon}$, and the value of domestic trade is $n_1 x_{11} = n_1 B_1 p^{-\sigma} P_1^{\sigma-\varepsilon}$.

The ratio of imports to domestic trade, denoted χ_{12}, is equal to $n_2 x_{21}/n_1 x_{11}$ which gives

$$\chi_{12} = (n_2/n_1)\left(\frac{1}{1+\phi}\right)^{\sigma-1}. \tag{23.26}$$

The term (n_2/n_1) represents the *odds* and the term $(1+\phi)^{1-\sigma}$ represents the *friction* due to trade costs. Recalling that $\sigma > 1$ it is clear that any increase in trade costs reduces the ratio of imports to domestic trade. Equation (23.26) is not suitable for empirical estimation because the number of varieties is rarely available in the data and when it is available is typically subject to large measurement errors. To get around this problem let v_i denote the value of sectorial GDP, $v_i \equiv p x n_i$, where we recall from (23.18) and (23.19) that x is the firm's total output. Now, noting that $n_2/n_1 = p x n_2 / p x n_1$ we can rewrite Eq. (23.26) as

$$\chi_{12} = \frac{v_2}{v_1}(1+\phi)^{1-\sigma}. \tag{23.27}$$

[1]This convenient term is used in Combes, Lafourcade, and Mayer (2005).

which is the simplest *odds and friction* specification. The equation written in this way is more suitable for empirical studies because the value of sectorial GDP is more easily measurable than the number of varieties.

23.3 Homogeneous Goods, Oligopoly, and Trade

23.3.1 A Cournot-Type Model

Brander (1981) and Brander and Krugman (1983) model increasing returns in a very simple way, assuming a cost function (equal in both countries) of the type

$$C(q) = F + cq, \tag{23.28}$$

where F is fixed cost and c the (constant) marginal cost. Transport costs are modelled according to the iceberg assumption, so that if a quantity x is exported from country 1 to country 2, the quantity gx arrives in country 2, where $0 \leq g \leq 1$ is the same for both countries. The higher g, the lower transport costs. The markets are located in the two countries and are really segmented as explained in the text. The two firms, one located in country 1 and the other in country 2, compete in the two markets (for the case in which they compete in a third market only, see Brander & Spencer, 1984, 1985, and below, Sect. 24.4.3.2) and behave as Cournot duopolists.

The demand functions are identical in the two countries, and for simplicity's sake we assume them to be normal (downward sloping) and linear, so that

$$p_1 = a - b(q_{11} + q_{21}), \tag{23.29}$$

$$p_2 = a - b(q_{12} + q_{22}), \tag{23.30}$$

where q_{ij} is the quantity offered by firm i in market j, and $a > 0, b > 0$. We can now specify the profit functions. For firm 1 we have

$$\pi_1 = \{[a - b(q_{11} + q_{21})]q_{11} + [a - b(q_{12} + q_{22})]q_{12}\} - [F + c(q_{11} + \frac{1}{g}q_{12})], \tag{23.31}$$

where we observe that, if the quantity *offered* in market 2 is q_{12}, the corresponding quantity *produced* must be $(1/g)q_{12}$, given transport costs. Similarly for firm 2 we have

$$\pi_2 = \{[a - b(q_{11} + q_{21})]q_{21} + [a - b(q_{12} + q_{22})]q_{22}\} - [F + c(q_{22} + \frac{1}{g}q_{21})]. \tag{23.32}$$

23.3.2 The Equilibrium Solution

Cournot behaviour implies that each firm maximises profit taking as given the quantities offered by the other firm. The first-order conditions for a maximum are

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_{11}} &= [-2bq_{11} - bq_{21} + a] - c = 0, \\
\frac{\partial \pi_1}{\partial q_{12}} &= [-2bq_{12} - bq_{22} + a] - c/g = 0, \\
\frac{\partial \pi_2}{\partial q_{21}} &= [-2bq_{21} - bq_{11} + a] - c/g = 0, \\
\frac{\partial \pi_2}{\partial q_{22}} &= [-2bq_{22} - bq_{12} + a] - c = 0,
\end{aligned} \tag{23.33}$$

whose solution will yield the optimal quantities q_{ij} provided that the second-order conditions are satisfied. The Hessian of firm 1's profit function is

$$\begin{bmatrix} -2b & 0 \\ 0 & -2b \end{bmatrix},$$

whose leading principal minors alternate in sign, starting from minus. Hence the second-order conditions are satisfied. The same holds for firm 2.

