Chapter 18
Conjoint Measurement Analysis

Conjoint Measurement Analysis plays an important role in marketing. In the design of new products it is valuable to know which components carry what kind of utility for the customer. Marketing and advertisement strategies are based on the perception of the new product's overall utility. It can be valuable information for a car producer to know whether a change in sportiness or a change in safety or comfort equipment is perceived as a higher increase in overall utility. The Conjoint Measurement Analysis is a method for attributing utilities to the components (part worths) on the basis of ranks given to different outcomes (stimuli) of the product. An important assumption is that the overall utility is decomposed as a sum of the utilities of the components.

In Sect. 18.1 we introduce the idea of Conjoint Measurement Analysis. We give two examples from the food and car industries. In Sect. 18.2 we shed light on the problem of designing questionnaires for ranking different product outcomes. In Sect. 18.3 we see that the metric solution of estimating the part-worths is given by solving a least squares problem. The estimated preference ordering may be nonmonotone. The nonmetric solution strategy takes care of this inconsistency by iterating between a least squares solution and the pool adjacent violators algorithm.

18.1 Introduction

In the design and perception of new products it is important to specify the contributions made by different facets or elements. The overall utility and acceptance of such a new product can then be estimated and understood as a possibly additive function of the elementary utilities. Examples are the design of cars, a food article or the program of a political party. For a new type of margarine one may ask

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whether a change in taste or presentation will enhance the overall perception of the product. The elementary utilities are here the presentation style and the taste (e.g. calory content). For a party program one may want to investigate whether a stronger ecological or a stronger social orientation gives a better overall profile of the party. For the marketing of a new car one may be interested in whether this new car should have a stronger active safety or comfort equipment or a more sporty note or combinations of both.

In Conjoint Measurement Analysis one assumes that the overall utility can be explained as an additive decomposition of the utilities of different elements. In a sample of questionnaires people ranked the product types and thus revealed their preference orderings. The aim is to find the decomposition of the overall utility on the basis of observed data and to interpret the elementary or marginal utilities.

Example 18.1 A car producer plans to introduce a new car with features that appeal to the customer and that may help in promoting future sales. The new elements that are considered are comfort/safety components (e.g. active steering or GPS) and a sporty look (leather steering wheel and additional kW of the engine). The car producer has thus four lines of cars.

car 1:	basic safety equipment	and	low sportiness
car 2:	basic safety equipment	and	high sportiness
car 3:	high safety equipment	and	low sportiness
car 4:	high safety equipment	and	high sportiness

For the car producer it is important to rank these cars and to find out customers' attitudes toward a certain product line in order to develop a suitable marketing scheme. A tester may rank the cars as described in Table 18.1.

The elementary utilities here are the comfort equipment and the level of sportiness. Conjoint Measurement Analysis aims at explaining the rank order given by the test person as a function of these elementary utilities.

Example 18.2 A food producer plans to create a new margarine and varies the product characteristics "calories" (low vs. high) and "presentation" (a plastic pot vs. paper package) (Backhaus, Erichson, Plinke, & Weiber, 1996). We can view this in fact as ranking four products.

product 1:	low calories	and	plastic pot
product 2:	low calories	and	paper package
product 3:	high calories	and	plastic pot
product 4:	high calories	and	paper package

Table 18.1 Tester's ranking of cars

Car	1	2	3	4
Ranking	1	2	4	3

Table 18.2 Tester's ranking of margarine

Product	1	2	3	4
Ranking	3	4	1	2

These four fictive products may now be ordered by a set of sample testers as described in Table 18.2.

The Conjoint Measurement Analysis aims to explain such a preference ranking by attributing *part-worths* to the different elements of the product. The part-worths are the utilities of the elementary components of the product.

In interpreting the part-worths one may find that for a test person one of the elements has a higher value or utility. This may lead to a new design or to the decision that this utility should be emphasised in advertisement schemes.

Summary

- ↪ Conjoint Measurement Analysis is used in the design of new products.
- ↪ Conjoint Measurement Analysis tries to identify part-worth utilities that contribute to an overall utility.
- ↪ The part-worths enter additively into an overall utility.
- ↪ The interpretation of the part-worths gives insight into the perception and acceptance of the product.

