

Chapter 6

Trusses

6

6 Trusses

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Objectives: A truss is a structure composed of slender members that are connected at their ends by joints. The truss is one of the most important structures in engineering applications. After studying this chapter, students should be able to recognise if a given truss is statically and kinematically determinate. In addition, they will become familiar with methods to determine the internal forces in a statically determinate truss.

6.1 Statically Determinate Trusses

A structure that is composed of straight slender members is called a *truss*. To be able to determine the internal forces in the individual members, the following assumptions are made:

1. The members are connected through smooth pins (frictionless joints).
2. External forces are applied at the pins only.

A truss that satisfies these assumptions is called an “ideal truss”. Its members are subjected to tension or to compression only (two-force members, see Section 2.4).

In real trusses, these ideal conditions are not exactly satisfied. For example, the joints may not be frictionless, or the ends of the members may be welded to a gusset plate. Even then, the assumption of frictionless pin-jointed connections yields satisfactory results if the axes of the members are concurrent at the joints. Also, external forces may be applied along the axes of the members (e.g., the weights of the members). Such forces are either neglected (e.g., if the weights of the members are small in comparison with the loads) or their resultants are replaced by statically equivalent forces at the adjacent pins.

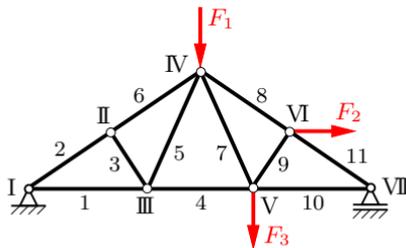


Fig. 6.1

In this chapter we focus on plane trusses; space trusses can be treated using the same methods. As an example, consider the truss shown in Fig. 6.1. It consists of 11 members which are connected with 7 pins (the pins at the supports are also counted). The members are marked with Arabic numerals and the pins with Roman numerals.

To determine the internal forces in the members we may draw a free-body diagram for every joint of the truss. Since the forces at the pins are concurrent forces, there are two equilibrium conditions at each joint (see Section 2.3). In the present example, we thus have $7 \cdot 2 = 14$ equations for the 14 unknown forces (11 forces in the members and 3 forces at the supports).

A truss is called *statically determinate* if all the unknown forces, i.e., the forces in the members and the forces at the supports, can be determined from the equilibrium conditions. Let a plane truss be composed of m members connected through j joints, and let the number of support reactions be r . In order to be able to determine the $m + r$ unknown forces from the $2j$ equilibrium conditions, the number of unknowns has to be equal to the number of equations:

$$2j = m + r. \quad (6.1)$$

This is a *necessary condition* for the determinacy of a plane truss. As we shall discuss later, however, it will not be sufficient in cases of improper support or arrangement of the members. If the truss is rigid, the number of support reactions must be $r = 3$.

In the case of a *space truss*, there exist three conditions of equilibrium at each joint, resulting in a total of $3j$ equations. Therefore,

$$3j = m + r \quad (6.2)$$

is the corresponding necessary condition for a space truss. If the truss is a rigid body, the support must be statically determinate: $r = 6$ (compare Section 3.2.2).

For the truss shown in Fig. 6.2a we have $j = 7$, $m = 10$ and $r = 2 \cdot 2$ (two pin connections). Hence, since $2 \cdot 7 = 10 + 4$, the necessary condition (6.1) is satisfied. The truss is, in addition, completely constrained against motion. Therefore, it is statically determinate.

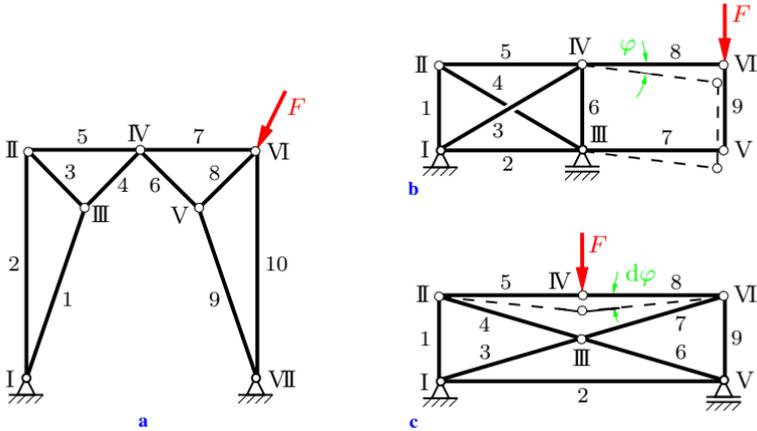


Fig. 6.2

A truss that is completely constrained against motion is called a *kinematically determinate* truss. In contrast, a truss that is not a rigid structure and therefore able to move is called kinematically indeterminate. This is the case if there are fewer unknowns than independent equilibrium conditions. If there are more unknowns than equilibrium conditions, the system is called statically indeterminate.