The four first-order conditions can be interpreted, as usual, as the equality between marginal revenue and marginal cost for each firm in each market. Note that marginal cost (and hence marginal revenue) for delivering an export unit (c/g) is higher than for a unit of domestic sales (c) owing to transport costs.

Equations (23.33) also define the reaction functions implicitly. For example, if we solve the first equation for q_{11}, we get the optimal quantity offered by firm 1 in market 1 in terms of the quantity offered by firm 2 in the same market. This reaction curve is

$$q_{11} = -\frac{1}{2}q_{21} + \frac{c-a}{2b}. \tag{23.34}$$

It can now be observed that the system of the four first-order conditions is separable: the first and third equation, in fact, only contain the two unknowns q_{11}, q_{21} and can be solved independently of the two other. Similarly, the second and fourth equations independently determine the unknowns q_{22}, q_{12}. This separability property depends on the constant marginal cost assumption, for if marginal cost were a function of output, q_{12} would enter the first equation, q_{11}would enter the second equation and so on; the four equations would all be linked. We also observe that the two subsystems are perfectly symmetric, so that the set of solutions to the first is also the set of solutions to the second, with $q_{11} = q_{22}$ and $q_{12} = q_{21}$. Hence we need consider only one subsystem, for example the first. This is a simple linear system, whose solution is

$$q_{11}^E = \frac{a + c/g - 2c}{3b}, \tag{23.35}$$

$$q_{21}^E = \frac{a + c - 2c/g}{3b}. \tag{23.36}$$

We are interested in a positive solution for q_{21}^E, the amount of "invasion" of country 2's firm into market 1, because $q_{21} = 0$ (and hence $q_{12} = 0$ as well, given the symmetry of the two subsystems) would mean no international trade. It is easy to see that for q_{21}^E to be positive we must have

$$g > \frac{2c}{a + c}, \tag{23.37}$$

which means that transport costs must be below a certain critical level before invasion will occur (recall that transport costs are inversely related to g). When transport costs tend to zero ($g \to 1$), the solution will tend to the Cournot solution

$$q_{11}^E = q_{21}^E = \frac{a - c}{3b}, \tag{23.38}$$

while for positive transport costs $q_{21}^E < q_{11}^E$, as can easily be determined from (23.35) and (23.36), namely the domestic firm has a higher share of the domestic market than the foreign firm. It is also easy to see that q_{11}^E decreases as g increases (i.e., as transport costs decrease), and that q_{21}^E increases as g increases. Hence the foreign firm's share of the domestic market increases, and that of the domestic firm decreases, as transport costs decrease, both approaching $1/2$. The opposite obviously holds when g decreases.

Since each firm has a smaller share of the foreign market than of the domestic market, marginal revenue is higher in the foreign market than in the domestic market, which we already knew from the first-order conditions. But there is more to it than that. Given the symmetry conditions, the overall quantity supplied to each market will be the same in both markets, hence the price also will be the same in both markets. If we now recall that a firm's mark-up over cost is defined as $(p - MC)/p$, where p is the selling price and MC the marginal cost, it follows that each firm's mark-up over cost is lower in its export market than in its domestic market. In fact, $(p - c/g)/p < (p - c)/p$ due to transport costs. Since the selling price is the same in both markets, and transport costs are borne by the exporting firm, the f.o.b. price of exports is below the domestic price, and—as Brander and Krugman (1983) note—there is *reciprocal* dumping.

23.3.3 Stability

Let us now come to stability. The usual way of modelling the dynamic process underlying the reactions is to introduce a lag. Given the quantity offered in period t by firm 2 in the market under consideration, firm 1 will use its own reaction curve to determine the quantity that it will offer in the next period. Firm 2 will act similarly. This amounts to considering the system of difference equations

$$q_{11,t+1} = -\frac{1}{2}q_{21,t} + \frac{c-a}{2b}, \tag{23.39}$$

$$q_{21,t+1} = -\frac{1}{2}q_{11,t} + \frac{c/g-a}{2b}. \tag{23.40}$$

The roots of the characteristic equation of this system are 1/2, −1/2 (for the procedure see Gandolfo, 2009, chaps. 9 and 10, sect. 10.1). Since they are both less than unity in absolute value, the equilibrium is dynamically stable.

