Chapter 4
The Heckscher-Ohlin Model

4.1 Basic Assumptions and Their Meaning

We shall first examine the Heckscher-Ohlin theory (Heckscher, 1919; Ohlin, 1933) in its simplest version, that is a model in which there are two countries, two final goods and two primary factors of production. Extensions will be examined later on, in this chapter. Given the great contribution made by P. A. Samuelson to the refinement and diffusion of this theory, many authors call it the Heckscher-Ohlin-Samuelson model.

This model stresses the differences in factor endowments as the cause of international specialization and trade. More precisely, the key element in the theory is that countries are endowed with factors in different proportions. This, gives rise to different relative marginal cost of production and will make that each country exports the commodity which uses the country's more abundant factor more intensively (the Heckscher-Ohlin theorem).

In addition to the usual basic assumptions (no transport costs, free trade, perfect competition, international immobility of factors, presence of only two commodities and two factors) there are the following:

1. The production functions exhibit positive but decreasing returns to each factor (i.e., positive but decreasing marginal productivities) and constant returns to scale (i.e., first degree homogeneity). They are internationally identical, but, of course, different between the two goods, that is the production function of good A is the same in country 1 and country 2, and is different from that of good B (which is identical in the two countries).
2. The structure of demand, that is the proportions in which the two goods are consumed at any given relative price, is identical in both countries and independent of the level of income.
3. Factor-intensity reversals are excluded (see below).

The first assumption, which embodies the usual properties of well-behaved production functions, and excludes the presence of international technological

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differences, is self-evident. The difference between the production functions of the two goods is of course necessary, otherwise it would not be possible to speak of two *different* goods.

The second assumption implies that tastes are internationally identical *and* represented by utility functions such that the income elasticity of demand is constant and equal to one for each good. Utility functions having this property belong to the class of *homothetic* utility functions (see any microeconomics textbook). This assumption serves to exclude the possibility that, although tastes are internationally identical, the two goods are consumed in different proportions in the two countries because of possible differences in income levels.

It is then clear that the first two assumptions serve to exclude any difference between the countries as regards technology and demand, so that one can concentrate on the differences in factor endowments.

The third assumption is necessary to determine univocally the *relative factor intensities* of the two goods. In general, given two factors (capital K and labour L) and two commodities A and B, we say that a commodity (for example A) uses a factor more intensively or is more intensive in a factor (for example capital) relative to the other commodity if the (K/L) input ratio in the former commodity is greater than the (K/L) input ratio in the latter.

Now, if production of each good took place according to only one technique with fixed and constant technical coefficients (L-shaped isoquants), it would be an easy matter to determine the relative factor intensities once and for all. But since we are dealing with production functions with a continuum of techniques[1] (smoothly continuous isoquants), different techniques will be chosen—in accordance with the standard cost minimization procedure—for each good at different factor-price ratios. As already clarified in the previous chapter, we follow common practice in talking of the price of a factor in the sense of price of the services or rental for the services of the factor, or unit factor reward. This warning is to be considered as implicitly recalled throughout the rest of the book.

It follows that the classification of goods according to their factor intensities becomes ambiguous. To remove this ambiguity we add the requirement that the classification must remain the same for any (admissible) factor-price ratio, namely—in our example—that commodity A is more capital-intensive relative to commodity B if the (K/L) input ratio in the former commodity is greater than the (K/L) input ratio in the latter *for all factor-price ratios*.

Conversely, when *factor-intensity reversal(s)* occur, it is not possible to rank the commodities unambiguously for all factor-price ratios, that is, the classification changes according to the value of the factor-price ratio. For example, it may happen that A is more capital-intensive relative to B for a certain range of factor-price ratios, whilst B becomes more capital-intensive relative to A for another range of factor-price ratios: a factor-intensity reversal has occurred.

[1]The same problem would arise in the presence of many techniques, but limited in number, of the fixed-coefficients type, such as are dealt with by activity analysis.

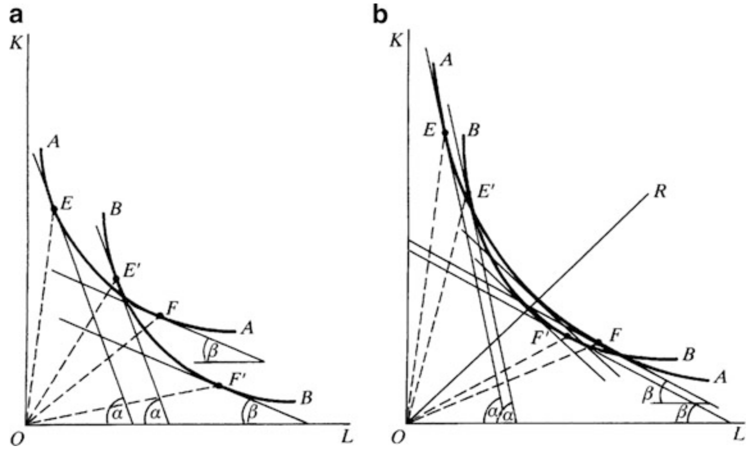


Fig. 4.1 Factor intensities: absence and presence of reversals

The condition which *excludes* factor-intensity reversals is that the representative isoquants of A and B, when drawn in the same diagram, do not cross more than once, as shown in Fig. 4.1a. Since with constant returns to scale all isoquants of the same production function have the same shape, the expansion path is linear and the input ratio, given the factor-price ratio, is the same for any output level, so that we can compare the representative isoquant of A with that of B, for example the unit isoquants. Note also that, owing to the assumption of internationally identical production functions, the following (Lerner-Pearce) diagram can refer equally well to either country.

Let us first consider Fig. 4.1a, where AA and BB indicate the unit isoquant of A and B respectively; these isoquants cross only once. If the factor-price ratio is, for example, equal (in absolute value) to $\tan\alpha$, then—by drawing a family of isocosts and following the usual cost minimization procedure (it goes without saying that the assumption of perfect domestic mobility of factors implies that the same factor-price ratio obtains in both industries)—we find the optimum input combinations: these are represented by point E in the A industry and by point E' in the B industry. The input ratios (K/L) in the two industries can be read off the diagram as the slopes of OE and OE' respectively, so that A is the capital-intensive commodity. At a different factor-price ratio, for example tan β, the new optimum input combinations will be represented by points F and F' in the A and B industries respectively, so that A is, again, the capital-intensive commodity (slope of OF > slope of OF'). An examination of Fig. 4.1a will show that this property holds for each and all factor-price ratios: commodity A is, therefore, unambiguously capital-intensive relative to commodity B. It goes without saying that, in parallel, commodity B is unambiguously labour-intensive relative to commodity A.

Let us then consider Fig. 4.1b, where the isoquants intersect twice. When the factor-price ratio is equal to $\tan\alpha$, the optimum input combinations in the two

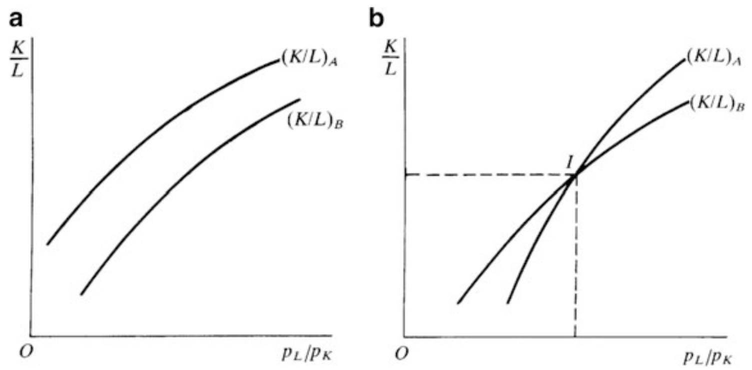


Fig. 4.2 Behaviour of the K/L ratio without and with factor-intensity reversal

industries are such that A is capital-intensive relative to B (slope of OE > slope of OE'), whilst the opposite is true when the factor-price ratio is equal to tan β (slope of OF' > slope of OF, so that now B is capital-intensive relative to A): a factor-intensity *reversal* has occurred. The reader can check as an exercise that such a reversal also occurs when one of the isoquants is internal to the other and they are tangent to each other at a point. This is not surprising, for a point of tangency is—loosely speaking—more similar to a multiple than to a single intersection. In mathematical terms, a point of tangency between two curves can be considered as the limit to which two (or more) intersection points tend when approaching indefinitely.

In Fig. 4.1b, the K/L ratio corresponding to which the reversal takes place is given by the slope of ray OR, along which the unit isoquant of A and the unit isoquant of B have the same slope, as can be seen from the fact that the two straight lines tangent to the isoquants along ray OR are parallel. A. P. Lerner (1952, p. 14) called this ray a "radiant of tangency", as all the A and B isoquants will have the same slope along it. It can be read off the diagram that the K/L ratio is greater in the A than in the B industry for all factor-price ratios such that the optimum input combinations lie above OR, and vice versa in the opposite case.The behaviour of the K/L ratio in the two sectors in the absence and presence of a factor intensity reversal is shown in Fig. 4.2. In all cases the K/L ratio is a monotonically increasing function of the factor-price ratio or relative price of factors (p_L/p_K), since producers will find it profitable to substitute capital for labour as the relative price of labour increases. This can be derived diagrammatically by considering the various points of tangency to the unit isoquant of isocosts with varying slope. But, whilst in the case of no factor-intensity reversals the two curves never intersect, in the case of a factor-intensity reversal they do.

In Fig. 4.2a, derived from Fig. 4.1a, the curve representing the K/L ratio in industry A—curve $(K/L)_A$—lies above the $(K/L)_B$ curve throughout: commodity A is always capital-intensive relative to commodity B.

In Fig. 4.2b, derived from Fig. 4.1b, the curves under consideration intersect in correspondence to the K/L ratio represented by the slope of OR, which in turn corresponds to the p_L/p_K ratio given by the common slope of the two isocosts tangent to the two unit isoquants along *OR*. To the left of the point of intersection I, that is for lower p_L/p_K ratios—corresponding to the part of Fig. 4.1b to the right of *OR*—commodity B is capital-intensive relative to A, whilst the opposite is true to the right of I (higher p_L/p_K ratios, corresponding to the part of Fig. 4.1b to the left of *OR*).

We have so far examined the case of a single reversal, corresponding to the fact that the A and B unit isoquants intersect twice, but it cannot be excluded that the unit isoquants intersect more than twice, giving rise to *more than one* factor-intensity reversal; in such a case, the two curves in Fig. 4.2 would intersect twice or more. In general, $n - 1$ factor-intensity reversals correspond to n intersections of the unit isoquants. The phenomenon of factor-intensity reversals is related to the elasticity of substitution between factors. In fact, the economic meaning of the circumstance that the isoquants cut twice is that the possibilities of factor substitution are different between the two sectors. Loosely speaking, the isoquants can cut twice when one is more curved (more convex to the origin) than the other, and the curvature of an isoquant is related to the elasticity of substitution (the more highly curved the isoquant is, the poorer substitutes the two factors of production are). This can be generalized to more than two intersections (see Sect. 20.1).

4.1.1 Relative Price of Goods and Relative Price of Factors

Although not immediately relevant, it is convenient to show now that, in the case of no factor-intensity reversal (also called the *strong* factor-intensity assumption), *a unique factor-price ratio corresponds to each commodity-price ratio*, *and vice versa*, i.e. there is a *one-to-one correspondence* between the relative price of goods and the relative price of factors.

Let us for example assume that the commodity-price ratio is $p_B/p_A = 4$, that is, four units of A exchange for one unit of B; in perfect competition, this implies that the production cost of one unit of B must be the same as that of four units of A. In fact, in the long run perfect competition leads to a situation in which the price of a commodity equals its production cost (see any microeconomics textbook). Since we have assumed $p_B/p_A = 4$ (the price of B is four times that of A) it follows that the production cost of one unit of B must be the same as that of four units of A.