We shall only consider trusses that satisfy the necessary conditions (6.1) or (6.2), respectively. Even then, a truss will not be statically determinate if the members or the supports are improperly arranged. Consider, for example, the trusses shown in Figs. 6.2b and 6.2c. With $j = 6$, $m = 9$ and $r = 3$ the necessary condition (6.1) is satisfied. However, the members 7 and 8 of the truss in Fig. 6.2b may rotate about a finite angle φ , whereas the members 5 and 8 of the truss in Fig. 6.2c may rotate about an infinitesimally small angle $d\varphi$. Each of the improperly constrained trusses is statically indeterminate.

6.2 Design of a Truss

In the following, we shall discuss three methods for designing a statically and kinematically determinate plane truss.

Method 1: We start with a single bar and add two members to form a triangle (see Fig. 6.3). This basic element represents a rigid body. It may be extended by successively adding two members at a time and connecting them in such a way that the structure remains rigid (one has to avoid having two members along a straight line; such an improper arrangement is indicated by the dashed line in Fig. 6.3). A truss designed from a basic triangle as described is called a *simple plane truss*.

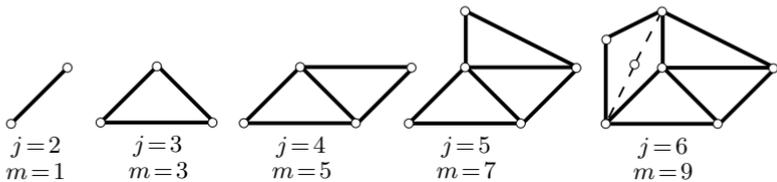


Fig. 6.3

As can be verified by inspection, the relation

$$2j = m + 3 \quad (6.3)$$

is satisfied by the trusses shown in Fig. 6.3. For every additional joint in a simple truss, there are two additional members. Therefore Equation (6.3) remains valid. If the truss is supported in such a way that there are $r = 3$ unknown reactions that completely constrain the truss, then it is statically determinate (see the necessary condition (6.1)).

Method 2: Two simple trusses are connected by three members (Fig. 6.4a). To ensure the rigidity of the system, the axes of the members must not be parallel or concurrent. The two simple trusses may also be connected by a joint and one member: the system in Fig. 6.4b has been obtained by replacing members 2 and 3 in Fig. 6.4a by the joint I.

If two simple trusses are connected through one joint only, the system is nonrigid. To obtain a statically and kinematically determinate system, an additional support must be introduced; an example is given in Fig. 6.4c. This system represents a three-hinged arch. It should be noted that instead of connecting the two simple

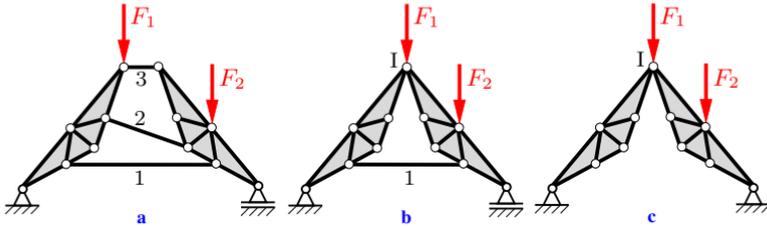


Fig. 6.4

trusses by one joint the trusses may be connected by two members that are not parallel and not concurrent.

It can easily be verified that for the three examples shown in Fig. 6.4, the necessary condition (6.1) is satisfied. Since these systems are completely constrained against motion, they are statically determinate.

Method 3: Consider a truss that has been designed according to the first or the second method. If we remove one member of the truss, the truss will become nonrigid. Therefore, we have to add one member at a different position in such a way that the truss will be rigid again. Since by doing so neither the number of the members nor the number of the joints is changed, condition (6.1) is still satisfied.

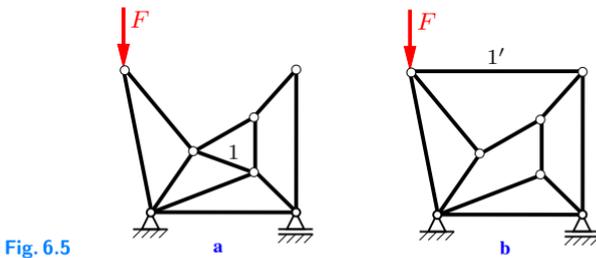


Fig. 6.5

An example of Method 3 is shown in Fig. 6.5. If we remove member 1 from the simple truss in Fig. 6.5a, the system is only partially constrained. Adding member 1' yields the statically determinate truss shown in Fig. 6.5b.