23.4 Vertically Differentiated Goods, Oligopoly, and Trade

The model that we present is based on the works of Gabszewicz, Shaked, Sutton, and Thisse (1981) and Shaked and Sutton (1982, 1983, 1984).

23.4.1 Consumers

There is a continuum of consumers who are assumed to have identical tastes, but different incomes, which are uniformly distributed over some interval $0 < a \leq I \leq b$. There are n vertically differentiated goods which are ranked according to quality in the same way by all consumers, say

$$0 < u_1 < \ldots < u_n, \tag{23.41}$$

where $u_k, k = 1, \ldots, n$, is the universally accepted measure of the quality of good k. Given that n may be large, for the moment we are in a context of monopolistic competition rather than of oligopoly, but the model will end up in an oligopolistic situation, as we shall see.

Given the difference in income, richer consumers are willing to pay more for a higher quality product. Each consumer makes indivisible and mutually exclusive purchases from among the n substitute goods, in the sense that any consumer either buys exactly one unit of the chosen good or buys nothing. The utility function of the representative consumer is denoted by $U(I, k)$, which indicates the utility achieved by consuming one unit of good k, and I units of income on "other things" (the latter are referred to by Gabszewicz et al. (1981) and by Shaked and Sutton (1983, 1984), as a Hicksian composite commodity). The utility obtained from consuming I units of income only is indicated by $U(I, 0)$.

These properties can be captured by a simple utility function of the form

$$U(I,k) = u_k \cdot I \quad \text{for } k = 1, 2, \ldots, n; \text{ and } U(I,0) = u_0 \cdot I, \tag{23.42}$$

where $u_0 > 0$ is conventionally taken to be smaller than u_1. If we denote by p_k the price of good k in terms of I, with the assumption that p_k increases as the quality increases, it is easy to see that a consumer having a given income $\overline{I}$ can obtain a utility

$$U(\overline{I} - p_k, k) = u_k \cdot (\overline{I} - p_k) \tag{23.43}$$

by devoting p_k units of income to the purchase of one unit of good k and $(\overline{I} - p_k)$ to "other things".

We can now define an income level I_k such that a consumer endowed with this income will be indifferent between good k at price p_k and good $k-1$ at price p_{k-1}. Using (23.43) and taking account of the second definition in (23.42), we have

$$u_k \cdot (I_k - p_k) = u_{k-1} \cdot (I_k - p_{k-1}), \tag{23.44}$$

$$u_1 \cdot (I_1 - p_1) = u_0 I_1, \tag{23.45}$$

respectively for $k > 1$ and $k = 1$.

If we define

$$r_{k-1,k} = \frac{u_k}{u_k - u_{k-1}}, \tag{23.46}$$

which is clearly greater than one, from (23.44) we get

$$\begin{aligned} I_k &= u_k p_k/(u_k - u_{k-1}) - u_{k-1} p_{k-1}/(u_k - u_{k-1}) \\ &= r_{k-1,k} p_k + (1 - r_{k-1,k}) p_{k-1} \\ &= p_{k-1} + (p_k - p_{k-1}) r_{k-1,k}, \end{aligned} \tag{23.47}$$

for all $k > 1$, and from (23.45)

$$I_1 = p_1 r_{0,1}$$

for the case of indifference between consuming no differentiated good and consuming the lowest quality of it.

It can easily be shown that a consumer with income above I_k will prefer the higher-quality good k at price p_k to the lower-quality good $k-1$ at price p_{k-1}, while a consumer with income below I_k will do exactly the opposite. Let us consider a consumer with income $I_k + \mathrm{d}I$, where $\mathrm{d}I \gtrless 0$, and I_k is as defined in Eqs. (23.44) and (23.45). Then the consumers' utility deriving from the consumption of good k or $k-1$ is respectively

$$U_k = u_k \cdot (I_k + \mathrm{d}I - p_k) = u_k \cdot (I_k - p_k) + u_k \cdot \mathrm{d}I,$$
$$U_{k-1} = u_{k-1} \cdot (I_k + \mathrm{d}I - p_{k-1}) = u_{k-1} \cdot (I_k - p_{k-1}) + u_{k-1} \cdot \mathrm{d}I,$$

from which, given Eqs. (23.44) and (23.45), we immediately obtain $U_k \gtrless U_{k-1}$ according as $(u_k - u_{k-1})\mathrm{d}I \gtrless 0$, i.e. according as $\mathrm{d}I \gtrless 0$, since $u_k - u_{k-1} > 0$ given (23.41). This result is of course a consequence of the fact that the utility function has been designed just to have the property that higher-income consumers are willing to spend more to get a higher-quality good.

