# Chapter 29
# Preference and Choice

**Sven Ove Hansson**

**Abstract** Preferences and choices have central roles in moral philosophy, economics, and the decision sciences in general. In a formal language we can express and explore the properties of preferences, choices, and their interrelations in a precise way, and uncover connections that are inaccessible without formal tools. In this chapter, the plausibility of different such properties is discussed, and it is shown how close attention to the logical details can help dissolve some apparent paradoxes in informal and semi-formal treatments.

## 29.1 Philosophical Problems of Preference

Comparative terms such as "better" and "equally good" have a prominent role both in everyday discussions and in specialized treatments of value in philosophy and economics. In spite of being understood by all of us, the meaning of these terms is in much need of clarification, as can be seen from the following three examples:

**The Paradox of the Outvoted Democrat, Wollheim's Paradox** (Wollheim [29]) Susan has worked hard in a campaign to save the regiment in her hometown from a close-down. When Parliament finally decides to close it down, she is very disappointed since she would very much prefer the regiment to be kept rather than being closed down. But she is also a strong supporter of democratic decision-making, and she would certainly not support the half-baked plans of some young officers to defy the decision and carry on as usual. Hence, she prefers that the

S. O. Hansson (✉)
Division of Philosophy, Royal Institute of Technology (KTH), Stockholm, Sweden
e-mail: soh@kth.se



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regiment be closed down rather than being kept. How can she at the same time prefer the closing down of the regiment to its continued existence and the other way around?

**Preference Holism** On the face of it, what we prefer and disprefer are small components of the world. I may prefer a cup of tea to a cup of coffee, or listening to Beethoven's second rather than his first symphony. But preferences referring to such small items presuppose that these items can be exchanged in isolation from the rest of the world, which may not always be the case. In a sense, such preferences are always conditional on what the rest of the world is like. I would not prefer having tea to having coffee if I had a medical condition that made tea poisonous to me. If, for some unknown reason, there will be more human suffering in the world if I listen to the second symphony rather than the first, then presumably I will prefer listening to the first. The only preferences that can hold unconditionally would be preferences that refer to the complete state of the world. But then, what can we mean by such holistic preferences and what is the relation between them and the more ordinary types of preferences that we express in our common lives?

**Preferences and Choices**

– Congratulations! You have won the lottery. You can now choose between a trip to London and one to Paris. Which do you choose?
– I choose to go to Paris.
– Why do you prefer a trip to Paris rather than one to London?'
– I don't.
– I'm sorry, I don't understand. I thought you said you have chosen Paris.
– Yes I did. I choose Paris but I prefer London.
– What do you mean? How can you choose one and prefer the other? Are you sure that Paris is your choice?
– Of course. What's the problem?

We expect preferences and choices to cohere. But since preferences and choices are quite different entities, it is not fully clear what it means for them to cohere. And if they do, what are the effects of their mutual coherence on the structure of our preferences, and on the structure of our choices?

With preference logic we can solve these and other problems of preference and choice. As a bonus, preference logic opens up new philosophical issues and insights that would not otherwise have been available to us.

## 29.2 The Basic Concepts of Preference Logic

Preference logic makes use of three comparative value concepts, namely "better" (strict preference), "equal in value to" (indifference), and "at least as good as" (weak preference). They are usually denoted by the symbols $>$, $\sim$, respectively $\geq$ (or by $P$, $I$, respectively $R$).

This formal language is idealized in several ways. In ordinary language we make a distinction between the subjective notion of preference and the presumably more objective notion of betterness. If you say that *Falstaff* is better than *Aida*, then you indicate a more objective or at least more generally applicable standpoint than if you say that you prefer *Falstaff* to *Aida*. In preference logic, no distinction is made between these two notions since they are assumed to have the same formal structure.

Furthermore, $A > B$ is taken to represent "$B$ is worse than $A$" as well as "$A$ is better than $B$" [3]. This is not in exact accordance with ordinary English. I consider the *Magic Flute* to be a better opera than *Idomeneo*, but it would be misleading to say that I consider *Idomeneo* to be worse than the *Magic Flute*. We tend to use "better" when focusing on the goodness of the higher-ranked of the two alternatives, and "worse" when emphasizing the badness of the lower-ranked one [7, p. 13]. This distinction is not made in preference logic.

