Chapter 2
The Classical (Ricardo-Torrens) Theory of Comparative Costs

2.1 Comparative Costs (Advantages) and International Trade

The classical theory of international trade is usually attributed to David Ricardo, who treated it in Chap. 7 of his *Principles* (Ricardo, 1817). But it is possible to find earlier statements of this theory in the work of Robert Torrens (1815): the reader interested in problems of historical priority should consult Viner (1937) and Chipman (1965a).

As far as the theory itself is concerned, we begin by observing that it affirms that the crucial variable explaining the existence and pattern of international trade is technology. A difference in *comparative costs* of production—the necessary condition for international exchange to occur—does, in fact, reflect a difference in the techniques of production. The theory also aims at showing that trade is beneficial to all participating countries.

If we simplify to the utmost, we can assume that there are two countries (England and Portugal in the famous example of Ricardo's), two commodities (cloth and wine), that all factors of production can be reduced to a single one, labour,[1] and that in both countries the production of the commodities is carried out according to fixed technical coefficients: as a consequence, the unit cost of production of each commodity (expressed in terms of labour) is constant.

It is clear that if one country is superior to the other in one line of production (where the superiority is measured by a lower unit cost) and inferior in the other line, the basis exists for a fruitful international exchange, as earlier writers, for example

[1]This is based on the classical labour theory of value. It is outside the scope of the present treatment to enter into the controversies concerning this theory, so that we shall simply observe that the validity of the classical theory of international trade is not based on the validity of the labour theory of value, as it is sufficient for unit costs of production to be measurable by a common unit across countries and to be constant.

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Table 2.1 Example of absolute advantage

Commodities	Unit costs of production in terms of labour	
	In England	In Portugal
Cloth	4	6
Wine	8	3

Adam Smith, had already shown. The simple example in Table 2.1 is sufficient to make the point; the reader should bear in mind that here as in the subsequent examples, the cost of transport is assumed to be absent, as its presence would complicate the treatment without altering the substance of the theory. As we see, the unit cost of manufacturing cloth is lower in England than in Portugal while the opposite is true for wine production. It is therefore advantageous for England to specialize in the production of cloth and to exchange it for Portuguese wine, and for Portugal to specialize in the production of wine and to exchange it for British cloth. Suppose, for example, that the (international) *terms of trade* (i.e., the ratio according to which the two commodities are exchanged for each other between the two countries, or international relative price) equals one, that is, international exchange takes place on the basis of one unit of wine for one unit of cloth. Then England with 4 units of labour (the cost of one unit of cloth) obtains one unit of wine, which otherwise—if produced internally—would have required 8 units of labour. Similarly Portugal with 3 units of labour (the cost of one unit of wine) obtains one unit of cloth, which otherwise—if produced internally—would have required 6 units of labour.

In this example we have reasoned in terms of *absolute* costs, as one country has an absolute advantage in the production of one commodity and the other country has an absolute advantage in the production of the other. That in such a situation international trade will take place and benefit all participating countries is obvious. Less so is the fact that international trade may equally well take place even if one country is superior to the other in the production of *both* commodities. The great contribution of the Ricardian theory was to show the conditions under which even in this case international trade is possible (and beneficial to both countries).

Now, this theory affirms that the necessary condition for international trade is, in any case, that a difference in *comparative costs* exists. Comparative cost can be defined in *two* ways: as the ratio between the (absolute) unit costs of the two commodities in the same country, or as the ratio between the (absolute) unit costs of the *same* commodity in the two countries. Following common practice, we shall adopt the former, but they are totally equivalent.

In fact, if we denote the unit costs of production of a good in the two countries by a_1, a_2 (where the letter refers to the good and the numerical subscript to the country: this notation will be constantly followed throughout the book) and the unit costs of the other good by b_1, b_2, then

$$(a_1/b_1 = a_2/b_2) \Longleftrightarrow (b_1/a_1 = b_2/a_2) \Longleftrightarrow (a_1/a_2 = b_1/b_2) \Longleftrightarrow (a_2/a_1 = b_2/b_1),$$

Table 2.2 Example of comparative advantage

	Unit costs of production in terms of labour	
Commodities	In England	In Portugal
Cloth	4	6
Wine	8	10

and similarly

$$(a_1/b_1 \gtrless a_2/b_2) \Longleftrightarrow (a_1/a_2 \gtrless b_1/b_2) \Longleftrightarrow (b_2/a_2 \gtrless b_1/a_1) \Longleftrightarrow (b_2/b_1 \gtrless a_2/a_1).$$

It therefore makes no difference whether the comparison is made between a_1/b_1 and a_2/b_2 or between a_1/a_2 and b_1/b_2, and so on.

The basic proposition of the theory under examination is that *the condition for international trade to take place is the existence of a difference between the comparative costs*. This is, however, a necessary condition only; the sufficient condition is that the international terms of trade lie *between* the comparative costs without being equal to either. When both conditions are met, it will be beneficial to each country to specialize in the production of the commodity in which it has the relatively greater advantage (or the relatively smaller disadvantage). Let us consider the following example (Table 2.2).
As England is superior to Portugal in the production of both commodities, it might seem that there is no scope for international trade, but this is not so. Comparative costs are $4/8 = 0.5$ and $6/10 = 0.6$ in England and Portugal respectively. England also has a relatively greater advantage (a *comparative advantage*) in the production of cloth: its unit cost, in fact, is lower in England than in Portugal by 33.3 % (2/6), while the unit cost of wine is lower in the former than in the latter country by 20 % (2/10). It can similarly be seen that Portugal has a relatively smaller disadvantage in the production of wine: its unit cost, in fact, is higher in Portugal than in England by 25 % (2/8), while the unit cost of cloth is higher in Portugal than in England by 50 % (2/4).