23.4.2 Firms, and Market Equilibrium

The behaviour of firms is based on a three-stage non-cooperative game. In the first stage firms decide whether or not to enter the industry. In the second stage each firm chooses the quality of its product (each firm is assumed to produce only one good). In the third stage each firm chooses its price, and only variable costs enter the pricing decision, given the assumption that all fixed costs have been incurred in the previous stages and are sunk costs. This three-stage process, as Shaked and Sutton (1982) observe, is meant to capture what happens in reality: the price can be varied easily, but a change in the specification of a product involves modification in the appropriate production facilities, and entry into the industry requires construction of a plant.

The solution that the authors seek is a perfect equilibrium, namely an n-tuple of strategies such that, after any stage, that part of the strategies pertaining to the game consisting of the remaining stages form a Nash equilibrium in that game. This allows Shaked and Sutton (1982, 1983, 1984) to study the three-stage game by first examining price competition in the third stage, taking qualities as given. This amounts to considering the short run. In the long run all stages of the game have to be considered, and the qualities are endogenously determined.

From our previous treatment of consumer's choice it follows that consumers are partitioned into segments or income brackets corresponding to the successive market shares of rival firms. More precisely, if we assume that each firm only produces one good, firm k will sell to consumers with income I_k to I_{k+1} for $k < n$ (with income I_k to b for firm n), where I_k, I_{k+1} are given by (23.47). Since each consumer buys one unit of the good, and there is a continuum of consumers, the number of units sold by firm k will be $(I_{k+1} - I_k)$. It is important to observe that a firm may be "just" excluded from the market in the sense that $I_k - I_{k-1} = 0$, so that this firm has a market share of zero, but a slight (infinitesimal) decrease in its price or a slight increase in the price set by any of its two neighbouring firms will make its market share positive.

Unit variable cost is assumed to be an increasing function of the quality but independent of the level of output, hence we denote it by c_k. Therefore the profit of any firm k becomes, for $k = 1, 2, \ldots, k-1$ and for $k = n$ respectively

$$\pi_k = (p_k - c_k)(I_{k+1} - I_k). \qquad \text{and} \qquad \pi_n = (p_n - c_n)(b - I_n). \tag{23.48}$$

If $p_k < c_k$, the firm will undergo losses and hence that quality will not be produced. Also note that for p_k sufficiently high the sales of the firm will be zero. Hence, we consider only the range in which $\pi_k > 0$.

The next question is whether an equilibrium exists. This will be a non-cooperative equilibrium (Nash equilibrium), namely a price vector such that, for any firm k, given the prices set by the other firms, the price fixed by firm k is its profit maximising price. To show that such equilibrium exists, Shaked and Sutton (1983, p. 1475) begin by proving the following

> Lemma I: For any given products $u_1, u_2, \ldots, u_n$ and corresponding prices $p_1, p_2, \ldots, p_n$, for all k, the profit of the kth firm is a single peaked function of its price.

The market share of any firm k is included between that of two neighbouring firms, $k+1$ and $k-1$. As p_k falls, it may happen that one (or both) neighbouring firm, say firm $k-1$, is driven out of the market, so that firm k will acquire firm $k-2$ as a new neighbour. We first consider the case in which the neighbours are firm $k-1$ and $k+1$. If we examine the profit function (23.48) we find that any turning point of π_k is a maximum, so that π_k is single peaked. In fact, we have

$$\pi_k' = (I_{k+1} - I_k) + (p_k - c_k)\frac{\mathrm{d}}{\mathrm{d}p_k}(I_{k+1} - I_k),$$

and from (23.47) we get

$$I_{k+1} - I_k = [p_k + (p_{k+1} - p_k)r_{k,k+1}] - [p_{k-1} + (p_k - p_{k-1})r_{k-1,k}],$$

whose derivative with respect to p_k is $(1 - r_{k,k+1} - r_{k-1,k})$. Thus we have

$$\begin{aligned} \pi_k' &= (p_k - c_k)(1 - r_{k,k+1} - r_{k-1,k}) + (I_{k+1} - I_k) \\ \pi_k'' &= 2(1 - r_{k,k+1} - r_{k-1,k}) < 0. \end{aligned} \tag{23.49}$$

Since the second derivative is always negative, any turning point of π_k is a global maximum, hence π_k is single peaked. It can easily be checked that such property also holds for $k = n$.