Preference logic is devoted to the preferences of rational individuals. Therefore, if a proposed principle for preference logic does not correspond to how we actually think and react, then this may either be because the principle is wrong or because we are not fully rational in some of the cases it covers.

## 29.3 The Set of Alternatives

The objects of preference are represented by the *relata* (*alternatives*) of the preference relation, i.e. $A$ and $B$ in $A > B$. They can be taken to be primitive objects with no further structure. However, in economics they are usually vectors that represent bundles of goods. In philosophical logic, they are usually sentences (or propositions) representing states of affairs. Sentences appear to be the best general-purpose representation that we have for the objects of our preferences. If Xiuxiu prefers fish to meat, then that can be expressed as a preference for (the state of affairs expressed in) "Xiuxiu eats fish" over (that expressed in) "Xiuxiu eats meat". Sentences can also be combined to form composite relata. Hence, a preference for drinking white wine ($w$) rather than red whine ($r$) when eating fish ($f$) can be expressed with conjunctive relata: $(w \wedge f) > (r \wedge f)$.

In order to specify a system of preference logic it is necessary to state what its relata are, in other words to identify its *alternative set*. Many problems in informal discussions on preferences can only be clarified if a precisely defined set of alternatives is introduced. (See Sect. 29.7.)

## 29.4 Constitutive Logical Properties

The following properties of the three comparative relations are taken to be part of the very meaning of preference and indifference:

(1) $A > B \rightarrow \neg(B > A)$ (asymmetry of strict preference)
(2) $A \sim B \rightarrow B \sim A$ (symmetry of indifference)
(3) $A \sim A$ (reflexivity of indifference)
(4) $A > B \rightarrow \neg(A \sim B)$ (incompatibility of preference and indifference)
(5) $A \geq A$ (reflexivity of weak preference)
(6) $(A \geq B) \leftrightarrow (A > B) \vee (A \sim B)$
(7) $(A > B) \leftrightarrow (A \geq B) \wedge \neg(B \geq A)$
(8) $(A \sim B) \leftrightarrow (A \geq B) \wedge (B \geq A)$

Using these properties, we can simplify the formal structure in either of two ways. One option is to use $>$ and $\sim$ as primitive notions, i.e. notions not defined in terms of any other notions. We can then define $\geq$ according to (6). The other option is to use $\geq$ as a primitive notion and define $>$ and $\sim$ according to (7) and (8). Formally, this works out equally well in both directions. If we use $>$ and $\sim$ as primitives, assume that they satisfy (1)–(4) and define $\geq$ according to (6), then the remaining properties (5), (7) and (8) all hold. Conversely, if we use $\geq$ as the sole primitive, assume that it satisfies (5) and define $>$ and $\sim$ according to (7) and (8), then the remaining properties (1)–(4) and (6) can easily be shown to hold [23].

The choice between these two ways to simplify the logic is fairly inconsequential. Using $\geq$ as the sole connective is preferable from the viewpoint of formal simplicity, but the use of $>$ and $\sim$ seems more conducive to conceptual clarity.

When $>$ and $\sim$ are defined from $\geq$ via (7) and (8) they are called the *strict part*, respectively the *symmetric part*, of $\geq$.

## 29.5 Completeness

In most applications of preference logic, it is taken for granted that the following property is satisfied:

$(A \geq B) \vee (B \geq A)$ (completeness or connectedness)

Given (2) and (6) it is equivalent with:

$$(A > B) \vee (A \sim B) \vee (B > A)$$

The assumption of completeness is often convenient since it provides us with a formal structure that is easier to work with. However, as shown in Box 29.1, in many situations it seems perfectly rational to have incomplete preferences.

**Box 29.1 Incomplete preferences**

*1. Lack of information*
Filomena does not know anything about *Falstaff* or *Aida*. Therefore she does not consider one of them to be better than the other and neither does she consider them to be of equal value.

