Chapter 1

1

Basic Concepts

1 Basic Concepts

Objectives: *Statics* is the study of forces acting on bodies that are in equilibrium. To investigate statics problems, it is necessary to be familiar with some basic terms, formulas, and work principles. Of particular importance are the *method of sections*, the *law of action and reaction*, and the *free-body diagram*, as they are used to solve nearly all problems in statics.

1.1 Force

The concept of *force* can be taken from our daily experience. Although forces cannot be seen or directly observed, we are familiar with their effects. For example, a helical spring stretches when a weight is hung on it or when it is pulled. Our muscle tension conveys a qualitative feeling of the force in the spring. Similarly, a stone is accelerated by gravitational force during free fall, or by muscle force when it is thrown. Also, we feel the pressure of a body on our hand when we lift it. Assuming that gravity and its effects are known to us from experience, we can characterize a force as a quantity that is comparable to gravity.

In statics, bodies at rest are investigated. From experience we know that a body subject *only* to the effect of gravity, falls. To prevent a stone from falling, to keep it in equilibrium, we need to exert a force on it, for example our muscle force. In other words:

> A force is a physical quantity that can be brought into equilibrium with gravity.

1.2 Characteristics and Representation of a Force

A single force is characterized by three properties: magnitude, direction, and point of application.

The quantitative effect of a force is given by its *magnitude*. A qualitative feeling for the magnitude is conveyed by different muscle tensions when we lift different bodies or when we press against a wall with varying intensities. The magnitude F of a force can be measured by comparing it with gravity, i.e., with calibrated or standardized weights. If the body of weight W in Fig. 1.1 is in equilibrium, then $F = W$. The "Newton", abbreviated N (cf. Section 1.6), is used as the unit of force.

From experience we also know that force has a *direction*. While gravity always has an effect downwards (towards the earth's center), we can press against a tabletop in a perpendicular or in an inclined manner. The box on the smooth surface in Fig. 1.2 will

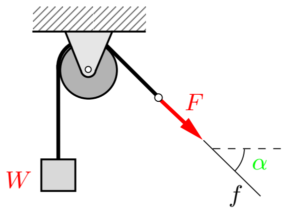

Fig. 1.1

move in different directions, depending on the direction of the force exerted upon it. The direction of the force can be described by its *line of action* and its sense of direction (orientation). In Fig. 1.1, the line of action f of the force F is inclined under the angle α to the horizontal. The sense of direction is indicated by the arrow.

Finally, a single force acts at a certain *point of application*. Depending on the location of point A in Fig. 1.2, the force will cause different movements of the box.

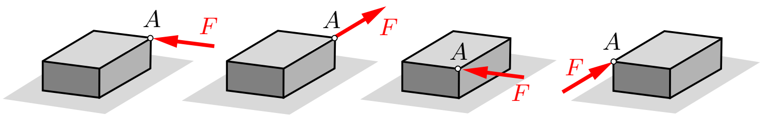

Fig. 1.2

A quantity determined by magnitude and direction is called a vector. In contrast to a free vector, which can be moved arbitrarily in space provided it maintains its direction, a force is tied to its line of action and has a point of application. Therefore, we conclude:

The force is a bound vector.

According to standard vector notation, a force is denoted by a boldfaced letter, for example by $\boldsymbol{F}$, and its magnitude by $|\boldsymbol{F}|$ or simply by F. In figures, a force is represented by an arrow, as shown in Figs. 1.1 and 1.2. Since the vector character usually is uniquely determined through the arrow, it is usually sufficient to write only the magnitude F of the force next to the arrow.

