Chapter 23
Coherence

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Abstract We encounter the notion of coherence in many branches of philosophy. This overview introduces some basic distinctions that can be used to characterize concepts of coherence. After that, two formal frameworks for the analysis of coherence are introduced. The first of these is based on the logic of support relations. It is used to show that coherentism and foundationalism may be combinable rather than antithetical. The second framework assumes that coherence comes in degrees and that it can be measured in probability-based units. The properties of such measures is discussed, and so are the difficulties in constructing a measure of coherence that satisfies intuitively reasonable constraints.

23.1 Coherence Is Everywhere

We encounter the notion of coherence in many branches of philosophy.

> In the theory of knowledge, coherentists claim that our beliefs all justify each other. Their adversaries, the foundationalists, maintain that a limited subset of the beliefs, the basic beliefs, provide the justification for all the others.
>
> According to Bayesian epistemologists, a rational subject's beliefs must be probabilistically coherent, that is, comply with the laws of probability.
>
> In the philosophy of science, internal tensions (incoherence) in a scientific theory or paradigm are seen as driving forces for its replacement by something better.
>
> In metaphysics, coherentists about truth claim that the truth of a proposition consists in its coherence with other propositions. According to its main rival, correspondence theory, the truth of a proposition is constituted by its correspondence to objective features of the world.

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S. O. Hansson, V. F. Hendricks (eds.), *Introduction to Formal Philosophy*, Springer Undergraduate Texts in Philosophy, https://doi.org/10.1007/978-3-319-77434-3_23

Consistency in logic and mathematics is often described as a form of coherence.

Ethicists such as John Rawls have emphasized that our ethical principles and our judgments in practical ethical issues should form a coherent system (be in a "reflective equilibrium").

In decision theory and action theory, it is usually assumed that a rational plan has to be coherent [17].

In recent years, legal scholars have increasingly emphasized that the law and its interpretation must form a coherent system, preferably based on some common principles.

In spite of the ubiquity of coherence in philosophy, surprisingly few attempts have been made to clarify the general meaning of this term in precise, formal terms.[1] Most formal treatments of coherence have focused on only one application area (usually epistemology), and consequently they lack in generality. The formalization of coherence is still at an early stage, and no consensus has been reached on what criteria a good model should satisfy, or how it should be constructed. In the following section, some distinctions that are essential for the formalization of coherence will be introduced. After that we will have a look at two formalizations, one that is quite general and a more specialized one that is often referred to in epistemology.

23.2 Distinctions That We Need

Some things come in degrees but are nevertheless often discussed in all-or-nothing terms. Temperature is one of these. Although it (literally) comes in degrees we can say: "Yesterday it was hot but today it is not." Coherence is another:

"Her talk was more coherent than his." (*comparative coherence*)
"Her talk was coherent. His was not." (*absolute coherence*)

A model of coherence can treat it in either of these two ways. The absolute version is simpler and may be more clear for some purposes, but of course the comparative version has room for more nuances.

Another important distinction is that between, on the one hand, cohering with something else, and on the other hand, being coherent in itself [3]:

"Her views on capital punishment do not cohere with her more general moral views." (*relational coherence*)
"Her moral standpoints are remarkably incoherent." (*systemic coherence*)

[1]This has been pointed out repeatedly, for instance by Bender [3], Bonjour [5], Bartelborth [2], Olsson [20, pp. 12–13] and Moretti and Akiba [18].

Furthermore, systemic coherence can be treated in two different ways that it is important to distinguish between, although the distinction is somewhat intricate. We normally see the coherence of a system as a matter of how well its parts hang or hold together. This interpretation is also recorded in dictionaries; to be coherent means to "stick or cling firmly together" according to the Oxford English Dictionary. Thus consider the following set of three sentences:

(1) {"Life is sacred", "All murderers should be executed", "It is soon five o'clock"}

This has the appearance of being an incoherent set, since the two first sentences do not fit well together. But now consider the following set:

(2) {"Life is sacred and all murderers should be executed", "It is soon five o'clock"}

This set consists of only two sentences, and there is no conflict between the two. If we consider the coherence of a set as something to be determined solely by relations among its elements, then (2) must be deemed much more coherent than (1). In similar fashion, any incoherent set could be made more coherent by merging its most diverging elements into a single element. But presumably most of us would see such an operation as a way to hide the incoherence rather than reduce it. The reason for this is that when we see a set such as (2), we do not accept its elements as representing the actual parts of that which the set represents.