In Fig. 4.3 (which, owing to the assumption of internationally identical production functions, equally applies to either country) we have drawn the isoquants $4A$ and $1B$. Since factor prices are equal in the two sectors and since the production cost of one units of B is the same as that of four units of A, it follows that the optimum (i.e., the minimum) isocost will be the same for $1B$ and $4A$. So we must find an isocost which is simultaneously tangent to isoquants $4A$ and $1B$; once found, (the absolute value of) its slope will give us the relative price of factors.

It can be clearly seen in Fig. 4.3a that only one such isocost (*CC*) exists in the case of a single intersection of the isoquants: therefore, a unique factor-price ratio corresponds to the given commodity-price ratio. It should be noted that the result does not change if we consider any couple of A and B isoquants standing in the ratio 4:1. For example, in Fig. 4.3a the unique isocost being simultaneously tangent to

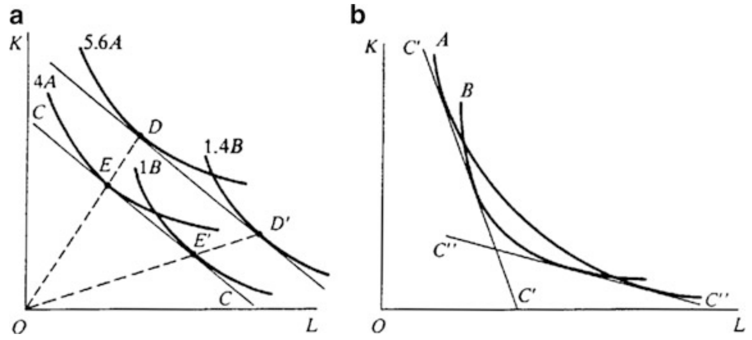


Fig. 4.3 Relative price of goods and relative price of factors

isoquants $5.6A$ and $1.4B$ (at D and D' respectively) is parallel to isocost CC (hence it represents the same factor-price ratio). This parallelism derives from the properties of homogeneous functions of the first degree. As we know (see Sect. 19.1.3) the isoquants of these functions have the same slope along any ray from the origin and, furthermore, their index is proportional to their distance from the origin (an isoquant twice as far from the origin represents a quantity twice as great). The space included between the two rays starting from the origin and passing through E and E' is called by Chipman (1966, p. 23) a "cone of diversification". Only one such cone exists in the absence of factor-intensity reversals; two or more of them will exist in the presence of reversals.

The correspondence between relative prices of factors and relative prices of goods is *one-to-one*, which means that a *unique* commodity-price ratio corresponds to each admissible factor-price ratio. In fact, the reasoning made above to pass from the relative price of goods to the relative price of factors can be inverted. Graphically, if we consider any family of isocosts with the same slope (for example that to which CC belongs), then each of them must necessarily determine a unique couple of isoquants simultaneously tangent to it and representing quantities of goods in the ratio of $4A$ to $1B$. On the contrary, in Fig. 4.3b, where the isoquants cut twice (that is, a factor intensity reversal is present, as explained above), there are two isocosts ($C'C'$ and $C''C''$) with the property of being simultaneously tangent to the isoquants $4A$ and $1B$: the factor-price ratio corresponding to the given commodity-price ratio is not unique.

We conclude this section by examining the behaviour of the relationship between the relative price of goods and the relative price of factors both with and without factor-intensity reversals. In the latter case such a relationship is monotonic, in the former it is not. Let us consider Fig. 4.4, which reproduces Fig. 4.3a, and assume that the relative price p_B/p_A shifts from 4 to 5, so that we must now find the isocost simultaneously tangent to the isoquants $5A$ and $1B$. As can be seen, a greater factor-price ratio p_L/p_K corresponds to the greater commodity-price ratio p_B/p_A, because $\tan\beta > \tan\alpha$. Since, as shown above, the correspondence is one-to-one, we can conclude that as the relative price of labour (p_L/p_K) increases, the relative

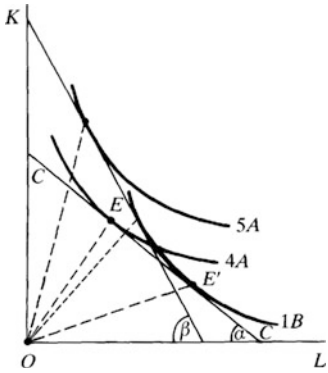


Fig. 4.4 Change in the factor-price ratio following a change in the commodity-price ratio

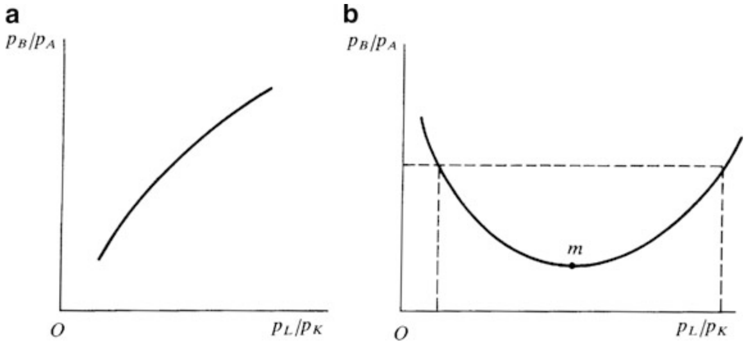


Fig. 4.5 Various relationships between relative price of factors and relative price of goods

price of commodity B (which is the labour-intensive commodity) increases. This relationship is drawn in Fig. 4.5a. We must note that it is monotonically *increasing* because we have assumed that B is the labour-intensive commodity; in the opposite case it would be monotonically *decreasing*; but in either case it is monotonic.

In the presence of factor-intensity reversals, the relationship under consideration, as we known, is no longer one-to-one, as two (or more, according to the number of reversals) factor-price ratios will correspond to any given commodity-price ratio. A case in which there is only one reversal is represented in Fig. 4.5b, where point m corresponds to the factor-price ratio at which the factor-intensity reversal occurs.

4.2 The Heckscher-Ohlin Theorem

The basic proposition of the Heckscher-Ohlin model is the following:

Theorem (Heckscher-Ohlin). *Each country exports the commodity which uses the country's more abundant factor more intensively.*

The concept of (relative) factor intensity has been clarified in Sect. 4.1; it is now the turn of the concept of (relative) *factor abundance*. The definition that immediately comes to mind is in *physical terms*: we say that a country (say country 1) is abundant in one factor (say capital) relative to the other, or that country 1 is relatively more endowed with capital than country 2, if the former country is endowed with more units of capital per unit of labour relative to the latter: $K_1/L_1 > K_2/L_2$, where K_1 is the total amount of capital available in country 1, etc.

An alternative definition is however possible, which makes use of the relative price of factors and is therefore called the *price definition*: country 1 is said to be capital abundant, relative to country 2, if capital is relatively cheaper (with respect to labour) in the former than in the latter country, at the (pre-trade) autarkic equilibrium, namely $p_{1K}/p_{1L} < p_{2K}/p_{2L}$, where p_{1K} is the price of capital in country 1, etc.

It is obvious that the physical definition reflects relative physical abundance, whilst the price definition reflects relative *economic* abundance. Since, thanks to the simplifying assumptions made at the beginning of Sect. 4.1, the Heckscher Ohlin theorem can be demonstrated with both the physical and the economic definition, we shall not claim the superiority of either one. Here we shall use the physical definition; the economic definition will be used in Sect. 4.5.1 where a brief discussion of the two definitions will also be given.

In the following treatment, we assume that commodity A is capital intensive relative to commodity B and that country 1 is capital abundant relative to country 2; it goes without saying that B is labour intensive relative to A and 2 labour abundant relative to 1. Thus we must prove that country 1 will export commodity A whilst country 2 will export commodity B.

The first step (a lemma) *in our proof is to show that—at the same commodity-price ratio—a country abundant in one factor has a production bias in favour of the commodity which uses that factor more intensively* namely, in our case, that country 1 has a production bias in favour of A whilst country 2 has a production bias in favour of B.

This can be shown by using the transformation curves or production-possibility frontiers (see Sect. 3.1) of the two countries; their relative position reflects the fact that country 1 is capital abundant relative to country 2 and that commodity A is capital intensive relative to commodity B (see Fig. 4.6). An alternative geometric treatment in terms of Edgeworth-Bowley boxes can be found in Lancaster (1957). It should be noted that it is not necessary for the two curves to intersect: what matters is that they have a *different* slope along any ray through the origin. If relative factor endowments were the same in both countries, then their transformation curves would have the *same* slope (that is, an identical opportunity cost) along any ray through the origin (in other words, they would be radial blow-ups of each other); similarly, the ratio of the outputs in the two sectors would be the same in both countries at any given common commodity-price ratio. In such a situation, given the assumption of identical structures of demand, there would be *no* scope for international trade.

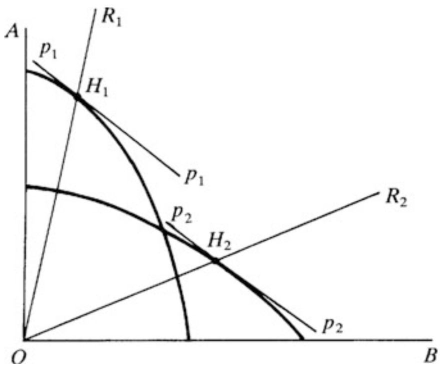


Fig. 4.6 Transformation curve and the Heckscher-Ohlin theorem

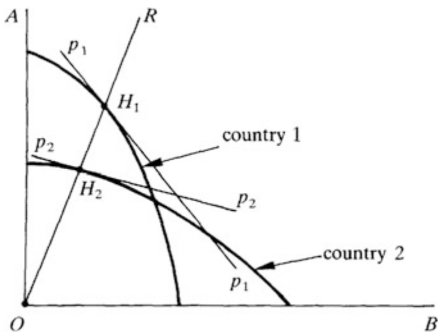


Fig. 4.7 Transformation curve and the Heckscher-Ohlin theorem: an alternative diagram

Let us consider a pre-trade (i.e. autarkic) situation and take a given commodity-price ratio which is identical in both countries (p_1p_1 and p_2p_2 are parallel, thus denoting the same price ratio p_B/p_A). Country 1 is at point H_1 on its own transformation curve and country 2 at point H_2. It can immediately be seen that, at the same relative price of goods, the ratio of the output of A to the output of B is greater in country 1 than in country 2 because the slope of OR_1 is greater than the slope of OR_2. This property holds for any common relative price of goods. An alternative way of looking at the same thing is based on Fig. 4.7. Let us consider, as before, a pre-trade situation and examine a given ratio of A–B, identical in both countries, for example, that represented by the slope of OR. Country 1 would then be at point H_1 on its transformation curve and country 2 at point H_2. The marginal rate of transformation is greater in country 1 than in country 2 (computed at H_1 and H_2 respectively). Commodity A would then be relatively cheaper in country 1 than in country 2, and vice versa for commodity B (we must bear in mind that in equilibrium the marginal rate of transformation coincides with the commodity-price ratio p_B/p_A). In other words, the *opportunity cost* of A in terms of B is lower in country 1, the capital-abundant country, has a production bias in favour of the capital-intensive commodity A, whilst the labour-abundant country 2 has a production bias in favour of the labour-intensive commodity B, in the sense that each country can expand its production of the commodity which is intensive in the country's abundant factor, at a lower opportunity cost than the other.

It is now easy to show that *each country exports the commodity which uses the country's more abundant factor more intensively.* This follow from the lemma and from the assumption that the structure of demand is identical in both countries (and independent of the level of income). In fact, with free trade and no transport costs, the commodity-price ratio (terms of trade) is the same in both countries. Now, according to the lemma, at the same relative price of goods country 1 (the capital-abundant country) will produce relatively more A (the capital-intensive commodity) and country 2 (the labour-abundant country) will produce relatively more B (the labour-intensive commodity): the ratio A/B is greater in country 1 than in country 2. But, given the assumption as to the structure of demand, at the same relative price of goods both countries wish to consume A and B in the same proportion: it follow that country 1 will export A (and import B, which will be exported by country 2) so that after trade the structure of the quantities of the goods available (the quantity available is given by domestic output plus imports or less exports) turns out be identical in both countries and equal to the structure of demand. This completes the proof.