18.2 Design of Data Generation

The product is defined through the properties of the components. A *stimulus* is defined as a combination of the different components. Examples 18.1 and 18.2 had four stimuli each. In the margarine example they were the possible combinations of the factors X_1 (calories) and X_2 (presentation). If a product property such as

$$X_3(\text{usage}) = \begin{cases} 1 \text{ bread} \\ 2 \text{ cooking} \\ 3 \text{ universal} \end{cases}$$

is added, then there are $3 \cdot 2 \cdot 2 = 12$ stimuli.

For the automobile Example 18.1 additional characteristics may be engine power and the number of doors. Suppose that the engines offered for the new car have 50, 70, 90 kW and that the car may be produced in 2-, 4-, or 5-door versions. These categories may be coded as

$$X_3(\text{power of engine}) = \begin{cases} 1 & 50\,\text{kW} \\ 2 & 70\,\text{kW} \\ 3 & 90\,\text{kW} \end{cases}$$

and

$$X_4(\text{doors}) = \begin{cases} 1 & 2\text{ doors} \\ 2 & 4\text{ doors} \\ 3 & 5\text{ doors} \end{cases}.$$

Both X_3 and X_4 have three factor levels each, whereas the first two factors X_1 (safety) and X_2 (sportiness) have only two levels. Altogether $2 \cdot 2 \cdot 3 \cdot 3 = 36$ stimuli are possible. In a questionnaire a tester would have to rank all 36 different products.

The *profile method* asks for the utility of each stimulus. This may be time consuming and tiring for a test person if there are too many factors and factor levels. Suppose that there are six properties of components with three levels each. This results in $3^6 = 729$ stimuli (i.e. 729 different products) that a tester would have to rank.

The *two factor method* is a simplification and considers only two factors simultaneously. It is also called trade-off analysis. The idea is to present just two stimuli at a time and then to recombine the information. Trade-off analysis is performed by defining the trade-off matrices corresponding to stimuli of two factors only.

The trade-off matrices for the levels X_1, X_2 and X_3 from the margarine Example 18.2 are given in Table 18.3. The trade-off matrices for the new car outfit are described in Tabel 18.4.

The choice between the profile method and the trade-off analysis should be guided by consideration of the following aspects:

1. requirements on the test person,
2. time consumption,
3. product perception.

Table 18.3 Trade-off matrices for margarine

X_3	X_1	
1	1	2
2	1	2
3	1	2

X_3	X_2	
1	1	2
2	1	2
3	1	2

X_1	X_2	
1	1	2
2	1	2

Table 18.4 Trade-off matrices for car design

X_4	X_3		
1	1	2	3
2	1	2	3
3	1	2	3

X_4	X_2	
1	1	2
2	1	2
3	1	2

X_4	X_1	
1	1	2
2	1	2
3	1	2

X_3	X_2	
1	1	2
2	1	2
3	1	2

X_3	X_1	
1	1	2
2	1	2
3	1	2

X_2	X_1	
1	1	2
2	1	2

The first aspect relates to the ability of the test person to judge the different stimuli. It is certainly an advantage of the trade-off analysis that one only has to consider two factors simultaneously. The two factor method can be carried out more easily in a questionnaire without an interview.

The profile method incorporates the possibility of a complete product perception since the test person is not confronted with an isolated aspect (2 factors) of the product. The stimuli may be presented visually in its final form (e.g. as a picture). With the number of levels and properties the number of stimuli rise exponentially with the profile method. The time to complete a questionnaire is therefore a factor in the choice of method.

In general the product perception is the most important aspect and is therefore the profile method that is used the most. The time consumption aspect speaks for the trade-off analysis. There exist, however, clever strategies on selecting representation subsets of all profiles that bound the time investment. We therefore concentrate on the profile method in the following.

Summary

↪ A stimulus is a combination of different properties of a product.

↪ Conjoint measurement analysis is based either on a list of all factors (profile method) or on trade-off matrices (two factor method).

↪ Trade-off matrices are used if there are too many factor levels.

↪ Presentation of trade-off matrices makes it easier for testers since only two stimuli have to be ranked at a time.