Therefore—provided that the terms of trade are greater than 0.5 and smaller than 0.6—British cloth will be exchanged for Portuguese wine to the benefit of both countries. Let us take an arbitrary admissible value of the terms of trade, say 0.55 (that is, international exchange takes place at the terms of 0.55 units of wine per one unit of cloth). In England, on the basis of the existing technology, one unit of cloth exchanges for 0.5 units of wine: 0.5 is, in fact, the comparative cost, and, according to the classical theory, the relative prices of goods, that is their exchange ratios, are determined by costs. For one unit of cloth, England can obtain, by way of international trade, 0.55 units of wine, more than the amount obtainable internally. Similarly in Portugal, to obtain one unit of cloth, 0.6 units of wine (0.6 is Portugal's comparative cost) are necessary, while by way of international trade only 0.55 units of wine are required. It is obvious that international trade is beneficial to both countries.

It is possible to arrive at the same conclusion by reasoning in terms of production costs. England with 4 units of labour (the cost of one unit of cloth) obtains, on the international market, 0.55 units of wine which, if produced internally, would have required $0.55 \times 8 = 4.4$ units of labour. Similarly Portugal with 5.5 units of labour (the cost of 0.55 units of wine, given by 0.55×10) obtains one unit of cloth, which would have required 6 units of labour if produced internally.

It can easily be shown that the terms of trade must be strictly located between the two comparative costs. If, in fact, the terms of trade were equal to either comparative cost, the concerned country would have no interest in trading, since the internal price ratio (given by the comparative cost) would be equal to the international one (the terms of trade). This would mean that the country in question would obtain the other commodity by way of trade at the same cost as it could be got internally. Assume, for example, that the terms of trade are 0.5, equal to the British comparative cost. Then England would obtain, on the international market, with 4 units of labour (the cost of one unit of cloth) 0.5 units of wine, which would have required $0.5 \times 8 = 4$ units of labour if produced internally. In other words, by exchanging cloth for wine on the international market England would obtain exactly the same amount of wine obtainable internally (0.5 units of wine per one unit of cloth): there is, then, no reason for engaging in international trade. It can similarly be seen that, if the terms of trade were 0.6, there would be no reason for Portugal to engage in international trade at all. We leave it to the reader to check, as an exercise, that if the terms of trade were to fall outside the interval between the comparative costs (that is, in our example, if they were smaller than 0.5 or greater than 0.6) then, by engaging in international trade, one of the two countries would suffer a loss.

2.2 Alternative Graphic Representations

We can now show two simple diagrams to represent the theory of comparative costs. Let x denote (the amount of) cloth and y (the amount of) wine and consider country 1. With any given quantity of labour L_1 it is possible to obtain an amount of cloth

$$x = \frac{1}{a_1} L_1,$$

where a_1 (see Sect. 2.1) is the unit cost of producing cloth—a constant because of the assumption of fixed technical coefficients.

Likewise, with the same amount of labour it is possible to obtain

$$y = \frac{1}{b_1} L_1$$

of wine.

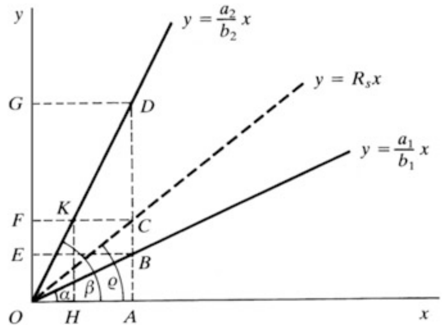

Fig. 2.1 Graphic representation of comparative costs

If we divide y by x we get

$$\frac{y}{x} = \frac{\frac{1}{b_1}L_1}{\frac{1}{a_1}L_1} = \frac{a_1}{b_1},$$

whence

$$y = \frac{a_1}{b_1}x. \tag{2.1}$$

We could have arrived at the same result by recalling that a_1/b_1 is the comparative cost, which (see Sect. 2.1) expresses the exchange ratio of the two commodities.

In an analogous way we get, for country 2, the relation

$$y = \frac{a_2}{b_2}x. \tag{2.2}$$

Equations (2.1) and (2.2) are represented in Fig. 2.1 as two straight lines starting from the origin. The elementary properties of straight lines tell us that $a_1/b_1 = \tan\alpha$ and $a_2/b_2 = \tan\beta$, that is, comparative costs are given by the slopes of the straight lines.

As the two lines do not coincide, there is a difference between the comparative costs: in fact, if these were equal ($a_1/b_1 = a_2/b_2$), the two lines would coincide. In this kind of diagram, therefore, the necessary condition for international trade is represented by the non-coincidence of the two lines.

Also the terms of trade can be represented as the slope of a straight line. In fact, if we denote these by R_s, then

$$\frac{y}{x} = R_s,$$

whence

$$y = R_s x, \tag{2.3}$$

which is a straight line through the origin with slope R_s. In Fig. 2.1 we have assumed that the sufficient condition for international trade is met, namely that line (2.3) falls strictly between lines (2.1) and (2.2); this amounts to saying that, having assumed $a_1/b_1 < a_2/b_2$, the inequality

$$\frac{a_1}{b_1} < R_s < \frac{a_2}{b_2} \tag{2.4}$$

holds. Of course, if $a_1/b_1 > a_2/b_2$, then the condition would be $a_1/b_1 > R_s > a_2/b_2$.