As a spin-off *the terms of trade* will be determined, in much the same way as in Sect. 3.3, Fig. 3.6 (and will lie between the autarkic commodity-price ratios of the two countries)—we call it a "spin-off" because the main point of the Heckscher-Ohlin theory is to prove the basic proposition on the pattern of trade rather than to determine the terms of trade. This not surprising, because—as we have already noted in Sect. 3.7, and as is now obvious from the treatment in the present chapter—the Heckscher-Ohlin theory can be considered, from the purely analytical point of view, as a particular case of the neoclassical theory in which production functions and structures of demand are assumed to be internationally identical.

4.3 Factor Price Equalization

We propose now to show that if there is incomplete specialization the Heckscher-Ohlin model gives rise to *factor-price-equalization* (henceforth FPE); this result is usually stated as follows:

Theorem (FPE). *International trade in commodities and incomplete specialization, under the assumptions of the Heckscher-Ohlin model and notwithstanding the international immobility of factors, equalizes relative and absolute factor prices across countries.*

It should be stressed that the equalization concerns not only *relative* factor prices (p_L/p_K), but also *absolute* factor prices, that is, $p_{1L} = p_{2L}, \quad p_{1K} = p_{2K}$. To prove FPE we shall assume that international trade *does not bring about complete specialization,* so that each country continues to produce both goods; it is important to stress that this assumption, which is *additional* to those at the basis of the Heckscher-Ohlin theorem, is necessary to demonstrate the theorem under consideration.

Let us first recall from Sect. 4.1 that, thanks to the assumption of no factor intensity reversals, there is a one-to-one correspondence between the relative price of goods and the relative price of factors, which is the same in both countries. Secondly, with free trade, no transport costs, etc., the same good must have the same price in both countries (the *law of one price*), so the relative price of goods is the same in both countries. It follows that the *relative price of factors* is identical in both countries.

To arrive at *absolute factor price equalization* (which is what interests us) some more groundwork is necessary.

As a consequence of the identity between the relative price of factors and of the assumptions on technology, the optimum input combination in each sector is the same in both countries (but for a factor of scale): in other words, $(K/L)_{1A} = (K/L)_{2A}$ and $(K/L)_{1B} = (K/L)_{2B}$, as can also be read off Fig. 4.2a. With constant returns to scale, marginal productivities depend solely on the factor input ratio (see Sect. 19.1.3) and are independent of scale. It follows that the marginal productivities of the two factors in the two sectors are identical in both countries, namely

$$\begin{aligned} MPK_{1A} &= MPK_{2A}, \\ MPL_{1A} &= MPL_{2A}, \\ MPK_{1B} &= MPK_{2B}, \\ MPL_{1B} &= MPL_{2B}, \end{aligned} \tag{4.1}$$

where *MPK* and *MPL* denote the marginal productivities of capital and labour respectively, and the subscripts refer to the countries and commodities as usual.

The importance of the assumption of incomplete specialization should be noted here. In fact, if specialization were complete (for example, country 1 produces exclusively commodity A and country 2 commodity B), the quantities MPK_{1B} and MPL_{1B} could not be defined in practice (because commodity B is not produced in country 1), neither could be MPK_{2A} and MPL_{2A} (because commodity A is not produced in country 2); therefore Eq. (4.1) could not be written and the rest of the proof would fall.

Now, under perfect competition the equilibrium condition *value of the marginal product of a factor = price of the factor* must hold. In symbols (remember that p_A and p_B are internationally identical) we have, with reference, for example, to capital,

$$\begin{aligned} p_A MPK_{1A} &= p_{1K}, \\ p_A MPK_{2A} &= p_{2K}, \\ p_B MPK_{1B} &= p_{1K}, \\ p_B MPK_{2B} &= p_{2K}, \end{aligned} \tag{4.2}$$

from which—since the marginal productivities obey (4.1)—it follows that $p_{1K} = p_{2K}$. In a similar way it can be shown that $p_{1L} = p_{2L}$. This completes the proof of FPE.

Better to appreciate the importance of this theorem, it is sufficient to realize that it shows that free trade in commodities is a perfect substitute for perfect international mobility of factors.[2] Note that, if perfect international factor mobility existed as well, then perfect competition would necessarily lead to the full international equalization of factor prices. But in our models of the pure theory of international trade we have assumed an absolute *international immobility of factors* (see Sect. 1.1), so that it might seem that no reason exists for the equalization of their prices, which would not be equal except by sheer chance.

Contrary to this impression, the theorem under consideration shows that FPE, far from being an improbable event, is a necessary consequence of international trade in the assumed conditions. This came as a surprise to the very writers who first gave a rigorous proof of this theorem: see Samuelson (1948, p. 169).

This explains the great deal of attention paid by international trade theorists to this theorem, which can also be given a graphic treatment.

4.3.1 A Graphic Treatment

For this purpose, we bring together in one diagram (the Samuelson-Johnson diagram) the relationships between (K/L) and p_L/p_K. In the upper half of Fig. 4.8 we have reproduced Fig. 4.2a, in the lower half, Fig. 4.5a turned upside down. Given the international identity of production functions etc., Fig. 4.8 can refer to either country.

Let us denote by $\varrho_1 \equiv (K/L)_1, \varrho_2 \equiv (K/L)_2$ the relative factor endowment in the two countries, where $\varrho_1 > \varrho_2$ owing to the assumption that country 1 is capital abundant relative to country 2. The introduction of ϱ_1 and ϱ_2 makes it possible to determine the *admissible range of variation* of relative factor prices (p_L/p_K) in *each country separately considered*. If we consider, for example, country 1, given its relative factor endowment ϱ_1, the relative price of factors can vary between p_1' and p_1''. Note that at point p_1', country 1 would be completely specialized in the production of A. In fact, in general the overall capital/labour ratio is a weighted average of the capital/labour ratios in the two industries, that is (omitting the country subscript)

[2] It is also possible to show that the opposite is true as well, i.e. that perfect international mobility of factors is a perfect substitute for free international trade. In other words, in a hypothetical model in which commodities are immobile (no international trade), but factors are perfectly mobile between countries, the equalization of factor prices (caused by their perfect mobility) will bring about the equalization of commodity prices across countries notwithstanding their immobility. See Mundell (1957) and Sect. 6.8, p. 137. See also Svensson (1984) for an examination of whether goods trade and factor mobility are necessarily substitutes or may be complements in particular cases.

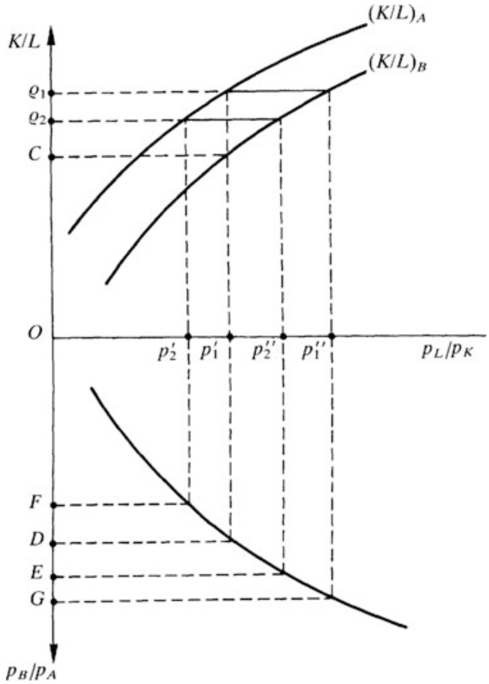


Fig. 4.8 The factor-price-equalization theorem

$$\frac{K}{L}=\frac{L_A}{L}\frac{K_A}{L_A}+\frac{L_B}{L}\frac{K_B}{L_B}, \tag{4.3}$$

where we have used the fact that $K_A + K_B = K$ by the assumption of full employment, which also ensures that the sum of the weights is one (because it also implies $L_A + L_B = L$). Now, if the relative price of factors is p_1', the capital/labour ratio in country 1's industry A is ϱ_1, whilst it would be $C < \varrho_1$ in industry B: but this is not possible, because (4.3) would not be satisfied (the sum of the weights is one); it is therefore necessary for the output of B to be zero in order for (4.3) to hold. It can be shown by similar reasoning that country 1 is completely specialized in B when the relative price of factors is p_1''.

A similar demonstration will show that country 2 is completely specialized in A when $p_L/p_K = p_2'$ and in B when $p_L/p_K = p_2''$; these values delimit the admissible range of variation of relative factor prices. It is now clear that only if the two ranges overlap and so admit of a common part (henceforth called "segment of equalization" for brevity) the equalization of relative factor prices (and so of absolute factor prices, if complete specialization does not occur) will be possible. This segment is $p_1' p_2''$ in our example; from the lower part of Fig. 4.8 it can be seen that the relative price of goods must fall in segment *DE*.

As can readily be seen from the diagram, the farther the relative factor endowments of the two countries are apart, the less probable is the presence of a segment of equalization. If ϱ_1 and ϱ_2 are so distant as to exclude the presence of such a segment, there will be complete specialization in at least one country and even the relative factor price equalization will be impossible. In general, various cases can be distinguished and classified as follows:

(a) A segment of equalization exists, and at the pre-trade equilibrium the relative prices of goods in the two countries are such that the corresponding relative prices of factors fall in this segment (in terms of Fig. 4.8, the relative prices of goods fall in *DE* in both countries before trade). In this case the equalization of the relative price of goods due to international trade brings about the equalization of the relative price of factors. To show this, we first observe that (terms of trade) that comes about as a consequence of international trade necessarily falls strictly between the two pre-trade equilibrium relative prices. In fact, if the terms of trade were equal to the pre-trade equilibrium relative prices of either country, this country would not obtain any benefit from trade and would not engage in international trade. If the terms of trade were lower than the smaller or higher than the greater pre-trade equilibrium price ratio, then one country would suffer a loss. As a matter of fact, we have shown during the analysis of the neoclassical theory (of which the Heckscher-Ohlin model can be considered as a particular case) that the terms of trade are always strictly included between the two autarkic equilibrium price ratios: see Figs. 3.6 and 3.7b.

    Given that the relative price of goods strictly falls between the two pre-trade equilibrium relative prices, the corresponding factor-price ratio must necessarily fall within the segment of equalization. Now, since specialization is not complete (the extreme points of the segment, which give rise to complete specialization, are in fact excluded), absolute factor price equalization will also occur.

(b) No segment of equalization exists. In this case complete specialization of at least one country is inevitable and even relative factor price equalization is excluded. In terms of Fig. 4.9, before trade, country 1's relative price of goods was in *DG* (for example at G') and country 2's was in FE (for example at F'), with the corresponding relative price of factors in $p_1' p_1''$ and $p_2' p_2''$ respectively. After the opening of trade, the (common) relative prices of goods will be included between G' and F': it may fall in $F'E$ or in ED, or in DG'.[3] If it falls in $F'E$, for example at point H, country 2 will produce both goods and the relative price of factors there will be p_{2H}. Country 1, on the contrary, will specialize completely in commodity A and the relative price of factors there will be p_1': it must, in fact, be stressed that, *when complete specialization obtains*, we can no longer use the one-to-one relation between relative factor prices and relative goods prices (which presupposes that both goods are produced domestically) and so—no matter what the terms of trade are—the relative price of factors will be that corresponding to the point of full specialization.

It can be checked by similar reasoning that, if the terms of trade fall in *ED*, country 1 will completely specialize in A and country 2 in B (the relative prices of factors will be p_1' and p_2'' respectively), whilst if they fall in DG', country 1 will produce both commodities and country 2 will completely specialize in B (the relative prices of factors will be: included between p_1' and $p_{1G'}$, and equal to p_2'', respectively).

[3]It cannot fall at F' or G' because, as stated repeatedly, the terms of trade cannot be equal to either pretrade autarkic equilibrium price ratio.