6.3

6.3 Determination of the Internal Forces

In the following, two methods to determine the internal forces in the individual members of a statically determinate truss will be discussed. In both methods, the conditions of equilibrium are applied to suitable free-body diagrams.

6.3.1 Method of Joints

The *method of joints* consists of applying the equilibrium conditions to the free-body diagram of each joint of the truss. It is a systematic method and can be used for every statically determinate truss.

In practice, it is often convenient first to identify those members of the truss that have a vanishing internal force. These members are called *zero-force members*. If the zero-force members are recognised in advance, the number of unknowns is reduced, which simplifies the analysis. It should be noted that the loading determines whether a member is a zero-force member or not.

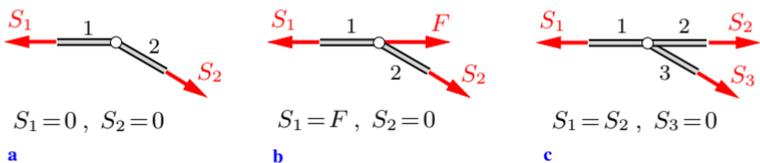


Fig. 6.6

The following rules are useful in identifying zero-force members.

1. If two members are not collinear at an *unloaded* joint (Fig. 6.6a), then both members are zero-force members.
2. Let two members be connected at a *loaded* joint (Fig. 6.6b). If the action line of the external force F coincides with the direction of one of the members, then the other member is a zero-force member.
3. Let three members be connected at an *unloaded* joint (Fig. 6.6c). If two members have the same direction, the third member is a zero-force member.

These rules can be verified by applying the equilibrium conditions to the respective joints.



Fig. 6.7

To free a member of a truss from its constraints we cut it at both ends, i.e., at the adjacent joints (Fig. 6.7a). The corresponding free-body diagram is shown in Fig. 6.7b. The members in a truss are subject to tension or to compression; Fig. 6.7b shows a bar under tension. According to Newton's third law (*actio = reactio*), forces S of equal magnitude and opposite directions act at the pins I and II. As can be seen from Fig. 6.7b an arrow that points away from the pin (pull) indicates tension in the member, whereas an arrow that points toward the pin (push) indicates compression.

It is not always possible to determine by inspection whether a member is subject to tension or compression. Therefore, we shall always assume that all the members of a truss are under tension. If the analysis gives a negative value for the force in a member, this member is in reality subject to compression.

The $m + r$ unknown forces can be determined from the $2j$ equilibrium conditions for the j joints. One may also apply the three equilibrium conditions for the complete truss. These equations are not independent of the equilibrium conditions at the joints. Therefore they may provide a check on the correctness of the analysis. In practice, it may be more convenient to determine first the support reactions from the free-body diagram of the complete truss. Then three other equilibrium equations within the method of joints will serve as checks.

The method of joints can also be used to determine the forces in a space truss where there are three equilibrium conditions at each joint. If the truss is a rigid body, there must be six support forces to ensure a statically determinate support. The six equilibrium conditions for the whole truss can be used as a check. On the other hand, if the support forces are computed in advance, six other equilibrium equations within the method of joints may serve as checks.

E6.1 **Example 6.1** The truss shown in Fig. 6.8a is loaded by an external force F .

Determine the forces at the supports and in the members of the truss.

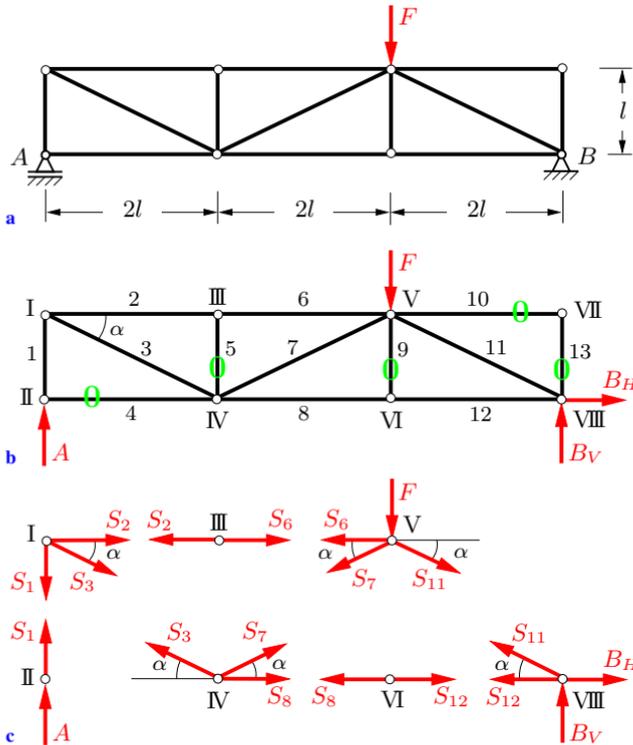


Fig. 6.8

Solution Fig. 6.8a represents a simple truss that is completely constrained against motion. Therefore, it is statically determinate.

