CHAPTER 23

Panel Models for the Analysis of Change and Growth in Life Course Studies

CHARLES N. HALABY

INTRODUCTION

Panel data figure prominently in research on the many aspects of the life course. The longitudinal structure of panel data, with the properties of many units (individuals, families, etc.) measured on several occasions spread over time, is ideal for observational studies of life course processes. Panel data have proven useful for research on subjects as fundamental as the causes and consequences of marital stability and dissolution (Biblarz & Raftery 1993; Thornton, Axinn, & Teachman, 1995), the social psychological development and well-being of children and adults (Booth & Amato, 1991; Chase-Lansdale, Cherlin & Kiernan, 1995; Moen, Robison, & Dempster-McClain, 1995; Nagin & Tremblay, 1999), and the evolution of conventional (Diprete & McManus, 1996) and deviant careers (Land & Nagin, 1996; Sampson & Laub, 1992), as well as for research on the issues surrounding the timing of all these processes and related transitions. There is now widespread agreement that panel data and the analytical advances they make possible are essential for rigorously addressing the types of questions that drive and are central to many life course studies.

Two classes of questions encompass many lines of empirical research and hold a privileged place in life course studies. One class, rooted in the traditional scientific interest in causal processes, focuses on assessing how events or changes in one area of social life may

CHARLES N. HALABY • Department of Sociology, University of Wisconsin, Madison, Wisconsin 53706

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bring about other significant changes along key dimensions of the life course. For example, Budig and England (2001) investigate the effect that children have on mothers' wages over a 12-year period of their career; similarly, McManus and DiPrete (2001) assess the impact of divorce and separation on the financial welfare of men. A second class of questions center not on estimating the effects of changes, but on describing how the trajectories that development and growth trace over time vary systematically across groups defined by different characteristics or by exposure to different "treatments" or life course experiences. For example, the Life Course Studies program at the Carolina Population Center explains that the The National Longitudinal Study of Adolescent Health "investigates key potential influences that shape trajectories of resilience and vulnerability from adolescence to young adulthood, with attention to potential sources of variation—such as gender, social contexts, race/ethnicity, and siblings."* These kinds of questions have moved to the forefront of life course studies and are also well-suited to panel analysis. At the same time, the distinction drawn here between classes of research is not sharp in theory or practice. Both types of analyses are joined, for example, in Cherlin, Chase-Lansdale, and McCrea's (1998) study of the effect of parental divorce on the mental health of children and on the role of social background in shaping the trajectory of mental health development during youth and early adulthood.

This paper reviews the core models and methods commonly used for assessing causal effects and charting trajectories of development and growth in panel data on the life course. This review is neither even-handed nor symmetrical. On the contrary, the purpose is to show how issues underlying the use of panel data to estimate causal effects extend to and illuminate the methods and limitations that accompany efforts to identify the forces that shape trajectories of development or growth. In the social sciences, panel models for estimating causal effects from observational data grow largely out of an econometric tradition, while models for tracing variation in development or growth trajectories grow out of a medical and biological research tradition that only recently has found application in life course studies. Because these divergent origins continue to separate applications of the two types of models, the way that econometric principles of causal analysis apply to growth modeling is not always explicitly acknowledged. I have found that the key principles motivating and governing the use of panel data for estimating causal effects are typically glossed over or lost entirely in treatments of the growth models used for investigating developmental trajectories. Consequently, important continuities that run through the formulation and testing of both types of models are obscured. Models aimed at estimating causal effects and charting growth trajectories rely ultimately on a common set of statistical principles. The goal of this review is to clarify these principles while acknowledging areas of uniqueness.

The substantive scope of this review is limited in a number of respects.† First, only panel models for metric response variables, not discrete or limited dependent variables, are considered. Second, since panel analyses for both causal effects and growth trajectories typically involve fitting only static models, this review does not consider models involving either "system" (i.e., lagged dependent variables) or "error" (serially correlated time-varying disturbances) dynamics. Third, only parametric models are discussed, since such models are

*This quote comes from the World Wide Web site http://www.cpc.unc.edu/projects/lifecourse/adhealth.html.

†The technical scope of this review is limited to applications in which the number of units (N) is large, the number of occasions (T) is small, and the data are balanced insofar as the number and spacing of observation occasions is the same for all units. For extensions to incomplete panels, including discussions of selection bias due to attrition or other sources of nonrandomly missing data, see Baltagi (1995, Chapters 9 and 10.5), Hsaio (1986, Chapter 8), and Wooldridge (1995).

most fully developed and most broadly applicable given available software. Accessible treatments of panel analysis that cast a broader net and include the many important subjects not covered here are available elsewhere.*

This chapter is organized into two main sections. The first section specifies the essential advantages that accompany even the simplest panel designs and identifies the main statistical principles that apply to the analysis of panel data. The second section uses the principles discussed in the first section to shed light on the issues that accompany the formulation and testing of growth models.

PANEL MODELS

Panel designs join the strengths of two fundamental observation schemes for making causal inferences about behavior (Holland, 1986). One observational protocol, typified by the static group comparison (Campbell & Stanley, 1963, p. 12), exposes different units to different values of a causal variable and compares their responses at a single point in time.† An advantage of this design is temporal homogeneity: confounding changes that might accompany the passage of time are ruled out as alternative causes. A disadvantage is that the units compared are different and, hence, heterogeneous with respect to unobserved properties that may confound the attribution of effect to the causal variable of interest. For this kind of design, "unit heterogeneity" is highly problematic, but temporal "instability" less so.

A second design involves the opposite observation scheme: at two different times the same unit is exposed to different values of the causal variable and the responses are compared. Because the unit is the same over time, many unobserved properties remain stable and, hence, are ruled out as explanations of change in the response variable. This design minimizes the threat of unit heterogeneity: one expects more similarity in the same unit observed at different times than in different units observed at the same time. A disadvantage is temporal instability: over time changes in unobserved extraneous variables are alternative explanations for change in the response variable.

Panel studies join these two designs and thereby achieve a measure of protection against the primary threats to causal inference in observational studies.‡ By observing many units at the same time on several different occasions, panel studies can effectively deal with the threats of unit heterogeneity and temporal instability. Unit homogeneity over time can be exploited to deal with the unobserved between-unit heterogeneity that is potentially a confounding factor in cross-sectional designs. Similarly, the temporal stability that comes with observing different units at the same time can be exploited to deal with the temporal instability in unobserved extraneous causes that may threaten inferences from longitudinal data.

*Standard econometric treatments of panel data include Baltaggi (1995), Hsaio (1986), and Maddalla (1986). For growth models as well as more general accounts of multilevel modeling, see Bryk and Raudenbush (1992), Goldstein (1995), Snijders and Bosker (1999). Treatments with a more biomedical emphasis include Diggle, Liang, and Zeger (1994) and Lindsey (1999).

†Randomly assigning values of the causal variable to units transforms the static group comparison into the classical experimental protocol known as the "posttest-only" control group design (Campbell & Stanley, 1963, p. 25). Randomization is the best method for dealing with unobserved unit heterogeneity, but is typically not available to life course researchers. Hence, the necessity for the methods discussed in this paper.

‡Another advantage of panel data that is not discussed here is the gain in efficiency: observing each of N units T times is usually more efficient (i.e., less error variation) than observing each of $(N \times T)$ units once.

Random Effects, Fixed Effects, and Unobserved Heterogeneity

To see the inference problems that panel data address, it pays to begin with a cross-sectional design. Suppose the aim is to estimate the effect of parental conflict on the mental health of children. Assume that data are available on many children $i = 1,2, \ldots, N$ at a single point in time t. Let y_{it} be the mental health of the ith child at time t and let x_{it} be a metric measure of the degree of parental conflict. Then the equation for the mental health of the ith child at time t can be written as

$$y_{it} = \alpha + \gamma x_{it} + \theta_i + e_{it} \tag{1}$$

where α is an intercept, γ is the parameter for the effect of parental conflict (x), and e_{it} is a transitory disturbance that represents unobserved time-varying causes of mental health that are independent of x, are serially uncorrelated, have mean zero, and constant variance over all units. The term θ_i ($\Sigma\theta_i = 0$), which represents an effect that is assumed to be uncorrelated with the transitory disturbances e_{it}, can be viewed either as a summary measure of unobserved, time-invariant, child-specific causes of mental health or simply as the unobserved permanent component of the ith child's mental health. The key point is that θ_i represents forces that shape mental health, but are possibly unknown to and certainly unmeasured by the researcher. The claim that θ_i represent time-invariant determinants of mental health (y_{it}) not only means that these are stable properties, but also that their effect on mental health is stable over time. Left open for now is the relationship of θ_i to the causal variable of primary interest, parental conflict.

A cross-section equation like Equation (1) is commonly fitted by least-squares estimation. Given our assumptions, the least-squares estimator $\hat{\gamma}_{LS}$ of the effect of parental conflict (x) on mental health (y) has expectation

$$E(\hat{\gamma}_{LS}) = \gamma + \lambda_{\theta x} \tag{2}$$

where $\lambda_{\theta x}$ is the parameter from an auxiliary regression of θ_i on x_{it}. The least-squares estimator captures two quantities: the effect γ of parental conflict on children's mental health and the expected mean difference $\lambda_{\theta x}$ in the permanent unobserved component of mental health θ_i for persons one unit apart on the metric measure of the causal variable, parental conflict.* The quality of the least-squares estimator $\hat{\gamma}_{LS}$ depends on this last term, which represents "unobserved heterogeneity bias."