We must now consider the case in which p_k falls sufficiently for driving the neighbouring firm $k-1$ out of the market, so that the new neighbours are firms $k-2$ and $k+1$. By the same procedure used above, it can be seen that with these new neighbours firm k's profit (say, $\hat{\pi}_k = (p_k - c_k)(I_{k+1} - I_{k-1})$) remains a single peaked function of p_k. We know that a zero market share of firm $k-1$ means $I_k - I_{k-1} = 0$, and we show that at the price at which this happens we also have

$$\hat{\pi}_k' > \pi_k', \tag{23.50}$$

so that $\hat{\pi}_k$ is a fortiori increasing at this point if π_k is increasing there. By the same procedure used for computing π_k' we get

$$\hat{\pi}_k' = (p_k - c_k)(1 - r_{k,k+1} - r_{k-2,k}) + (I_{k+1} - I_{k-1}). \tag{23.51}$$

By using $I_k = I_{k-1}$ and the fact that the definition (23.46) implies $r_{k-2,k} < r_{k-1,k}$, we can easily see from (23.49) and (23.51) that (23.50) does indeed hold. This completes the demonstration of the lemma.

The lemma implies that each firm's profit function is quasi-concave, hence (Friedman, 1977, p. 157) a noncooperative price equilibrium $p_1, p_2, \ldots, p_n$ exists for any set of products $1, 2, \ldots, n$.

The next step of the analysis is to prove that, under normal conditions (namely when all consumers strictly rank the goods in the same way, as assumed at the beginning) the market has the finiteness property. This means that, at any Nash equilibrium involving a number of products drawn from the existing interval of qualities, there is an upper bound B to the number of firms which enjoy positive market shares and prices exceeding unit variable cost. This can be shown quite simply for the particular case studied by Gabszewicz et al. (1981) and Shaked and Sutton (1982) , in which variable cost is assumed to be zero and the distribution of income is not much dispersed, namely $b < 4a$. In this case at most two products (the top two) have a positive market share at equilibrium. The first order condition for a maximum implies $\pi_k' = 0$, hence from (23.49) we have, letting $c_k = 0$,

$$\begin{aligned} p_k(1 - r_{k,k+1}) - p_k r_{k-1,k} + I_{k+1} - I_k &= 0, \\ -p_n r_{n-1,n} + b - I_n &= 0. \end{aligned} \tag{23.52}$$

From the definition of I_k given in (23.47) we get $p_k r_{k-1,k} = I_k + (r_{k-1,k} - 1)p_{k-1}$, and by substituting this into (23.52) we get

$$\begin{aligned} p_k(1 - r_{k,k+1}) - (r_{k-1,k} - 1)p_{k-1} + I_{k+1} - 2I_k &= 0, \\ -p_{n-1}(r_{n-1,n} - 1) + b - 2I_n &= 0, \end{aligned}$$

whence

$$
\begin{aligned}
I_{k+1} &= 2I_k + p_k(r_{k,k+1} - 1) + (r_{k-1,k} - 1)p_{k-1} > 2I_k, \\
b &= 2I_n + p_{n-1}(r_{n-1,n} - 1) > 2I_n,
\end{aligned}
\tag{23.53}
$$

where the inequalities follow from the fact that the r's, as defined in (23.46), are greater than one. From (23.53) we get, by letting $k + 1 = n$ in the first equation,

$$4I_{n-1} < b. \tag{23.54}$$

Now by assumption $b < 4a$, which in conjunction with (23.54) implies

$$I_{n-1} < a. \tag{23.55}$$

This inequality means that I_{n-1} is already lower than the lower bound to the distribution of income. Since I_{n-1} is the income at which a consumer endowed with this income is indifferent between good $n - 1$ at price p_{n-1} and good $n - 2$ at price p_{n-2} (see Sect. 23.4.1), it follows that no such income exists, and hence the two top firms (n and $n - 1$) cover the market. For further reference note that this result can be strengthened to the case in which $2a < b < 4a$ (Shaked & Sutton, 1982). For the general case of the finiteness property see Shaked and Sutton (1983).