*2. Insufficiently specified alternatives*
When asked which he prefers, £500 or that his daughter gets a better grade in math, Ali has no answer to give. The reason is that the comparison is insufficiently specified. Does he have an offer to bribe the teacher? Or is he offered an extra course for his daughter that he only has to pay if she gets a better grade?

*3. Costliness of acquiring preferences*
There are about 90 brands of cheese in the local grocery store. In order to make her preference relation complete over all of these brands, Alice would have to buy samples of all of them and engage in extensive comparative testing. Since she is not very fond of cheese, making her preferences complete would not be worth the effort or the costs.

*4. Morally questionable preferences*
José has a nightmare in which a terrorist forces him to choose which of his three children will be killed. If he makes no choice, then all three of them will be killed. When he wakes up, José realizes that in such a situation, he would have to make a choice. However, he feels that he would be a worse person if he knew beforehand what his preferences would then be.

## 29.6 Transitivity

By far the most discussed logical property of preferences is the following:

$(A \geq B) \wedge (B \geq C) \rightarrow (A \geq C)$ (transitivity of weak preference)

It logically implies a whole herd of similar but logically weaker properties such as the following [23]:

$(A \sim B) \wedge (B \sim C) \rightarrow (A \sim C)$ (transitivity of indifference)
$(A > B) \wedge (B > C) \rightarrow (A > C)$ (transitivity of strict preference)
$(A \sim B) \wedge (B > C) \rightarrow (A > C)$ (IP-transitivity)
$(A > B) \wedge (B \sim C) \rightarrow (A > C)$ (PI-transitivity)
There is no series $A_1, \ldots, A_n$ of alternatives such that $A_1 > \ldots > A_n > A_1$ (acyclicity).

Transitivity is often taken to be an obvious requirement of rationality. If I consider $A$ to be at least as good as $B$, and $B$ at least as good as $C$, could there be any reason for me not to consider $A$ to be at least as good as $C$? Well in fact there could. Box 29.2 exhibits the major types of examples that have been used as arguments against transitivity.[1]

**Box 29.2 Intransitive preferences**

*1. Indistinguishable differences add up and become distinguishable*
Aaron cannot taste the difference between wines $A$ and $B$ or between wines $B$ and $C$, but he is able to taste the difference between $A$ and $C$, and he likes $A$ better [5, p. 34].

*2. Negligible differences add up and become relevant*
A self-torturer has a pain-inducing device implanted in her body. The device has 1001 settings, from 0 (off) to 1000. Each increase leads to a noticeable but negligible increase in pain. Each time that she advances the dial by one setting she receives £10,000, but there is no way for her to retreat. In the end the pain is so unendurable that she would gladly relinquish her fortune and return to 0 [17].

*3. A trifle does not affect comparisons between disparate objects*
A boy is indifferent between receiving a bicycle or a pony, and he is also indifferent between receiving a bicycle with a bell and a pony. However, he prefers receiving a bicycle with a bell to receiving just a bicycle [13].

*4. Diverging preferences over several dimensions are reduced to one dimension*
In an experiment performed in the 1950s, 62 college students were asked several questions about which of two potential marriage partners they preferred. The questions were so arranged that all three pairwise combinations of the following three persons were covered: $A$ who was described as very intelligent, plain looking, and well off, $B$ who was portrayed as intelligent, very good looking, and poor, and $C$ who was reported to be fairly intelligent, good looking, and rich. 17 of the students exhibited the circular preference pattern $A > B > C > A$. This can be explained by these students always choosing the partner who was superior in two out of the three criteria [14]. This appears to be a mechanism at play in many situations where preferences over several dimensions have to be reduced to a single dimension [15].

[1] On preference transitivity, see also Chap. 31.

Sometimes preferences can be constructed from numerical values. Let $u$ be a function that assigns a real number $u(A)$ to each element $A$ of the alternative set. Then $u$ is a *numerical representation* of $\geq$ if and only if it holds for all $A$ and $B$ that:

$$A \geq B \text{ if and only if } u(A) \geq u(B)$$

It has been shown that a preference relation has a numerical representation if and only if it is both complete and transitive. (This only holds under some rather technical conditions, but these conditions are satisfied whenever the alternative set is either finite or countably infinite. [6, pp. 27–29], [20, pp. 109–110])

## 29.7 The Outvoted Democrat

We are now equipped to deal with the paradox of the outvoted democrat that was mentioned above. Let $r$ denote that the regiment in Susan's hometown stays in place and $\neg r$ that it does not. Susan wants to keep the regiment, so she prefers $r$ to $\neg r$. But a decision to the contrary has been made, and since she wants democratic decisions to be implemented she also prefers $\neg r$ to $r$.