(c) A segment of equalization exists, but the pre-trade equilibrium relative prices of goods are *not* such as to make both countries' relative prices of factors fall within it: in terms of Fig. 4.8, $(p_B/p_A)_2$ is, for example, included in *FD*, whilst $(p_B/p_A)_1$ is, for example, included in EG. After trade, the terms of trade will be included between these price ratios as usual, but the outcome will be different, depending on where the terms of trade themselves happen to fall. If they fall in *DE* (excluding the extreme points D and E), both the relative and the absolute factor prices will be equalized as in case (a). But they may equally well fall in *FD* or in *EG*: in both instances the result will be the same as in case (b), that is one country will completely specialize (both cannot, however) and factor price equalization will be impossible.

Since in case (c)—differently from cases (a) and (b)—it is important to know the exact position of the terms of trade (an information that we can get only by exactly knowing the demand side), we must conclude that also in case (c) the result is, in general, ambiguous.

We conclude this section with three observations. Firstly, the essential role played by the assumption of absence of complete specialization in the factor price equalization theorem must be stressed again. Secondly, the presence or absence of a segment of equalization is related to the spread between the relative factor endowments of the two countries: as we have seen, the more distant ϱ_1 and ϱ_2 the more probable—*ceteris paribus*—the absence of such a segment and the complete specialization of at least one country. Thirdly, it is always possible, even in the absence of full factor price equalization, to state that international trade brings about a *tendency* to relative factor price equalization: it can in fact be readily seen from Fig. 4.9 that, *after* trade, the relative factor prices will, in any case, be closer than before trade.

4.3.2 The Factor Price Equalization Set

The FPE theorem has been proven in Sect. 4.3.1 under the assumption of incomplete specialization. In this section we address a related question, namely, under what conditions the free trade equilibrium is one of incomplete specialization. We have seen above that the degree of international specialization is positively related to differences in relative factors endowments. The conditions we search for will therefore concern such differences. To investigate this matter we follow the thought experiment proposed by Samuelson (1949, 1953) and known as the *integrated world equilibrium* approach.

Consider first a world economy constituted by only one country with endowments given by $\overline{K}$ and $\overline{L}$. The equilibrium for this economy, necessarily a closed economy, is identified by the equilibrium price of factors (p_K^*, p_L^*) and equilibrium value of output (A_W^*, B_W^*). We refer to this equilibrium as the integrated equilibrium. Imagine splitting this single-country world economy into a two-country world economy with free trade between them. The split is operated by arbitrarily allocating a portion of the world endowments to each country so as to exhaust world endowments. Clearly, there is an infinity of possible such allocations. We search for

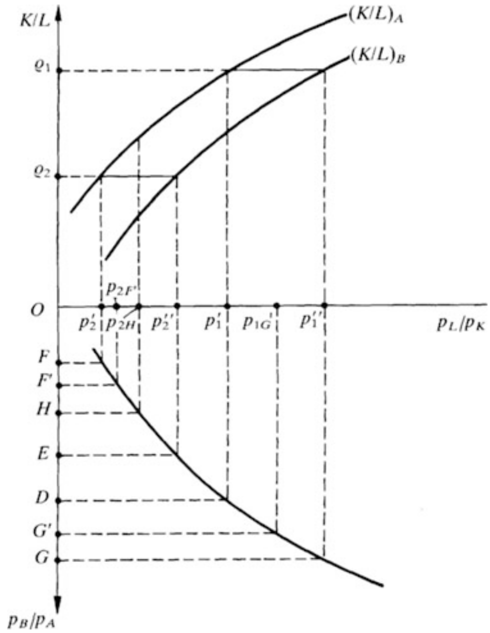


Fig. 4.9 A case of no factor price equalization

allocations such that the equilibrium factor prices of the two-country world economy are the same as those of the integrated equilibrium and such that the outputs of A and B in each country are positive and sum up to A_W^* and B_W^*, respectively. If such allocations exist they will be, by construction, allocations characterized by FPE and incomplete specialization. We therefore will have found the conditions on countries' factor endowments under which a free trade world economy implies incomplete specialization and FPE.

This thought experiment has a simple graphical representation[4] which we depict in Fig. 4.10.The base and height of the rectangle represent world endowments $\overline{L}$ and $\overline{K}$, respectively. The diagonal represents the vector of world endowments. Recall that in the integrated economy $\overline{L}$ and $\overline{K}$ are employed to produce the quantities A_W^* and B_W^*.

The next step is to find the sectorial employment vectors of the integrated equilibrium. The elements of such vectors are the employment of L and K in each industry, denoted L^A, $K^A$and L^B , K^B. Naturally, the sum of the sectorial employment vectors gives the endowment vectors, that is, $L^A + L^B = \overline{L}$ and $K^A + K^B = \overline{K}$. Recall that the slope of a vector is given by the ratio of its elements, thus, for instance, the slope of the sectorial employment vector measured on the L-axis

[4]This diagram is commonly attributed to Dixit and Norman (1980, pp. 109ff.), but earlier presentations can be found in Travis (1964, pp. 15ff.) and Lancaster (1957, pp. 31ff.).

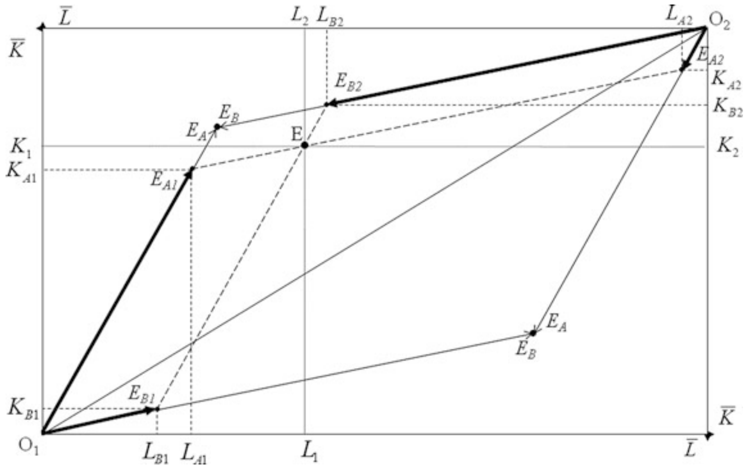


Fig. 4.10 The factor price equalization set

for industry A is given by K^A/L^A which is the capital intensity in industry A. The integrated equilibrium yields the equilibrium values of factor prices from which, as studied in Sect. 4.1.1, we can determine the factor intensity in each industry. Given the factor intensities we can draw the sectorial employment vectors by drawing two segments (from whichever corner) whose slopes correspond to the factor intensities and whose length is such that the sum of them give the vector of world endowments. In Fig. 4.10, the vectors O_1E_A and O_1E_B (or, equivalently, O_2E_A and O_2E_B) represent the sectorial employment vectors. Recalling the parallelogram rule of vectorial sum it is immediate that the sum $O_1E_A + O_1E_B$ gives the world endowment vector. The length of each sectorial employment vector is proportional to factor inputs and, therefore, reflects the quantity of output; the longer the sectorial employment vector the larger the sector output.

We now split the world into two countries and assume free trade between them. We refer to this equilibrium as to the free trade equilibrium. Any point in the rectangle represents a possible division of the world economy into two countries but not all the divisions will give rise to the free trade equilibrium we require. We require the free trade equilibrium to have the same factor prices and, therefore, the same factor intensities as the integrated equilibrium. We also require both countries to produce both goods.

To find any of the divisions satisfying the requirements we begin by arbitrarily assigning to each country a share of the integrated equilibrium outputs. This assures incomplete specialization. Let s_{A1}, s_{B1}, $s_{A2} = 1 - s_{A1}$, and $s_{B2} = 1 - s_{B2}$ be the arbitrarily chosen shares. The next step is to find the sectorial employment vectors for each country, denoted E_{A1} and E_{B1} and E_{A2} and E_{B2}. They must have the same slope as the sectorial employment vectors of the integrated equilibrium (because we require same factors price) and their length must be such that the corresponding factor inputs give outputs equal to $A_1^* = s_{A1} A_W^*$, $B_1^* = s_{B1} B_W^*$, $A_2^* = (1 - s_{A1}) A_W^*$, $B_2^* = (1 - s_{B1}) B_W^*$. Given these requirements, each country's employment vectors are a fraction of the integrated equilibrium employment vectors. Precisely, $E_{A1} = s_{A1} E_A$, $E_{B1} = s_{B1} E_A$, $E_{A2} = (1 - s_{A1}) E_A$, $E_{B2} = (1 - s_{B1}) E_B$. Let O_1 be the origin of measures for country 1 and O_2 that for country 2 in the two-country world economy. These vectors are shown in Fig. 4.10 as fractions of the world sectorial employment vectors. The sum of $O_1 E_{A1}$ and $O_1 E_{B1}$ gives the vector $O_1 E$ which represents total employment in country 1. Because of full employment, this vector necessarily represents the endowment vector for country 1 consistent with the arbitrarily chosen shares. Analogously, $O_2 E$ represents the resulting endowment vector for country 2. The vectors $O_1 E$ and $O_2 E$ have precisely the properties we have required; they represent an allocation of world endowments such that the resulting free trade equilibrium yields the same factor prices as the integrated equilibrium (i.e., FPE) and such that each country is incompletely specialized. It is easy to verify that the set of all such possible allocations is constructed as the sum of all possible fractions of the world employment vectors and, therefore, is represented graphically by the area demarcated by the parallelogram composed by the four vectors $O_1 E_A$, $O_2 E_B$, $O_2 E_A$, $O_1 E_B$. Any point inside the parallelogram represents a division of the integrated world economy such that factor prices equalize and countries are incompletely specialized. The borders of the parallelogram belong to the FPE set but imply that at least one country is completely specialized.

An alternative but equivalent way of constructing the FPE set is the following. Pick a point in the rectangle and consider the vector drawn from O_1 to the chosen point. If this vector can be decomposed into two vectors which are portions (including 0 and 1) of the integrated equilibrium employment vector then the chosen point belongs to the FPE set. If such decomposition is impossible then the chosen point does not belong to the FPE set. Naturally, drawing the vector from O_1 or O_2 is equivalent. Clearly, only points in the parallelogram allow to draw vectors that can be decomposed into portions of the integrated equilibrium employment vectors.

The conclusion of the analysis can be summarized in the following

Theorem (Factor Price Equalization Set). *The Factor-Price Equalization Set is the set of all weighted sums of the integrated-equilibrium employment vectors, where the weights take values between zero and one.*

4.4 The Factor Content of Trade and the Heckscher-Ohlin-Vanek Theorem

The Heckscher-Ohlin theorem states that each country exports the commodity which uses the country's more abundant factor more intensively. This theorem may be reformulated in terms of the *factor content of trade*. This reformulation, due to Vanek (1968), is instructive as it allows seeing the Heckscher-Ohlin theory in a different perspective and permits discussing some generalization in a simple way.

