Chapter 26
Percussion Instruments

This chapter is about percussion instruments. The percussion family includes well-known instruments like drums, cymbals, and bells—and also hollow logs, tin cans, and truck springs. Anything that can be beaten or rattled to make a noise is potentially a percussion instrument. Therefore, this could be a very long chapter. To bring a little order to the zoo, this chapter starts with a definition.

Percussion Instruments Defined Percussion instruments are free vibrators. Unlike sustained tone instruments, where power is continually fed into the instrument to keep the tone sounding, a percussion instrument gets all its energy with an initial strike. After the initial impulse the percussion instrument vibrates in its natural modes of vibration. Eventually friction and loss of energy to radiation damp these vibrations so that they can no longer be heard.

It is a fact of mechanics that no real physical system has modes of vibration with frequencies in a precise harmonic relationship. Therefore, a corollary of this definition is that all percussive sounds are more or less inharmonic, or aperiodic. That is an absolutely firm conclusion. On the other hand, the phrase “more or less” gives a lot of latitude. Some percussive systems, such as the percussive strings of guitars and pianos, have modes of vibration that are nearly harmonic. Such systems can play melodies and close harmony just like sustained-tone instruments. Other percussion instruments make sounds that are grossly inharmonic.

Some percussion instruments, such as marimba bars and chime tubes, start with a very inharmonic system but incorporate refinements intended to emphasize their tonal character and enable them to play melodies and even close harmony. Other percussion instruments, like snare drums, go the other way and incorporate features designed to make them less tonal.

To bring further order to the zoo, this chapter continues by organizing percussive instruments according to type of vibrator. It deals with bars, membranes and plates.

W.M. Hartmann, *Principles of Musical Acoustics*, Undergraduate Lecture Notes in Physics, DOI 10.1007/978-1-4614-6786-1_26,

26.1 Bars, Rods, and Tubes

A bar is a length of solid metal or wood with a rectangular cross section. Rods (solid) and tubes (hollow) have circular cross sections. Musically useful vibrations of bars, rods, and tubes have vibrations that are transverse to the length. The ends may be free or clamped. For instance, a marimba bar has both ends free, but a clock chime rod has one end clamped.

The modes of vibration of a free bar, rod, or tube can be described by the number of nodes. The first mode has two nodes, the second mode has three, and so on, as shown for a bar in Fig. 26.1.

The frequencies of these modes of vibration are given by the formula

$$f = 0.113\,\frac{v_L\,t}{L^2}\,(2n+1)^2, \tag{26.1}$$

where v_L is the speed of sound in the bar material, t is the thickness of the bar, and L is its length. The quantity n numbers the modes, and $(2n + 1)$ takes on the values 3.011, 5, 7, 9, ... for mode numbers 1, 2, 3, 4, ... Thus, it is the sequence of odd integers starting with 3, except that the first value is a little more than 3. It should be evident that the frequencies are not at all in a harmonic relationship. Exercise 1 shows that the second mode of vibration has a frequency that is 2.76 times the first, i.e., $f_2/f_1 = 2.76$. The third mode has a frequency that is 5.40

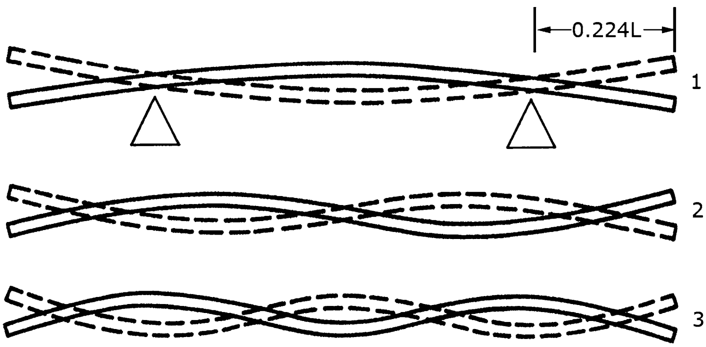

Fig. 26.1 The first three modes of vibration of a uniform rectangular bar with free ends. *Triangles* show mounting points that encourage the first mode and discourage all the others by damping their vibrations

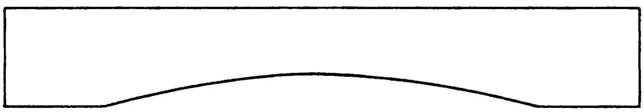

Fig. 26.2 How a bar can be undercut to lower the frequency of the first mode

times the first. It is typical of systems that have stiffness (like bars and rods) for the mode frequencies to grow more rapidly than the mode number. Here the frequencies grow as the square of the mode number. One says that compared to a harmonic relationship, the mode frequencies are "stretched." In musical applications, the pitch that is assigned to a bar corresponds to the frequency of the first mode.