The way one treats the connection between person-specific effects like θ_i and the observed causal variable(s) has important implications for the kind and quality of estimators that are employed. There are two alternative approaches. One approach is to assume that the unobserved person-specific causes θ_i are mean independent of the causal variable. This would be plausible if, for example, values of the causal variable were randomly assigned to units. It then would be sensible to treat the θ_i as "random effects", which, like e_{it}, are uncorrelated with the causal variable, have mean zero, and constant variance. Under this assumption, Equation (1) is a random effects model with composite random disturbance $u_{it} = \theta_i + e_{it}$. This implies $\lambda_{\theta x} = 0$, so that the unobserved person effects would not be a source of heterogeneity bias and the least-squares estimator $\hat{\gamma}_{LS}$ would be appropriate.

*The best way to look at this term is to simply view it an indicator of an association between the unobservable causes given by θ_i and the causal variable of interest as given by x.

An alternative approach is called for if the unobserved person effects are correlated with the causal variable, for then $\lambda_{\theta x} \neq 0$ and the least-squares estimator employed under a random effects assumption would suffer from unobserved heterogeneity bias. For the case of assessing the effect of parental conflict on children's mental health, an alternative approach is certainly advisable: it is not plausible to think of parental conflict as if it were randomly assigned to children and therefore independent of all unobservable child-specific forces that shape mental health. For example, θ_i may include a child's persistent exposure to parental alcoholism or abusive child-rearing practices, forces that one would expect to be associated with the level of parental conflict. In this case, avoiding bias in the estimate of the causal parameter γ necessarily means taking account of the relationship between θ_i and parental conflict. To this end, θ_i may be treated not as a random variable, but as "fixed effects," as person-specific constants that shift the mean of mental health and that need to be dealt with in estimating γ. In this case, Equation (1) is a fixed-effects model.

Under a fixed-effects model, there are two approaches to adjusting the least-squares estimator to account for the correlation of θ_i with the causal variable. The first is the conventional cross-sectional solution, which amounts to measuring time-invariant person variables that are summarized by θ_i and that will control for the correlation when entered into Equation (1). For example, if θ_i is determined, except possibly for a random error, by

$$\theta_i = \Sigma\phi_k w_{ik} \tag{3}$$

where the w_k are $k = 1, \ldots, K$ measured variables (e.g., exposure to parental alcoholism, abusive child-rearing practices, etc.), then substitution into Equation (1) gives

$$y_{it} = \alpha + \gamma x_{it} + \Sigma\phi_k w_{ik} + e_{it} \tag{4}$$

which will render the least-squares estimator $\hat{\gamma}_{LS}$ unbiased by regression adjusting for the correlation of x with the w_k. Controlling for the measured covariates w_k is intended to validate the random effects assumption, thereby rendering the least-squares estimator unbiased. This solution is problematic, however, because it relies on the untestable assumption that the w_k exhaust the variation in θ_i that is associated with x_{it}. With cross-sectional data, then, the random effects assumption is required at some point if the least-squares estimator is to be unbiased.

Now suppose that observations on mental health (y) and parental conflict (x) at a second point in time become available. An equation for each time point $t = 1,2$ can be written as

$$y_{i1} = \alpha + \gamma x_{i1} + \theta_i + e_{i1} \tag{5}$$

$$y_{i2} = \alpha + \gamma x_{i2} + \theta_i + e_{i2} \tag{6}$$

where a period difference in intercepts has been suppressed. If θ_i and x_{it} are correlated, estimation of the pooled equations again yields the biased estimator $\hat{\gamma}_{LS}$ with expectation $(\gamma + \lambda_{\theta x})$. Averaging these equations over time yields the so-called "between" regression of $\bar{y}_i$ on $\bar{x}_i$.

$$\bar{y}_i = \alpha + \gamma\bar{x}_i + \theta_i + \bar{e}_i. \tag{7}$$

Least-squares estimation of this equation yields the "between" estimator $\hat{\gamma}_b$ of γ. The expectation of this estimator is

$$E(\hat{\gamma}_b) = \gamma + \lambda_{\theta\bar{x}_i}. \tag{8}$$

so it too is biased by the relationship of θ_i to the over time mean of the causal variable $\bar{x}_{i.}$. This result will prove useful in understanding the models and methods discussed later. For now it shows that the bias in the least-squares estimator of the pooled regression of Equations (5) and (6) can be traced to between-unit variation in $\bar{x}_{i.}$, since Equation (7) averages across all within-unit over time variation in x_{it}.*

All this suggests that an effective means of dealing with heterogeneity bias would be to exploit the within-unit over time variation that panel data make available. To this end, transform x and y to deviations from their unit-specific over time means (i.e., time-demean the data) by subtracting Equation (7) from Equations (5) and (6). The resulting model is

$$(y_{i1} - \bar{y}_{i.}) = \gamma(x_{i1} - \bar{x}_{i.}) + (e_{i1} - \bar{e}_{i.}) \tag{9}$$

$$(y_{i2} - \bar{y}_{i.}) = \gamma(x_{i2} - \bar{x}_{i.}) + (e_{i2} - \bar{e}_{i.}) \tag{10}$$

where the θ_i that were the source of bias in the least-squares estimator have been eliminated. Applying least squares to the pooled equations yields the unbiased and consistent *fixed effects* or *within* estimator $\hat{\gamma}_{FE}$ and yields standard errors and tests statistics that are valid.†

An alternative approach to exploiting within-person variation is to estimate by least squares a model in first differences. Subtracting the time 1 from the time 2 equation yields

$$(y_{i2} - y_{i1}) = \gamma(x_{i2} - x_{i1}) + (e_{i2} - e_{i1}) \tag{11}$$

where again the person effects have been eliminated. The first-differenced estimator $\hat{\gamma}_{FD}$ is unbiased and consistent and the least-squares standard errors and test statistics are all valid.‡ For the two-period case, the first-differenced estimator and fixed-effects estimator are identical ($\hat{\gamma}_{FE} = \hat{\gamma}_{FD}$) and so too are their standard errors and test statistics.

Another "within" estimator that is equivalent to the fixed-effects and first differenced estimators for the two-period case and has a particularly simple form occurs when x is an indicator variable scored 1 for exposure to some event between time 1 and time 2 and 0 otherwise.¶ For example, if x indicates not the level of parental conflict but the occurrence of divorce (see Cherlin et al., 1998; McManus & Diprete, 2001), applying least squares to

$$y_{i2} - y_{i1} = \alpha + \gamma x_i + (e_{i2} - e_{i1}) \tag{12}$$

yields the so-called *difference-in-differences* estimator of the effect γ on children's mental health:

$$\hat{\gamma}_{dd} = (\bar{y}_{2|x=1} - \bar{y}_{1|x=1}) - (\bar{y}_{2|x=0} - \bar{y}_{1|x=0}) \tag{13}$$

*The correlation between $\bar{x}_{i.}$ and θ_i completely accounts for the correlation between x_{it} and θ_i; controlling for $\bar{x}_{i.}$ renders $\lambda_{\theta x} = 0$. Equivalently, heterogeneity bias cannot be traced to within-unit over time variation in x_{it} around its mean $\bar{x}_{i.}$, since the person effects θ_i are orthogonal to $(x_{it} - \bar{x}_{i.})$.

†This supposes that the residual degrees of freedom correctly account for the estimation of N unit means $\bar{y}_{i.}$. Standard least-squares routines will yield degrees of freedom equal to $NT - k$ rather than the correct $(NT - k - N)$, where k is the number of regression coefficients. In this case, the reported standard errors must be multiplied by the square root of $(NT - k)/(NT - k - N)$.

‡The residual degrees of freedom are automatically adjusted because N observations are lost by the differencing procedure.

¶An excellent treatment of panel methods for estimating the effect of events is Allison (1994).

which is the difference in the over time mean change in children's mental health between the group that experienced parental divorce ($x = 1$) and the group that did not ($x = 0$). This estimator is unbiased and the least-squares standard errors and test statistics are all valid. Estimators based on within-unit over time variation are unbiased and consistent because they eliminate possible heterogeneity bias caused by unobserved individual effects. Denoting all these estimators as $\hat{\gamma}_w$ we have

$$E(\hat{\gamma}_w) = \gamma \tag{14}$$

in contrast to the between-unit estimator $\hat{\gamma}_b$

$$E(\hat{\gamma}_b) = \gamma + \lambda_{\theta\bar{x}} \tag{15}$$

This suggests that the difference between these two types of estimators ($\hat{\gamma}_w - \hat{\gamma}_b$) gives evidence of a correlation between the explanatory variable and the person effects, that is, evidence of heterogeneity bias. This result hinges on the fact that the within estimators are unbiased and consistent whether or not person effects are correlated with the causal variable, whereas the between estimator is only unbiased and consistent if the correlation is zero.