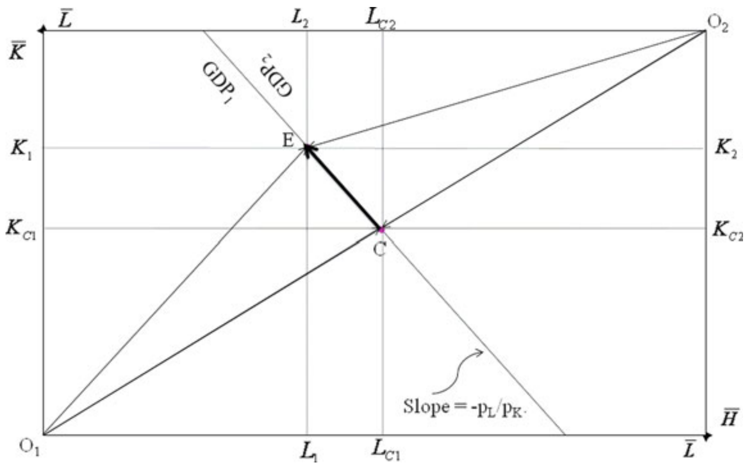


Fig. 4.11 The factor content of trade

The factor content of trade is defined as the quantity of factors used to produce the goods exported *minus* the quantity of factors used to produce the goods imported. We refer to these quantities as the factor services embodied in net trade. Thus, for instance, for a country who exports ten units of A and imports ten units of B, the factor content of trade is given by the factor services embodied in the ten units of A exported minus the factor services embodied in the ten units of B imported. Noting that net exports are given by production minus consumption, the factor content of trade may equivalently be defined as the vector representing the factor services embodied in the goods produced by the country minus the vector representing the factor services embodied in the production of the goods consumed by the country. The reformulation under examination predicts that in free and balanced trade the sum given by the capital services embodied in exports minus the capital services embodied in imports is positive for the capital abundant country and negative for the labour abundant country; signs reversed for L. More generally, we have the following:

Theorem (Heckscher-Ohlin-Vanek). *Each country is the net exporter of the services of its abundant factor and the net importer of the services of its scarce factor.*

Net exports are given by production minus consumption. Therefore, the factor content of trade is simply the vector representing the factor services embodied in the goods produced by the country minus the vector representing the factor services embodied in the production of the goods consumed by the country. The factor content of trade vector has a simple graphical representation.Figure 4.11 represents the free trade world economy discussed in Sect. 4.3.2, where O_1E and O_2E are the endowment vectors of country 1 and 2, respectively, and point E is assumed to be within the Factor Price Equalization set (not shown in the figure). Comparing the slopes O_1E and O_2E with respect to the $L-$axis shows that country 1 is relatively capital abundant. Since there is full employment, the endowment vectors O_1E and O_2E are also the total employment vectors. A line emanating from E whose slope is given by the relative price of L represents the GDP line (or budget constraint line) of each country since it is obtained by multiplying factor endowments by factor prices. The diagonal represents the vector of factor services embodied in the production the goods consumed by the world economy.

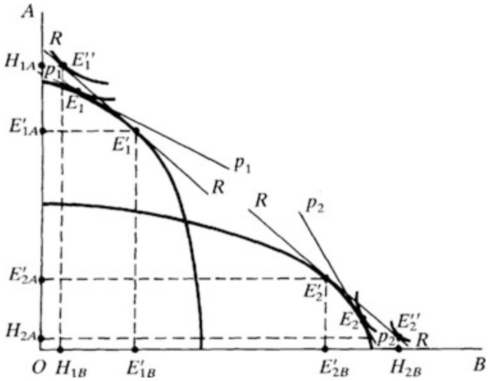
Since preferences are identical and homothetic and since trade is balanced, each country will consume a share of world production of goods equal to its share of world income. Therefore, the vector representing factor services embodied in the goods consumed by a country will necessarily lie on the diagonal and will necessarily be a fraction of it. Its length is given by the intersection of the GDP line with the diagonal. In Fig. 4.11, O_1C and O_2C represent such vectors for country 1 and 2, respectively. The vector CE is the factor content of trade vector for country 1. Its elements are $(L_1 - L_{C1}) < 0$ and $(K_1 - K_{C1}) > 0$. The first element is negative and the second is positive reflecting the fact that country 1 is relatively well endowed of K. The vector EC is the factor content of trade vector for country 2. In conclusion, we have found that, as stated in the Heckscher-Ohlin-Vanek theorem, each country is the net exporter of the services of its abundant factor and the net importer of the services of its scarce factor.

It is interesting to note that the H-O-V theorem does not require any information about the output of goods in each country or about the direction of trade. This will be important when discussing a generalization of the Heckscher-Ohlin theory in which there are more goods than factors. It will also be important when addressing the empirical verifications of the Heckscher-Ohlin theory.

4.5 Extensions and Qualifications

This section aims at analysing the consequences of dropping some of the basic assumptions examined in Sect. 4.1, in particular that concerning the structure of demand, that concerning the absence of factor-intensity reversals, and that concerning the presence of only two commodities and two factors. The assumption of internationally identical production functions cannot be dropped without altering the essence of the Heckscher-Ohlin theory. A list of studies on the extensions and qualifications of the Heckscher-Ohlin model includes Baldwin (2008); Bhagwati (1972); Bhagwati and Srinivasan (1983); Brecher and Choudhri (1982); Chacholiades (1978); Davis et al. (1997); Deardoff (1982); Dixit and Woodland (1982); Ethier (1982, 1984); Feenstra (2004); Hamilton and Svensson (1984); Harkness (1978, 1983); Harrod (1953); Helpman (1984a); Herberg et al. (1982); Horiba (1974); Johnson (1957); Leontief (1956); Maskus (1985); Neary (1984, 1985b); Pearce (1952); Samuelson (1967); Sarkar (1984); Takayama (1972).

Fig. 4.12 Non-identical structures of demand



4.5.1 Non-identical Structures of Demand

If we drop the assumption of internationally identical structures of demand, the Heckscher-Ohlin proposition is no longer necessarily true. In fact, if a country has a strong preference for the commodity which uses the country's more abundant factor more intensively (remember that we are using the physical definition of factor abundance), it may happen that, when trade opens up, each country exports the other commodity, namely the one which is intensive in the country's less abundant factor. This is illustrated in Fig. 4.12, where the transformation curves and the social indifference curves of the two countries are brought together, in the same way as in Fig. 3.15.[5] The pre-trade equilibriumpoints are E_1 and E_2 for country 1 and country 2 respectively; the corresponding relative prices of goods are measured by the (absolute value of the) slope of $p_1 p_1$ and $p_2 p_2$. After trade begins, an intermediate price ratio (terms of trade) will obtain, for example, that measured by the slope of RR. The production point will be E'_1 for country 1 which, however, given its strong preference for commodity A, will consume at E''_1, importing $E'_{1A}H_{1A}$ of A and exporting $E'_{1B}H_{1B}$ of B. Therefore, country 1 will import the commodity intensive in capital (the country's more abundant factor) and export the commodity intensive in labour (the country's less abundant factor). Similarly it can be seen that country 2 will produce at point E'_2 and consume at point E''_2, importing $E'_{2B}H_{2B}$ (equal to $E'_{1B}H_{1B}$) of B and exporting $E'_{2A}H_{2A}$ (equal to $E'_{1A}H_{1A}$) of A: commodity B is intensive in labour (country 2's more abundant factor) and A in capital (country 2's less abundant factor). Thus the Heckscher-Ohlin proposition is contradicted.

[5]We refer the reader to that chapter for the problems related to the use of social indifference curves. With the occasion, we point out that Fig. 4.12 makes it possible to show the gains from trade in the same way as in Fig. 3.15. In the case of the Heckscher-Ohlin model with identical structures of demand, we can use the same diagram with the proviso that an *identical* family of social indifference curves (which, in addition, must be homothetic) must be used for both countries.

It should, however, be noted that this result *may*, and need not, occur: it is, in fact, possible—as the reader can ascertain graphically by experimenting with different families of social indifference curves—that the basic proposition remains valid even with different structures of demand, provided that, in each country, these are not too much biased towards the commodity which uses the country's more abundant factor more intensively. We can therefore conclude that the assumption of identical structures of demand is a sufficient, but not a necessary, condition for the validity of the Heckscher-Ohlin theorem.

It is important to stress that the possible invalidity of this theorem, because of different structures of demand, *does not invalidate* the factor price equalization theorem, which continues to hold within the limits clarified in the previous section. The latter theorem, in fact, does not depend on the assumption of identical demand structures, and as long as no factor-intensity reversal occurs and specialization is incomplete, the theorem under consideration remains valid.

However, the possible invalidity of the Heckscher-Ohlin theorem when demand structures are different, has led various authors to investigate the possibility of reformulating the theorem without that assumption. The answer is that it can be done, *provided that the Heckscher-Ohlin theorem is reformulated in terms of the price definition of factor abundance* (see Sect. 4.2). The reason is intuitive: in country 1, in the pre-trade autarkic equilibrium situation, the strong bias of tastes towards the capital-intensive commodity A implies that this factor, notwithstanding its relative abundance in physical terms, will be relatively scarce (less abundant) in economic terms, namely, will have a greater relative price than in country 2, where exactly the opposite situation obtains. Thus we shall have

$$p_{1K}/p_{1L} > p_{2K}/p_{2L}, \text{namely } p_{1L}/p_{1K} < p_{2L}/p_{2K}, \tag{4.4}$$

and so, in terms of the price definition of factor abundance, country 1 is *labour*-abundant relative to country 2. More rigorously, (4.4) can be arrived at by way of the one-to-one correspondence between relative factor prices and relative prices of goods. Figure 4.12 tells us that, in the pre-trade equilibrium situation, $(p_B/p_A)_1 < (p_B/p_A)_2$. Therefore—see Fig. 4.5a—we have $(p_L/p_K)_1 < (p_L/p_K)_2$, as was to be shown.

In conclusion, the Heckscher-Ohlin theorem is valid independently of the structure of demand (thus assumption 2 of Sect. 4.1 can be dropped), *if* the price definition of factor abundance is adopted. This is one of the motives which have induced some writers to prefer the price to the physical definition. It is interesting to point out that Ohlin himself used the price definition of abundance, though hinting at a physical definition: "... the real problem is to demonstrate what lies behind such inequality in prices, or, more precisely, to show in what way *differences in equipment* come to be expressed in differences in money *costs and prices*" (Ohlin, 1933, p. 13; p. 7 of the 1967 edition. Our italics).

However, arguments for the physical definition are not lacking. Relative factor abundance in physical terms is observable at any moment (provided of course that the factors can be measured unambiguously, but this is a general problem). On the contrary, relative factor abundance in price terms is not observable, as it is defined with reference to a hypothetical pre-trade autarkic equilibrium situation. Some authors (see, for example, Leamer, 1984, p. 2) even think that hypotheticals such as autarkic prices, that have no observable counterpart, are to be excluded from discussion.

4.5.2 Factor-Intensity Reversals

To investigate the consequences of the presence of factor-intensity reversals it is expedient to use the diagram which brings together the relationships between the capital/labour ratio and the factor-price ratio, and between the latter and the commodity-price ratio. We have reproduced Fig. 4.2b in the upper half of Figs. 4.13 and 4.5b, turned upside down, in the lower half.

Various cases must now be distinguished, according to the position of the relative factor endowments of the two countries. If these endowments are such that, in the interval between them, no factor-intensity reversal occurs, as is the case of ϱ_1 and ϱ_2, then the Heckscher Ohlin theorem remains valid, for any factor intensity reversal occurring *outside* the $\varrho_1 - \varrho_2$ interval is irrelevant: in the relevant stretch, commodity A is unambiguously capital-intensive relative to commodity B (in terms of Fig. 4.1b, only the part of the diagram to the left of the radiant of tangency must be considered). The factor price equalization theorem also remains valid (within the limits in which it is valid in general: existence of a segment of equalization, etc.).

If, on the contrary, relative factor endowments are separated by a point of factor-intensity reversal (as is the case of ϱ_1 and ϱ_2' in Fig. 4.13), then exportables have the same kind of factor intensity in both countries, so that the Heckscher Ohlin theorem is no longer valid or, to be precise, remains valid for one country only. Let us assume that the pre-trade equilibrium relative prices of commodities are $(p_B/p_A)_1$ and $(p_B/p_A)_2$; as we know, the terms of trade will fall at an intermediate point, for example R_s. Country 1 will export commodity A and country 2 commodity B[6]: now, as can be seen from the diagram, in country 1 the capital-intensive commodity is A and, in country 2, the capital-intensive commodity is B (owing to the factor-intensity reversal). Thus the Heckscher Ohlin theorem is valid for country 1, the capital-abundant country relative to country 2, but not for country 2, which is the relatively labour-abundant country. In this case also the factor price equalization theorem is invalid ,as no segment of equalization exists; besides, it can be seen that the relative price of factors moves in the same direction in both countries: from *OD*

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[6]This cannot be directly seen from the diagram, but from an inspection of the transformation curves. More simply, as $(p_B/p_A)_1 > R_s$, country 1 will find it profitable, when trade begins, to give up A in exchange for B and similarly, as $(p_B/p_A)_2 < R_s$, country 2 will give up B in exchange for A.