18.3 Estimation of Preference Orderings

On the basis of the reported preference values for each stimulus conjoint analysis determines the part-worths. Conjoint analysis uses an additive model of the form

$$Y_k = \sum_{j=1}^{J} \sum_{l=1}^{L_j} \beta_{jl} \boldsymbol{I}(X_j = x_{jl}) + \mu, \text{ for } k = 1, \dots, K \text{ and } \forall\ j \sum_{l=1}^{L_j} \beta_{jl} = 0. \tag{18.1}$$

X_j $(j = 1, \dots, J)$ denote the factors, x_{jl} $(l = 1, \dots, L_j)$ are the levels of each factor X_j and the coefficients β_{jl} are the part-worths. The constant μ denotes an overall level and Y_k is the observed preference for each stimulus and the total number of stimuli are:

$$K = \prod_{j=1}^{J} L_j.$$

Equation (18.1) is without an error term for the moment. In order to explain how (18.1) may be written in the standard linear model form we first concentrate on $J = 2$ factors. Suppose that the factors engine power and airbag safety equipment have been ranked as follows:

			Airbag	
			1	2
Engine	50 kW	1	1	3
	70 kW	2	2	6
	90 kW	3	4	5

There are $K = 6$ preferences altogether. Suppose that the stimuli have been sorted so that Y_1 corresponds to engine level 1 and airbag level 1, Y_2 corresponds to engine level 1 and airbag level 2, and so on. Then model (18.1) reads:

$$\begin{aligned}
Y_1 &= \beta_{11} + \beta_{21} + \mu \\
Y_2 &= \beta_{11} + \beta_{22} + \mu \\
Y_3 &= \beta_{12} + \beta_{21} + \mu \\
Y_4 &= \beta_{12} + \beta_{22} + \mu \\
Y_5 &= \beta_{13} + \beta_{21} + \mu \\
Y_6 &= \beta_{13} + \beta_{22} + \mu.
\end{aligned}$$

Now we would like to estimate the part-worths β_{jl}.

Table 18.5 Ranked products

			X_2 (calories)	
			Low	High
			1	2
X_1 (usage)	Bread	1	2	1
	Cooking	2	3	4
	Universal	3	6	5

Example 18.3 In the margarine example let us consider the part-worths of X_1 = usage and X_2 = calories. We have $x_{11} = 1$, $x_{12} = 2$, $x_{13} = 3$, $x_{21} = 1$ and $x_{22} = 2$. (We momentarily re-labeled the factors: X_3 became X_1.) Hence $L_1 = 3$ and $L_2 = 2$. Suppose that a person has ranked the six different products as in Table 18.5.

If we order the stimuli as follows:

$$\begin{aligned} Y_1 &= \text{Utility}\,(X_1 = 1 \wedge X_2 = 1) \\ Y_2 &= \text{Utility}\,(X_1 = 1 \wedge X_2 = 2) \\ Y_3 &= \text{Utility}\,(X_1 = 2 \wedge X_2 = 1) \\ Y_4 &= \text{Utility}\,(X_1 = 2 \wedge X_2 = 2) \\ Y_5 &= \text{Utility}\,(X_1 = 3 \wedge X_2 = 1) \\ Y_6 &= \text{Utility}\,(X_1 = 3 \wedge X_2 = 2)\,, \end{aligned}$$

we obtain from Eq. (18.1) the same decomposition as above:

$$\begin{aligned} Y_1 &= \beta_{11} + \beta_{21} + \mu \\ Y_2 &= \beta_{11} + \beta_{22} + \mu \\ Y_3 &= \beta_{12} + \beta_{21} + \mu \\ Y_4 &= \beta_{12} + \beta_{22} + \mu \\ Y_5 &= \beta_{13} + \beta_{21} + \mu \\ Y_6 &= \beta_{13} + \beta_{22} + \mu. \end{aligned}$$

Our aim is to estimate the part-worths β_{jl} as well as possible from a collection of tables like Table 18.5 that have been generated by a sample of test persons. First, the so-called metric solution to this problem is discussed and then a non-metric solution.

Metric Solution

The problem of conjoint measurement analysis can be solved by the technique of Analysis of Variance (ANOVA). An important assumption underlying this technique is that the "distance" between any two adjacent preference orderings corresponds to the same difference in utility. That is, the difference in utility between the products ranked 1st and 2nd is the same as the difference in utility between the products

Table 18.6 Metric solution for car example

			X_2 (airbags)			
			1	2	$\bar{p}_{x_{1\bullet}}$	β_{1l}
X_1(engine)	50 kW	1	1	3	2	−1.5
	70 kW	2	2	6	4	0.5
	90 kW	3	4	5	4.5	1
$\bar{p}_{x_{2\bullet}}$			2.33	4.66	3.5	
β_{2l}			−1.16	1.16		

ranked 4th and 5th. Put differently, we treat the ranking of the products—which is a cardinal variable—as if it were a metric variable.