When two sets, such as (1) and (2), have the same contents, we tend to assume that they also have the same degree of coherence. The underlying assumption is that coherence is a property of the contents of the set, rather than a property of the collection of elements that is used to present the contents. In order to determine the coherence of the contents, we identify its "actual" constituent parts (which may be different from those that were presented to us). We then investigate to what extent these "actual" constituents hang together. When thinking in this way we apply a *presentation-insensitive* notion of systemic coherence.

But we should not exclude the possibility of evaluating the coherence of a particular presentation of a set. I once listened to an exposition of a new legislation that was correct but nevertheless confusing because of the disorganized order of presentation. I could then have said: "The material he presented in his talk was coherent, but the presentation was incoherent." Such a comparison would involve both a presentation-insensitive and a *presentation-sensitive* notion of systemic coherence.

Finally, we have to distinguish between the different *types of cohesive and repulsive forces* that operate in different systems whose coherence we want to analyze. In logic and mathematics, coherence depends on the forces of logical implication [7]. In other areas there is a wider variety of forces conferring or constraining coherence. In epistemology, different types of inferential relations (in a wide sense) can be at play, giving rise for instance to explanatory, evidential, justificatory, or probabilistic coherence [22, p. 144], [4, p. 96]. Ethical coherence can be construed for instance in terms of derivability from common underlying principles, presence of redundant support from several moral principles, or absence of conflicting statements.

23.3 The Support Relations Model

In this section we are going to introduce a simple model of systemic coherence [11]. Its assumption is that we have a system to be evaluated with respect to its coherence, and that this system is represented by a set. (If we are studying presentation-insensitive coherence, then this set has to be composed of the "actual" components of the system.) A support relation S represents the coherence-conferring forces in the system. If a and b are elements of the set, then aSb denotes that a supports b.[2] For illustration we can use diagrams in which the elements are represented by points and S by arrows. An arrow from a to b denotes that a supports b. See Fig. 23.1.

This is in several respects a highly simplified representation of the forces that make coherent systems stick together. First, support is treated as an all-or-nothing affair although we know that support comes in degrees. Secondly, only positive contributions to coherence are covered. In our example (1) above, what made the set incoherent was not just the lack of coherence-conferring relations among the elements but the presence of a conflictual relationship between the first two elements. Thirdly, the model cannot deal adequately with cases where two or more elements in combination provide a support that none of them confers alone, as the first two sentences do to the third in the following example:

(3) {"Amy's father is Chinese", "Amy's mother is British", "Amy is bilingual"}

These limitations can easily be removed. We can replace the binary relation by a function s on pairs of elements, such that $s(a, b)$ is a number representing the degree to which a supports b. Negative values can represent disconnecting forces. In this way we get rid of the first two limitations. To get rid of the third we just need to extend the function to cover expressions such as $s(\{a, b\}, c)$ in which the first argument is a set of sentences.

However, simple binary all-or-nothing support relations are suitable for illustrating certain essential properties of support relations, and they will therefore be used here. One of their major advantages is that they provide us with a convenient representation not only of the interconnected relations in a coherent set but also of the one-sided support relations from the base (basic beliefs) to the rest of the set

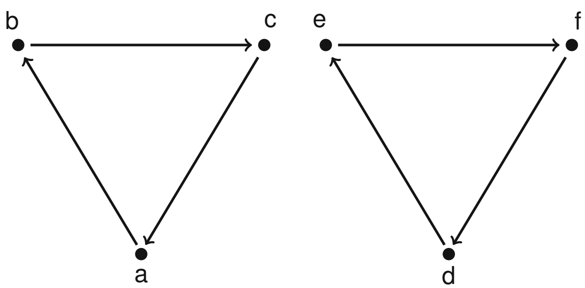


Fig. 23.1 Support relations among the six elements of the set $\{a, b, c, d, e, f\}$. Universal supportedness and Universal supportingness are both satisfied

[2] S is irreflexive, i.e. $\neg(xSx)$ holds for all x.

that are assumed to hold in a foundationalist framework. We can therefore use this simple model to clarify the relationship between coherentism and foundationalism.

Some more notation is needed. The set whose coherence or foundations we are going to investigate needs a name. We can call it E. We also need quantifiers. Unless otherwise specified, $\forall x$ means "for all $x \in E$" and $\exists x$ means "for some $x \in E$". We will also have use for the ancestral S^* of S. It denotes a chain of S-relations that connects two elements of E, hence:

aS^*b holds if and only if either aSb or there is a finite series of elements $x_1, \ldots, x_n$ such that aSx_1, x_kSx_{k+1} for all $1 \le k < n$, and x_nSb.