Extensions and Specification Tests

The analysis above generalizes to more than two periods and regression adjustment for additional measured covariates. Consider a model of the form

$$y_{it} = \alpha + \sum_{t=2}^{T} \delta_t + \sum_k \beta_k w_{kit} + \sum_p \phi_p z_{ip} + \gamma x_{it} + \theta_i + e_{it} \tag{16}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. This model includes $(T - 1)$ terms δ_t for time-specific effects, a term θ_i ($\Sigma\theta_i = 0$) for person effects, and a transitory disturbance e_{it} that obeys the earlier assumptions. The causal variable of interest is x_{it}, which may be metric or categorical and has an effect on y given by γ. Two distinct sets of explanatory variables are entered as controls: the w_{kit} ($k = 1,\ldots, K$ variables), which vary over time and across units and the z_{pi} ($p = 1,\ldots, P$ variables), which vary only between units because they represent time-invariant characteristics (e.g., gender, social origins).

Including measured time-invariant variables like z_p may account for the correlation between θ_i and the explanatory variables. If the unobserved θ_i are assumed to be uncorrelated with the observed regressors, nothing is gained in terms of bias control by distinguishing "within" and "between" unit variation in the estimation of the parameters. An unbiased and consistent estimator of γ (and the other parameters) can be obtained by treating the θ_i as random effects and applying least squares to the pooled panels of NT observations. There is, however, a gain in efficiency, as well as valid standard errors and test statistics, to be realized by taking account of the positive serial correlation in the errors of Equation (16) that is induced by the fact that $u_{it} = \theta_i + e_{it}$ and $u_{is} = \theta_i + e_{is}$, $s \neq t$ both contain the common θ_i. Hence, a better estimation procedure would be generalized least squares (henceforth, GLS), which would yield a consistent and efficient random effects estimator of the parameters, as well as valid standard errors and test statistics. Denote the GLS random effects estimator of γ as $\hat{\gamma}_{GLS}$.

Most statistical software for GLS estimation assumes that the unobserved person effects are uncorrelated with the explanatory variables. If this assumption is false, the GLS estimator is biased and inconsistent. Hence, the person effects should be treated as fixed and the longitudinal structure of the data exploited by using the same methods identified for the two-period case. First differencing Equation (16) gives

$$(y_{it} - y_{it-1}) = \alpha + \sum_{3}^{T}\delta_t + \sum_k \beta_k(w_{kit} - w_{kit-1}) + \gamma(x_{it} - x_{it-1}) + (e_{it} - e_{it-1}) \qquad (17)$$

where one term for period effects is lost. Applying least squares to the pooled data yields the unbiased and consistent first-differenced estimator of the parameters. Alternatively, applying the fixed-effects transformation (i.e., time-demean the data) to Equation (16) yields

$$(y_{it} - \bar{y}_i) = \alpha + \sum_{t=2}^{T}\delta_t + \sum_k \beta_k(w_{kit} - \bar{w}_{ki}) + \gamma(x_{it} - \bar{x}_i) + (e_{it} - \bar{e}_i) \qquad (18)$$

which can be estimated by least squares. Denote the fixed-effects estimator of γ as $\hat{\gamma}_{FE}$.

The fixed-effects estimator and the first-differenced estimator are unbiased and consistent, although for $T > 2$ they are not the same. The standard errors and test statistics that accompany the fixed-effects estimator are valid if the idiosyncratic transitory errors e_{it} are constant variance and serially uncorrelated; this holds as well for the first-differenced estimator if the disturbances $(e_{i2} - e_{i1})$ in the *transformed* equation are constant variance and serially uncorrelated. Under these assumptions, these estimators are fully efficient for a fixed effects model. The efficiency of both estimators depends directly on the over time variation in the explanatory variables. For example, the standard error of $\hat{\gamma}_{FE}$ depends on the independent variation in x_{it} about its time mean $\bar{x}_i$, since one cannot get precise estimates of the effect of a change in x if not much change actually occurred. For causes that change slowly, longer intervals between time periods may yield more efficient estimators, although this must be weighed against the increase in error variation from extraneous, unmeasured transitory causes. For example, if past research suggests that levels of hostility and conflict in a household are relatively stable over time, efficient estimation of their effect on children's mental health would call for longer intervals between panels.

The fixed effect estimator deserves special attention because it is more commonly used in applied work. A key issue concerns its performance compared to the GLS estimator if θ_i is uncorrelated with the explanatory variables. The fixed effects estimator is still unbiased and consistent, although less efficient than using GLS to estimate a random effects model. Yet when N is large and there is plenty of time variation in the explanatory variables, not much may be lost by using fixed-effects estimation when GLS estimation of a random effects model is best. However, if the random effects assumption is wrong, the GLS estimator is biased and inconsistent, while fixed-effects estimation is unbiased and efficient. This kind of trade off clearly favors the fixed-effects estimator, which is why Allison (1994; see also Nickell, 1981, p. 1418), for example, was led to conclude that "the [fixed-effect] estimator is nearly always preferable [to the GLS random effects estimator] for estimating effects … with nonexperimental data" (p. 181).

The choice of model and estimators need not be made blindly. As indicated for the two-period case with a single explanatory variable, the difference $(\hat{\gamma}_w - \hat{\gamma}_b)$ between the "between" and "within" estimators is evidence of heterogeneity bias, so that large values of this statistic would lead to rejection of the hypothesis that the person-specific effects are uncorrelated with the regressors. This same principle carries over to the contrast ($\hat{\gamma}_{FE} - \hat{\gamma}_{GLS}$) between the GLS and fixed-effects estimator (Arellano, 1993; Baltagi, 1995; Hausman, 1978; Peracchi, 2001, p. 406). In models with several explanatory variables, the magnitude of the difference between the GLS estimates and the fixed-effect estimates is an indication of the heterogeneity bias induced in the GLS random effects estimator when the person effects are correlated with the explanatory variables. A statistic that summarizes the differences between the two sets of estimates is the basis for the most important specification test in panel data applications: the Hausman (1978) χ^2 test of the hypothesis that the person effects and the explanatory variables are uncorrelated. A small value of the Hausman (1978) χ^2 statistic fails to reject the null hypothesis and favors GLS estimation of a random effects model; a large value favors fixed-effects estimation of a fixed-effects model. If efficiency is not problematic (e.g., N is large and intervals between periods are long), it is conceivable that one might forgo the random effects model straight away in favor of fixed-effects estimation.* But there is little to recommend using GLS random effects estimation without a Hausman test for correlated person effects.

A comparison of Equations (16) and (18) (or 17) shows that one consequence of applying the fixed-effects (or first-difference) transformation is that measured time-invariant explanatory variables like z_p are swept away along with the individual effects, so that the parameters ϕ_p cannot be estimated.† This occurs because the effects of observed time-invariant explanatory variables cannot be separately identified from the effects of the unobserved time-invariant θ_i. Is the inability to identify parameters like ϕ_p a disadvantage of within estimators? Perhaps, but the disadvantage is only compelling if a fixed effects model is not warranted in the first place. Otherwise, the loss of time-invariant explanatory variables can hardly be construed as a serious cost, especially if research interest is largely confined to assessing how *changes* in explanatory variables bring about *changes* in a response variable. Indeed, to view the loss of information about the parameters of time-invariant explanatory variables as a serious disadvantage of within estimators is to misconstrue the principal purpose of panel data. Researchers who choose GLS estimation of random effects models solely for the efficiency gains that might come with exploiting between-unit variation and who ignore unobserved heterogeneity bias might as well settle for cross-section data and avoid the extra cost of collecting panel data. As Wooldridge (2000) correctly note, "In most applications, the only reason for collecting panel data is to allow for the unobserved effects [θ_i] to be correlated with the explanatory variables" (p. 421). Allison (1994) expresses similar sentiments on this issue.

Measured time-invariant explanatory variables are irrelevant for fixed effects estimation, but nevertheless figure prominently in the Hausman specification test for correlated person effects and, hence, in the evidence favoring a fixed or random effects model. Although the Hausman test is based only on estimates of the parameters of time-varying explanatory variables, time-invariant explanatory variables help determine the outcome of the test through their effect on the GLS estimator and its variance. Hence, important measured time-invariant explanatory variables must always be included in the random effects model estimated by GLS. Failure to do so will usually have a huge impact on the Hausman statistic, as it should, since it is sensitive to the omission of all time-invariant correlated effects, whether observed (but omitted) or strictly unobserved.

As an illustration that also shows the power of the test, Table 23-1 gives the results of fitting earnings equations to 1980–1987 data from the National Longitudinal Survey of Youth

*Even if fixed effects estimation is the default approach, a Hausman test may be informative about omitted causes of the response variable and the sources of heterogeneity bias.

† This conclusion will be qualified when growth models are discussed.

TABLE 23-1. Generalized Least-squares and Fixed-effect Parameter Estimates and Hausman Test Statistics for Short and Long Versions of Earnings Equations; Full-time Employed Males, 1980–1987 ($N = 544$ and $T = 8$).