The members of the truss are numbered in the free-body diagram of the complete truss (Fig. 6.8b). Zero-force members are identified by inspection and marked with zeroes: member 4 (according to Rule 2), the members 5 and 9 (Rule 3) and the members 10 and 13 (Rule 1).

To further reduce the number of unknown forces, we compute the support forces by applying the equilibrium conditions

to the whole truss:

$$\rightarrow: \quad \underline{\underline{B_H = 0}},$$

$$\overset{\curvearrowright}{A}: \quad -4lF + 6lB_V = 0 \quad \rightarrow \quad \underline{\underline{B_V = \frac{2}{3}F}},$$

$$\overset{\curvearrowleft}{B}: \quad -6lA + 2lF = 0 \quad \rightarrow \quad \underline{\underline{A = \frac{1}{3}F}}.$$

Fig. 6.8c shows the free-body diagrams of the joints. As previously stated, we assume that every member is subjected to tension. Accordingly, all of the corresponding arrows point away from the joints. Zero-force members are omitted in the free-body diagrams. Therefore, joint VII need not be considered. Applying the equilibrium conditions to each joint yields

$$\text{I)} \quad \rightarrow: \quad S_2 + S_3 \cos \alpha = 0,$$

$$\quad \downarrow: \quad S_1 + S_3 \sin \alpha = 0,$$

$$\text{II)} \quad \uparrow: \quad S_1 + A = 0,$$

$$\text{III)} \quad \rightarrow: \quad S_6 - S_2 = 0,$$

$$\text{IV)} \quad \rightarrow: \quad S_8 + S_7 \cos \alpha - S_3 \cos \alpha = 0,$$

$$\quad \uparrow: \quad S_7 \sin \alpha + S_3 \sin \alpha = 0,$$

$$\text{V)} \quad \rightarrow: \quad S_{11} \cos \alpha - S_6 - S_7 \cos \alpha = 0,$$

$$\quad \downarrow: \quad S_7 \sin \alpha + S_{11} \sin \alpha + F = 0,$$

$$\text{VI)} \quad \rightarrow: \quad S_{12} - S_8 = 0,$$

$$\text{VIII)} \quad \rightarrow: \quad B_H - S_{11} \cos \alpha - S_{12} = 0,$$

$$\quad \uparrow: \quad B_V + S_{11} \sin \alpha = 0.$$

These are 11 equations for the 8 unknown forces in the members and the 3 forces at the supports. Since the support forces have been computed in advance and are already known, the analysis is simplified, and three equations may be used as a check on the correctness of the results. Using the geometrical relations

$\sin \alpha = l/\sqrt{5}l^2 = 1/\sqrt{5}$, $\cos \alpha = 2l/\sqrt{5}l^2 = 2/\sqrt{5}$, we obtain

$$\underline{\underline{S_1 = -\frac{1}{3}F}}, \quad \underline{\underline{S_2 = S_6 = -\frac{2}{3}F}}, \quad \underline{\underline{S_3 = \frac{\sqrt{5}}{3}F}},$$

$$\underline{\underline{S_7 = -\frac{\sqrt{5}}{3}F}}, \quad \underline{\underline{S_8 = S_{12} = \frac{4}{3}F}}, \quad \underline{\underline{S_{11} = -\frac{2}{3}\sqrt{5}F}}.$$

It is useful to present the results in dimensionless form in a table, including negative signs:

Table of Forces

i	1	2	3	4	5	6	7	8	9	10	11	12	13
$\frac{S_i}{F}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{\sqrt{5}}{3}$	0	0	$-\frac{2}{3}$	$-\frac{\sqrt{5}}{3}$	$\frac{4}{3}$	0	0	$-\frac{2}{3}\sqrt{5}$	$\frac{4}{3}$	0

The negative values for the members 1, 2, 6, 7 and 11 indicate that these members are under compression.

E6.2 **Example 6.2** Fig. 6.9 shows a spatial truss loaded by two external forces F at the joints IV and V.

Compute the forces in the members 1-6.