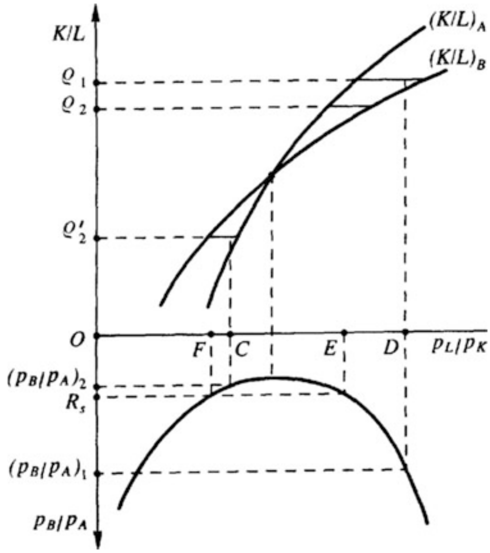

Fig. 4.13 Factor-intensity reversals, H-O and FPE

to *OE* in country 1 and from *OC* to *OF* in country 2. In Fig. 4.13 these movements bring the relative prices of factors nearer, because $DE > FC$, but in general, movements of this kind may equally well bring them farther apart. Therefore, as the relative price of factors moves in the same direction in both countries (either decreasing, as in Fig. 4.13, or increasing), it is no longer possible to state that, in general, there will be at least a tendency towards relative factor price equalization.

In Fig. 4.13 we have examined the case of a single reversal but, as we know, there may be two or more reversals. We give a list of results (which can be derived by graphic analysis):

(a) If there is an odd number of reversals occurring in the interval between the two countries' relative factor endowments, the same conclusions hold as shown above, with reference to Fig. 4.11;

(b) If there is an even number of reversals occurring in the interval between the two countries' relative factor endowments, then each commodity can be unambiguously classified as intensive in a given factor. However, the pattern of trade may not conform to the Heckscher-Ohlin theorem (for example, it may happen that the labour-abundant country exports the capital-intensive commodity). When this occurs, the relative prices of factors will move in opposite directions. On the contrary, when the pattern of trade conforms to the Heckscher-Ohlin theorem, the relative prices of factors will move towards each other, but will never coincide.

4.5.3 The Heckscher-Ohlin-Vanek Generalization

The model studied above is often referred to as the two-by-two version of the Heckscher-Ohlin theory since it counts only two goods and two factors. The two-by-two version is somewhat special since the dimensionality is low (two-by-two) and the number of goods equals the number of factors. In this section we investigate whether the results of the theory are robust to a generalization that allows for many goods and many factors.

Any such generalization gives only three possible dimensional structures: (a) more factors than goods, (b) equal number of goods and factors, (c) more goods than factors. The first dimensional structure is well illustrated by the *specific factor model* that for its importance deserves a separate discussion that we postpone to Sect. 6.2. The second and third dimensional structures may be treated together for our purposes. In what follows let N be the number of goods and M the number of factors and assume that $N \geqslant M \geqslant 2$ with $M > 2$ if $N = M$. This generalization is often called the Heckscher-Ohlin-Vanek generalization.

Consider first the effects of such generalization for the FPE theorem. The FPE set can be constructed using the same logic as in Sect. 4.3.2. Indeed, neither the requirements nor the logic of construction of the FPE depend on the number of goods and factors as long as $N \geqslant M$. Beginning by the integrated world equilibrium we note that it is unaffected by the existence of more goods than factors. The integrated equilibrium system of equations counts the same number of unknowns as there are equations regardless of the number of goods and factors. There will be N efficiency conditions (price = marginal cost), M equilibrium conditions in factor markets, and $N - 1$ equilibrium conditions in commodity markets. These equilibrium conditions determine M factor prices, N commodity outputs, and $N-1$ commodity prices. The FPE is again given by the sums of all possible fractions of the sectorial employment vectors of the integrated equilibrium where the fractions are arbitrarily chosen shares of the integrated equilibrium outputs. It is useful to provide a graphical representation of the FPE set in this context where $N \geqslant M$. To this purpose consider the simple case where $N = 3$ and $M = 2$. Let A, B and D be goods (the letter C is reserved for consumption) and, as usual, let L and K be factors.The FPE is represented in Fig. 4.14 where the world sectorial employment vectors are O_1A, O_1B, and O_1D. The last two vectors are also represented by the vectors AB' and $B'O_2$. We recall that these sectorial employment vectors are drawn using the information on factor intensities and on total output of goods. Indeed, since output is proportional to inputs, the length of each sectorial employment vector represents (in the space of factors) the total industry output in the integrated equilibrium.

The next step is to choose arbitrarily a partition of the integrated equilibrium outputs keeping factors prices the same as in the integrated equilibrium. The vectors O_1A_1, O_1B_1, and O_1D_1 represent one such partition since they are fractions of the integrated equilibrium sectorial employment vectors. The last two vectors are also represented by the vectors A_1B_1' and $B_1'E$. The corresponding partition

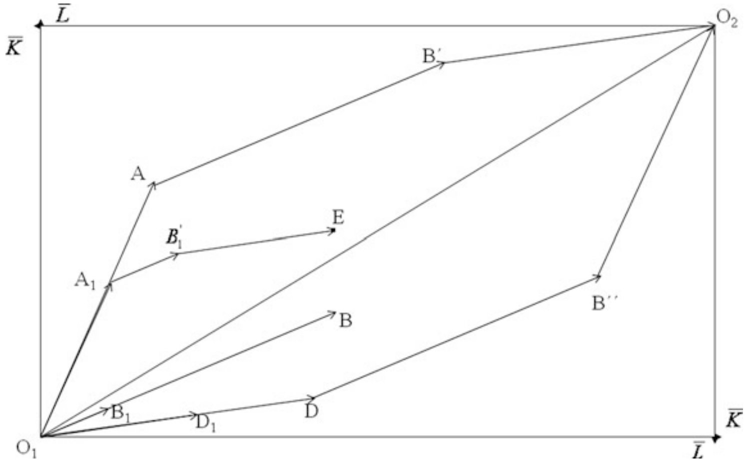

Fig. 4.14 The factor price equalization set: 3 goods and 2 factors

of world endowments is necessarily given by the vector obtained from the sum of O_1A_1, O_1B_1, and O_1D_1, represented by point E in Fig. 4.14. This partition of world endowments between countries satisfies FPE by construction. It is quite clear that the set of all FPE-compatible partitions is obtained from the sum of all possible arbitrarily chosen fractions of the vectors O_1A, O_1B, and O_1D. Graphically the FPE is represented by the area demarcated by the parallelogram $O_1AB'O_2B''DO_1$. In conclusion, the M-by-N generalization where $N \geqslant M$ has no effect on the validity of the FPE theorem.

Coming to the Heckscher-Ohlin theorem, the major nuisance resulting from having more goods than factors is that the model no longer determines the quantities of goods produced in each country. This is clear by noting that while the integrated equilibrium counts the same number of equations and unknowns (regardless of the number of goods and factors) the two-country free-trade equilibrium does not. The latter is composed by N efficiency conditions (price = marginal cost), $2M$ equilibrium conditions in factor markets (M conditions in each country), and $N-1$ equilibrium conditions in commodity markets. The endogenous variables are M factor prices, $2N$ commodity outputs (N in each country), and $N-1$ commodity prices. We therefore have $2M+2N-1$ equations and $M+2N+N-1$ unknowns. Since $N > M$ the free trade equilibrium counts more unknowns than equations, therefore it cannot determine the equilibrium values of all the endogenous variables. In particular, this means that when passing from the integrated equilibrium to the two-country free-trade equilibrium there is an infinity of production structures for the two countries that is consistent with the integrated equilibrium factor prices. This indeterminacy in production is particularly disturbing for the Heckscher-Ohlin theorem since it does not allow to relate output proportions to endowment proportions. Therefore, it is not possible to say which goods are exported by each country.

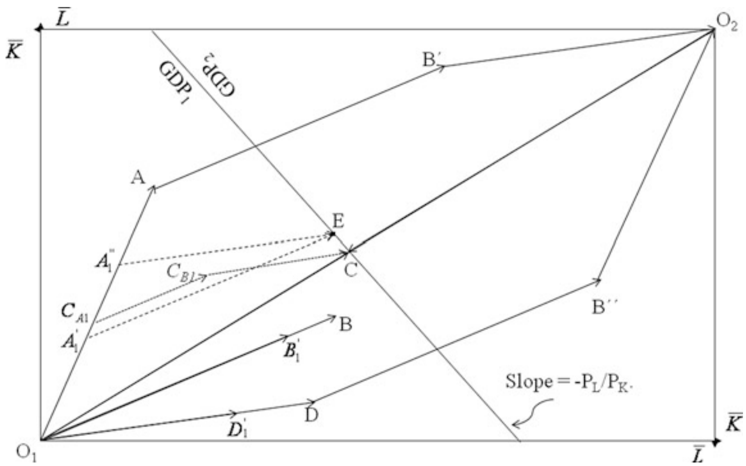

Fig. 4.15 The factor content of trade: 3 goods and 2 factors

However, the Heckscher-Ohlin-Vanek theorem (Sect. 4.4) remains intact.We show this in Fig. 4.15. Consider the endowment point E in the FPE set. This partition of world endowments is compatible with many production structures. In the figure we represents two extreme cases of production structures for country 1 (the choice of the country is irrelevant). The first case has output of A represented by O_1A_1', output of B represented by $O_1B_1' = A_1'E$, and output of D equal to 0. The second case has output of A represented by O_1A_1'', output of B equal to 0, and output of D equal to $O_1D_1' = A_1''E$. The sum of each of these two sets of vectors gives the endowment vector for country 1. As explained above, there is an infinity of equally possible alternatives (one of them is represented by the vectors O_1A_1, O_1B_1, and O_1D_1 in Fig. 4.14).

This example shows the indeterminacy of the production pattern when there are more goods than factors. It becomes impossible to associate relative outputs of goods with relative factor endowments. In addition, it is possible that the *K-abundant* country imports a *K-intensive* good. To see this recall from Sect. 4.4 that the consumption vector lies on the main diagonal and is identified by the GDP line; this is represented by point C in the figure. Then consider the consumption vectors for country 1 represented by O_1C_{A1}, $C_{A1}C_{B1}$, and $C_{B1}C$. Each sectorial consumption vector is in the same proportion to the corresponding integrated equilibrium sectorial employment vector as the vector of total consumption of the country is to the vector of total consumption of the world economy. Therefore, $O_1C_{A1}/O_1A = C_{A1}C_{B1}/O_1B = C_{B1}C/O_1D = O_1C/O_1O_2$. These proportions are required by the assumption of identical and homothetic preferences, which implies that each country consumes a share of output in world output (for each good) equal to its share of income in world income. It is clear that in the first case of production structure (represented by O_1A_1', $O_1B_1' = A_1'E$ and output of $D = 0$) country 1 imports good A in spite of the fact that A is *K-intensive* and country 1 is *K-abundant*. The Heckscher-Ohlin theorem stated in terms of production and export pattern of goods does not survive the $N \geqslant M$ generalization, but the Heckscher-Ohlin-Vanek theorem does. Indeed, the factor content of trade vector for country 1 is the vector CE, and for country 2 it is EC. It is clear that, in spite of the indeterminacy of the production pattern and in spite that the K-abundant country may end up importing the most L-intensive good, each country exports the services of its relatively abundant factor.