26.1.1 Useful Bars

The marimba, xylophone, and vibraphone are instruments that use rectangular bars to make a tone. These instruments are intended to play melodies and close harmony, and so it is evident that something must be done about all that inharmonicity. In fact, three things are done. First, the bar is undercut, as shown in Fig. 26.2. This lowers the frequencies of all the modes, but especially of the first mode, where all the bending takes place near the middle of the bar. By lowering the frequency of the first mode, the ratio f_2/f_1 can be made 3.0 or 4.0 instead of 2.76. This allows the second mode to create the third or fourth harmonic of the tone. Second, the bar can be mounted so as to favor the first mode of vibration. Figure 26.1 shows that the nodes on the extreme left or right of the bar move toward the outside of the bar for higher mode numbers. If the bar is mounted where the first mode has a node and is not vibrating anyway, the first mode is not damped by the mounting as much as the higher modes. Finally, resonating tubes (pipes open at one end and closed at the other) can be placed under the bars to emphasize the first mode frequency. In the end, the tone produced by these instruments is rather like a decaying sine tone because of the emphasis given to the first mode, but the excitation of all the other modes at the onset makes the sound interesting.

26.1.2 Useful Tubes

Tubes are used to make chimes (Fig. 26.3). Orchestral chimes are a set of long brass tubes that hang in a rack at the back of the orchestra in the percussion section. Wind chimes hang from trees in the back yard to entertain(?) the neighbors. Because these tubes are cylindrical and hollow, you might imagine that the physics of open pipes from Chap. 8 would be relevant here, as they are for the vibraphone resonators. Do not be fooled. The vibration of a chime bar has nothing at all to do with the vibration of air in a pipe. In a chime, the vibration is in the walls of the tube itself, and the relevant speed of sound is the speed of propagation of a displacement of the brass walls. This is very different from a wind instrument or resonant pipe, where the vibration is in the air column and the relevant speed of sound is the speed of sound in air.

The modes of a chime are similar to the modes of a solid bar or rod (see Fig. 26.1) and the frequency ratios of the modes are similar too (see Eq. (26.1)).

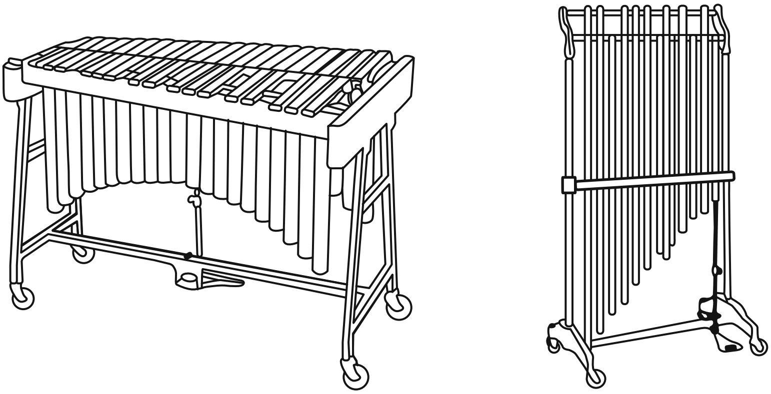

Fig. 26.3 (*Left*) A vibraphone has undercut bars and resonating tubes under the bars. Disks within the tubes are rotated by a motor to tune and detune the resonators. This imparts a tremolo to the sound, favored in jazz applications. Chimes, on the *right*, also have metal tubes but they are used in an entirely different way

The suspension of the chime tube, and the reduced internal friction, allows higher frequency modes to be sustained longer in the chime compared to the solid bar.

What is interesting about the chime is that its nominal frequency is not even close to any of the mode frequencies. Instead, the pitch of the chime comes from modes 4, 5, and 6. According to Eq. (26.1) the frequencies are proportional to 9^2, 11^2 and 13^2, or 81, 121, and 169. These numbers are in the ratios 2, 2.99, and 4.17. These are close enough to the ratio 2, 3, 4 that the chime creates a tone with a missing fundamental, as described in Chap. 13 on pitch perception. The frequencies of modes 4, 5, and 6 can be brought into an even better 2, 3, 4 ratio by loading an end of the chime tube with a solid plug, and manufacturers regularly do this.