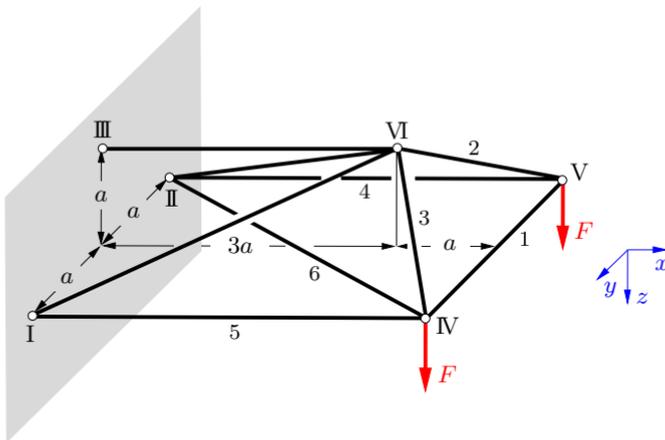


Fig. 6.9

Solution We free the joints V and IV by passing imaginary cuts through the bars, and we assume that the members 1-6 are in tension. The vector equations of equilibrium for these joints are given by

$$\text{V:} \quad S_1 \mathbf{e}_y + S_2 \mathbf{e}_{\text{V/VI}} - S_4 \mathbf{e}_x + F \mathbf{e}_z = \mathbf{0},$$

$$\text{IV:} \quad -S_1 \mathbf{e}_y + S_3 \mathbf{e}_{\text{IV/VI}} - S_5 \mathbf{e}_x + S_6 \mathbf{e}_{\text{IV/II}} + F \mathbf{e}_z = \mathbf{0}.$$

The initially unknown unit vectors can be determined from the vectors connecting adjacent joints, e.g., for $\mathbf{e}_{\text{V/VI}}$ we obtain

$$\mathbf{e}_{\text{V/VI}} = \frac{1}{\sqrt{a^2 + a^2 + a^2}} \begin{pmatrix} -a \\ a \\ -a \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}.$$

Similarly, the other unit vectors are

$$\mathbf{e}_{\text{IV/VI}} = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \quad \mathbf{e}_{\text{IV/II}} = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix},$$

$$\mathbf{e}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Introducing these into the two vector equations we get the six scalar equations

$$\begin{aligned} \text{V:} \quad & -S_2 \frac{1}{\sqrt{3}} - S_4 = 0, & \text{IV:} \quad & -S_3 \frac{1}{\sqrt{3}} - S_5 - S_6 \frac{2}{\sqrt{5}} = 0, \\ & S_1 + S_2 \frac{1}{\sqrt{3}} = 0, & & -S_1 - S_3 \frac{1}{\sqrt{3}} - S_6 \frac{1}{\sqrt{5}} = 0, \\ & -S_2 \frac{1}{\sqrt{3}} + F = 0, & & -S_3 \frac{1}{\sqrt{3}} + F = 0. \end{aligned}$$

Their solution yields the forces

$$\begin{aligned} \underline{\underline{S_2}} &= \underline{\underline{\sqrt{3}F}}, & \underline{\underline{S_1}} &= \underline{\underline{-F}}, & \underline{\underline{S_4}} &= \underline{\underline{-F}}, \\ \underline{\underline{S_3}} &= \underline{\underline{\sqrt{3}F}}, & \underline{\underline{S_6}} &= \underline{\underline{0}}, & \underline{\underline{S_5}} &= \underline{\underline{-F}}. \end{aligned}$$

6.3.2 Method of Sections

It is not always necessary to determine the forces in all of the members of a truss. If several forces only are of interest, it may be advantageous to use the *method of sections* instead of the method of joints. In this case, the truss is divided by a cut into two parts. The cut has to be made in such a way that it either goes through *three* members that do not all belong to the same joint, or passes through *one* joint and *one* member. If the support reactions are computed in advance, the free-body diagram for each part of the truss contains only three unknown forces that can be determined by the three conditions of equilibrium.

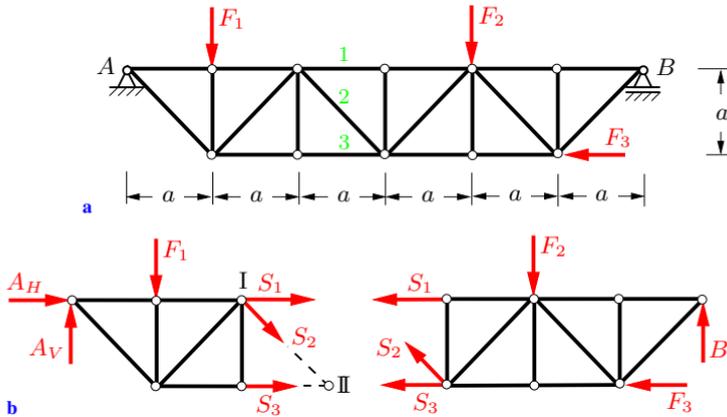


Fig. 6.10

To illustrate the method, we consider the truss shown in Fig. 6.10a with the objective of determining the forces in members 1-3. As a first step, the reactions at the supports A and B are computed by applying the conditions of equilibrium to the free-body diagram of the whole truss (not shown in the figure). In the second step, we pass an imaginary section through the members 1-3, cutting the truss into two parts. Fig. 6.10b shows the free-body diagrams of the two parts of the truss. The internal forces in members 1-3 act as external forces in the free-body diagrams; they are assumed to be tensile forces.