In order to make sense of this we must observe that $r$ and $\neg r$ are insufficient to describe the alternatives. Susan's preferences also refer to the decision that has been made. Let $Dr$ denote that a democratic decision has been made in favour of $r$, and similarly $D\neg r$ that a democratic decision has been made in the other direction. Then instead of merely $r$ and $\neg r$ we have to consider the four alternatives $Dr \wedge r$, $Dr \wedge \neg r$, $D\neg r \wedge r$, and $D\neg r \wedge \neg r$. We can expect her preferences to be as in Fig. 29.1.

Our next task is to derive preferences for $r$ and $\neg r$ from these preferences over composite states of affairs. It is reasonable to assume that these should be preferences *ceteris paribus* or all things being equal.

What does it mean to prefer $r$ to $\neg r$ "everything else being equal"? Since $D\neg r$ holds in the actual world, one interpretation is "when $D\neg r$ is not changed". Susan prefers $D\neg r \wedge \neg r$ to $D\neg r \wedge r$, i.e. she prefers $\neg r$ to $r$ when $D\neg r$ is kept constant. This accounts for her preference for $\neg r$ over $r$.

But there is also another interpretation. Alternatively, the background factor to be kept constant is whether or not the democratic decision is respected. Let us introduce the predicate $R$ such that $Rr$ means that the democratic decision with respect to $r$ or $\neg r$ is respected. We can now rewrite the four alternatives. $Dr \wedge r$ is equivalent with $Rr \wedge r$, $D\neg r \wedge r$ with $\neg Rr \wedge r$, etc. This gives rise to the reformulation of the preference ordering that is shown in Fig. 29.2.

$$Dr \wedge r$$
$$\vee$$
$$D\neg r \wedge \neg r$$
$$\vee$$
$$D\neg r \wedge r$$
$$\vee$$
$$Dr \wedge \neg r$$

**Fig. 29.1** The outvoted democrat's preferences, as expressed with the predicate $D$ for "A democratic decision has been made to the effect that...". $\vee$ denotes transitive strict preference

$$Rr \wedge r$$
$$\vee$$
$$Rr \wedge \neg r$$
$$\vee$$
$$\neg Rr \wedge r$$
$$\vee$$
$$\neg Rr \wedge \neg r$$

**Fig. 29.2** The same preferences, expressed with the predicate $R$ for "The democratic decision concerning ... is respected"

Since Susan prefers $Rr \wedge r$ to $Rr \wedge \neg r$, and she also prefers $\neg Rr \wedge r$ to $\neg Rr \wedge \neg r$, there can be no doubt that with this construction, she prefers $r$ to $\neg r$. In this way, the ambiguity of the phrase "everything else being equal" makes it possible for the outvoted democrat to strictly prefer, at the same time, $r$ to $\neg r$ and $\neg r$ to $r$, without being inconsistent [8].

## 29.8 Preference Holism

In the example of the outvoted democrat, we had preferences over complete alternatives (such as $Dr \wedge \neg r$) but also preferences over smaller units (such as $r$). More generally speaking, what is the relation between preferences on these two levels? In most philosophical discussions, the complete alternatives are much larger entities than in the voting example. Usually, they are taken to be possible worlds, i.e. sets of sentences that represent everything that can be said about the state of the world [18].

The most common approach is to assume that there is an underlying, holistic preference relation over the complete alternatives (possible worlds) from which preferences over smaller things are derivable as *ceteris paribus* preferences. This should of course not be seen as a faithful representation of actual deliberative or evaluative processes. Instead, the holistic preference relation should preferably be conceived as a reconstruction used to describe a coherence requirement on preferences. With this caveat, how can preferences over sentences be reconstructed as derivable from underlying preferences over possible worlds?