Inequality (2.4) is the same as

$$\tan\alpha < \tan\varrho < \tan\beta, \tag{2.5}$$

which has an obvious graphic interpretation. If this condition is satisfied, international trade will take place, and it will be profitable for country 1 to specialize in the production of x and for country 2 to specialize in the production of y. In terms of the diagram, in fact, the propositions so far examined are equivalent to saying (a) that the country whose line representing its comparative cost lies between the line representing the terms of trade and the horizontal axis will find it profitable to specialize in the production of (and in any case to export) the good measured on this axis, and (b) that the country whose comparative-cost line lies between the terms-of-trade line and the vertical axis will find it profitable to specialize in the production of (and in any case to export) the good measured on this axis.

To show this, let us suppose that, given the terms of trade R_s, a quantity OA of x is exchanged for OF of y. It is easy to see that the amount OA is exported by country 1 (and so imported by country 2) while the amount OF is exported by country 2 (and so imported by country 1). The proof is straightforward, and in the course of this proof we shall also have occasion to show a measure of the gains from trade accruing to each country. Now, at the domestic price ratio, country 1 would have obtained $OE = AB$ of y for OA of x, whilst it can obtain $OF = AC$ by way of international trade. It is therefore profitable for country 1 to engage in international trade following the pattern just described (that is, to export x and to import y). The gains from trade accruing to this country can be measured, for example, in terms of y: they are given by segment BC, namely by the additional quantity of y that country 1 obtains in exchange for the same quantity of x. Let us consider country 2 which, at the domestic price ratio, would have had to give up $OG = AD$ of y to obtain OA of x, whilst it has to give up $OF = AC$ by way of international trade. It is therefore profitable to country 2 to engage in international trade with the pattern just described, and the benefit accruing to this country, measured in terms of y, is given by segment DC.

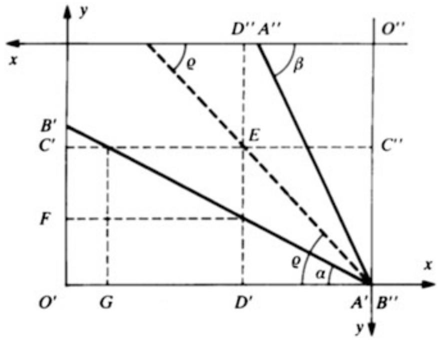

Fig. 2.2 Transformation curve and comparative costs

The gains from trade can also be measured in terms of x, but the measures are equivalent as can be shown by transforming them into each other by using the internal price ratio of the country concerned. For example, country 2 by trading OF of y on the international market obtains $OA = FC$ of x instead of $OH = FK$: the benefit in terms of x is, therefore, KC. But if we consider the right-angled triangle KCD we obtain $DC = KC \cdot \tan C\hat{K}D = KC \cdot \tan \beta$, where $\tan C\hat{K}D = \tan \beta =$ comparative cost or domestic exchange ratio of the two goods in country 2.

An alternative diagram of the theory of comparative costs is based on the concept of *transformation curve* (or *production-possibility frontier*) studied in microeconomic theory (see also below, Sect. 3.1). In our simplified model, in which there is only one factor of production and the technical coefficients are fixed, the transformation curve is linear (the general case will be treated in Sect. 3.1). It is in fact given, for country 1, by the equation

$$a_1 x + b_1 y = \overline{L}_1, \tag{2.6}$$

where $\overline{L}_1$ is the total amount of labour existing in country 1. Equation (2.6) is the equation of a monotonically decreasing straight line in the (x, y) plane, since we can write it as

$$y = -\frac{a_1}{b_1} x + \frac{\overline{L}_1}{b_1}. \tag{2.7}$$

In absolute value, the slope of this line equals the comparative cost in country 1. Comparative cost and marginal rate of transformation (or opportunity cost: see Sect. 3.1) are therefore one and the same thing.

In a similar way, we obtain the transformation curve of country 2. Consider then Fig. 2.2, where we have brought together the transformation curves of the two countries.

The line $A'B'$ is the transformation curve of country 1, i.e. the diagram of (2.7); in absolute value, $\tan \alpha$ equals the comparative cost of country 1. The line $A''B''$ is the transformation curve of country 2, rotated anticlockwise by 180° and placed so that point B'' coincides with point A'; it goes without saying that $O''B''$ and $O'B'$ are parallel. The absolute value of $\tan\beta$ equals the comparative cost in country 2.

Let us take an arbitrary admissible value of the terms of trade, say $\tan\varrho$, and assume that international trade occurs at point E, whose coordinates are the quantities exchanged. Country 1 specializes completely in the production of commodity x, of which it produces the amount $O'A'$; of this, a part is consumed domestically ($O'D'$), whilst the remaining part ($D'A'$) is exported in exchange for the quantity $O'C' = ED' = C''B''$ of commodity y. Note that, since the terms of trade are measured by $\tan\varrho$, and since (by considering the right-angled triangle $ED'A'$) we have $ED' = D'A' \cdot \tan\varrho$, it follows that by giving $D'A'$ of x, ED' of y can be obtained, and vice versa. This means that the trade balance is necessarily in equilibrium. In fact, balance-of-trade equilibrium, or value of exports = value of imports, requires

$$p_x D'A' = p_y ED'$$

or

$$\frac{p_x}{p_y} D'A' = ED', \tag{2.8}$$

which is indeed true, since commodities are exchanged at a relative price (p_x/p_y) given by the terms of trade, namely $p_x/p_y = \tan\varrho$.

Similarly, country 2 completely specializes in y and produces the amount $O''B''$ of this commodity, consuming $O''C''$ domestically and exporting $C''B''$ in exchange for $O''D'' = D'A'$ of commodity x. This result (complete specialization in both countries) is the normal outcome of trade in the Ricardian model. This may not be the outcome when one country (say country 1) is small with respect to the other, so that this country's production of x is not sufficient to fully satisfy, in addition to its own domestic demand, also the demand for this commodity by country 2. In such a case country 2 will not specialize completely in commodity y and will continue to produce both y and x.