Both parts of the truss in Fig. 6.10b are rigid bodies in equilibrium. Therefore, either part may be used for the analysis. In practice, the part that involves a smaller number of forces will usually lead to a simpler calculation. We shall apply the equilibrium conditions to the free-body diagram on the left-hand side of Fig. 6.10b. It is advantageous to use moment equations about the points of intersection of two unknown forces. Each of the corresponding equations contains *one* unknown force only and can be solved immediately:

$$\begin{aligned} \curvearrowleft \text{I} : & -2a A_V + a F_1 + a S_3 = 0 \quad \rightarrow \quad S_3 = 2A_V - F_1, \\ \curvearrowleft \text{II} : & -3a A_V - a A_H + 2a F_1 - a S_1 = 0 \\ & \rightarrow \quad S_1 = 2F_1 - 3A_V - A_H, \\ \uparrow : & A_V - F_1 - \frac{1}{2}\sqrt{2} S_2 = 0 \quad \rightarrow \quad S_2 = \sqrt{2}(A_V - F_1). \end{aligned}$$

Since the support reactions have been computed in advance, the forces in members 1-3 are now known.

In many cases, the method of sections can be applied without having to determine the forces at the supports. Consider, for example, the truss in Fig. 6.11a. The forces in members 1-3 can be obtained immediately from the equilibrium conditions for the part of the truss on the right as shown in Fig. 6.11b.

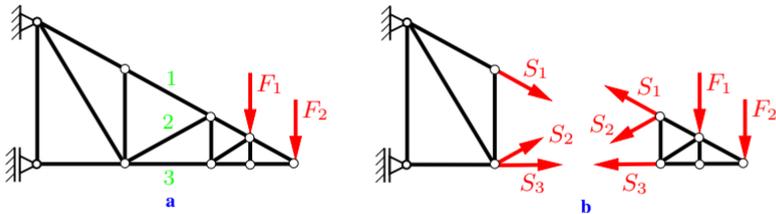


Fig. 6.11

The method of sections is also applicable to spatial trusses. Since there are six equilibrium conditions for a rigid body in the case of a spatial problem, the truss has to be divided by a cut through six members, or through three members and a pin.

E6.3

Example 6.3 A truss is loaded by two forces, $F_1 = 2F$ and $F_2 = F$, as shown in Fig. 6.12a.

Determine the force S_4 .

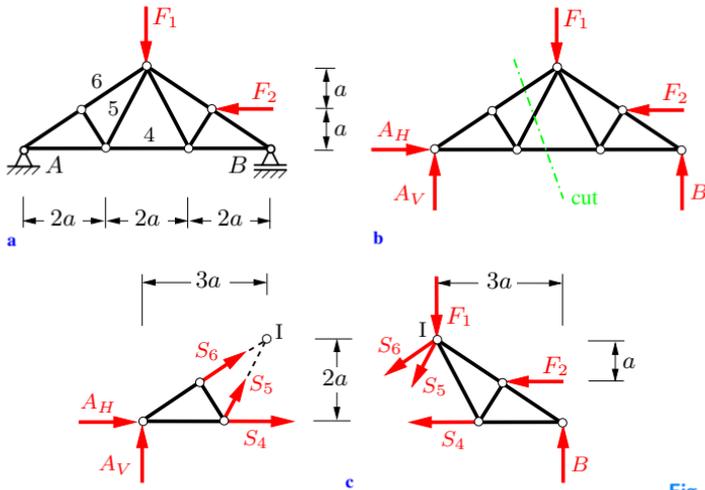


Fig. 6.12

Solution First, we determine the forces at the supports. Applying the equilibrium conditions to the free-body diagram of the whole truss (Fig. 6.12b) yields

$$\begin{aligned} \curvearrowleft A: \quad & -3a F_1 + a F_2 + 6a B = 0 \quad \rightarrow \quad B = \frac{3F_1 - F_2}{6} = \frac{5}{6} F, \\ \curvearrowleft B: \quad & -6a A_V + 3a F_1 + a F_2 = 0 \quad \rightarrow \quad A_V = \frac{3F_1 + F_2}{6} = \frac{7}{6} F, \\ \rightarrow: \quad & A_H - F_2 = 0 \quad \rightarrow \quad A_H = F_2 = F. \end{aligned}$$