Coherentism has been explicated by Ernest Sosa as meaning that "a body of knowledge is a free-floating raft every plank of which helps directly or indirectly to keep all the others in place, and no plank of which would retain its status with no help from the others" [30, p. 24]. It follows from this that nothing is unsupported and that everything supports something else. In the formal language, this means that the following two conditions should be satisfied:

$(\forall x)(\exists y)(ySx)$ (*Universal supportedness*)
$(\forall x)(\exists y)(xSy)$ (*Universal supportingness*)

These are two reasonable conditions, but they are not sufficient to define even a minimal notion of coherence. This can be seen from Fig. 23.1. In this diagram, both conditions are satisfied, but it would be strange to claim that the set $\{a, b, c, d, e, f\}$ is coherent. As several authors have pointed out, a coherent system should not have any isolated subsystem or part that is unconnected with the rest of the system [4, p. 97], [31]. This is ensured by the following simple condition:

$(\exists x)(\forall y)(xS^*y)$ (*Non-fragmentation*)

Figure 23.2 shows a case in which *Universal supportedness*, *Universal supportingness*, and *Non-fragmentation* are all satisfied. The combination of these three conditions ensures at least a minimal degree of coherence.[3]

Let us now turn to foundationalism. According to Ernest Sosa, it means that "every piece of knowledge stands at the apex of a pyramid that rests on stable and secure foundations whose stability and security does not derive from the upper stories or sections" [30, p. 24]. This condition refers to a proper, non-empty subset of E. Let us call it B. Then $E \setminus B$ denotes the superstructure, i.e. the set of

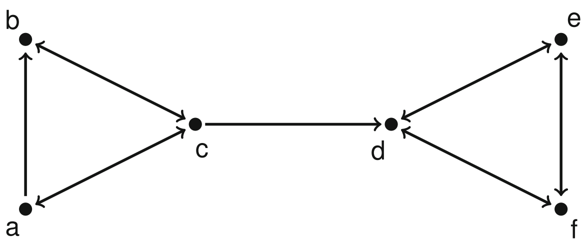


Fig. 23.2 In this case Universal supportedness, Universal supportingness and Non-fragmentation are all satisfied

[3] Alternative, stronger conditions are discussed in [11].

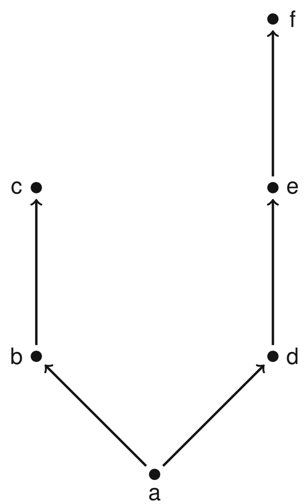
elements of E that are not also elements of B. The quotation from Sosa provides us with two conditions on B. First, it should not be supported by any element of the superstructure, i.e. $(\forall y \in E \setminus B)(\forall x \in B)\neg(ySx)$. Secondly, it should support all the elements of the superstructure. However, this support may be indirect. Therefore we do not need to require that $(\forall y \in E \setminus B)(\exists x \in B)(xSy)$; it is sufficient to require that $(\forall y \in E \setminus B)(\exists x \in B)(xS^*y)$. All this adds up to the following combined requirement:

There is a set B with $\varnothing \neq B \subset E$ such that
$(\forall y \in E \setminus B)(\forall x \in B)\neg(ySx)$ and
$(\forall y \in E \setminus B)(\exists x \in B)(xS^*y)$. (*Base*)

This is a reasonable definition of a base and therefore of foundationalism. It says, essentially, that the base supports the rest of the system but is not supported by it. See Fig. 23.3 in which this criterion is satisfied. This figure also illustrates that the base will not in all cases be uniquely defined. There are in fact no less than eleven alternative bases in this diagram, namely $\{a\}$, $\{a, d\}$, $\{a, d, e\}$, $\{a, d, e, f\}$, $\{a, b\}$, $\{a, b, d\}$, $\{a, b, d, e\}$, $\{a, b, d, e, f\}$, $\{a, b, c\}$, $\{a, b, c, d\}$, and $\{a, b, c, d, e\}$. This may appear disturbing, since we presumably want the base to be well-defined. But the problem can be solved fairly easily. One of the alternative bases, namely $\{a\}$, is uniquely inclusion-minimal, i.e., it is a subset of all the others. We can take it to be the genuine base, of which the other ten are mere extensions.