As can be seen, point E lies *beyond* both transformation curves, and so it represents a basket of goods that neither country could have obtained in autarky. Consider, for example, country 1. In autarky, together with $O'D'$ of x this country could have obtained $O'F$ of y (less than the amount $O'C'$ that it obtains through international trade). The gains from trade accruing to this country can be measured, in terms of y, by $C'F$ (in terms of x they are measured by GD'). The gains from trade accruing to country 2 can be found in a similar way.

It is also obvious from the diagram that the closer the terms-of-trade line is to a country's transformation curve, the smaller that country's share of the gains; this share drops to zero when the terms-of-trade line coincides with that country's transformation curve (and all the gains go to the other country). This is an alternative way of showing the result already demonstrated in the previous treatment.

2.3 A Modern Interpretation in Terms of Optimization

The theory of comparative costs has been taken up again by modern scholars in terms of optimization. The general treatment will be given in Sects. 18.1 and 18.2; here we shall limit ourselves to a reformulation in these terms of the simple problem treated in the previous section.

We recall from that treatment that the benefits from international trade can be seen as an increase in the quantity of goods, and so in the real income (output) which can be obtained from the given amount of labour (by assumption, equal to the total amount available). It follows that the optimum can be interpreted as the maximization of real income given a certain input of labour; such an optimum, however, can be seen either from the point of view of the single country or from the point of view of the world as a whole (consisting, in our simple model, of two countries only).

2.3.1 Maximization of Real Income in Each Country

Let us begin by examining the optimum as the maximization of real national income in each country separately considered. Let p_x and p_y be the absolute prices (expressed in terms of some external unit of measurement, for example, gold). The generic value of monetary national income is $Y = p_x x + p_y y$, where x and y are the outputs of the two goods. If we divide Y by the price of either good, for example by p_y, we obtain real national income Y_R measured in terms of y.

Since, as we shall see presently, the relative price in the problem is given, the result would not change if we measured real income in terms of good x. On the other hand, since p_x and p_y are given, we could just as well consider Y, which would then be national income at constant prices. Thus there is no loss of generality by considering good x as the *numéraire* (unit of measurement).

We thus have the following two problems of constrained maximization:

$$\max Y_{1R} = \left(p_x/p_y\right) x_1 + y_1 \quad \text{sub } a_1 x_1 + b_1 y_1 \leq \overline{L}_1, \quad x_1 \geq 0, \quad y_1 \geq 0, \tag{2.9}$$

and

$$\max Y_{2R} = \left(p_x/p_y\right) x_2 + y_2 \quad \text{sub } a_2 x_2 + b_2 y_2 \leq \overline{L}_2, \quad x_2 \geq 0, \quad y_2 \geq 0, \tag{2.10}$$

where for each country the constraints are the respective transformation curve (the $\leq$ sign means that, in principle, all points internal to the curve are also admissible) and the non-negativity of the outputs.

The exchange ratio or relative price of the two goods, p_x/p_y, is to be taken as given, determined on the international market (in the same way in which, in Sect. 2.2, we considered the terms of trade as exogenously given). In fact, owing to

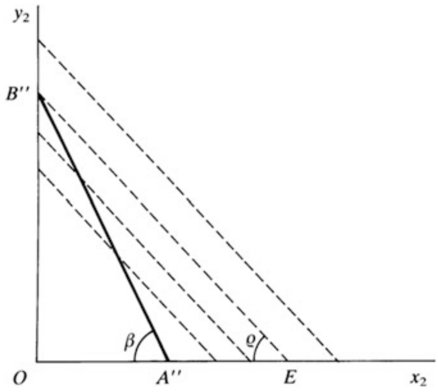

Fig. 2.3 Transformation curve and maximization of real income

the assumptions of perfect competition and of absence of transport costs, with free trade the domestic price ratio must necessarily be equal to the international terms of trade.

The data are then completed by assumption (2.4) of Sect. 2.2.

With these premises, problems (2.9) and (2.10)—which are linear programming problems of the simplest sort—can find an easy graphic solution. In fact, the function to be maximized can be represented by a family of parallel straight lines with a negative slope, each of which represents the locus of all combinations of x and y yielding the same real income (a *budget line* or, as we prefer to call it, an *isoincome line*: this terminology has the same derivation as isocost, isoquant, etc.); furthermore, the farther any such line is from the origin, the higher the corresponding real income. As a matter of fact, from the equation $Y_R = (p_x/p_y)\, x + y$ we get

$$y = -\left(p_x/p_y\right) x + Y_R, \tag{2.11}$$

which, if we consider Y_R as a parameter, defines a family of straight lines with the properties stated.

The graphic solution of our problem then consists in finding the highest isoincome attainable without going beyond the transformation curve of the country concerned, and remaining in the first quadrant (non-negativity constraints). If we consider, for example, country 2, we can draw Fig. 2.3, where $\tan \varrho =$ international relative price (terms of trade) and $\tan \beta$ = *marginal rate of transformation* = a_2/b_2; given the assumptions, $\tan \varrho < \tan \beta$.

It can easily be seen that, given the constraint, the highest isoincome attainable is $B''E$; consequently, the constrained-optimum point is B''. Country 2 thus maximizes its real national income by specializing entirely in the production of good y.

In a similar way it can be shown that country 1 maximizes its real national income by specializing entirely in the production of good x.

The reader will remember that complete specialization is indeed the outcome of the theory of comparative costs. This theory therefore implies the maximization of the real national income of each country separately considered.