In Cartesian coordinates (see Fig. 1.3 and Appendix A.1), the force vector can be represented using the unit vectors $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$

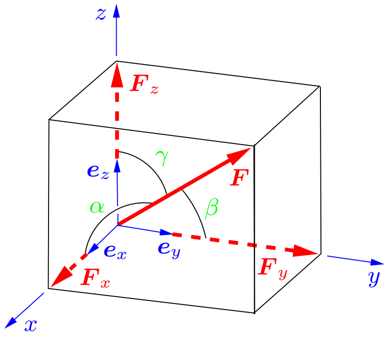

Fig. 1.3

by

$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z = F_x\,\boldsymbol{e}_x + F_y\,\boldsymbol{e}_y + F_z\,\boldsymbol{e}_z\,. \tag{1.1}$$

Applying Pythagoras' theorem in space, the force vector's magnitude F is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}\,. \tag{1.2}$$

The direction angles and therefore the direction of the force follow from

$$\cos\alpha = \frac{F_x}{F}\,, \quad \cos\beta = \frac{F_y}{F}\,, \quad \cos\gamma = \frac{F_z}{F}\,. \tag{1.3}$$

1.3 The Rigid Body

1.3

A body is called a *rigid body* if it does not deform under the influence of forces; the distances between different points of the body remain constant. This is, of course, an idealization of a real body composed of a reasonably stiff material which in many cases is fulfilled in a good approximation. From experience with such bodies it is known that a single force may be applied at any point on the line of action without changing its effect on the body as a whole (*principle of transmissibility*).

This principle is illustrated in Fig. 1.4. In the case of a deformable sphere, the effect of the force depends on the point of

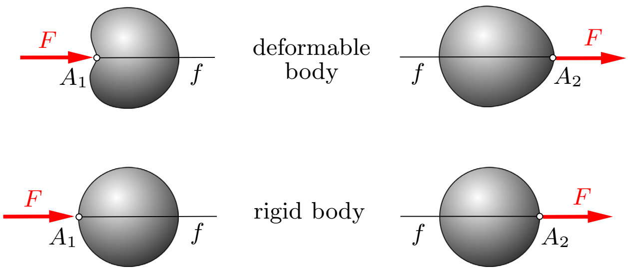

Fig. 1.4

application. In contrast, for a rigid sphere the effect of the force F on the entire body is the same, regardless of whether the body is pulled or pushed. In other words:

> The effect of a force on a rigid body is independent of the location of the point of application on the line of action. The forces acting on rigid bodies are "sliding vectors": they can arbitrarily be moved along their action lines.

A parallel displacement of forces changes their effect considerably. As experience shows, a body with weight W can be held in equilibrium if it is supported appropriately (underneath the center of gravity) by the force F, where $F = W$ (Fig. 1.5a). Displacing force F in a parallel manner causes the body to rotate (Fig. 1.5b).

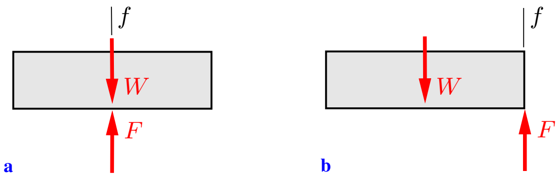

Fig. 1.5

1.4 Classification of Forces, Free-Body Diagram

1.4

A single force with a line of action and a point of application, called a *concentrated force*, is an idealization that in reality does not exist. It is almost realized when a body is loaded over a thin wire or a needlepoint. In nature, only two kinds of forces exist: volume forces and surface or area forces.

A *volume force* is a force that is distributed over the volume of a body or a portion thereof. Weight is an example of a volume force. Every small particle (infinitesimal volume element $\mathrm{d}V$) of the entire volume has a certain small (infinitesimal) weight $\mathrm{d}W$ (Fig. 1.6a). The sum of the force elements $\mathrm{d}W$, which are continuously distributed within the volume yields the total weight W. Other examples of volume forces include magnetic and electrical forces.

Area forces occur in the regions where two bodies are in contact. Examples of forces distributed over an area include the water pressure p at a dam (Fig. 1.6b), the snow load on a roof or the pressure of a body on a hand.

A further idealization used in mechanics is the *line force*, which comprises forces that are continuously distributed along a line. If a blade is pressed against an object and the finite thickness of the blade is disregarded, the line force q will act along the line of contact (Fig. 1.6c).