	Short model			Long model		
Independent variables	Fixed-effect estimates	GLS random effect estimates	Difference	Fixed-effect estimates	GLS random effect estimates	Difference
Constant	1.33 (58.85)	1.28 (48.92)			0.486 (4.75)	
Year						
1981	0.113 (5.24)	0.109 (5.04)	0.003	0.113 (5.24)	0.110 (5.11)	0.002
1982	0.165 (7.62)	0.158 (7.26)	0.007	0.165 (7.62)	0.161 (7.40)	0.005
1983	0.208 (9.46)	0.199 (9.01)	0.009	0.208 (9.46)	0.202 (9.18)	0.006
1984	0.273 (12.26)	0.261 (11.67)	0.013	0.273 (12.26)	0.265 (11.92)	0.008
1985	0.323 (14.40)	0.311 (13.86)	0.012	0.323 (14.40)	0.316 (14.10)	0.008
1986	0.382 (16.83)	0.369 (16.26)	0.013	0.382 (16.83)	0.373 (16.54)	0.009
1987	0.441 (19.23)	0.425 (18.56)	0.017	0.441 (19.23)	0.430 (18.87)	0.011
Schooling (years)	—	—		—	0.071 (8.21)	—
Black (= 1)	—	—		—	−0.122 (2.63)	—
Married (= 1)	0.057 (3.08)	0.079 (4.71)	−0.023	0.057 (3.08)	0.077 (4.57)	−0.020
Occupational status (SEI/10)	0.012 (2.19)	0.024 (4.74)	−0.012	0.012 (2.19)	0.016 (3.17)	−0.004
Union (= 1)	0.086 (4.42)	0.116 (6.37)	−0.030	0.086 (4.42)	0.118 (6.53)	−0.032
Hausman χ^2			81 ($p < 0.0000$)			37.14 ($p < 0.0001$)

Note: Appearing in parentheses below the coefficients are the *t*-ratios.

Sample hereafter, NLSY.* The data are annual observations for $N = 545$ full-time working males who completed their schooling by 1980. The left-hand panel gives the GLS estimates, the fixed effect estimates, and the difference between these estimates for a model that includes only time-varying explanatory variables (year, occupational socioeconomic status, union membership, and marital status).† As the "difference" column shows, the GLS estimates of the coefficients of socioeconomic status, union membership, and marital status are considerably (i.e., 35–200%)

*These data were previously analyzed by Vella and Verbeek (1998). They are discussed by Wooldridge (2000) and available for downloading at http://ideas.uqam.ca/ideas/data/bocbocins.html.

†All the models and tests of this section were carried out using the *xtreg* command in Stata 7. This command will fit the between-, fixed-, and random-effects models and compute the relevant Hausman test.

larger than their fixed-effect counterparts. The largest difference is for socioeconomic status, with the GLS estimate twice the fixed effect estimate. The Hausman statistic is $\chi^2 = 81$ ($p < 0.0000$), so the null hypothesis is rejected in favor of the conclusion that important correlated individual effects have been omitted from the model. The right-hand panel of Table 23-1 gives the results when years of schooling and race, two time-invariant variables, are added to the random effects specification. The Hausman χ^2 statistic has fallen dramatically to $\chi^2 = 37$, a drop due largely to the decrease in the difference between the GLS and fixed-effects estimates of the coefficient of socioeconomic status, the one time-varying regressor that is most strongly related to schooling and race. Still, the Hausman statistic remains large enough to recommend fixed effects estimation.

The loss of information about the role of time-invariant explanatory variables in the process of change over time is hardly complete under fixed-effect estimation. Time-invariant explanatory variables may affect the rate of change in the response variable and may condition the effect of time-varying explanatory variables on the response variable; both types of interactions are estimable in a fixed-effects framework. Time-invariant variables with time-varying parameters are easily handled because neither the fixed effects nor first difference transformation eliminates them. For example, including interactions of parental schooling with a linear term for age in a fixed-effects model of child development would identify differences in the effect of parental schooling as children aged, even though the actual baseline effect of parental schooling on the level of child development could not be identified. Similarly, including in a model for marital satisfaction a term for the interaction of race with a time-varying explanatory variable like economic welfare would identify race differences in the effect of changes in economic welfare, even though race differences in marital satisfaction would not be identified.

To illustrate, Table 23-2 gives the results of GLS and fixed-effect estimation of an earnings model that includes terms for race, schooling, and the interaction of each of these with a linear term for year.* A couple of points are noteworthy. First, adding the time-varying interaction terms has virtually no effect on the Hausman test, which changes from 36.64 (not shown) to 37.23. Second, the "difference" column shows that the two estimators yield virtually identical coefficients for the interaction terms. These coefficients indicate that the rate of change in wage is about 1.8% less for Blacks than for others and that each year of schooling increases the rate of change in wage by 0.36%. Finally, the equations of Table 23-2 are, in effect, growth models, although I have not formulated them from within the usual statistical framework for analyzing variation in growth trajectories. I now turn to a discussion of that framework.

GROWTH MODELS

Background and Fundamentals

Growth modeling is a specialized application of panel data methods that has gained some currency in life course research. Cherlin et al. (1998) used growth models to describe the relationship between divorce and mental health for a cohort of children and McLeod and Shanahan (1996) used growth models to describe the relationship between poverty and

*Because all men remained in the labor force and in the sample throughout the 8-year period, the year-to-year change in labor force experience is constant and equal to the year-to-year change in period. For the fixed effects model, this means that the linear period effect is perfectly collinear with the linear effect for labor force experience, so that both variables yield exactly the same estimates and other statistics.

TABLE 23-2. Generalized Least-squares and Fixed-effect Parameter Estimates of Race and Schooling Differences in the Rate of Change in Earnings; Full-time Employed Males, 1980–1987 ($N = 544$ and $T = 8$)

Independent variables	*Fixed-effect estimates*	*GLS random effect estimates*	*Difference*
Constant	1.36 (69.06)	0.65 (5.65)	
Year	0.019 (1.16)	0.018 (1.14)	0.000
Year × Black	−0.019 (2.53)	−0.018 (2.51)	−0.000
Year × Schooling	0.004 (2.73)	0.004 (2.64)	0.000
Schooling (years)	—	0.058 (5.93)	—
Black (= 1)	—	−0.058 (1.10)	—
Married (= 1)	0.053 (2.89)	0.073 (4.39)	−0.020
Occupational status (SEI/10)	0.012 (2.27)	0.016 (3.23)	−0.004
Union (= 1)	0.088 (4.53)	0.119 (6.63)	−0.031
Hausman χ^2			37.23 ($p < 0.0000$)

Note: Appearing in parentheses below the coefficients are the *t*-ratios.

mental health for a birth cohort observed on three occasions. Although new additions to the sociological literature, growth models have a long history in the biological and medical sciences, where they have been used to analyze how the parameters governing growth trajectories generated by developmental or aging processes may vary between populations defined by different treatments or characteristics. The early history of growth modeling involved the application of standard multivariate analysis of variance methods to balanced data. But the modeling technology used in sociological applications is tied more directly to the development of methods for analyzing growth processes when data are unbalanced by variation across units in the timing or spacing of measurements. For example, the NLSY wage data used above would be unbalanced if not all men appeared in the sample every year or if not all men worked every year. Methods for unbalanced panels were consolidated by Laird and Ware's (1982) exposition of what they termed "two-stage" random-effects regression models, but what now fall under the rubric of mulitlevel (Goldstein, 1995) or hierarchical models (Bryk and Raudenbush, 1992).* The hierarchical linear model approach to growth modeling that is elaborated below and which I favor for its close connection to econometric panel models can

*An alternative approach to developmental or growth trajectories in given by Nagin (1999).

be cast in terms of covariance structure analysis and developed in parallel fashion (McArdle & Epstein, 1987; Willett & Sayer, 1994).*

Growth modeling typically involves a somewhat different emphasis from and some extensions beyond the panel models discussed above. Rather than emphasizing the use of within-unit over time variation to avoid bias in the estimation of parameters governing the effect of changes in explanatory variables on changes in the mean of a response variable, growth modeling attends mainly to describing and quantifying between-unit variation in the time trajectory of a response variable. Such between-unit variation in patterns of change is conceptualized in terms of variation in the mean of the response variable and variation in its rate of change over time.

The first "stage" or "level" of a growth model is an equation for the measurements on a response variable y_{it} that is observed for the ith unit over a temporally ordered set of measurement occasions $t = 1,2, \ldots, m$.† The argument of a growth model, the dimension with respect to which "growth" in y_{it} is assessed, is not always obvious or natural, but must meet certain mild restrictions. In the general model

$$y_{it} = f(T_{it}) + e_{it} \tag{19}$$

the main restrictions on T are that it be measured on a metric scale and that it be monotonically non-decreasing and sometimes increasing over measurement occasions. Laird and Ware (1982, p. 972) model growth in children's pulmonary capacity as a function of height, so T_{it} is the height of the ith child at the tth measurement occasion. In life course applications, T is typically the date or time of observation or the age of the unit. In cohort studies like Cherlin et al. (1998) and McLeod and Shanahan (1996), the distinction between observation date and age is eclipsed because the two are perfectly collinear. Since data structures in which T_{it} is fixed across units at each measurement occasion are quite common, assume that T is either the time of observation or age in a cohort.