2.3.2 *Maximization of Real World Income*

The same problem of maximizing real income can be formulated from the point of view of the world as a whole. Real world income in terms of good y is

$$Y_{RM} = (p_x/p_y)(x_1 + x_2) + (y_1 + y_2) = (p_x/p_y)x_M + y_M, \tag{2.12}$$

where x_M and y_M are the quantities of the two goods globally produced in our two-country world. In order to proceed in the same way as before, it is necessary to determine the world transformation curve.

The world transformation curve is defined as that curve which—for the world as a whole and within the limits of total existing resources—gives the maximum producible quantity of y for any given quantity of x to be produced, and vice versa. This transformation curve must, therefore, be derived from a maximization procedure. Let us note that, in general, any transformation curve is the outcome of a maximization procedure and is, therefore, a locus of points sharing the property of efficiency in production. In the case of a single country and fixed technical coefficients the procedure is trivial: given for example the quantity x_1, the labour required to produce it is $x_1 a_1$. As the total amount of labour is $\overline{L}_1$, we are left with $\overline{L}_1 - x_1 a_1$ to produce y, the maximum output of which is $y_1 = (\overline{L}_1 - x_1 a_1)/b_1$, which is Eq. (2.7) already examined in Sect. 2.2.

Also at the world level the derivation of the world transformation curve is a fairly simple matter, thanks to the assumption of fixed technical coefficients.

With reference to Fig. 2.4, let us begin by determining the extreme points (intercepts): these are A and B. Segment OA represents the maximum possible output of x, obtained when all world resources are employed to produce this good. It is obvious that this segment is the sum of segments $O'A'$ and $O''A''$ in Fig. 2.2; algebraically we have $OA = \overline{L}_1/a_1 + \overline{L}_2/a_2$. Similarly the maximum world output of y turns out to be $OB = O'B' + O''B'' = \overline{L}_1/b_1 + \overline{L}_2/b_2$.

To find the other points of the world transformation curve, let us suppose we start from point A and forgo one unit of good x: a certain amount of labour will then become available for employment in the production of good y. As we are reasoning at world level, we must determine—on the basis of technology—which country it is better to perform these operations in, so as to optimize the result, that is to obtain the maximum amount of y_M for the one unit of x_M we have forgone.

Now, if we forgo one unit of x in country 1, we free an amount of labour equal to a_1 which, if employed in that country to produce y, will allow an increase in the output of y equal to a_1/b_1 (that is, obviously, country 1's marginal rate of transformation). If we carry out the same operations in country 2, we get a_2/b_2 more

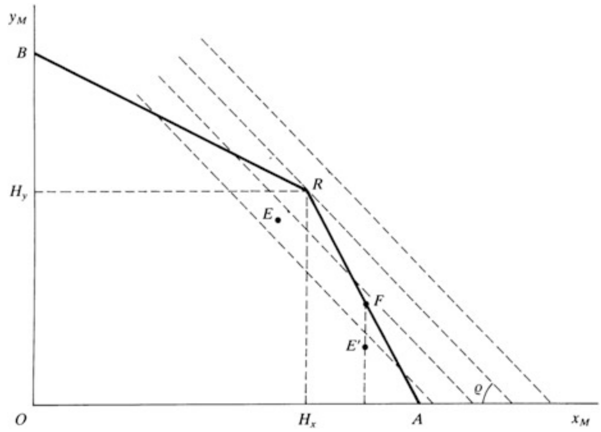

Fig. 2.4 World transformation curve and maximization of real world income

of y for one unit less of x. As we have assumed (see above) that $a_1/b_1 < a_2/b_2$, the operations under consideration are better carried out in country 2, and since the marginal rate of transformation is constant, this continues to hold for further decreases in x_M.

Therefore, starting from A, the best course of action is that country 1 continues to produce only good x, whilst the world output of y will be maximized by "transforming" x into y in country 2, according to this country's transformation curve.

We shall therefore move along segment *AR*, whose slope equals that of country 2's transformation curve: actually, this segment is nothing more than the transformation curve of country 2 drawn with reference to the auxiliary origin H_x.

When it arrives at point *R*, country 2 will produce exclusively good y, whilst country 1 will still be entirely specialized in the production of good x: this point corresponds to the Ricardian situation and is therefore called the *Ricardo point* by Dorfman, Samuelson, and Solow (1958, p. 35). From this point, further reductions in x_M and increases in y_M can only take place in country 1, along its transformation curve (this is *RB*, with reference to the auxiliary origin H_y), whilst country 2 will produce exclusively good y, as shown above.

The world transformation curve is thus the kinked curve *ARB*. The reader might like to check that the same curve would be obtained by starting from point *B*.

If we now draw the isoincome lines representing real world income as defined in Eq. (2.12), we obtain a family of straight lines with the usual properties. The highest isoincome attainable is the one passing through the Ricardo point: it is therefore demonstrated that the solution found by the theory of comparative costs implies the maximization of real world income.

The above treatment also enables one to give an answer to the objections of Pareto (1906) and successive authors to the theory of comparative costs. According to Pareto, it is possible for international trade to give rise to a worse situation than the autarkic one, for example when the quantity of a good increases but the quantity of the other decreases. If we interpret this criticism in terms of Fig. 2.4, we see that the coordinates of point R represent greater quantities of both goods with respect to, say, point E (inside the transformation curve), but not with respect to all internal points. At point E', for example, the quantity of x is greater, but that of y is smaller, than at point R. In a case like this it is not possible, according to Pareto, to establish whether one point is preferable to the other without introducing utility, and when this is done, it may well be that point E' will yield a greater utility than point R. It is however possible to rebut Pareto's criticism without having to introduce assumptions on the utility function. In fact, the efficiency properties of the world transformation curve allow us to state that, for any internal point, it is possible to find a point on the frontier which denotes a better situation (in the example above, the latter is point F, where the quantity of x is the same as, but the quantity of y is greater than, at point E'). Therefore international trade will always be preferable to autarky provided that it gives rise to points on the world transformation curve; this will indeed be the case for any admissible terms of trade.