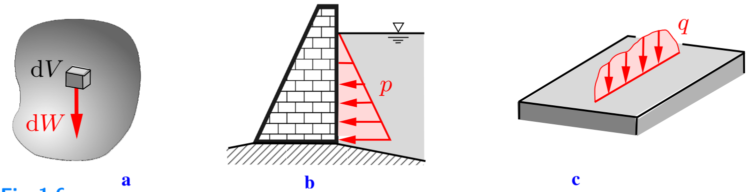

Fig. 1.6

Forces can also be classified according to other criteria. *Active forces* refer to the physically prescribed forces in a mechanical system, as for example the weight, the pressure of the wind or the snow load on a roof.

Reaction forces are generated if the freedom of movement of a body is constrained. For example, a falling stone is subjected only to an active force due to gravity, i.e., its weight. However, when the stone is held in the hand, its freedom of movement is constrained; the hand exerts a reaction force on the stone.

Reaction forces can be visualized only if the body is separated from its geometrical constraints. This procedure is called *freeing* or *cutting free* or *isolating* the body. In Fig. 1.7a, a beam is loaded by an active force W. Supports A and B prevent the beam from moving: they act on it through reaction forces that, for simplicity, are also denoted by A and B. These reaction forces are made visible in the so-called *free-body diagram* (Fig. 1.7b). It shows the forces acting on the body instead of the geometrical constraints through the supports. By this "freeing", the relevant forces become accessible to analysis (cf. Chapter 5). This procedure is still valid when a mechanical system becomes movable (dynamic) due to freeing. In this case, the system is regarded as being frozen when the reaction forces are determined. This is known as the *principle of solidification* (cf. Section 5.3).

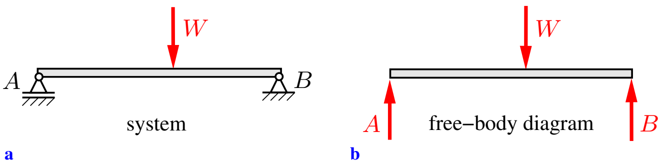

Fig. 1.7

A further classification is introduced by distinguishing between *external forces* and *internal forces*. An *external force* acts from the outside on a mechanical system. Active forces as well as reaction forces are external forces. *Internal forces* act between the parts of a system. They also can be visualized only by imaginary cutting or *sectioning* of the body. If the body in Fig. 1.8a is sectioned by an imaginary cut, the internal area forces p distributed over the cross-section must be included in the free-body diagram; they replace the initially perfect bonding between the two exposed surfaces (Fig. 1.8b). This procedure is based on the hypothesis, which is confirmed by experience, that the laws of mechanics are equally valid for parts of the system. Accordingly, the system initially consists of the complete body at rest. After the cut, the system consists of two parts that act on each other through area forces in such a way that each part is in equilibrium. This procedure, which enables calculation of the internal forces, is called the *method of sections*. It is valid for systems in equilibrium as well as for systems in motion.

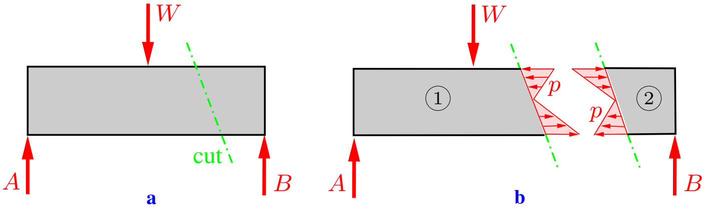

Fig. 1.8

Whether a force is an external or an internal force depends on the system to be investigated. If the entire body in Fig. 1.8a is considered to be the system, then the forces that become exposed by the cut are internal forces: they act between the parts of the system. On the other hand, if only part ① or only part ② of the body in Fig. 1.8b is considered to be the system, the corresponding forces are external forces.