A general form for Equation (19) is

$$y_{it} = \delta_t + e_{it} \tag{20}$$

where δ_t ($t = 1, \ldots, m$) are unrestricted time effects and e_{it} are time-varying disturbances that are assumed to be normal with mean zero and constant variance σ_e^2. Simple assumptions about disturbances are the rule when there are few time points or occasions per unit. Simple is also the rule when choosing one function $f(T_{it})$ that can serve as the trajectory for all units and that imposes a smooth structure on the time path of y_{it}. To be sure, neither simple nor smooth is necessary: Snijders and Bosker (1999) explore many elaborate and very flexible growth functions $f(T_{it})$ that researchers might find useful. In practice, linear models or models that are linear in some transformation of T are most common (Cherlin et al., 1998; McLeod & Shanahan, 1995), with quadratic functions also receiving attention (Bryk & Raudenbush, 1992; Horney et al., 1995).

The "level 1" specification for a linear growth model can be written as‡

$$y_{it} = \beta_{0i} + \beta_{1i}T_{it} + e_{it} \tag{21}$$

*Each approach has its advantages. Hierarchical linear modeling is better for handling unbalanced data, while covariance structure analysis allows a more flexible treatment of the error covariance structure (Willett & Sayer, 1994, p. 368).

†Growth modeling can easily handle unbalanced data, but to maintain continuity and minimize notation, I again assume the data are balanced, so that $m_i = m$ and the timing of occasions is constant over units.

‡Throughout I adhere loosely to the conventional notation of hierarchical models (e.g., Snijders & Bosker, 1999).

where β_{0i} is an individual-specific time-invariant intercept and β_{1i} is an individual-specific time-invariant slope parameter for the rate of change in y_{it}. As the model stands, the parameter β_{0i} gives the mean of y_{it} for the ith unit when $T = 0$. When $T = 0$ is not substantively meaningful, centering T around a sensible reference point is a common procedure. Hence, the level 1 equation might be written as

$$y_{it} = \beta_{0i} + \beta_{1i}(T_{it} - T_0) + e_{it} \tag{22}$$

where T_0 is a useful reference point, perhaps the value of T at the start of the observation period. Centering gives the intercept a meaningful interpretation, for now β_{0i} is the level of the response variable for the ith unit at time (or age) $T_{it} = T_0$. Cherlin et al. (1998) observe a cohort of children beginning at age 7 and so used the age-centering transformation $(T_{it} - 7)$ in order to interpret β_{0i} as the level of a child's mental health at the start of the study. Since centering aids interpretation, but otherwise leaves the fundamental statistical properties of the growth model unchanged, assume that T is centered appropriately so that $T_{i1} = 0$ means that β_{0i} gives the "baseline" value of y at the start of the observation period.

In addition to a level 1 equation for the measurements y_{it}, growth models consist of a set of level 2 regressions for explaining between-unit variation in the level 1 parameters β_{0i} and β_{1i}. Although the goal is to identify the contribution of measured conditions to between-unit variation in these parameters, the simplest model omits explanatory variables in order to assess the overall amount of variation. One possible starting point for a stage-2 model is

$$\beta_{0i} = \gamma_{00} + u_{0i} \tag{23}$$

$$\beta_{1i} = \gamma_{10} \tag{24}$$

where u_{0i} is a random person effect that has mean zero, variance τ_0^2, and is independent of the time-varying level 1 disturbance e_{it}. For the time being, I have specified the slope parameter β_{1i} for the rate of change in y as just γ_{10}; there is no random person-specific slope effect.

The connection of this formulation to earlier models becomes apparent when the two levels are combined by substitution to form the full model:

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + u_{0i} + e_{it} \tag{25}$$

which is exactly like the models considered earlier with one explanatory variable T, a random time-varying disturbance e_{it}, and a time-invariant person effect u_{0i}. The intercept giving the mean of y_{it} at baseline is $(\gamma_{00} + u_{0i})$, with γ_{00} giving the "fixed" component that applies to all persons and u_{0i} giving the part that applies only to the ith person. The error component u_{0i} is then just an alternative expression of the person effects (i.e., θ_i) that in earlier models were the source of unobserved heterogeneity. In most applications, u_{0i} is assumed to be random and independent of T without much discussion. In cohort studies in which T is age, this assumption is automatically met: the person-specific effect is by design uncorrelated with T_{it}, since the latter does not vary between units. The same is true if T_{it} tracks the occasion of measurement in a balanced design. Hence, under the assumptions set down above, heterogeneity bias in the estimator of the parameter γ_{10} for the rate of change in y_{it} is not an issue: The fixed effects and GLS random effects estimator of γ_{10} are identical, so a Hausman test would yield $\chi^2 = 0$.

A natural extension of the level 2 model of Equations (23) and (24) is

$$\beta_{0i} = \gamma_{00} + u_{0i} \tag{26}$$

$$\beta_{1i} = \gamma_{10} + u_{1i} \tag{27}$$

where now a random person-specific effect u_{1i} has been added to the equation for the slope β_{1i}. Like u_{0i}, u_{1i} is assumed to have mean zero, variance τ_1^2, and to be independent of e_{ij}; it is also customary to allow for a covariance between the person effects, say, τ_{01}. Joining this level 2 model to the level 1 model of Equation (21) yields

$$y_{it} = \gamma_{00} + \gamma_{10}T_{ij} + u_{0i} + u_{1i}T_{it} + e_{it} \tag{28}$$

The new feature of this model is the term $u_{1i}T_{ij}$, where u_{1i} appears as a random coefficient of the time variable.* The rate of change in y_{it} is now $(\gamma_{10} + u_{1i})$, with γ_{10} giving the mean $(E(\beta_{1i}) = \gamma_{10})$ or "fixed" component that applies to all units and u_{1i} giving the random part that applies only to the ith unit. This random slope effect is a source of unobserved heterogeneity that was not present in the panel models previously considered; it is the one fundamental innovation that renders the typical growth model statistically distinctive from earlier models. This random component of the rate of change in y_{it} varies across units, but for each unit is constant over time, just like the person effects u_{0i} for the mean level of y_{it}.†

The model of Equation (28) is a linear random effects growth model. The mean trend line of the response variable across all units is

$$E(y_{it}) = \gamma_{00} + \gamma_{10}T_{it} \tag{29}$$

with variation around this mean in person-specific trend lines. Variation in the level of y_{i1} at baseline (i.e., $T = 0$) is generated by u_{0i} and measured by the level 2 variance τ_0^2 and variation in the slope is generated by u_{1i} and measured by the level 2 variance τ_1^2. All the panel models considered earlier had assumed, in effect, that $u_{1i} = 0$ for all units and hence $\tau_1^2 = 0$.

A major part of the attraction of random effects growth models is the capacity to assess the unexplained level 1 and level 2 variation. As Snijders and Bosker (1999) observe for hierarchical modeling more generally, "[The] partitioning of unexplained variability over the various levels is the essence of hierarchical random effects models" (p. 48). Given the special emphasis on the level 2 variation captured by τ_0^2 and τ_1^2, growth modeling generally calls for formal tests on these variance parameters. The null hypothesis that all units share a common intercept (i.e., $\beta_{01} = \beta_{02} = \ldots = \beta_{0n} = \gamma_{00}$) implies $u_{01} = u_{02} = \ldots = u_{0n} = 0$, which can be formulated as a test of $\tau_1^2 = 0$. Similarly, a test of the hypothesis of a common underlying rate of change (i.e., $\beta_{11} = \ldots = \beta_{1n} = \gamma_{10}$) and no person-specific random slope effects amounts to a test of $\tau_1^2 = 0$. Failure to reject the joint null $\tau_0^2 = \tau_1^2 = 0$ would lead one to constrain the u_{0i} and u_{1i} to zero, yielding a model with no random effects at all. Alternatively, rejecting the null would invite single parameter tests of each separate variance component. Judging from extant empirical applications, models with no explanatory level 2 variables yield estimated intercept variances that are almost always statistically significant, with $\tau_0^2 = 0$ easily rejected. In contrast, point estimates of τ_1^2 are usually considerably smaller than their intercept counterparts and less often statistically significant. For the NLSY wage data, fitting the model of Equation (28) yields the estimates (SEs) $\hat{\gamma}_{00} = 1.43$ (0.014) and $\hat{\gamma}_{10} = 0.063$ (0.003) for the baseline mean and slope, respectively

*This new terms renders the disturbance variance dependent on T and, hence, heteroscedastic.

†If the variable coefficient u_{1i} is correlated with T_{it}, the usual GLS estimator of the mean rate parameter γ_{10} will be biased and inconsistent. But our design assumptions imply that T_{it} does not vary between units and, hence, cannot be correlated with u_{1i}.

and $\hat{\tau}_0^2 = 0.168$ and $\hat{\tau}_1^2 = 0.003$ for the variance parameters. A likelihood ratio test of the null hypothesis $\tau_0^2 = \tau_1^2 = 0$ yields highly significant $\chi^2 = 2156$ and each individual variance estimate is highly significant in its own right. Hence, both the baseline mean of wages and the rate of growth in wages from 1980–1987 vary across workers.