4.6 Empirical Studies

4.6.1 Leontief's Paradox

The empirical relevance of the Heckscher-Ohlin theorem has been the subject of very many studies, beginning with the pioneering one of Leontief (1953). By applying his *input–output* analysis[7] to the 1947 input–output table of the US economy, Leontief computed the total (direct and indirect) input requirements of capital and labour per unit of the composite commodity "US exports" and per unit of the composite commodity "US competitive import replacements"; in both cases the unit was one million dollars' worth of commodities at 1947 prices and composition. By "competitive import replacements" Leontief refers to "imports of commodities which can be and are, at least in part, actually produced by domestic industries", so that by replacing a unit of imports with a unit of domestic production, it is possible to find out "whether it is true that the United States exports commodities the domestic production of which absorbs relatively large amounts of capital and little labour and imports foreign goods and services which—if we had produced them at home—would employ a great quantity of indigenous labour but a small amount of domestic capital" (1953, p. 75). The principal findings of this analysis are summarized in Table 4.1, adapted from Leontief (1953):
As can be seen from the last column, it turned out that the United States exported labour-intensive commodities and imported capital-intensive ones. Now, since the United States was generally considered to be a capital abundant country relative to all its trading partners (remember that the data refer to 1947), Leontief's results were in sharp disagreement with the Heckscher-Ohlin theorem (according to which the US ought to have exported capital-intensive commodities), whence the "paradox", as it came to be known in the literature.

[7] For an explicit treatment of intermediate goods in the pure theory of international trade see below, Sect. 6.4.

Table 4.1 Domestic capital and labour requirements per million dollars of US exports and of competitive import replacements (of average 1947 composition)

	Capital (dollars, in 1947 prices)	Labour (man-years)	K/L
Exports	2,550,780	182.313	13,991
Import replacements	3,091,339	170.004	18,184

4.6.2 Explaining the Paradox

Leontief's analysis gave rise to wide debate, concerning both its statistical and theoretical aspects, and to a host of successive empirical studies, which still continue, with conflicting results. It would be impossible to survey this enormous amount of literature here, so we shall focus on some aspects only. Surveys of the initial debate aroused by Leontief's original analysis and of the empirical studies carried out up to the early 1960s are contained in Bhagwati (1964, pp. 21ff.) and Chipman (1966, pp. 44ff.). For subsequent surveys, see Stern (1975), Deardoff (1984), Kohler (1988), Leamer and Levinsohn (1995), Baldwin (2008). See also Leamer (1984), for an original treatment.

By simplifying to the utmost, it is possible to divide the attempts at explaining Leontief's paradox into two groups. The first includes all those works which maintain that serious mistakes or, at the very least, inaccuracies, were made in passing from the theoretical formulation to its empirical testing, so that the latter is vitiated and cannot be considered as a refutation of the Heckscher-Ohlin theorem. The second includes all those works which maintain that one or more of the basic assumptions are not fulfilled in reality, so that the theorem itself loses all validity: the empirical analysis must necessarily confirm this invalidity.

It is self-evident that, whilst the attempts that belong to the first group attempt to rescue the theorem, those belonging to the second are destructive of the theorem itself.

4.6.2.1 Mistakes in Calculations?

Considering the *first group*, we begin with the argument (set forth by Leontief himself, 1953, pp. 87ff.) according to which American labour was—at that time—more efficient than rest-of-the-world labour, so that, when the former was converted into equivalent units of the latter, the United States became a labour abundant country relative to the rest of the world. According to Leontief, it was plausible to assume a coefficient of conversion of three: "...in any combination with a given quantity of capital, one man-year of American labour is equivalent to, say, three man-years of foreign labour. Then, in comparing the relative amounts of capital and labour possessed by the United States and the rest of the world (...) the total number of American workers must be multiplied by three (...). Spread thrice as thinly as the unadjusted figures suggest, the American capital supply per 'equivalent worker' turns out to be comparatively smaller, rather than larger, than that of many other countries" (1953, pp. 87–88).

One must, of course, avoid the logical mistake of attributing the greater efficiency of American labour to the greater amount of capital per man employed in the United States, for by so doing one would commit a tautology; such greater efficiency is, in fact, attributed by Leontief to entrepreneurship, superior organization etc. in the United States relative to other countries. These elements, however, increase not only the productivity of labour but also that of capital, and so if these were to increase by the same proportion, the relative factor abundance would not change. Therefore, Leontief concludes (1953, p. 90), "... entrepreneurship, superior organization, and favourable environment must have increased—in comparison with other countries—the productivity of American labor much more than they have increased the efficiency of American capital".

It should however be noted that subsequent studies did not confirm the coefficient of conversion of three that Leontief assumed. For example, Kreinin (1965) interviewed managers and engineers of about 2,000 US firms operating both at home and abroad, through questionnaires. These aimed at determining the amount of labour time required to produce one unit of the same output—with the same equipment and organization of labour—in plants in the United States and abroad. Most persons interviewed did in fact judge US labour more efficient than its foreign counterpart, but by 20 or 25 %; the resulting coefficient of conversion of 1.20 or 1.25 was far below the coefficient of 3 that, according to Leontief, would have made the USA a relatively labour abundant country.

Other researchers observed, in criticizing Leontief's study, that it is wrong to consider two factors of production (physical capital and labour) only. For example, according to Diab (1956) and Vanek (1959), one must consider at least another factor, *natural resources*: for instance, the same equipment and the same workers with the same organization operating in the oil extractive industry will obtain better results in Venezuela or in the Arabian countries than in the United States, for the very simple reason that US oil-fields are less rich. Therefore if one neglects the natural resources factor, incorrect results will be obtained, whilst the paradox will disappear if this factor is taken into account. And in fact Vanek (1959), in addition to the data given by Leontief (Table 4.1 above), computed the input of (goods having a high content of) natural resources required to produce one unit of exports and one unit of import replacements: this input turned out to be \$340,000 and \$630,000 at 1947 prices, respectively. Therefore the United States imported goods intensive in natural resources (no matter whether this intensity was computed relative to capital or to labour), which was the relatively less abundant factor there, and exported goods intensive in capital and labour relative to natural resources (the first two factors being more abundant relative to the third). It followed that the Heckscher-Ohlin theorem, far from being refuted, was fully confirmed.

Other authors stress the importance of the *human capital* factor, which is that embodied in skilled workers, managers, engineers etc. as distinct from general or unskilled labour. Leaving aside the practical problems of the various methods of measuring human capital (capitalization of wage differentials; years of education;

professional qualifications; etc.), the consideration of this factor lends support to the hypothesis that US exports are intensive in human capital (a relatively abundant factor in that country) with respect to import replacements, in accordance with the Heckscher-Ohlin theorem: see, for example, Stern and Maskus (1981), who also cite similar results of previous studies; see also Lane (1985) and Charos and Simos (1988).

An important contribution is that of Casas and Choi (1984, 1985a), who were the first to point out that the Heckscher-Ohlin theorem—as all the theorems in the pure theory of international trade—implicitly presupposes a situation of balanced trade. Since in reality the trade balances are never in equilibrium, the paradoxical empirical results can be due to the non-verification of this essential condition. And in fact they maintain that the same data used by Leontief would have shown, under balance-of-trade equilibrium, that US exports were indeed more capital intensive than import replacements.

Finally it must be pointed out that, according to some writers (e.g. Clifton & Marxsen, 1984; Leamer, 1980; Williams, 1970), the test used by Leontief and subsequent writers is incorrect; by employing a revised test, they have shown that the pattern of US trade in 1947 was indeed in accordance with the Heckscher-Ohlin theorem (Williams, Leamer) which, in addition, turns out to be valid for many other countries (though not for all) in more recent times (Clifton and Marxsen). See also Leamer (1984) for an original study according to which "what emerges from the data analysis is a surprisingly good explanation of the main features of the trade data in terms of a relatively brief list of resource endowments" (p. 187). However, contrary to this result, Bowen, Leamer, and Sveinkauskas (1987), using the data on foreign trade of 27 countries in 1967, found that the Heckscher-Ohlin proposition was not confirmed.

4.6.2.2 Wrong Assumptions in the Model?

Let us now pass to the studies which belong to the *second group*, and begin with non-identical structures of demand. As we know (see Sect. 4.5.1) if the United States had tastes strongly biased in favour of the capital-intensive goods (the supposedly abundant factor), this might imply an import of these goods, whence the paradox. However, a study by Houthakker (1957) gives evidence for the contrary, namely for a similarity of the demand functions in different countries. Besides, it is a general phenomenon that, as per-capita income increases, society tends to spend more on labour-intensive goods such as services. It follows that, at the time considered by Leontief, the structure of US demand should have been biased in favour of labour-intensive goods relative to the rest of the world, that is, in exactly the opposite direction to that required for the paradox to occur.

Another important strand in the Leontief paradox problem is that consisting of those studies which aim to show that the phenomenon of factor-intensity reversals, far from being an exception, is the norm. The first systematic study in this sense is due to Minhas (1962) who, by using constant elasticity of substitution (CES) production functions, found that factor intensity reversals were quite frequent in reality. However, subsequent studies gave conflicting results (for example, Philpot, 1970, obtained results contrary to Minhas', whilst Yeung & Tsang, 1972, observed the presence of reversals), so that it is not possible to draw definite conclusions. It should however be noted that, as Fisher and Hillman (1984) have shown, the possible presence of factor intensity reversals at the level of single products or industries has no direct relevance for the aggregate ($2 \times 2 \times 2$) version of the Heckscher-Ohlin theorem.

In the traditional Heckscher-Ohlin theorem it is assumed that all countries produce (or can produce) the same goods. This is in disagreement with facts, as we shall see in Chap. 7; here we only wish to point out that, according to Brecher and Choudhri (1984), if one introduces new products in the Heckscher-Ohlin model, it is possible to give a satisfactory explanation of the Leontief paradox. We have so far examined some of the explanations of Leontief's paradox on the assumption that it exists. But this may not be so correct, since subsequent studies carried out with reference to both the United States and other countries have not systematically confirmed the presence of the paradox. As regards the United States, Stern and Maskus's (1981), already cited, confirmed the presence of Leontief's paradox by using the 1958 input-output table, whilst the paradox disappeared when the 1972 table was used. It should however be remembered that Stern and Maskus also take account of human capital (see above), so that their results are not directly comparable with Leontief's.

Wood (1994) argues that, contrary to the findings of most previous empirical tests, Heckscher-Ohlin theory provides an accurate explanation of the pattern of trade. The crucial point of his claim is that in testing this theory one should only consider internationally *immobile* factors, as this is the framework of the theory. Now, since capital is internationally mobile, all empirical tests that take capital into account and treat it as an immobile factor like land, do in fact mis-specify the theory. What Wood does is to examine the pattern of North-South trade in manufactures using a Heckscher-Ohlin model in which the factors of production are simply skilled and unskilled labour, which have a very low mobility between the North (the industrial countries) and the South (the developing countries). The empirical results are quite good, since he finds that the North (abundant in skilled labour) exports skill-intensive manufactures to the South (which is abundant in unskilled labour) in exchange for unskilled-labour-intensive manufactures. The importance of capital mobility in interpreting Leontief paradoxes is also stressed by Gaisford (1995). The role of capital mobility in the context of the Heckscher-Ohlin model is treated in Sect. 6.8.1.

The Heckscher-Ohlin theory assumes identical technology between countries. Yet, it is generally recognised that technology and factor supply differences can jointly determine comparative advantage. Harrigan (1997) proposes an empirical model aimed at jointly estimating the impact of different technologies and different factor endowments on international specialization and trade. He assumes Hicks-neutral technology differences across countries in addition to different factor endowments. The empirical estimation based on a data set of ten industrial countries

Table 4.2 Additional information on trade and endowments

Trade of goods and factor services	Values
Exports	\$16, 678.4 million
Imports (competitive)	\$6, 175.7 million
Net exports of capital services (K_T)	\$23, 450 million
Net exports of labour services (L_T)	1.990 million man-years
Capital/labour intensity of trade (K_T/L_T)	\$11, 783 per man-year

over 20 years for seven different manufacturing sectors show that technology differences are an important determinant of specialization, and that factor supplies alone cannot explain which industrial countries produce which goods.