Georg Henrik von Wright, one of the pioneers of preference logic, attempted to explain the notion of *ceteris paribus* by means of counting differences in terms of logically independent atomic states of the world [28]. He assumed that there are $n$ logically independent states of affairs $p_1 \ldots p_n$. It then holds for each world $w$ and each atom $p_k$ that $w$ contains either $p_k$ or $\neg p_k$. The similarity between worlds is measured by counting the number of atoms about which they agree.

Unfortunately, this simple construction does not work. Its major weakness is that the choice of atomic states is logically arbitrary. Consider the following two ten-atom worlds:

$$w_1 = \text{Cn}(\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\})$$

$$w_2 = \text{Cn}(\{\neg p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\})$$

where Cn is the consequence operator that takes us from any set of sentences to the set of all its logical consequences. $w_1$ and $w_2$ appear to be very similar, and it seems as if a comparison between them can be used for a *ceteris paribus* comparison between $p_1$ and $\neg p_1$. But now consider the sentences $r_2, \ldots r_{10}$, so defined that for each of them, $r_k \leftrightarrow (p_1 \leftrightarrow p_k)$. We can then rewrite $w_1$ and $w_2$ in the following alternative way:

$$w_1 = \text{Cn}(\{p_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{10}\})$$

$$w_2 = \text{Cn}(\{\neg p_1, \neg r_2, \neg r_3, \neg r_4, \neg r_5, \neg r_6, \neg r_7, \neg r_8, \neg r_9, \neg r_{10}\})$$

Written in this way, $w_1$ and $w_2$ seem to be quite dissimilar. Since there are no objectively given logical atoms, logic cannot help us to choose between these two ways to compare the two sets. A measure of similarity that can be used to (re)construct *ceteris paribus* preferences will have to make use of more information than what is inherent in the logic.

## 29.9 Choice Functions

To investigate the relationship between preference and choice we need a formal representation also of the latter concept. The standard representation is a *choice function*. A choice function is defined over a set $\mathcal{A}$ of alternatives, and for each subset of that set it chooses, intuitively speaking, the most choiceworthy alternatives. Formally, $C$ is a choice function for $\mathcal{A}$ if and only if it is a function such that for each subset $\mathcal{B}$ of $\mathcal{A}$, $C(\mathcal{B})$ is a subset of $\mathcal{B}$ that is non-empty if $\mathcal{B}$ is non-empty.

$C(\mathcal{B})$ can have more than one element. Since the alternatives are taken to be mutually exclusive, this does not mean that the agent chooses more than one alternative, only that there is more than one alternative that she is willing to choose. Which of these alternatives she ends up with is a matter of picking rather than choosing [26]. Hence, if $a$, $b$, and $c$ are three marriage partners, then $C(\{a, b, c\}) = \{a, b\}$ does not indicate a bigamous proclivity but equal propensities to choose $a$ or $b$.[2]

Among the rationality properties that have been proposed for choice functions, the following two are arguably the most important ones [22]:

*Chernoff* (property $\alpha$) [4]
If $\mathcal{B}_1 \subseteq \mathcal{B}_2$ then $\mathcal{B}_1 \cap C(\mathcal{B}_2) \subseteq C(\mathcal{B}_1)$.

Property $\beta$
If $\mathcal{B}_1 \subseteq \mathcal{B}_2$ and $X, Y \in C(\mathcal{B}_1)$, then: $X \in C(\mathcal{B}_2)$ if and only if $Y \in C(\mathcal{B}_2)$

Suppose that we are choosing the best novelists from different categories. According to Chernoff, if one of those chosen in the category of European novelists is French, then (s)he is also one of those chosen in the category of French novelists. According to property $\beta$, if one of those chosen in the category of European novelists is French, then all those chosen in the category of French novelists must also be among those chosen in the category of European novelists.

Although these choice principles hold in many cases, it is not difficult to find examples in which they do not seem to hold, see Box 29.3.

[2]Obviously, choice functions can be defined so that picking is not needed: A *monoselective* choice function [11] is one that selects a single element out of any non-empty set to which it can be applied.