This completes the study of the third stage of the game. In the second stage (choice of quality) a Nash equilibrium exists that involves two distinct qualities produced by two firms that both earn positive profits. The entry of further firms would lead to a configuration in which all firms would earn zero profits. It can also be shown that the top quality firm enjoys a greater revenue than its rival, and that the revenues of both firms increase as the quality of the better product improves. Finally, the examination of the first stage of the game, which involves the decision whether to enter or not the market (it is at this stage that the fixed costs are assumed to be incurred), allows to conclude that a perfect equilibrium exists in which two firms enter, produce distinct products, and have positive profits. No perfect equilibrium exists in which more than two firms enter. The proofs of the results concerning the second and first stage are rather lengthy, hence we refer the reader to Shaked and Sutton (1982).

23.4.3 International Trade

The extension of this model to international trade is straightforward. If we start from two identical economies, then in autarky each will support the same B goods by the finiteness property. When free trade (no transport costs are assumed) is opened, the combined world economy will have the same properties of the two identical autarkic economies (same income distribution, etc.). Hence it will support the same B goods, and international trade will be generated by the fact that consumers in both countries demand the same B goods as before, which will be produced partly in one country and partly in the other. Consumers will be better off as shown in the text.

When the two countries are not identical, then the combined world economy can support more than the number of goods supported by each in isolation. Gabszewicz et al. (1981) study the simplified case (see above) in which each of the two autarkic economies supports two goods, which means that $2a < b < 4a$ in both countries; all four goods are assumed to be different. The income distributions in the two countries are different, but not too much: more precisely, there exists an overlapping interval a_1, a_2 such that

$$\frac{a_1}{2} < a_2 < a_1 < \frac{b_1}{2} < b_2 < b_1,$$

where (a_1, b_1) and (a_2, b_2) are the intervals over which the income distributions in the two countries are defined. Now, the market share of the highest-quality good n will extend below $b_1/2$, that of good $(n-1)$ will extend below a_1, and that of the third good $(n-2)$ will extend below a_2 (see Gabszewicz et al., 1981). Hence, these three goods will cover the market.

For further developments of this approach to international trade see Motta (1992).

23.5 Horizontal Differentiation, Oligopoly, and Trade

The Eaton and Kierzkowski (1984) model assumes that there are only two basic commodities, one homogeneous (A) and the other horizontally differentiated (B). The homogeneous commodity plays a secondary role: it only serves to allow consumers to spend income when they do not purchase the differentiated commodity. Thus, the analysis can be concentrated on the latter.

23.5.1 Demand for Characteristics

The demand for the differentiated commodity follows Lancaster's approach, with the simplifying assumption that such commodity only contains one characteristic Z (which can be measured by a real number). Each consumer has an ideal model of good B represented by a value of Z, say θ_i, where i denotes the consumer. The consumer's utility declines as the model actually consumed becomes more distant from the ideal. Hence consumer i will buy an alternative model only if the price of this non-ideal alternative is sufficiently lower than the price of the ideal model. Finally, if the price for all available varieties of the commodity exceeds a certain upper limit, the consumer will not buy this commodity, and concentrate expenditure on the homogeneous good A.

The formalisation corresponding to these assumptions is the utility function (suggested by Salop, 1979)

$$U(Y, p_i, \theta_i, Z_i, \overline{p}) = \max[Y - (p_i + |\theta_i - Z_i|), Y - \overline{p}], \tag{23.56}$$

where Z_i is the model consumed by individual i, p_i the price paid for it, Y income, and $\overline{p}$ the maximum price. This utility function has the following characteristics: At most one unit of the differentiated commodity will be purchased. The maximum price that the individual is willing to pay is $\overline{p}$, provided that the model corresponds exactly to the ideal, namely $Z_i = \theta_i$. When there is no such correspondence, the individual will be willing to purchase the non-ideal model at a price not higher than $\overline{p} - |\theta_i - Z_i|$; clearly, this price is the lower, the greater the distance from the ideal. In general, the consumer will choose the model for which $p_i + |\theta_i - Z_i|$ is at a minimum, if this amount is less than or equal to $\overline{p}$. Hence if there is no model for which this is true, namely if $p_i + |\theta_i - Z_i| > \overline{p}$, i.e. $p_i > \overline{p} - |\theta_i - Z_i|$ for all existing models, the consumer will not purchase the differentiated commodity.