As regards other countries, studies carried out in the years 1959–1962 by various authors (for a survey, see Bhagwati, 1964, pp. 24–25) with reference to Japan, India, East Germany, and Canada, in some cases confirmed Leontief's paradox and in others did not; similarly the article by Clifton and Marxsen (1984) already cited, shows that the pattern of trade in various countries (Australia, Ireland, Japan, Korea and New Zealand, besides the United States) conforms to the Heckscher-Ohlin theorem, whilst that of other countries (Israel, Kenya, and the United Kingdom) does not.

4.6.3 What Paradox? There Is No Paradox

In this section we reconsider the Leontief paradox in the light of the Heckscher-Ohlin-Vanek model. We follow the line of thought in Leamer (1980) since it is particularly instructive.

We have shown in Table 4.1 above the nature of the paradox: Leontief's (1953) study shows that the capital intensity of US exports is lower than the capital intensity of US imports. This result, at first sight, would imply that the US were a labour abundant country in 1947, and this is at odds with the sound opinion that the US were a capital abundant country. But this is not all. According to Leamer, Leontief reported additional findings as complementary information. These findings are summarized in Table 4.2, adapted from Leamer (1980, Table 2).

This table shows that the US had a trade balance surplus and was a net exporter of the services of both factors.

Leamer (1980, Table 3) supplements this information with results based on Travis (1964),[8] summarized here in Table 4.3. This table shows that the capital intensity in net exports was higher than in production and that the capital intensity in production was higher than in consumption.

[8]Net exports data are taken from Table 4.2. Production data are drawn from Travis (1964, Table 7 on p. 108). Consumption data are calculated using the identity Consumption = Production—Net Exports.

Table 4.3 Capital intensity of consumption, production, and trade

	Production	Net exports	Consumption
Capital ($ million)	328.519	23, 450	305, 069
Labour (million man-years)	47.273	1.99	45.28
Capital/labour ($ per man-year)	6, 949	11, 783	6, 737

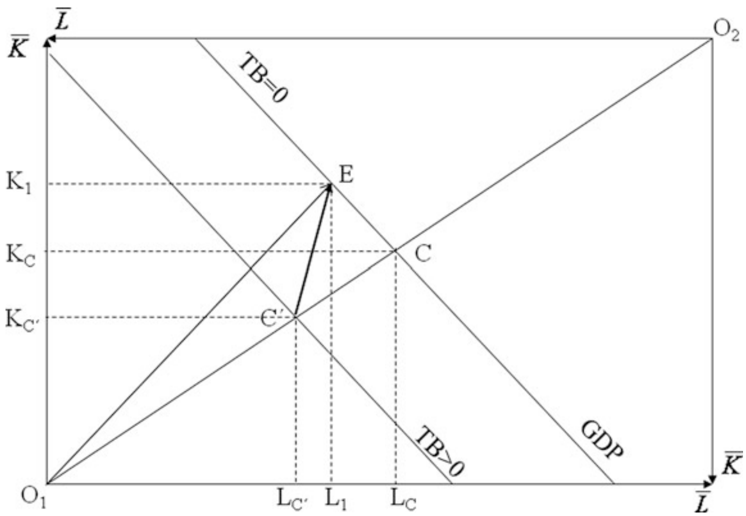

Fig. 4.16 The Leontief paradox reconsidered

Leamer made use of Tables 4.2 and 4.3. To understand Leamer's reasoning we have to bear in mind the Heckscher-Ohlin-Vanek model (see Sects. 4.4 and 4.5.3). Let us begin by identifying the factor content of trade vector in a situation of trade surplus.

A trade surplus takes place when production exceeds consumption. Thus, a country experiencing a trade surplus is consuming less than what it could, it is in fact saving part of its income. This implies that the vector of factor services embodied in consumption is smaller than the vector of factor services embodied in the maximum level of consumption that the country may achieve. In Fig. 4.16, the vector O_1C represents the latter and the vector O_1C' represents the former.

The two vectors have identical slope given the homotheticity of preferences. The line TB $= 0$ indicates the value of consumption corresponding to the situation of equilibrium of the trade balance. This line corresponds to the GDP line since all income is spent. The line TB > 0 indicates the value of consumption corresponding to the situation of trade balance surplus. With E being the endowment point, the vector $C'E$ is the factor content of trade vector for the country with the trade balance surplus (country 1).[9] Clearly, the country is a net exporter of the services of both factors. The slopes of the vectors $C'E$, O_1E, and O_1C represent, respectively, the capital intensity in net exports, in production, and in consumption. Remarkably, their ranking is precisely as found in the data reported in Table 4.3: namely, the slope of $C'E$, is larger than the slope of O_1E, which is larger than the slope of O_1C. It is surprising that for so long Leontief's findings did not stimulate investigation in the direction of comparing the factor content of consumption, trade, and production. The reason is that Leontief and many other scholars after him were not using the correct theoretical framework. They were thinking in terms of the two-by-two version of the Heckscher-Ohlin theory according to which we should find that the capital intensity in exports exceeds the capital intensity in imports for a capital abundant country. Yet, when there are more goods than factors the ordering of exports and imports by factor intensities is compatible with either ordering of relative factor abundance. In such a context, we have seen above (Fig. 4.14) a case where the capital abundant country imports a capital intensive good.

Therefore, the finding that the capital intensity in exports is smaller than in imports for a capital abundant country is not, per se, an invalidation of the Heckscher-Ohlin theory. We have seen above that a more robust prediction of the theory is formulated in terms of the Heckscher-Ohlin-Vanek theorem according to which, in balanced trade, each country exports the services of its abundant factor. Of course, if we observe the country being a net exporter of both factors it must be that it has a (sufficiently large) trade surplus. In this case we should observe the capital intensity in net trade to be larger than the capital intensity in consumption. Further, regardless of the trade balance, for a capital abundant country we should observe the capital intensity in production to exceed that in consumption, which is exactly the point made by Leamer. In conclusion, when using the correct theoretical framework, the paradox disappears.

4.6.4 Factor Content of Trade Studies

Seeing the Heckscher-Ohlin theory through the lenses of the factor content of trade has changed the way empirical research is conducted. We discuss in this section the logic of some of these verifications. The empirical verification consist in computing the factor content of trade from trade and technology data and comparing it with the factor content of trade resulting from the difference between factor content of production and the factor content of consumption. As discussed in Sect. 4.4, the two vectors should coincide; indeed, mathematically, from the former we obtain the latter.

It is convenient to discuss the matter by means of an example. Let the endowment of capital and labour in a country be, respectively, $K_1 = 10$ and $L_1 = 5$, and let

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[9] Obviously, country 2 is running a trade balance deficit (its consumption exceeds production) and its factor content of trade vector is EC'.

the GDP of this country be, for instance, 8 % of the world GDP. Let $\bar{K} = 100$ and $\bar{L} = 80$ be world endowments. The country in question is therefore relatively well endowed with K. Let K_1^T and L_1^T denote the factor content of trade obtained from trade and technology data. K_1^T and L_1^T are obtained by multiplying the vector of net exports by the factor input per unit of output for each good. This should be equivalent to the factor content of trade obtained from endowments and consumption. Let $\hat{K}_1^T$ and $\hat{L}_1^T$ denote the latter. With reference to Fig. 4.11, we have $\hat{K}_1^T \equiv K_1 - K_{C1}$ and $\hat{L}_1^T \equiv L_1 - L_{C1}$. In our example they are $\hat{K}_1^T = 10 - 0.08 \times 100 = 2$, and $\hat{L}_1^T = 5 - 0.08 \times 80 = -1.4$. The two computations should give identical results, that is, we should find $\hat{K}_1^T = K_1^T$, and $\hat{L}_1^T = L_1^T$. This is in a nutshell the logic of the empirical studies based on the factor content of trade. The results of many such studies have shown that the two computations do not give identical results. In many cases even the signs do not match, that is K_1^T and $\hat{K}_1^T$ have opposite sign, likewise for L. Furthermore, K_1^T and L_1^T are often very small in absolute magnitude with respect to $\hat{K}_1^T$, and $\hat{L}_1^T$. This means that the observed volumes of trade are very small with respect to the volumes that we would expect to observe given factor endowment differences. This phenomenon is been dubbed by Trefler (1995) the "mystery of the missing trade". We discuss here three different ways to reconcile theory with data.

The first approach, suggested in Trefler (1993), consists in estimating the technological difference needed for the theory to fit the data perfectly and then verify the plausibility of these estimates against an alternative and independent indicator of technological differences. Thus, returning to our example, the first step is to multiply the endowment of each factor by parameters π_{Ki} and π_{Li} which reflect the productivity of that factor in country i. The resulting factor content of trade in our example becomes $\hat{K}_1^T = \pi_{K1}10 - 0.08 * (\pi_{K1}K_1 + \pi_{K2}K_2)$, and $\hat{L}_1^T = \pi_{L1}5 - 0.08 * (\pi_{L1}L_1 + \pi_{L2}L_2)$, and analogously for country 2. The equations $K_1^T = \hat{K}_1^T$, and $L_1^T = \hat{L}_1^T$ allow estimating the productivity parameters of each factor in country 1 relative to the same factor in country 2. Having done this estimation, the second step is to compare these estimates with alternative measures of productivity differences. Trefler does this by comparing the estimates of labour productivity differences with observed real wage differences between countries. One expects to see that higher wages correspond to higher estimates of productivity (in our example, if one finds $\pi_{L1} > \pi_{L2}$ then one should also observe the wage in country 1 to be higher than in country 2). Indeed the correlation between estimated productivity and wages found by Trefler is extremely high (he reports an estimated coefficient of 0.9). This result lends support to the technology-amended version of the Heckscher-Ohlin theory but is only indirect evidence. Evidence is indirect since the factor content of trade equation holds as an identity given the degrees of freedom generated by the insertion of the productivity parameters. A more direct evidence can be found in the second approach, proposed in Trefler (1995).

The second approach consists, broadly speaking, in restricting the technology differences to be uniform in the sense that all factors are assumed to be proportionally more productive in a country with respect to the other country.[10] Then the factor content of trade equation is no longer an identity. Using the second approach Trefler finds that nearly one half of the "missing trade" is explained by uniform productivity differences between countries. This result too, and even more directly than the first approach, gives support to a technology amended version of the H-O theory.

A third approach is suggested by Davis and Weinstein (2001). Their starting point is that in the presence of barriers to international trade there is no complete convergence of commodity prices; neither in absolute nor in relative terms. Therefore, as is clear from the study of Sect. 4.3.1, there is no complete equalization of factor prices either. In particular, in each country the relative price of the relatively scarce factor will be higher than what it would be if commodity prices had converged completely (see Fig. 4.8 in Sect. 4.3.1). As a consequence, the factor intensity for each industry will differ between countries: the K-abundant country will use more K-intensive techniques than the L-abundant country in all industries. This becomes important when computing K_1^T and L_1^T. In computing the factor content of exports and imports one should apply the techniques prevailing in each country. Davis and Weinstein find that the factor content of trade equation fits the data well when account is taken of the different techniques between countries. It is worth mentioning that this result does not require assuming different technologies between countries and, in this sense, it represents an even stronger evidence in support of the Heckscher-Ohlin-Vanek model.

4.6.5 Concluding Remarks

The first confrontation of the Heckscher-Ohlin theory with data has been rather traumatic since Leontief's results, at first sight, appeared as paradoxical. A number of explanations have been proposed for the paradox but the empirical performance of the theory remained far from satisfactory. Since the 1980s however the theory has fared pretty well. First, the correct interpretation of Leontief's result showed that there was no paradox after all. Second, factor content of trade studies have provided solid empirical support for the theory. The fact that the empirical performance of the Heckscher-Ohlin model improves when taking account of technology and demand differences between countries is particularly in line with what the neoclassical theory (see Sect. 1.2 and Chap. 3) had conjectured already one and a half century ago. Overall, this ancient theory proves to be very relevant to explain contemporary patterns of trade.

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[10]In our example, for instance, if L is δ % more productive in country 1 than in country 2, so is K in exactly the same proportion δ. These are called Hicks neutral technology differences. For a more detailed definition see Sect. 13.5.1.

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