But now consider Fig. 23.4. Condition *Base* is satisfied here as well. But in this case there is no uniquely inclusion-minimal base, i.e., no base that is a subset of all the others. (To see this, just note that both $\{a, b\}$ and $\{a, b, c\}$ can serve as bases.) From this example we can conclude that contrary to common assumptions, the base of a foundationalist system need not be well-defined.

Fig. 23.3 A case in which Base is satisfied



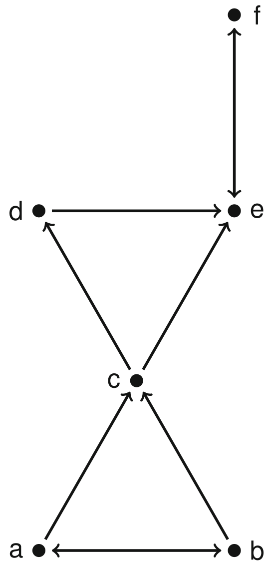


Fig. 23.4 Universal supportedness, Universal supportingness, Non-fragmentation, and Base are all satisfied. The example therefore has both coherentist and foundationalist properties

Figure 23.4 has another, even more important property. We have already seen that it satisfies *Base*. We can easily verify that it also satisfies our three conditions for coherentism, namely *Universal supportedness*, *Universal supportingness*, and *Non-fragmentation*. This example shows that the minimal conditions of coherentism and those of foundationalism are compatible with each other. This opens up the possibility of considering structures of support that are intermediate between, or combine, the classic notions of coherentism and foundationalism. The traditional dichotomy can then be replaced by descriptive models that recognize a wider variety of types of support structures. This more nuanced picture emerges, characteristically, from endeavours to express these notions in formal language so that their implications can be spelt out in full detail.

23.4 Probabilistic Measures of Coherence

Let us now turn to another formal representation of coherence that has attracted much attention lately, namely probabilistic coherence measures. Just as in the previous approach, it is assumed that we have a collection of objects, and that the coherence we are looking for is a property of this collection. In this more specified approach, it is assumed that the elements are propositions to which we can assign probabilities. However, the collection is not represented in the formal language by a set but by a sequence. The reason for this is that duplicates may be of interest. The collection $\langle a, a, b\rangle$ contains two reports that a, whereas $\langle a, b\rangle$ only has one such report. Consequently, these two collections may differ in their degrees of probabilistic coherence. Since the sets $\{a, a, b\}$ and $\{a, b\}$ are identical, they cannot be used to represent such distinctions.

A probabilistic coherence measure is a numerical function that takes us from such a sequence of propositions to a number that represents its degree of coherence. The measure is supposed to depend only on the probabilities of the elements of the sequence and of their Boolean combinations [20, p. 100].

Lewis [15, p. 338] proposed a definition of coherence in terms of probability. Beginning in 1999, veritably dozens of probabilistic coherence measures have been put forward. The following two are probably the ones most often referred to:

Shogenji's [28] coherence measure:

$$C_S(\langle A_1, \ldots, A_n \rangle) = \frac{P(A_1 \& \ldots \& A_n)}{P(A_1) \times \ldots \times P(A_n)}$$

Olsson's [19] coherence measure:

$$C_O(\langle A_1, \ldots, A_n \rangle) = \frac{P(A_1 \& \ldots \& A_n)}{P(A_1 \vee \ldots \vee A_n)}.$$

Shogenji's measure is the ratio between how probable it is that all the sentences are true and how probable this would have been if they had been probabilistically independent of each other. Olsson's measure has been called a measure of overlap. It is the ratio between the probability of all sentences being true and the probability of at least one of them being so. Other measures have been proposed for instance by Fitelson [9, 10], Meijs [16], Douven and Meijs [8], and Siebel and Wolff [29]. Bovens and Hartmann [6] have proposed an arguably somewhat more sophisticated approach in which the numerical measure is replaced by an incomplete ordering. Then sets can be incomparable in terms of their degrees of probabilistic coherence. However, consideration of the two measures already mentioned is sufficient to give a picture of the general nature of probabilistic coherence measures.