Introducing a mean utility μ Eq. (18.1) can be rewritten. The mean utility in the above Example 18.3 is $\mu = (1+2+3+4+5+6)/6 = 21/6 = 3.5$. In order to check the deviations of the utilities from this mean, we enlarge Table 18.5 by the mean utility $\bar{p}_{x_{j\bullet}}$, given a certain level of the other factor. The metric solution for the car example is given in Table 18.6.

Example 18.4 In the margarine example the resulting part-worths for $\mu = 3.5$ are

$$\begin{array}{llll} \beta_{11} = & -2 & \beta_{21} = & 0.16 \\ \beta_{12} = & 0 & \beta_{22} = & -0.16\,. \\ \beta_{13} = & 2 & & \end{array}$$

Note that $\sum_{l=1}^{L_j} \beta_{jl} = 0$ $(j = 1, \ldots, J)$. The estimated utility $\hat{Y}_1$ for the product with low calories and usage of bread, for example, is:

$$\hat{Y}_1 = \beta_{11} + \beta_{21} + \mu = -2 + 0.16 + 3.5 = 1.66.$$

The estimated utility $\hat{Y}_4$ for product 4 (cooking ($X_1 = 2$) and high calories ($X_2 = 2$)) is:

$$\hat{Y}_4 = \beta_{12} + \beta_{22} + \mu = 0 - 0.16 + 3.5 = 3.33.$$

The coefficients β_{jl} are computed as $\bar{p}_{x_{jl}} - \mu$, where $\bar{p}_{x_{jl}}$ is the average preference ordering for each factor level. For instance, $\bar{p}_{x_{11}} = 1/2 * (2 + 1) = 1.5$.

The fit can be evaluated by calculating the deviations of the fitted values to the observed preference orderings. In the rightmost column of Table 18.8 the quadratic deviations between the observed rankings (utilities) Y_k and the estimated utilities $\hat{Y}_k$ are listed.

Table 18.7 Metric solution for Table 18.5

			X_2 (calories)			
			Low	High		
			1	2	$\bar{p}_{x_{1\bullet}}$	β_{1l}
X_1 (usage)	Bread	1	2	1	1.5	−2
	Cooking	2	3	4	3.5	0
	Universal	3	6	5	5.5	2
$\bar{p}_{x_{2\bullet}}$			3.66	3.33	3.5	
β_{2l}			0.16	−0.16		

Table 18.8 Deviations between model and data

Stimulus	Y_k	$\hat{Y}_k$	$Y_k - \hat{Y}_k$	$(Y_k - \hat{Y}_k)^2$
1	2	1.66	0.33	0.11
2	1	1.33	−0.33	0.11
3	3	3.66	−0.66	0.44
4	4	3.33	0.66	0.44
5	6	5.66	0.33	0.11
6	5	5.33	−0.33	0.11
$\sum$	21	21	0	1.33

The technique described that generated Table 18.7 is in fact the solution to a least squares problem. The conjoint measurement problem (18.1) may be rewritten as a linear regression model (with error $\varepsilon = 0$):

$$Y = \mathcal{X}\beta + \varepsilon \tag{18.2}$$

with $\mathcal{X}$ being a design matrix with dummy variables. $\mathcal{X}$ has the row dimension $K = \prod_{j=1}^{J} L_j$ (the number of stimuli) and the column dimension $D = \sum_{j=1}^{J} L_j - J$. The reason for the reduced column number is that per factor only $(L_j - 1)$ vectors are linearly independent. Without loss of generality we may standardise the problem so that the last coefficient of each factor is omitted. The error term ε is introduced since even for one person the preference orderings may not fit the model (18.1).

Example 18.5 If we rewrite the β coefficients in the form

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} = \begin{pmatrix} \mu + \beta_{13} + \beta_{22} \\ \beta_{11} - \beta_{13} \\ \beta_{12} - \beta_{13} \\ \beta_{21} - \beta_{22} \end{pmatrix} \tag{18.3}$$

and define the design matrix $\mathcal{X}$ as

$$\mathcal{X} = \left(\begin{array}{c|cc|c} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ \hline 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right), \tag{18.4}$$

then Eq. (18.1) leads to the linear model (with error $\varepsilon = 0$):

$$Y = \mathcal{X}\beta + \varepsilon. \tag{18.5}$$

The least squares solution to this problem is the technique used for Table 18.7.