Then we pass an imaginary section through the members 4-6 (Fig. 6.12c). The unknown force S_4 follows from the moment equation about point I (intersection of the action lines of the forces S_5 and S_6) of the free-body diagram on the left-hand side of Fig. 6.12c:

$$\begin{aligned} \curvearrowleft I: \quad & 2a S_4 + 2a A_H - 3a A_V = 0 \\ & \rightarrow \quad \underline{\underline{S_4}} = \frac{1}{2}(3A_V - 2A_H) = \underline{\underline{\frac{3}{4} F}}. \end{aligned}$$

The corresponding moment equation for the free-body diagram on the right-hand side may be used as a check:

$$\begin{aligned} \hat{\curvearrowright} : \quad & -2a S_4 + 3a B - a F_2 = 0 \\ \rightarrow \quad & S_4 = \frac{1}{2}(3B - F_2) = \frac{3}{4}F. \end{aligned}$$

6.4 Supplementary Problems

Detailed solutions to most of the following examples are given in (A) D. Gross et al. *Formeln und Aufgaben zur Technischen Mechanik 1*, Springer, Berlin 2011 or (B) W. Hauger et al. *Aufgaben zur Technischen Mechanik 1-3*, Springer, Berlin 2011.

Example 6.4 The truss shown in Fig. 6.13 carries the two forces $F_1 = F$ and $F_2 = 3F$.

Calculate the forces in the members 1, 2 and 3.

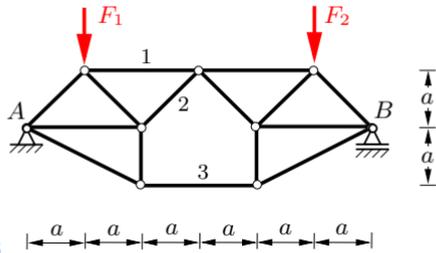


Fig. 6.13

Results: see (B) $S_1 = -2F/3$, $S_2 = -\sqrt{2}F/3$, $S_3 = F$.

Example 6.5 Determine the forces in the members 1, 2 and 3 of the truss shown in Fig. 6.14.

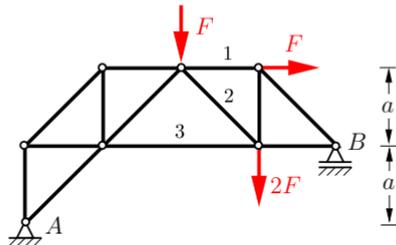


Fig. 6.14

Results: see (A) $S_1 = -3F/2$, $S_2 = -\sqrt{2}F/2$, $S_3 = 3F$.

E6.6

Example 6.6 The structure in Fig. 6.15 consists of a hinged beam AB and five bars. It is subjected to a line load q_0 .

Determine the forces in the bars.

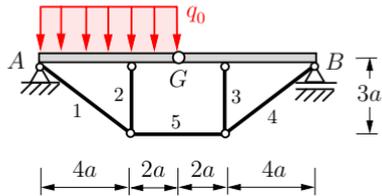


Fig. 6.15

Results: see (B) $S_1 = S_4 = 15 q_0 a / 4$, $S_2 = S_3 = -9 q_0 a / 4$,
 $S_5 = 3 q_0 a$.

E6.7

Example 6.7 Determine the forces in the members 1-7 of the truss shown in Fig. 6.16.

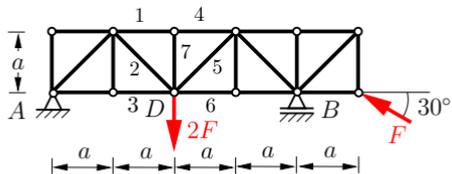


Fig. 6.16

Results: see (A) $S_1 = S_4 = -\frac{9}{4} F$, $S_2 = \frac{9}{8} \sqrt{2} F$, $S_7 = 0$,
 $S_3 = \left(\frac{9}{8} - \frac{\sqrt{3}}{2}\right) F$, $S_5 = \frac{7}{8} \sqrt{2} F$, $S_6 = \left(\frac{11}{8} - \frac{\sqrt{3}}{2}\right) F$.

E6.8

Example 6.8 The truss shown in Fig. 6.17 carries the forces $F_1 = 10 \text{ kN}$ and $F_2 = 20 \text{ kN}$.

Calculate the forces in all the members.

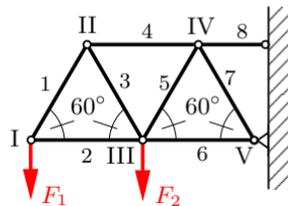


Fig. 6.17

Results: see (A) $S_1 = S_4 = -S_3 = 11.6 \text{ kN}$, $S_2 = -5.8 \text{ kN}$,
 $S_5 = -S_7 = 34.6 \text{ kN}$, $S_6 = -28.9 \text{ kN}$, $S_8 = 46.2 \text{ kN}$.