The models considered to this point are all "unconditional": no explanatory variables have been introduced to account for the components of variation in y_{it}. Time-varying and time-invariant explanatory variables enter growth models in formally distinct ways. Time-varying explanatory variables are entered at level 1 (with slopes then fixed at level 2) (see Bryk & Raudenbush, 1992, p. 151), since this part of the model speaks to variation in y_{it} over time. In analogous fashion, time-invariant explanatory variables are accommodated at level 2, since this part of the model addresses between-unit variation in *time-invariant* intercepts and slopes. I begin with time-varying explanatory variables in order to underscore the connection between growth models and other panel models.

Time-Varying Explanatory Variables

Let x_{it} be a time-varying explanatory variable like economic welfare that is believed to affect the time path of a response variable y_{it} like marital satisfaction. The level 1 model then may be expanded as follows:

$$y_{it} = \beta_{0i} + \beta_{1i}T_{it} + \beta_{xi}x_{it} + e_{it} \tag{30}$$

with the coefficient β_{xi} specified as fixed rather than random at the second level:

$$\beta_{0i} = \gamma_{00} + u_{0i} \tag{31}$$

$$\beta_{1i} = \gamma_{10} + u_{1i} \tag{32}$$

$$\beta_{xi} = \gamma_x \tag{33}$$

A preliminary model of exactly this form is specified by Horney et al. (1995) in their study of the evolution of criminal careers, with x_{it} representing changing employment and personal circumstances that affect the propensity to offend. The combined model is

$$y_{it} = \gamma_{00} + \gamma_{1,0}T_{it} + \gamma_x x_{it} + u_{0i} + u_{1i}T_{it} + e_{it} \tag{34}$$

where interest generally centers on estimating γ_x as well as $\sigma_{0|x}^2$ and $\sigma_{1|x}^2$, which indicate the variation remaining to be explained by the introduction of variables at level 2. Introducing time-varying explanatory variables may account not just for level-1 variation in y_{it}, but also between-person variation in intercepts and slopes, so that all three conditional variance components, $\sigma_{e|x}^2$, $\sigma_{0|x}^2$, $\sigma_{1|x}^2$ are smaller than their unconditional counterparts in the "empty" model with only T. For example, adding socioeconomic status, marital status, and union membership to a linear wage growth model yields variance estimates $\hat{\sigma}_{e|x}^2 = 0.1063$ (vs. $\hat{\sigma}_e^2 = 0.1066$), $\hat{\sigma}_{0|x}^2 = 0.1544$ (vs. $\hat{\sigma}_0^2 = 0.1685$), and $\hat{\sigma}_{1|x}^2 = 0.0031$ (vs. $\hat{\sigma}_1^2 = 0.0032$), all smaller than their unconditional counterparts in an "empty" model.*

*The parameter estimates given here are the only ones reported for the fitted models. The rest of the parameter estimates are available upon request.

For the model of Equation (34), the GLS (or maximum likelihood) random effects estimator of γ_x, the parameter for the effect of changes in x_{it} on changes in y_{it}, will be biased and inconsistent if x_{it} is correlated with the unobserved random person effects u_{0i} and u_{1i}.* In order to highlight the connection between estimation in a growth modeling context and in the context considered earlier, a less general formulation is useful. As Bryk and Raudenbush (1992) advise, fitting models for mean effects is a prudent first step toward fitting models with random or non-random slopes. In that spirit, continue to let Equation (30) be the level 1 model and specify the level 2 model as

$$\beta_{0i} = \gamma_{00} + u_{0i} \tag{35}$$

$$\beta_{1i} = \gamma_{10} \tag{36}$$

$$\beta_{xi} = \gamma_x \tag{37}$$

Substitution yields the combined model

$$y_{it} = \gamma_{00} + \gamma_{1,0}T_{it} + \gamma_x x_{it} + u_{0i} + e_{it} \tag{38}$$

which is exactly like the typical panel model with unobserved individual effects. In a growth modelling framework, dealing with correlated individual effects that might bias the estimation of γ_x is mainly a matter of controlling for between-unit variation in x_{it} by explicitly modeling the dependence of the unobserved random intercept effect on $\bar{x}_{i.}$.† Hence, the random effect u_{0i} can be written in terms of the auxiliary regression:

$$u_{0i} = \gamma_{0\bar{x}}\bar{x}_{i.} + v_{0i} \tag{39}$$

where v_{0i} is a residual that is uncorrelated by construction with $\bar{x}_{i.}$. Upon substitution into Equation (38), the full combined model becomes

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + \gamma_x x_{it} + \gamma_{0\bar{x}}\bar{x}_{i.} + v_{0i} + e_{it} \tag{40}$$

Introducing $\bar{x}_{i.}$ to the intercept equation will account for the correlation of x_{it} with u_{0i}, the original source of unobserved hetergeneity bias.

The growth model of Equation (40) blurs the distinction between random and fixed effect models: it is a random effects model that yields the same estimators of key parameters as a fixed effects model. Generalized least-squares (or maximum likelihood) estimation of this random effects model will yield a consistent estimator of γ_x (and γ_{10}) even if x_{it} is correlated with the original person effect u_{0i}. In fact, GLS estimation yields exactly the same parameter estimates, standard errors and test statistics as least-squares applied to

$$(y_{it} - \bar{y}_{i.}) = \gamma_{10}(T_{it} - \bar{T}_i) + \gamma_x(x_{it} - \bar{x}_{i.}) + (e_{it} - \bar{e}_{i.}) \tag{41}$$

*HLM5, a Windows program that I highly recommend and which I used to fit the wage growth equations, estimates the level 2 parameters (i.e., γ_{00}, γ_{10}, etc.) by generalized least-squares and the variance parameters by maximum likelihood. Except for those with random slope effects, all the growth models of this section could be fit with identical results using the *xtreg* command in Stata7.

†This method was first introduced in the econometrics literature by Mundlak (1978). This method is typically used, for example, to control for "contextual" effects in multilevel models of student outcomes observed across many schools (Bryk & Raudenbush, 1992, p. 71). In those applications x_{it} might be a measure of the socioeconomic status of the tth student in the ith school with $\bar{x}_{i.}$ the mean socioeconomic status of all students in the ith school.

Hence, controlling for $\bar{x}_{i.}$ in Equation (40) has the same effect as the mean-deviation transformation (Baltagi, 1995, p. 117): it eliminates the correlation of x_{it} with the source of unobserved heterogeneity u_{0i}.*

Generalized least-squares estimation of the random effects model

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + \gamma_x(x_{it} - \bar{x}_{i.}) + (\gamma_{0\bar{x}} + \gamma_x)\bar{x}_{i.} + v_{0i} + e_{it} \tag{42}$$

also yields the within estimator of the parameters. This model is statistically equivalent to the model of Equation (40): the point estimates of all the parameters are identical, as are the estimates of the variances (σ_e^2 and τ_0^2) and standard errors.† To be sure, the estimated coefficient of $\bar{x}_{i.}$ will usually differ between the two models, since it represents different parameters. In Equation (42), the parameter $(\gamma_{0\bar{x}} + \gamma_x)$ can be shown to be the coefficient of $\bar{x}_{it}$ from the "between" regression of $\bar{y}_{i.}$ on $\bar{x}_{i.}$ and $\bar{T}_{i.}$. In contrast, $\gamma_{0\bar{x}}$, the coefficient of $\bar{x}_{it}$ in Equation (40), is the *difference* in the between and within estimators of γ_x. Hence, $\gamma_{0\bar{x}}$ reflects the extent to which x_{it} is correlated with the unobserved person effect u_{0i} (see Equation [39]), so that fitting Equation (40) yields a specification test. The estimated coefficient $\hat{\gamma}_{0\bar{x}}$ is an indication of the heterogeneity bias in the standard GLS random effects estimator of γ_x if $\bar{x}_{i.}$ were omitted from Equation (40). Indeed, squaring the ratio of $\hat{\gamma}_{0\bar{x}}$ to its standard error essentially yields a Hausman χ^2 test of correlated individual effects (Arrelano, 1993; Baltagi, 1995, p. 69; Hausman, 1978, p. 1263).‡ If $\gamma_{0\bar{x}} = 0$ cannot be rejected, then $u_{0i} = v_{0i}$ and the model becomes

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + \gamma_x x_{it} + u_{0i} + e_{it} \tag{43}$$

for which GLS random effects estimation will yield consistent and efficient estimators. If $\gamma_{0\bar{x}} = 0$ is rejected, then random effects estimation of Equation (40) (or 42) will yield the unbiased and consistent within estimator of γ_x.

The results of fitting Equations (38) and (40) to the NLSY wage data are given in columns 1 (standard GLS) and 2 (GLS fixed effect) of Table 23.3. The estimates for socioeconomic status, marital status, and union membership are virtually identical to those given earlier (Table 23.2).¶ Before we saw that the difference between these estimates indicated that the time-varying explanatory variables are correlated with the person effects, making GLS estimates biased. This same result is indicated in column 2 by the coefficients of the time means (i.e., $\bar{x}_{i.}$, here called "heterogeneity terms") for the explanatory variables. As before, these coefficients, which are all significant, indicate that the GLS estimator overstates the effects of changes in socioeconomic status, marital status, and union membership on changes in hourly wage. Occupational socioeconomic status is most strongly correlated with omitted time-invariant person effects, as found earlier. The hypothesis that all three coefficients for heterogeneity bias are zero is easily rejected (Wald $\chi^2 = 71$), as is the hypothesis that the intercept variance $\tau_0^2 = 0$ ($\chi^2 = 1600$).