**Box 29.3 Violations of the choice axioms**

*1. A choice can aim at another position than the top position*
If the host offers Hao to take a fruit from a bowl with a big apple, a small apple, and an orange, then he will choose the big apple. However, if there is only a small and a big apple then he will (out of politeness) choose the smaller one [1] (This violates Chernoff.).

*2. The alternative set carries information about the alternatives*
An acquaintance whom Elena meets in the street offers her to come home to him for tea. In the choice between having tea at his house and going home she intends to opt for the former. But then he adds an additional option, namely to have some cocaine at his house. Among the three alternatives that she now has, she chooses to go home [25] (This violates Chernoff.).

In exactly the same situations, her friend Graciela would be indecisive in the first case (i.e. both having tea and going home are in her choice set), whereas she would choose to go home in the second case. (This violates property $\beta$.)

## 29.10 Preference-Based Choice

Preferences often have the function of guiding our choices. Sometimes it is even maintained that choice is nothing else than revealed preference [19, 21]. In order to clarify what it means for choices to be determined by preferences we can consider a choice function $C$ that is derived from a preference relation $\geq$ as follows:

$$C(\mathcal{B}) = \{X \in \mathcal{B} \mid (\forall Y \in \mathcal{B})(X \geq Y)\}$$

A choice function $C$ is *relational* if and only if it is based on some preference relation $\geq$ in this way. The formal connections are quite neat. It is possible to base a choice function on a given preference relation $\geq$ if and only if $\geq$ satisfies completeness and acyclicity. All such choice functions (i.e. all relational choice functions) satisfy Chernoff. A relational choice function satisfies property $\beta$ if and only if the underlying preference relation is transitive [22].

## 29.11 Choice-Based Preference

Conversely, we can take choice as primary and define preferences in terms of a choice function. The obvious way to do this is to identify preference with "choice from two-member sets" [2], as follows:

$p \geq q$ if and only if $p \in C(\{p, q\})$

If this construction is applied to a choice function that has in its turn been derived from a preference relation in the way shown above, then the original preference relation will be recovered [23].

The definition of preference as (hypothetical) choice is popular among economists. This approach makes it possible to take an agnostic stance on mental processes, and treat preference relations merely as technical means to express well-organized propensities to choose.

From a philosophical point of view this interpretation of preferences is far from unproblematic. We often entertain preferences in matters in which we have no choice. Consider Vladimir who has bought a lottery ticket. He would prefer winning a luxury cruise to the Bahamas rather than winning a gift voucher worth £20,000 in his local grocery store. Since one cannot, by definition, choose to win, preferences such as these cannot be choice-guiding. Indeed, if he were given a choice between the cruise and the voucher, he would choose the voucher. The act of choosing something may have negative characteristics (such as shame at choosing something useless) that the event of winning it does not have [9, p. 22]. Furthermore, the first example in Box 29.3 shows that even in matters where we have a choice, the definition of preference as binary choice gives rise to difficulties in "interpreting preference thus defined as preference in the usual sense with the property that if a person prefers $x$ to $y$ then he must regard himself to be better off with $x$ than with $y$" [24, p. 15].

## 29.12 A Central Dilemma in Preference Logic

There are two properties that we have a strong tendency to ascribe to preferences, and yet turn out to be difficult to reconcile [9, pp. 20–23]. One of these is *pairwiseness*: Whether a preference statement such as $A \geq$ B, $A > B$, or $A \sim B$ holds should depend exclusively on the properties of $A$ and $B$, and not be influenced by other elements of the set of alternatives. It should therefore make no difference if we compare $A$ and $B$ when deliberating on the elements of the set $\{A, B\}$ or when deliberating on the elements of the set $\{A, B, C, D, E\}$. The other property is *choice-guidance*, which means that the logical properties of preferences should be compatible with their use as guides for choosing among the elements of the alternative set.

In combination, these two principles imply the further principle of *binary choice*, i.e. that comparisons of only two alternatives at a time are sufficient to determine a choice among all the alternatives. The examples in Box 29.3 show that this principle is not always plausible.

The tension between pairwiseness and choice-guidance is a central dilemma in the theory of preference. It is also an example of a philosophical insight that could only be gained with the help of formal representations of preferences.

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