As stated in Section 1.3, a force acting on a rigid body can be displaced along its line of action without changing its effect on the body. Consequently, the principle of transmissibility can be used in the analysis of the external forces. However, this principle can generally not be applied to internal forces. In this case, the body is sectioned by imaginary cuts, therefore it matters whether an external force acts on one or the other part.

The importance of internal forces in engineering sciences is derived from the fact that their magnitude is a measure of the stress in the material.

1.5 Law of Action and Reaction 1.5

A universally accepted law, based on everyday experience, is the *law of action and reaction*. This axiom states that a force always has a counteracting force of the same magnitude but of opposite direction. Therefore, a force can never exist alone. If a hand is pressed against a wall, the hand exerts a force F on the wall (Fig. 1.9a). An opposite force of the same magnitude acts from the wall on the hand. These forces can be made visible if the two bodies are separated at the area of contact. Note that the forces act upon two different bodies. Analogously, a body on earth has a certain weight W due to gravity. However, the body acts upon the earth

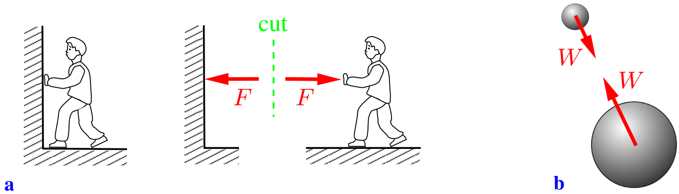

Fig. 1.9

with a force of equal magnitude: they attract each other (Fig. 1.9b). In short:

> The forces that two bodies exert upon each other are of the same magnitude but of opposite directions and they lie on the same line of action.

This principle, which Newton succinctly expressed in Latin as

> actio = reactio

is the third of Newton's axioms (cf. Volume 3). It is valid for long-range forces as well as for short-range forces, and it is independent of whether the bodies are at rest or in motion.

1.6

1.6 Dimensions and Units

In mechanics the three basic physical quantities length, time and mass are considered. Force is another important element that is considered; however, from a physical point of view, force is a derived quantity. All other mechanical quantities, such as velocity, momentum or energy can be expressed by these four quantities. The geometrical space where mechanical processes take place is three-dimensional. However, as a simplification the discussion is limited sometimes to two-dimensional or, in some cases, one-dimensional problems.

Associated with length, time, mass and force are their dimensions $[l]$, $[t]$, $[M]$ and $[F]$. According to the international SI unit system (Système International d'Unités), they are expressed using the base units meter (m), second (s) and kilogram (kg) and the derived unit newton (N). A force of 1 N gives a mass of 1 kg the acceleration of $1\,\mathrm{m/s^2}$: $1\,\mathrm{N} = 1\,\mathrm{kg\,m/s^2}$. Volume forces have the dimension force per volume $[F/l^3]$ and are measured, for example, as a multiple of the unit $\mathrm{N/m^3}$. Similarly, area and line forces have the dimensions $[F/l^2]$ and $[F/l]$ and the units $\mathrm{N/m^2}$ and N/m, respectively.

The magnitude of a physical quantity is completely expressed by a number and the unit. The notations $F = 17\,\mathrm{N}$ or $l = 3\,\mathrm{m}$ represent a force of 17 newtons or a distance of 3 meters, respectively. In numerical calculations units are treated in the same way as numbers. For example, using the above quantities, $F \cdot l = 17\,\mathrm{N} \cdot 3\,\mathrm{m} = 17 \cdot 3\,\mathrm{Nm} = 51\,\mathrm{Nm}$. In physical equations, each side and each additive term must have the same dimension. This should always be kept in mind when equations are formulated or checked.

Very large or very small quantities are generally expressed by attaching prefixes to the units meter, second, newton, and so forth: k (kilo $= 10^3$), M (mega $= 10^6$), G (giga $= 10^9$) and m (milli $= 10^{-3}$), μ (micro $= 10^{-6}$), n (nano $= 10^{-9}$), respectively; for example: $1\,\mathrm{kN} = 10^3\,\mathrm{N}$.