Shogenji's measure has been criticized for yielding counter-intuitive results for agreeing reports. Suppose that we initially have one report saying that a coin that was thrown yesterday yielded heads. This is represented by a sequence $\langle A \rangle$ with only one element, namely the proposition A with probability 0.5. Shogenji's measure yields the coherence $C_S(\langle A \rangle) = 1$. But then we receive another report saying the same thing. This increases the degree of coherence to $C_S(\langle A, A \rangle) = 2$. A third report will yield $C_S(\langle A, A, A \rangle) = 4$, etc. According to Fitelson [9] who pointed out this, the measure behaves counterintuitively since we should expect coherence to remain constant when more and more agreeing reports are received. Olsson's measure fares much better here; indeed we have $C_O(\langle A \rangle) = 1$, $C_O(\langle A, A \rangle) = 1$, etc. for any number for agreeing reports.

On the other hand, Olsson's measure has another disputable feature. Based on an insight first reported by Bovens and Hartmann [6], Koscholke and Schippers [14] showed that according to this measure, a set's degree of coherence cannot be increased by adding another proposition to the set. Examples are easily found in which this runs counter to intuition:

m Haris and Rosarita are going to marry.
n Haris and Rosarita have no romantic relationship.
r Rosarita is a refugee for whom a marriage is the only chance not to be sent back to a war zone.
h Haris is a pro-refugee activist.

It would seem plausible to claim in this case that the set $\{m, n, r, h\}$ is more coherent than its subset $\{m, n, r\}$, which is in its turn more coherent than $\{m, n\}$.

Shogenji's and Olsson's measures have a feature in common for which they have been much criticized: They both yield different results for logically equivalent sets. Akiba [1] said: "Obviously the coherence of two beliefs B_1 and B_2 should be no different from the coherence of one conjunctive belief $B_1\&B_2$." Therefore, he said, the two sets $\{B_1, B_2\}$ and $\{B_1\&B_2\}$ should have the same degree of coherence. This, however, is not the case. Let B_1 denote that the next throw of a fair dice will yield an odd number and B_2 that it will yield a prime. Then we have $C_S(\{B_1, B_2\}) = 4/3$ and $C_S(\{B_1\&B_2\}) = 1$. Similarly, $C_O(\{B_1, B_2\}) = 1/2$ and $C_O(\{B_1\&B_2\}) = 1$. Hence neither of these two measures satisfies Akiba's criterion.

In reply, Olsson has questioned Akiba's assumption that sets with the same contents should always have the same degree of coherence, irrespective of how these contents are distributed among sentences [20, p. 102]. Responding to this, Moretti and Akiba [18] showed that a wide variety of probabilistic coherence measures assign different degrees of coherence to logically equivalent sets of propositions. They call this the "problem of belief individuation". However, the seeming problem can be resolved with help of the distinction between presentation-sensitive and presentation-insensitive coherence that was introduced above. Different presentations of one and the same information can differ in their degrees of coherence, even though the coherence of the respective information is the same. Shogenji's and Olsson's coherence measures are presention-sensitive, and there is nothing wrong in them being so.[4] A presentation-insensitive measure would have to replace a given presentation by some standard presentation of the same information before measuring the probabilistic relations among the elements.

A considerable number of probabilistic coherence measures have been proposed. For instance, Koscholke [13] reviewed eighteen of them. Schippers [24, 25] has shown that it is impossible to construct a coherence measure that satisfies a small set of intuitively plausible properties. Based on this, he proposed that a pluralist approach to coherence measures may be appropriate [25, p. 972].

An issue of much interest for coherentist epistemology is whether a probabilistic coherence measure can be truth-conducive. By this is meant that a more coherent sequence is more likely to be true [12]. Several impossibility results have been put forward, indicating that no informative coherence measure can be truth-conducive [6, 12, 20, 21]. The debate is still on-going, and attempts to save

[4] Olsson introduced his measure in a framework where coherence is a property of reports, typically coming from different sources. (Bovens and Hartmann did the same.) Presentation-sensitivity is thus explicitly assumed.

truth-conduciveness have been made [23, 26, 27]. However, it is difficult to see how truth-conduciveness could be achieved with presentation-sensitive measures (like the ones currently under discussion). Expectedly, a truth-conducive measure should treat two sequences equally if they have the same relation to the truth, which they have if they convey the same information.

As mentioned at the outstart, the formal treatment of coherence is still at a surprisingly early stage, given the importance of this concept in several branches of informal philosophy. There is a need for new and innovative measures and models.

Acknowledgements I would like to thank Erik Olsson for very useful comments on an earlier version of this text.

References and proposed readings

Asterisks (∗) indicate recommended readings.