In practice we have more than one person to answer the utility rank question for the different factor levels. The design matrix is then obtained by stacking the above design matrix n times. Hence, for n persons we have as a final design matrix:

$$\mathcal{X}^* = 1_n \otimes \mathcal{X} = \left.\begin{pmatrix} \mathcal{X} \\ \vdots \\ \vdots \\ \mathcal{X} \end{pmatrix}\right\} n - \text{times}$$

which has dimension $(nK)(L-J)$ (where $L = \sum_{j=1}^{J} L_j$) and $Y^* = (Y_1^\top, \dots, Y_n^\top)^\top$. The linear model (18.5) can now be written as:

$$Y^* = \mathcal{X}^*\beta + \varepsilon^*. \tag{18.6}$$

Given that the test people assign different rankings, the error term ε^* is a necessary part of the model.

Example 18.6 If we take the β vector as defined in (18.3) and the design matrix $\mathcal{X}$ from (18.4), we obtain the coefficients:

$$\begin{aligned} \hat{\beta}_1 &= 5.33 = \hat{\mu} + \hat{\beta}_{13} + \hat{\beta}_{22} \\ \hat{\beta}_2 &= -4 = \hat{\beta}_{11} - \hat{\beta}_{13} \\ \hat{\beta}_3 &= -2 = \hat{\beta}_{12} - \hat{\beta}_{13} \\ \hat{\beta}_4 &= 0.33 = \hat{\beta}_{21} - \hat{\beta}_{22} \\ \sum_{l=1}^{L_j} \hat{\beta}_{jl} &= 0. \end{aligned} \tag{18.7}$$

Solving (18.7) we have:

$$\begin{aligned}
\hat{\beta}_{11} &= \hat{\beta}_2 - \tfrac{1}{3}\left(\hat{\beta}_2 + \hat{\beta}_3\right) &&= -2\\
\hat{\beta}_{12} &= \hat{\beta}_3 - \tfrac{1}{3}\left(\hat{\beta}_2 + \hat{\beta}_3\right) &&= 0\\
\hat{\beta}_{13} &= -\tfrac{1}{3}\left(\hat{\beta}_2 + \hat{\beta}_3\right) &&= 2\\
\hat{\beta}_{21} &= \hat{\beta}_4 - \tfrac{1}{2}\hat{\beta}_4 = \tfrac{1}{2}\hat{\beta}_4 &&= 0.16\\
\hat{\beta}_{31} &= -\tfrac{1}{2}\hat{\beta}_4 &&= -0.16\\
\hat{\mu} &= \hat{\beta}_1 + \tfrac{1}{3}\left(\hat{\beta}_2 + \hat{\beta}_3\right) + \tfrac{1}{2}(\hat{\beta}_4) &&= 3.5.
\end{aligned} \tag{18.8}$$

In fact, we obtain the same estimated part-worths as in Table 18.7. The stimulus $k = 2$ corresponds to adding up β_{11}, β_{22}, and μ (see (18.3)). Adding $\hat{\beta}_1$ and $\hat{\beta}_2$ gives:

$$\hat{Y}_2 = 5.33 - 4 = 1.33.$$

Nonmetric Solution

If we drop the assumption that utilities are measured on a metric scale, we have to use (18.1) to estimate the coefficients from an adjusted set of estimated utilities. More precisely, we may use the monotone ANOVA as developed by Kruskal (1965). The procedure works as follows. First, one estimates model (18.1) with the ANOVA technique described above. Then one applies a monotone transformation $\hat{Z} = f(\hat{Y})$ to the estimated stimulus utilities. The monotone transformation f is used because the fitted values $\hat{Y}_k$ from (18.2) of the reported preference orderings Y_k may not be monotone. The transformation $\hat{Z}_k = f(\hat{Y}_k)$ is introduced to guarantee monotonicity of preference orderings. For the car example the reported Y_k values were $Y = (1,\ 3,\ 2,\ 6,\ 4,\ 5)^{\top}$. The estimated values are computed as:

$$\begin{aligned}
\hat{Y}_1 &= -1.5 - 1.16 + 3.5 = 0.84\\
\hat{Y}_2 &= -1.5 + 1.16 + 3.5 = 3.16\\
\hat{Y}_3 &= -0.5 - 1.16 + 3.5 = 2.84\\
\hat{Y}_4 &= -0.5 + 1.16 + 3.5 = 5.16\\
\hat{Y}_5 &= 1.5 - 1.16 + 3.5 = 3.34\\
\hat{Y}_6 &= 1.5 + 1.16 + 3.5 = 5.66.
\end{aligned}$$

If we make a plot of the estimated preference orderings against the revealed ones, we obtain Fig. 18.1.