2.4 Generalizations

In Sects. 2.1–2.3 we have considered the simple case of international trade concerning two goods and two countries. In this section we first examine the extension of the Ricardian theory to n countries trading two goods and then the general case of n countries and m goods. Further treatment of the classical theory is contained in Allen (1965), Bhagwati et al. (1998), Chacholiades (1978), Edgeworth (1894), Graham (1923), Haberler (1936), Hartwick (1979), Jones (1961), McKenzie (1954a, 1954b, 1955), Ricardo (1817), Whitin (1953). In the Appendix, Sect. 18.3, we study the generalization to a continuum of goods. Before moving to these generalizations we mention other advancements in research concerning the sources of the differences in comparative costs between countries. One traditional source, probably the most direct, is the technology in the strict sense of the engineering aspects of the production process. But other sources are definitely to be considered. As a matter of fact, anything that contributes to determining the unit cost of production is a potential source of comparative cost/advantage. Among such sources one may list the quality of institutions, of commercial laws, of infrastructures, the features of the labour market, the effectiveness of law enforcement and cultural traits of economic agents. For developments in these directions see, e.g., Cuñat and Melitz (2007), Levchenko (2007), Nunn (2007), Costinot (2009), Belloc and Bowles (2013), and Belloc (2006) for a review of the role of institutions.

2.4.1 Two Goods and n Countries

A necessary condition for international trade to take place when there are n countries is that at least two of these have different comparative costs, for it is self-evident that, if all had the same comparative cost, there would be no incentive to engage in international trade, exactly as in the two-country case. Once this condition is satisfied, it is not very relevant whether all countries have different comparative costs or whether there exist subsets of countries with the same comparative cost; to simplify the treatment, we shall adopt the former assumption. No loss of generality is involved in assuming that the countries can be ordered in such a way that

$$\frac{a_1}{b_1} < \frac{a_2}{b_2} < \ldots < \frac{a_n}{b_n}. \tag{2.13}$$

Now, once the necessary condition is met, the sufficient condition is that the terms of trade are strictly included between the two extreme comparative costs,

$$\frac{a_1}{b_1} < R_s < \frac{a_n}{b_n}. \tag{2.14}$$

A new complication should be noted: even if (2.14) is satisfied, R_s may happen to coincide with some intermediate comparative cost. In this case, the country concerned will not participate in international trade, which will involve the remaining $n - 1$ countries. In any case we shall find a certain number of countries with a comparative cost lower than R_s while the remaining ones will have a comparative cost higher than R_s, namely

$$\frac{a_1}{b_1} < \ldots < \frac{a_i}{b_i} \leq R_s \leq \frac{a_{i+1}}{b_{i+1}} < \ldots < \frac{a_n}{b_n}, \tag{2.15}$$

where $i = 2, 3, \ldots, n - 1$ denotes any country other than the first and the last. If the equality sign holds in the weak inequality $a_i/b_i \leq R_s$, then country i will not engage in international trade.

Once condition (2.15) is satisfied, international trade will take place between the countries with a comparative cost lower than R_s, on the one hand, and the countries with a comparative cost higher than R_s, on the other. The former group of countries will specialize entirely in the production of x, whilst the latter will specialize entirely in the production of y: therefore, x will be exported by the former to the latter group, and vice versa for y.

This result can be given a simple graphic interpretation in terms of the world transformation curve. When there are n countries, a world transformation curve can be constructed by way of the same procedures explained in the case of two countries: starting, for example, from the point where the world produces exclusively good x, the best course of action will be to "transform" good x into good y along country

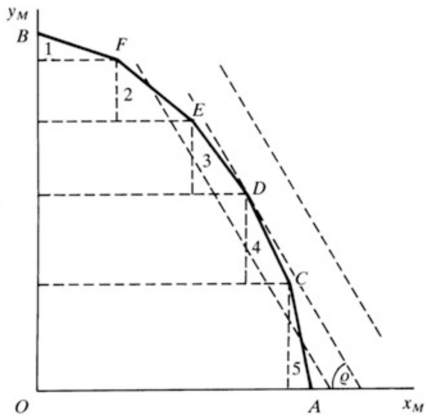

Fig. 2.5 The world transformation curve with n countries

n's transformation curve, then along country $(n-1)$'s and so on (the reasoning is altogether similar to that employed in Sect. 2.3).

If we assume, for example, that there are five countries, we get Fig. 2.5, where the numbers denote the transformation curves of the various countries stacked one on top of the other in the usual manner. In the diagram, given for example the terms of trade measured by $\tan \varrho$, the maximization of real world income $Y_{RM} = (p_x/p_y)(x_1 + x_2 + \ldots + x_5) + (y_1 + y_2 + \ldots + y_5) = (p_x/p_y)x_M + y_M$ is obtained at point D, so that countries 1,2,3 specialize entirely in the production of good x, and countries 4,5 in the production of good y. It is in fact easy to see that $a_1/b_1 < a_2/b_2 < a_3/b_3 < R_s < a_4/b_4 < a_5/b_5$.

In the particular case in which $a_i/b_i = R_s$, the isorevenue line will be tangent to a facet of the polygonal curve $ACDEFB$ (the facet corresponding to country i's transformation curve) and the solution will be indeterminate. In such a case, as we said, country i will not participate in international trade and will produce the same output combination as before, when no international trade existed: this will enable us to determine the precise point on the facet under consideration. The result is that country i will not necessarily specialize, whilst all the remaining countries will, as explained above.