Table 1.1

	U.S. Customary Unit		SI Equivalent
Length	1 ft		0.3048 m
	1 in	(12 in = 1 ft)	25.4 mm
	1 yd	(1 yd = 3 ft)	0.9144 m
	1 mi		1.609344 km
Force	1 lb		4.4482 N
Mass	1 slug		14.5939 kg

In the U.S. and some other English speaking countries the U.S. Customary system of units is still frequently used although the SI system is recommended. In this system length, time, force and mass are expressed using the base units foot (ft), second (s), pound (lb) and the derived mass unit, called a *slug*: 1 slug = 1 $\mathrm{lb\,s^2/ft}$. As division and multiples of length the inch (in), yard (yd) and mile (mi) are used. In Table 1.1 common conversion factors are listed.

1.7

1.7 Solution of Statics Problems, Accuracy

To solve engineering problems in the field of mechanics a careful procedure is required that depends to a certain extent on the type of the problem. In any case, it is important that engineers express themselves clearly and in a way that can be readily understood since they have to present the formulation as well as the solution of a problem to other engineers and to people with no engineering background. This clarity is equally important for one's own process of understanding, since clear and precise formulations are the key to a correct solution. Although, as already mentioned, there is no fixed scheme for handling mechanical problems, the following steps are usually necessary:

1. Formulation of the engineering problem.
2. Establishing a mechanical model that maps all of the essential characteristics of the real system. Considerations regarding the quality of the mapping.
3. Solution of the mechanical problem using the established model. This includes:
   - Identification of the given and the unknown quantities. This is usually done with the aid of a sketch of the mechanical system. Symbols must be assigned to the unknown quantities.
   - Drawing of the free-body diagram with all the forces acting on the system.
   - Formulation of the mechanical equations, e.g. the equilibrium conditions.

- Formulation of the geometrical relationships (if needed).
- Solving the equations for the unknowns. It should be ensured in advance that the number of equations is equal to the number of unknowns.
- Display of the results.

4. Discussion and interpretation of the solution.

In the examples given in this textbook, usually the mechanical model is provided and Step 3 is concentrated upon, namely the solution of mechanical problems on the basis of models. Nevertheless, it should be kept in mind that these models are mappings of real bodies or systems whose behavior can sometimes be judged from daily experience. Therefore, it is always useful to compare the results of a calculation with expectations based on experience.

Regarding the accuracy of the results, it is necessary to distinguish between the numerical accuracy of calculations and the accuracy of the model. A numerical result depends on the precision of the input data and on the precision of our calculation. Therefore, the results can never be more precise than the input data. Consequently, results should never be expressed in a manner that suggests a non-existent accuracy (e.g., by many digits after the decimal point).

The accuracy of the result concerning the behavior of the real system depends on the quality of the model. For example, the trajectory of a stone that has been thrown can be determined by taking air resistance into account or by disregarding it. The results in each case will, of course, be different. It is the task of the engineer to develop a model in such a way that it has the potential to deliver the accuracy required for the concrete problem.

1.8

1.8 Summary

- Statics deals with bodies that are in equilibrium.
- A force acting on a rigid body can be represented by a vector that can be displaced arbitrarily along its line of action.
- An active force is prescribed by a law of physics. Example: the weight of a body due to earth's gravitational field.
- A reaction force is induced by the constrained freedom of movement of a body.
- Method of sections: reaction forces and internal forces can be made visible by virtual cuts and thus become accessible to an analysis.
- Free-body diagram: representation of all active forces and reaction forces which act on an isolated body. Note: mobile parts of the body can be regarded as being "frozen" (principle of solidification).
- Law of action and reaction: actio = reactio.
- Basic physical quantities are length, mass and time. The force is a derived quantity: $1\,\text{N} = 1\,\text{kg}\,\text{m}/\text{s}^2$.
- In mechanics idealized models are investigated which have the essential characteristics of the real bodies or systems. Examples of such idealizations: rigid body, concentrated force.