1. Akiba, K. (2000). Shogenji's probabilistic measure of coherence is incoherent. *Analysis, 60*, 356–359.
2. Bartelborth, T. (1999). Coherence and explanations. *Erkenntnis, 50*, 209–224.
3. Bender, J. W. (1989). Coherence, justification, and knowledge: The current debate. In J. W. Bender (Ed.), *The current state of the coherence theory* (pp. 1–14). Dordrecht: Kluwer.
4. Bonjour, L. (1985). *The structure of empirical knowledge*. Cambridge, MA: Harvard University Press.
5. ∗Bonjour, L. (1999). The dialectic of foundationalism and coherentism. In J. Greco & E. Sosa (Eds.), *The Blackwell guide to epistemology* (pp. 117–142). Malden: Blackwell. [A rigorous but informal introduction to coherence and foundationalism.]
6. ∗Bovens, L., & Hartmann, S. (2003). *Bayesian epistemology*. Oxford: Oxford University Press. [An excellent introduction that connects the issue of probabilistic coherence with other issues in epistemology and the philosophy of science.]
7. Daya, K. (1960). Types of coherence. *Philosophical Quarterly, 10*, 193–204.
8. Douven, I., & Meijs, W. (2007). Measuring coherence. *Synthese, 156*, 405–425.
9. Fitelson, B. (2003). A probabilistic theory of coherence. *Analysis, 63*, 194–199.
10. Fitelson, B. (2004). Two technical corrections to my coherence measure. Online paper, http://fitelson.org/coherence2.pdf.
11. ∗ Hansson, S. O. (2007). The false dichotomy between coherentism and foundationalism. *Journal of Philosophy, 104*, 290–300. [The support relations model.]
12. Klein, P., & Warfield, T. A. (1994). What price coherence? *Analysis, 54*, 129–132.
13. ∗Koscholke, J. (2016). Evaluating test cases for probabilistic measures of coherence. *Erkenntnis, 81*, 155–181. [Evaluates eighteen probabilistic coherence measures against a number of test cases.]
14. Koscholke, J., & Schippers, M. (2016). Against relative overlap measures of coherence. *Synthese, 193*, 2805–2814.
15. Lewis, C. I. (1946). *An analysis of knowledge and valuation*. La Salle: Open Court.
16. Meijs, W. (2006). Coherence as generalized logical equivalence. *Erkenntnis, 64*, 231–252.
17. Millgram, E. (2000). Coherence: The price of the ticket. *Journal of Philosophy, 97*(2), 82–93.
18. Moretti, L., & Akiba, K. (2007). Probabilistic measures of coherence and the problem of belief individuation. *Synthese, 154*, 73–95.
19. Olsson, E. J. (2002). What is the problem of coherence and truth? *Journal of Philosophy, 94*, 246–272.

20. ∗ Olsson, E. J. (2005). *Against coherence. Truth, probability, and justification.* Oxford: Clarendon Press. [An excellent overview with a particular emphasis on probabilistic measures and truth conduciveness.]
21. Olsson, E. J. (2005). The impossibility of coherence. *Erkenntnis, 63*, 387–412.
22. Rescher, N. (1979). *Cognitive systematization: A systems-theoretic approach to a coherentist theory of knowledge*. Oxford: Blackwell.
23. Roche, W. (2014). On the truth-conduciveness of coherence. *Erkenntnis, 79*, 647–665.
24. Schippers, M. (2014). Probabilistic measures of coherence: From adequacy constraints towards pluralism. *Synthese, 191*(16), 3821–3845.
25. ∗ Schippers, M. (2015). Towards a grammar of Bayesian coherentism. *Studia Logica, 103*, 955–984. [Proposes a set of intuitively reasonable constraints on a probabilistic coherence measure, and shows that for purely mathematical reasons, no measure can satisfy them all.]
26. Schippers, M. (2016). The problem of coherence and truth redux. *Erkenntnis, 81*, 817–851.
27. Schupbach, J. N. (2008). On the alleged impossibility of Bayesian coherentism. *Philosophical Studies, 141*, 323–331.
28. Shogenji, T. (1999). Is coherence truth-conducive? *Analysis, 59*, 338–345.
29. Siebel, M., & Wolff, W. (2008). Equivalent testimonies as a touchstone of coherence measures. *Synthese, 161*, 167–182.
30. Sosa, E. (1980). The raft and the pyramid: Coherence versus foundations in the theory of knowledge. *Midwest Studies in Philosophy, 5*, 3–25.
31. Spohn, W. (1999). Two coherence principles. *Erkenntnis, 50*, 155–175.