2.4.2 m Goods and n Countries

Let us begin by examining the case of m goods and two countries. For this purpose, it is expedient to adopt the alternative definition of comparative cost (see Sect. 2.1), namely the ratio between the absolute unit costs of the same good in the two countries. Without loss of generality, we can order the comparative costs in an increasing manner (namely in order of diminishing country 1 comparative advantage), that is

$$\frac{a_2}{a_1} > \frac{b_2}{b_1} > \frac{c_2}{c_1} > \ldots > \frac{m_2}{m_1}. \tag{2.16}$$

For motives that will become clear further on, it is expedient to introduce the ratio between the two countries' unit money wage rates, both expressed in a common monetary unit, say gold (as the exchange rate is assumed to be perfectly rigid, it can be set at one without loss of generality). Let this ratio be $\omega = w_1/w_2$.

It can then be shown that the condition for international trade to take place is that ω is strictly included between the two extreme comparative costs, i.e.

$$\frac{a_2}{a_1} > \omega > \frac{m_2}{m_1}. \tag{2.17}$$

It can also be shown that all goods with a comparative cost lower than ω will be exported by country 2, which will specialize entirely in their production, whilst all goods having a comparative cost higher than ω will be exported by country 1, which will specialize entirely in their production. In the particular case in which there is a good having a comparative cost exactly equal to ω, this good will, in general, be produced by both countries and will not be internationally traded.

To prove these statements, we begin by observing that, given the money wage rates w_1 and w_2, the (monetary) unit cost of production and so the (monetary) price of the various goods in the two countries, before international trade is opened, will be

$$\begin{aligned} p_{A_1} &= w_1 a_1, & p_{A_2} &= w_2 a_2, \\ p_{B_1} &= w_1 b_1, & p_{B_2} &= w_2 b_2, \\ &\ldots\ldots\ldots\ldots & &\ldots\ldots\ldots\ldots \\ p_{M_1} &= w_1 m_1, & p_{M_2} &= w_2 m_2. \end{aligned} \tag{2.18}$$

Now, given the assumptions of free trade, perfect competition and no transport costs, each good will be bought where it costs least. Therefore if—for example—we have $p_{C_1} < p_{C_2}$, country 2 will buy good C from country 1 (which will become an exporter of this good) instead of producing it internally, and vice versa. Furthermore, since, in the pure of theory of international trade, imports must be paid for by exports, each country must be able to export some good. It is now obvious that, if it were

$$\omega \geq \frac{a_2}{a_1}, \tag{2.19}$$

country 2 would produce all goods at a lower price than country 1, which could not then engage in international trade, being unable to export anything. In fact, since $\omega = w_1/w_2$ by definition, from Eq. (2.19) we get

$$1 \geq \frac{a_2 w_2}{a_1 w_1}, \tag{2.20}$$

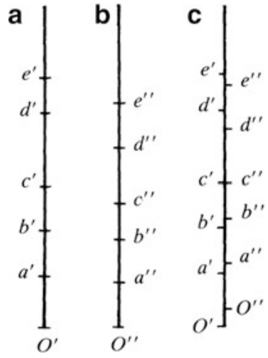

Fig. 2.6 Exchange of more than two goods between two countries

whence, given Eqs. (2.18),

$$p_{A_1} \geq p_{A_2}, \tag{2.21}$$

so that country 1 produces good A at a price higher than (or at most equal to) country 2. Now, account being taken of Eq. (2.16), if (2.19) holds, it will also be true that ω is higher than all other comparative costs and so, by similar reasoning, that the price of $B, C, \ldots$ is higher in country 1. There is, therefore, no scope for international trade.

In a similar way it can be proved that if $\omega \leq m_2/m_1$, country 2 produces good M at a price higher than (or at most equal to) country 1 etc., so that, also in this case, there can be no international trade.

If, on the contrary, inequality (2.17) holds, by considering the left-hand side of it we get

$$p_{A_1} < p_{A_2}, \tag{2.22}$$

whilst by considering the right-hand side we have

$$p_{M_2} < p_{M_1}, \tag{2.23}$$

so that there exists at least one good (A) which country 1 can export and at least one good (M) exportable by country 2.

If we now indicate by the subscript Ω a generic good and by θ the corresponding technical coefficient, it can easily be seen that $\theta_2/\theta_1 < \omega$ is equivalent to $p_{\Omega_2} < p_{\Omega_1}$ (good Ω will be exported by country 2), whilst $\theta_2/\theta_1 > \omega$ is equivalent to $p_{\Omega_2} > p_{\Omega_1}$ (good Ω will be exported by country 1). This demonstrates the second part of the proposition.

In conclusion, given ω, we can divide all goods into two groups: one comprising the goods exported from country 1 to country 2 (these are the goods having a comparative cost lower than ω) and the other comprising the goods exported from country 2 to country 1 (those with a comparative cost higher than ω).

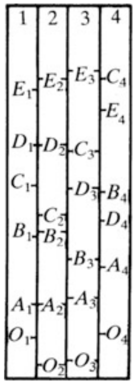

Fig. 2.7 Exchange of more than two goods among more than two countries

This treatment is amenable to a simple graphic representation, provided by Edgeworth. In Fig. 2.6a start from origin O' and draw segments representing the *logarithms* of the technical coefficients (unit costs in terms of labour) of the various goods in country 1, that is, $O'a' = \log a_1$, $O'b' = \log b_1$ and so on up to good E (we have considered only five goods, but they can be of any number). Similarly, in Fig. 2.6b draw segments representing the logarithms of the technical coefficients in country 2 ($O''a'' = \log a_2$, etc.).