Example 6.9 A rope is guided around the smooth pin C of the truss shown in Fig. 6.18. One end of the rope is connected to a rigid wall; the other end carries a box of weight W .

Calculate the force S_{11} in member 11.

Result: $S_{11} = (1 - \sqrt{2}/2)W$.

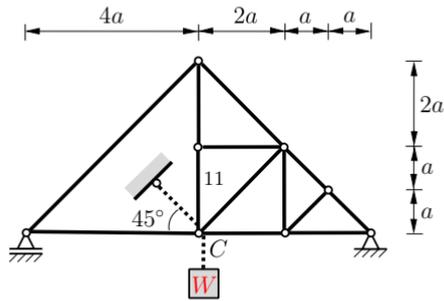


Fig. 6.18

Example 6.10 The truss shown in Fig. 6.19 carries the forces $F_1 = 10$ kN and $F_2 = 20$ kN (given: $a = 3$ m).

Calculate the forces in the members 6 and 8.

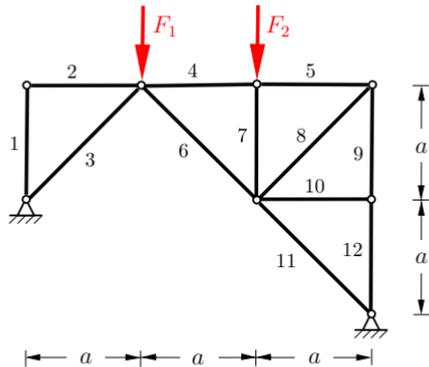


Fig. 6.19

Results: $S_6 = 0$, $S_8 = 10\sqrt{2}$ kN.

E6.11

Example 6.11 Determine the support reactions and the forces in the members of the space truss shown in Fig. 6.20.

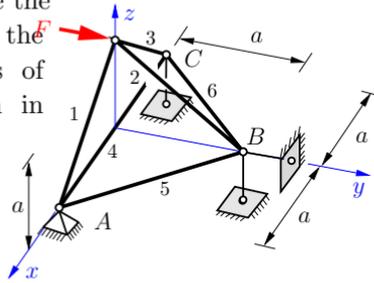


Fig. 6.20

Results: see (A) $A_x = A_y = 0, \quad A_z = C_z = -F/2,$
 $B_z = -B_y = F, \quad S_1 = S_3 = F/\sqrt{2},$
 $S_2 = -\sqrt{2}F, \quad S_4 = -F/2, \quad S_5 = S_6 = 0.$

E6.12

Example 6.12 Calculate the forces in all the members of the space truss in Fig. 6.21.

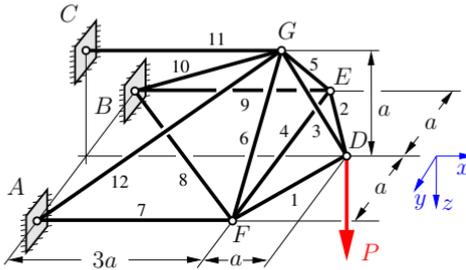


Fig. 6.21

Results: see (A) $S_1 = S_2 = -P/\sqrt{2}, \quad S_3 = \sqrt{2}P,$
 $S_4 = -S_7 = -S_9 = P/2, \quad S_5 = S_6 = S_8 = 0,$
 $S_{10} = S_{12} = -\sqrt{11}P/2, \quad S_{11} = 4P.$

6.5 Summary

- A truss is a structure that consists of straight members connected at joints.
- A truss is statically determinate if the forces in the members and at the supports can be determined from the equilibrium conditions. This is the case if the number of the unknown forces equals the number of the independent equilibrium conditions and the system is rigid.
- A truss is kinematically determinate if it is completely constrained against motion. It is kinematically indeterminate if it is nonrigid and therefore can undergo a finite or an infinitesimal motion.
- The internal forces in the members and the support reactions can be determined with the method of joints:
 - ◊ Draw free-body diagrams of all the joints. Introduce the external loads, the support reactions and the forces in the members. Assume that the force in each member is a tensile force.
 - ◊ Write down the equilibrium conditions for the joints (2 equations for a plane truss, 3 equations for a spatial truss).
 - ◊ Solve the system of equations.
 - ◊ The system of equations has a unique solution if the determinant of the matrix of the coefficients is not equal to zero. Then the truss is statically and kinematically determinate.
- It is usually more practical to apply the method of sections instead of the method of joints if only several forces are to be determined.