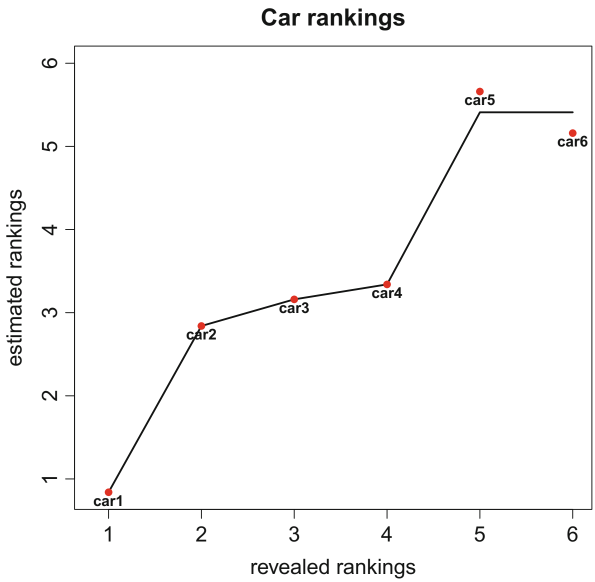

Fig. 18.1 Plot of estimated preference orderings vs. revealed rankings and PAV fit `MVAcarrankings`

We see that the estimated $\hat{Y}_6 = 5.16$ is below the estimated $\hat{Y}_5 = 5.66$ and thus an inconsistency in ranking the utilities occurs. The monotone transformation $\hat{Z}_k = f(\hat{Y}_k)$ is introduced to make the relationship in Fig. 18.1 monotone. A very simple procedure consists of averaging the "violators" $\hat{Y}_6$ and $\hat{Y}_5$ to obtain 5.41. The relationship is then monotone but the model (18.1) may now be violated. The idea is therefore to iterate these two steps. This procedure is iterated until the stress measure (see Chap. 17)

$$\text{STRESS} = \frac{\sum_{k=1}^{K} (\hat{Z}_k - \hat{Y}_k)^2}{\sum_{k=1}^{K} (\hat{Y}_k - \bar{\hat{Y}})^2} \tag{18.9}$$

is minimised over β and the monotone transformation f. The monotone transformation can be computed by the so-called pool-adjacent-violators (PAV) algorithm.

Summary

↪ The part-worths are estimated via the least squares method.

↪ The metric solution corresponds to analysis of variance in a linear model.

↪ The non-metric solution iterates between a monotone regression curve fitting and determining the part-worths by ANOVA methodology.

↪ The fitting of data to a monotone function is done via the PAV algorithm.

18.4 Exercises

Exercise 18.1 *Compute the part-worths for the following table of rankings*

		X_2	
		1	2
X_1	1	1	2
	2	4	3
	3	6	5

.

Exercise 18.2 *Consider again Example 18.5. Rewrite the design matrix $\mathcal{X}$ and the parameter vector β so that the overall mean effect μ is part of $\mathcal{X}$ and β, i.e. find the matrix $\mathcal{X}'$ and β' such that $Y = \mathcal{X}'\beta'$.*

Exercise 18.3 *Compute the design matrix for Example 18.5 for $n = 3$ persons ranking the margarine with X_1 and X_2.*

Exercise 18.4 *Construct an analog for Table 18.8 for the car example.*

Exercise 18.5 *Compute the part-worths on the basis of the following tables of rankings observed on $n = 3$ persons.*

		X_2			X_2			X_2	
X_1	1	1	2	X_1	1	3	X_1	3	1
	2	4	3		4	2		5	2
	3	6	5		5	6		6	4

Exercise 18.6 *Suppose that in the car example a person has ranked cars by the profile method on the following characteristics:*

$$
\begin{aligned}
X_1 &= \text{motor}\\
X_2 &= \text{safety}\\
X_3 &= \text{doors}
\end{aligned}
$$

X_1	X_2	X_3	Preference
1	1	1	1
1	1	2	3
1	1	3	2
1	2	1	5
1	2	2	4
1	2	3	6

X_1	X_2	X_3	Preference
2	1	1	7
2	1	2	8
2	1	3	9
2	2	1	10
2	2	2	12
2	2	3	11

X_1	X_2	X_3	Preference
3	1	1	13
3	1	2	15
3	1	3	14
3	2	1	16
3	2	2	17
3	2	3	18

There are $k = 18$ *stimuli.*
Estimate and analyse the part-worths.