Then put the two diagrams together in Fig. 2.6c *in such a way that the distance between the two origins represents the logarithm of the parameter* ω, that is $O'O'' = \log \omega$, stipulating that O'' will be above O' if $\omega_2 > \omega_1$ and so $\omega > 1$ (whence $\log \omega > 0$), and below it in the opposite case. Once the figure has been drawn, we can immediately check whether (2.17) is met and determine the point where the succession of goods is divided between those exported by country 1 and those exported by country 2. In fact, if we consider the inequality $a_1/a_2 < \omega$ and take the logarithms, we get

$$\log a_1 < \log a_2 + \log \omega, \tag{2.24}$$

the graphic counterpart of which is

$$O'a' < O''a'' + O'O'', \tag{2.25}$$

which is certainly satisfied since a' is below a''. It follows that the relative position of the various points in Fig. 2.6c will immediately tell us the division of the goods in the two groups: good A and good B will be exported by country 1; good C (for which $c_1/c_2 = \omega$) will not be traded internationally; goods D and E will be exported by country 2.

Edgeworth's ingenious diagram was extended by Viner to any number of countries, thus enabling us to examine the exchange of n goods among m countries graphically. In Fig. 2.7, adapted from Viner (1937, p. 465), we consider five commodities and four countries; the diagram is drawn accordingto the same principles

Table 2.3 Pattern of trade of five goods among four countries

	Country 1	Country 2	Country 3	Country 4
Exports	A	C	B	D,E
Imports	B,C,D,E	B,D,E	A,C,D,E	A,B,C

as Fig. 2.6 and the distances between the origins represent the relative money wage rates of the various countries. From an inspection of the figure the pattern of trade immediately results (see Table 2.3). Note, finally, that country 2 may either export, import, or not trade in commodity *A* as this commodity is on the margin of trade for that country.

2.5 The Problem of the Determination of the Terms of Trade

In the previous treatment we have determined the limits within which the terms of trade must lie, but—as the reader may have noticed—we have not specified how, and at what value, the terms of trade themselves are determined within these limits.

As a matter of fact, it is a generally accepted opinion that the Ricardian theory of comparative costs as such is incapable of determining the terms of trade and only determines the limits within which they must lie. This would constitute a serious limitation to this theory seen as a model aimed at the explanation of international trade, for any such model ought to explain not only the causes and pattern of trade, but also the terms of trade. The limitation, on the contrary, would be almost irrelevant if one believes that the Ricardian theory must be seen from the normative, rather than the positive, point of view. According to Bhagwati (1964, p. 4), for example, the Ricardian theory is more plausibly seen "as a highly simplified model which was intended to be, and served as, an eminently successful instrument for demonstrating the welfare proposition that trade is beneficial" rather than "as a serious attempt at isolating the crucial variables which can be used to 'explain' the pattern of trade". In our opinion, both elements are present in the theory under consideration, and we have treated it in this sense in the present chapter.

In order to solve the problem of the determination of the terms of trade—the accepted opinion goes on—it is necessary to introduce the demand side in addition to the productive side focused on by the original formulation of the theory of comparative costs.

The first precise reasoning in this sense was J.S. Mill's equation of international demand, according to which the terms of trade are determined so as to equate the value of exports and the value of imports. As Mill (1848, chap. XVIII, sect. 4, pp. 592–593) writes,

> The law which we have now illustrated, may be appropriately named, the Equation of International Demand. It may be concisely stated as follows. The produce of a country exchanges for the produce of other countries, at such values as are required in order that the whole of her exports may exactly pay for the whole of her imports. This law of International

> Values is but an extension of the more general law of Value, which we called the Equation of Supply and Demand. We have seen that the value of a commodity always so adjusts itself as to bring the demand to the exact level of the supply. But all trade, either between nations or individuals, is an interchange of commodities, in which the things that they respectively have to sell constitute also their means of purchase: the supply brought by the one constitutes his demand for what is brought by the other. So that supply and demand are but another expression for reciprocal demand: and to say that value will adjust itself so as to equalize demand with supply, is in fact to say that it will adjust itself so as to equalize the demand on one side with the demand on the other.

We find here, in a nutshell, the elements that were to be taken up again and further developed by Alfred Marshall in his theory of international reciprocal demand curves, leading to the neoclassical theory of international trade, that will be treated in the next chapter. In fact, from the point of view of the history of economic thought, J.S. Mill cannot be considered entirely as a member of the classical school, as in his writings many elements are present which later were to characterize the neoclassical school.

Actually, there is no dearth of attempts (for surveys of the earlier literature see Viner, 1937; Chipman, 1965a; Takayama, 1972, chap. 5) at introducing demand in the theory of comparative costs, leaving all its other hypotheses unaltered. We shall examine in the Appendix (see Sect. 18.3) an elaboration of the Ricardian model (with a continuum of goods and the presence of demand functions) due to Dornbusch, Fischer, and Samuelson (1977) .

We must at this point ask ourselves what is the validity of the received opinion. It obviously leads to considering the classical theory of comparative costs, enriched by the introduction of demand functions, as a particular case of the neoclassical theory, which would occur when one assumed fixed-coefficient production functions. This has been challenged by those who maintain that such a view would misrepresent the classical theory, whose vision is completely different from the neoclassical one.

In particular, Negishi (1982) maintains that, contrary to the received opinion, the *original* Ricardian theory is perfectly able to determine the terms of trade without having recourse to demand factors, but by using solely cost-price relations. This would be possible, according to Negishi (p. 200), by making use of "the classical theory of wages, the rate of profit, and the role of exporters and importers, which have been missing in the standard interpretation of the classical theory of international trade". For an examination of this interesting thesis, we refer the reader to the Appendix, Sect. 18.4.

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