*The estimator of γ_x obtained by including $\bar{x}_{i.}$ as a regressor also has another interpretation: it is the instrumental variables estimator that results when $(x_{it} - \bar{x}_{i.})$ is used as an instrument for x_{it} in the least-squares regression of y_{it} on T_{it} and x_{it}.

†These equivalencies also hold when a random slope effect u_{1i} is included; they do not hold when the coefficient of x_{it} is itself random.

‡Relevant here is the omitted variable interpretation of the Hausman test (Maddalla 2000 p. 498). In particular, if $\bar{x}_{i.}$ in Equation (40) is replaced by the residuals from the regression of x_{it} on $(x_{it} - \bar{x}_{i.})$, the parameter estimates and standard errors would be exactly identical. Since the coefficient of the residuals indicates heterogeneity bias, so too does the coefficient of $\bar{x}_{i.}$, since they are the same.

¶The very slight difference is totally due to the different time functions.

TABLE 23-3. Wage Models; Full-time Employed Males, 1980–1987 (N = 544 and T = 8)

	Models						
Independent variables	1	2	3	4	5	6	7
Constant	1.31	0.904	0.907	0.985	0.702	0.702	0.539
	(55)	(17)	17)	(15)	(5.8)	(5.4)	(4.2)
Year	0.057	0.059	0.059	0.036	0.019	0.019	0.033
	(22)	(22.7)	(17)	(3.2)	(1.2)	(0.9)	(1.5)
Level 1 effects							
Occupational status	0.024	0.012	0.008	0.006			0.006
	(4.8)	(2.3)	(1.4)	(1.1)			(1.1)
Married (= 1)	0.081	0.059	0.059	0.058			0.057
	(4.8)	(3.2)	(3.1)	(3.0)			(3.0)
Union (= 1)	0.116	0.086	0.084	0.086			0.087
	(6.4)	(4.4)	(4.3)	(4.5)			(4.5)
Level 2 effects (intercept)							
Black (= 1)					−0.062	−0.062	−0.051
					(1.1)	(1.0)	(0.9)
Schooling (years)					−0.062	0.062	0.051
					(6.2)	(5.6)	(4.2)
Level 2 effects (slope)							
Black (= 1)					−0.019	−0.019	−0.017
					(2.6)	(1.9)	(1.7)
Schooling					0.003	0.004	0.001
					(2.9)	(2.1)	(0.4)
Heterogeneity terms (intercept)							
Occupational status (SEI/10)		0.103	0.106	0.073			0.031
		(7.0)	(7.2)	(4.2)			(1.6)
Union (= 1)		0.264	0.268	0.299			0.268
		(5.1)	(5.2)	(4.9)			(4.3)
Married (= 1)		0.140	0.142	0.198			0.182
		(3.2)	(3.2)	(3.8)			(3.5)
χ^2		71	75	49			33
Heterogeneity terms (slope)							
Occupational status (SEI/10)				0.010			0.009
				(3.6)			(2.8)
Union (= 1)				−0.010			−0.008
				(1.0)			(0.8)
Married (= 1)				−0.016			−0.019
				(1.9)			(2.2)
χ^2				22			17
Variance components							
τ^2_0	0.123	0.109	0.143	0.141	0.117	0.157	0.134
	(1695)	(1600)	(871)	(878)	(1710)	(972)	(830)
τ^2_1			0.003	0.003		0.003	0.003
			(183)	(165)		(180)	(162)
σ^2_e	0.125	0.125	0.106	0.106	0.125	0.107	0.106
Deviance −2log(L)	4496	4428	4245	4224	4484	4304	4186

Note: Appearing in parentheses below the coefficients and variance components are the t-ratios and χ^2 statistics, respectively.

The next logical direction in which the level 2 model might be revised is given by

$$\beta_{0i} = \gamma_{00} + \gamma_{0\bar{x}}\bar{x}_{i.} + v_{0i} \tag{44}$$

$$\beta_{1i} = \gamma_{10} + u_{1i} \tag{45}$$

$$\beta_{xi} = \gamma_x \tag{46}$$

where the rate of change is now subject to the random person effect u_{1i}. The full combined model is

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + \gamma_x x_{it} + \gamma_{0\bar{x}}\bar{x}_{i.} + u_{1i}T_{it} + v_{0i} + e_{it} \tag{47}$$

which is the final form of the models fit by Horney et al. (1995) in their study of criminal careers. GLS random effects estimation will yield a consistent estimator of γ_x if either $\tau_1^2 = 0$ or, failing that, x_{it} is uncorrelated with the individual time-invariant person slope effects u_{1i}. One signal of the latter problem would be a sharp difference between the fixed effects estimates of γ_x and the estimates when a random slope effect is added to the model. The results of fitting this model to the NLSY data are given in column 3 of Table 23.3. The hypothesis $\tau_1^2 = 0$ is easily rejected $\tau_1^2 = 0.003$, $\chi^2 = 183$, $p < 0.0001$), so there remains significant variation in wage growth rates even after taking account of within and between variation in occupational status, marital status, and union membership. Virtually all the coefficients unchanged from their previous values, with one exception: the coefficient of occupational status drops sharply to 0.008 from 0.012. This suggests correlated unobservable random slope effects, especially for occupational status.

One way to assess whether estimates of γ_x reflect unobservable slope effects is to model the correlation between u_{1i} and the explanatory variables. Hence, suppose that unobservable slope effects are related to the means of the explanatory variables as follows:

$$u_{1i} = \gamma_{1\bar{x}}\bar{x}_i + v_{1i} \tag{48}$$

The level 2 regressions for the intercept β_{0i} and slope β_{1i} of the growth function then become

$$\beta_{0i} = \gamma_{00} + \gamma_{0x}\bar{x}_{i.} + v_{0i} \tag{49}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{1x}\bar{x}_{i.} + v_{1i} \tag{50}$$

The full combined model is then

$$y_{it} = \gamma_{00} + \gamma_{10}T_{it} + \gamma_x x_{it} + \gamma_{0\bar{x}}\bar{x}_{i.} + \gamma_{1\bar{x}}\bar{x}_{i.}T_{it} + v_{0i} + v_{1i}T_{it} + e_{it} \tag{51}$$

where we see that introducing $\bar{x}_{it}$ to the slope equation yields an "interaction" term $\gamma_{1x}\bar{x}_{i.}T_{it}$ in the full model. The rate of growth in y_{it} is now $(\gamma_{10} + \gamma_{1\bar{x}}\bar{x}_{i.} + v_{1i})$; $\gamma_{1\bar{x}}$ gives the expected difference in the rate of growth for persons who are one unit apart on the mean of the explanatory variable $\bar{x}_{i.}$. A χ^2 test of $\gamma_{1\bar{x}} = 0$ can be used to check for an association between the explanatory variables and the rate of change in the response variable. Applied to the NLSY wage data, this test yields $\chi^2 = 22$ (see column 4 in Table 23-3), which is significant ($p < 0.001$). Judging by the *t*-statistics, the wage trajectory varies most with mean occupational status, which appears to be associated with a heightened rate of wage growth. The coefficients for union membership and marital status indicate that both are associated with a slower rate of wage growth, though only the latter is marginally significant. These results indicate that the GLS estimates of the γ_x parameters of the model of Equation (47) are biased by correlated slope effects, especially with respect to occupational status. Hence, comparing the estimates of the γ_x parameters in column 4 and column 3 shows that controlling for correlated slope effects has a proportionately larger impact on the coefficient of occupational status, which drops to 0.006 from 0.008, than on the coefficients of either marital status or union membership, which barely change. On the whole, this final model gives reliable evidence that changes in union membership and marital status, but probably not occupational status, yield changes in mean hourly wage. Note that the slope variance $\tau_1^2 = 0.003$ (Table 23-3, model 4) remains highly significant $\chi^2 = 165$), so that neither between nor within variation in this set of explanatory variables does much to account for individual differences in wage trajectories.

Time-Invariant Explanatory Variables

The principal reason behind including time-varying covariates in growth models is the same as in other contexts: to estimate the effect of changes in the explanatory variables on changes over time in the mean of the response variable.* Yet most applications of growth modeling omit time-varying covariates altogether: instead the focus is on describing between-unit variation in the parameters governing growth by expanding the level 2 model for the intercept and slope to include time-invariant properties of the units. This describes the Cherlin et al. (1998) study of the effect of divorce on children's mental health. Their growth models include no time-varying covariates at all in the level 1 regression; time-invariant properties are introduced to the level 2 intercept and slope equations. Similarly, McLeod and Shanahan's (1996) models for childhood depression and antisocial behavior are formulated exclusively in terms of time-invariant explanatory variables that appear in the level 2 equations. Such models follow the same principles already discussed for time-varying explanatory variables, though with limitations.