23.5.2 The Production Side

Let us now consider the production side. There are increasing returns to scale, and the total cost of producing an amount x of a particular model is assumed to be $K + cx$, where the marginal cost c is constant and the fixed cost K is a sunk cost, namely it must be incurred by the firm at the moment of the choice of the model to produce, before the levels of output and price are determined. Hence when the firm sets these levels, the cost K is sunk and the model is already determined. Finally, a single firm can produce only a single model of the commodity.

In spite of its apparent simplicity, this model gives rise to a rich taxonomy, according to the number of firms (one, hence monopoly, or two, hence duopoly) and to the categories of consumers (the types of consumers are distinguished according to the type of ideal model). Here we consider only one case, referring the reader to Eaton and Kierzkowski (1984) for the others. It is the case of two types of consumers and one firm.

There are n_1 consumers of type 1, all having θ_1 as the ideal model, and n_2 of type 2, with ideal model θ_2. The single firm can produce a single model (say Z_1), which can be assumed to be closer to the ideal θ_1 without loss of generality. Assuming that price discrimination is not possible, the firm must decide whether:

(a) Not to produce at all; or
(b) To produce and sell to just the type of consumers whose ideal is closer to Z_1. This means charging the limit price for type 1 consumers, $\overline{p} - |\theta_1 - Z_1|$. Since θ_1 is closer to Z_1 than θ_2, this lim it price will certainly be higher than $\overline{p} - |\theta_2 - Z_1|$, the limit price for type 2 consumers, who will not buy the commodity. Hence the firm's current profit will be

$$\pi_1 = (\overline{p} - |\theta_1 - Z_1| - c)n_1 - K; \tag{23.57}$$

or

(c) To sell to both types of consumers, charging the limit price for type 2 consumers, $\overline{p} - |\theta_2 - Z_1|$. Since this is lower than the limit price for type 1 consumers, these will also buy the commodity, and the firm's current profit will be

$$\pi_{1,2} = (\overline{p} - |\theta_2 - Z_1| - c)(n_1 + n_2) - K. \tag{23.58}$$

If $\overline{p} - |\theta_1 - Z_1| < c$, the firm will not produce, since the highest selling price, which is the limit price for type 1 consumers, is below marginal cost. Excluding this case, alternative (c) or (b) will be selected according as $\pi_{1,2} \gtrless \pi_1$, from which, after simple manipulations,

$$(\overline{p} - c)(1 - \lambda) \gtrless |\theta_2 - Z_1| - |\theta_1 - Z_1|\,\lambda, \tag{23.59}$$

where λ is defined as the proportion of type 1 consumers in the overall market, namely $n_1/(n_2 + n_2)$. It is easy to see that selling to the broader set of consumers is the superior alternative when $\overline{p} - c$ is high, λ is low, and the two ideal qualities are not substantially different.

23.5.3 International Trade

The extension of this model to international trade is straightforward, if we assume that in the home country, where the producing firm is located, there are only type 1 consumers, say n_1 (with ideal model θ_1), while in the foreign country (denoted by an asterisk) there are only type 2 consumers, say n_2^*, whose ideal model is θ_2. With no loss of generality we can assume $\theta_2 > \theta_1$. In autarky, the firm produces exactly model θ_1 and charges the maximum price $\overline{p}$. When trade is opened, the firm will consider exporting to the foreign market. If we assume no transport costs and no segmentation, the firm can sell to both types of markets at the (lower) price that type 2 consumers are willing to pay, namely, since $Z_1 = \theta_1$, at the price $\overline{p} - |\theta_2 - \theta_1| = \overline{p} - (\theta_2 - \theta_1)$. If we compare the firm's profits in the two situations, we find that the firm will begin to export to the foreign market if

$$[\overline{p} - (\theta_2 - \theta_1) - c](n_1 + n_2^*) > (\overline{p} - c)n_1. \tag{23.60}$$

We see that the more similar are the demand patterns and the larger is the foreign country, the more likely is that trade will take place. Similarity in demand patterns is again, contrary to conventional theory, a cause of trade. Another important difference with respect to the traditional theory is that trade is indifferent to the foreign country, that will receive no gain. This occurs because the sole producer of the differentiated product will be able to fix the price at a level that will leave foreign consumers indifferent between consuming only the homogeneous good (as before trade) and both the differentiated and homogeneous good. The domestic consumers will benefit from lower prices and the domestic producer's profit will be larger. Hence, the domestic country's welfare improves while the foreign country receives no benefit from trade.

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