The typical level 2 equations for a model with time-invariant explanatory variables are

$$\beta_{0i} = \gamma_{00} + \gamma_{0z} z_i + u_{0i} \tag{52}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{1z} z_i + u_{1i} \tag{53}$$

where z_i is a metric or categorical variable believed to influence both the baseline level of the response variable and its rate of growth.†‡ Combining this level 2 model with a linear level 1 model for trend (ignoring time-varying explanatory variables) yields the full equation for y_{it}:

$$y_{it} = \gamma_{00} + \gamma_{1,0} T_{it} + \gamma_{0z} z_i + \gamma_{1z} z_i T_{it} + u_{0i} + u_{1i} T_{it} + e_{it} \tag{54}$$

*Our aim in including $\bar{x}_{i.}$ in the level 2 regressions discussed above was largely a matter of controlling unobserved heterogeneity bias that threatened the estimation of γ_x; it was not a matter of "explaining" between-unit variation in the intercept and slope of the growth path.

†The analogy to contextual effects in research on schools or neighborhoods is again relevant. Hence, z_i might indicate for the *i*th school whether it is public or private or the proportion of teachers with advanced degrees; it would be student invariant, since it would take on the same value for all students in the *i*th school.

‡The same variables are usually included in both level 2 equations (e.g., Cherlin et al., 1998). At the very least, explanatory variables appearing in the slope equation would also appear in the intercept equation, the principal being the usual one of only including interactions after main effects have been accounted for.

The interpretations of the intercept and slope parameters governing the differences associated with z_i are identical to those discussed above for the case of the over time mean $\bar{x}_{i.}$.*

The time invariance of z_i has rather different implications for the estimation of the mean parameter γ_{0z} and rate parameter γ_{1z}. Because z_i is time invariant, the formulation used to deal with unobserved heterogeneity in the case of a time-varying explanatory variables is not available: there is no difference between z_i and its mean over time, no within-variation in z_i, so the parameter γ_{0z} for the effect of z_i on the mean of y_{it} cannot be distinguished from the person-specific effects u_{0i} with which z_i is perfectly correlated. This is not to say that an estimate $\hat{\gamma}_{0z}$ cannot be produced, but only that its relationship to the true parameter is clouded by unobserved heterogeneity. Hence, the GLS random effects estimator, $\hat{\gamma}_{0z}$ say, will be a function of the true parameter γ_{0z} and a parameter λu_{0iz}, say, for the correlation of z with the unobserved individual effects. There is no way within a random effects framework to identify these two parameters short of assuming that z_i is uncorrelated with u_{0i}.† Since such an assumption may be too strong with observational data, consistent estimation of γ_{0z} for the effect of z_i on the mean of y_{it} at baseline is problematic. Yet this is not of great consequence for most growth modeling applications: most researchers are not very interested in the effect of explanatory variables on the baseline mean of the response variable. Indeed, estimating such effects is mainly a cross-sectional exercise for which panel data are largely irrelevant.

In most growth modeling, the parameter of theoretical interest is γ_{1z} for the interaction of z_i and T_{it}, since this shows how the rate of change or trajectory of the response variable varies with z_i. For some models, this parameter can be consistently estimated without threat of heterogeneity bias, as in the model of Table 23.2. For example, when the slope equation is specified as non-random ($\tau_1^2 = 0$), the combined model becomes

$$y_{it} = \gamma_{00} + \gamma_{1,0}T_{it} + \gamma_{0z}z_i + \gamma_{1z}z_iT_{it} + u_{0i} + e_{it} \tag{55}$$

Although the term z_iT_{it} varies between-units as well as overtime, the "main effect" term for z_i partials out the between-variation and controls for unobserved heterogeneity generated by u_{0i}. The upshot is that GLS random effects estimation of Equation (55) yields the within estimator of the key parameter γ_{1z} and exactly the same standard errors and test statistics as fixed effects estimation.‡

Column 5 of Table 23.3 gives the fitted model (Equation [55]) showing the effects of race and schooling on the wage growth trajectory for the NLSY data. The estimated slope parameters indicate that Black wages grew at an annual rate of 1.9% less than that of others and that each year of schooling increased the rate of growth in wages by about 0.3% (equivalently, the annual rate of return to schooling increased by 0.3% over this 8-year period). Column 6 gives the estimates of Equation (54), where the random slope effect u_{1i} is included so that τ_1^2 is no longer fixed at zero. The point estimates are strikingly similar to their fixed effect counterparts in column 5, although the *t*-statistics are smaller because the fixed effect estimator understates the true standard errors when, as is the case here, $\tau_1^2 > 0$.

*The intercept is $(\gamma_{00} + \gamma_{0z}z_i + u_{0i})$, with γ_{0z} capturing the mean difference in the response variable at baseline for persons one unit apart on (or in different categories of) the z metric; the rate of change in y_{it} is $(\gamma_{10} + \gamma_{1z}z_i + u_{1i})$, with γ_{1z} capturing differences in the rate of change in y_{it} for different values of z. To the extent that z_i is a source of between-unit variation, the expectation is $\tau_{0|z}^2 < \tau_0^2$ and $\tau_{1|z}^2 < \tau_1^2$. Again, a likelihood ratio test of $\tau_{0|z}^2 = \tau_{1|z}^2 = 0$ would be appropriate.

†Instrumental variable estimation would be an appropriate alternative (Hausman & Taylor, 1981).

‡Note the parallel to the case of time-varying explanatory variables: just as consistent estimation of γ_x, the parameter for the effect of changes in x_{it} on changes in the response variable y_{it}, calls for controlling $\bar{x}_{i.}$, so too consistent estimation of γ_{1z}, the parameter for between-group variation in growth trajectories, calls for controlling z_i.

The GLS estimates of the γ_{1z} parameters for race and schooling differences in the rate of wage growth are consistent if both variables are uncorrelated with the unobserved random slope effect u_{1i}. In general, the prospects for obtaining consistent estimators of γ_{1z} parameters are less bleak than those for the γ_{0z} parameters: the existence of time-invariant, person-specific random *slope* effects is not nearly as theoretically or empirically compelling, especially over long periods, as person-specific random intercept effects. Hence, assuming that time-invariant covariates are uncorrelated with individual slope effects like u_{1i} is weaker than assuming they are uncorrelated with baseline effects like u_{0i}. There is also likely to be considerably less heterogeneity to begin with in random slope effects than in random intercept effects, so bias induced by the former in estimates of γ_{1z} parameters is likely to be much less than that induced by the latter in estimates of γ_{0i} parameters.*

Most applications of growth modeling focus on time invariant explanatory variables and exclude time-varying explanatory variables altogether. This practice might be justified by an interest in estimating reduced-form models for the "total effects" of time-invariant background variables, although such justification is rarely expressed. In any event, there is no reason why the two types of explanatory variables cannot be mixed just as in any panel analysis. Indeed, omitting relevant level 1 time-varying explanatory variables can impact estimates of the γ_{1z} rate parameters. As an illustration, column 7 of Table 23.3 gives a fitted model that joins the model of column 4 for the effects of changes in the time-varying explanatory variables on changes in wages, to the model of column 6 for the effects of time-invariant variables on the wage trajectory. The estimates of the level 1 γ_x parameters for occupational status, marital status, and union membership are virtually identical to their previous values; similarly, the estimates of the γ_{0z} parameters for the intercept effects of race and schooling are hardly changed. The estimates of the level 2 γ_{1z} slope parameters for race and schooling have diminished, especially that for schooling, which now appears to have no net association with wage growth when the time-varying explanatory variables are controlled.

CONCLUSION

The purpose of this review has been to give an integrated account of the considerations and methods that underlie the use of panel data in life course studies aimed at (1) the estimation of the effect of a change in an explanatory variable on the change in a response variable and (2) the analysis of variation in growth trajectories. Issues of bias and consistency due to unobserved heterogeneity have been a central theme because in static models they take priority over and are separable from questions of efficiency and the estimation of random components of variation. Methods for dealing with complex error structures, including those characterized by heterogeneity over time or serial correlation have not been discussed here, but are available for growth models as for more standard panel models (Bryk & Raudenbush, 1992; Goldstein, 1995; Wooldridge, 2000, Chs 12 and 13). Yet such methods assume less practical significance with the arrival of routine procedures for the robust estimation of standard errors. In contrast, the effect of unobserved heterogeneity on estimators is a persistent and core issue in the treatment of models that extend beyond those examined here and, hence, forms something of an organizing principle for those who wish to explore other applications of panel analysis to life course phenomena. For example, guarding against unobserved heterogeneity,

*As observed earlier, in observational studies τ_1^2 typically constitutes a much smaller fraction of the total variation in the response variable than does τ_0^2.

along with the attendant issues of fixed as compared to random effects, is at the heart of questions pertaining to the estimation of panel models for limited dependent variables (Maddalla, 1986) and for dynamic social processes (Nickell, 1981).

The effort to deal with unobserved heterogeneity has not always been at the forefront of the concerns motivating the choice of statistical methods by life course researchers. The irony in this is that life course research is uniquely positioned to take advantage of the simple procedures outlined here, since the use of longitudinal data is in a real sense constitutive of the mission of life course studies. Yet life course researchers have only recently begun to exploit the power of panel methods for illuminating the social and psychological changes that accompany the transitions that mark the stages of the life course and that drive the evolution of developmental trajectories. As the study of the life course matures, research increasingly will move away from the ad hoc application of regression methods that fail to exploit the potential of longitudinal data and toward the more systematic and theoretically grounded approaches to panel data that the methods reviewed in this chapter make possible.

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