26.2 Membranes

Drum heads are membranes stretched over a hollow frame. A few examples appear in Fig. 26.4. They are always circular, or nearly. The modes of vibration of a drum head are displacements perpendicular to the membrane surface, as a function of position on that surface. Therefore, the modes are two dimensional. That makes them different from the modes that have been considered for a stretched string, or for air in a pipe, or for a solid bar, all of which are one dimensional. For a one-dimensional systems there is a mode number, always giving some indication of the number of nodes in the standing wave pattern. For instance, index n describes the number of nodes in Fig. 26.1. To describe a mode in two dimensions requires *two* mode numbers.

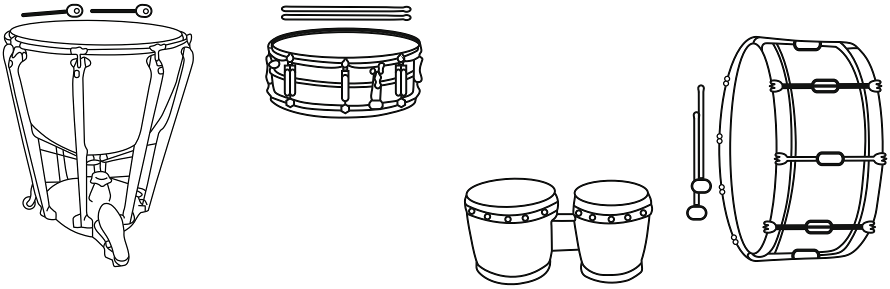

Fig. 26.4 A few drums: from left to right: Timpani or kettledrum, Snare drum, Bongo drums (played with the fingers), Bass drum

It is useful to study the membrane that has the greatest possible symmetry, a membrane that is a perfect circle, has uniform density, and is stretched with uniform tension. A well-tuned timpani (kettledrum) with a mylar head approximates such an ideal system. The modes are described by a pair of numbers $[m, n]$, where m gives the number of nodal lines and n gives the number of nodal circles, including the outer rim.

26.2.1 Chladni Patterns

You can see the individual modes of vibration of a drum head by forming Chladni patterns. The first step is to put the membrane in a horizontal orientation and drive it with a steady sine tone from a loudspeaker underneath. Because it has only a single frequency, the sine tone will excite no more than one mode. By tuning the sine tone you can excite the different modes in turn. The next step is to sprinkle aluminum filings onto the membrane. (You could also use glitter, or any dark granular material that is not magnetic and does not clump.) The filings make it easy to know when the sine tone frequency has hit one of the modal frequencies because the membrane will vibrate a lot, and the filings will dance around on top of it. After a short time the filings will settle on the nodal lines and circles. That makes it easy to identify the mode, and a frequency counter attached to the sine tone generator indicates the frequency of the mode.

If you get the chance to do this experiment, it is likely that your drum head will not have absolutely uniform tension. Then the patterns you find will be distortions of the patterns in Fig. 26.5. However, they should be identifiable. Even though a nodal line may curved and a nodal circle may turn into an oval, or acquire cusps, you can tell what the pattern is trying to be.

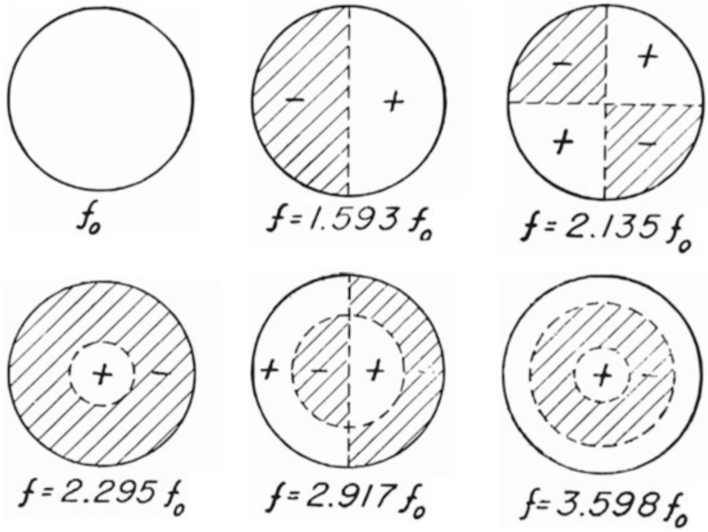

Fig. 26.5 The modes of a uniformly stretched circular membrane are given by two numbers indicating the number of nodal lines and the number of nodal circles. Numbers below the mode drawings show mode frequencies relative to the frequency of the lowest mode, called "[0,1]." Other modes shown are, in order, [1,1], [2,1], [0,2], [1,2], [0,3]

26.2.2 Timpani

Timpani are the most important drums in the orchestra. They are tuned. Although they do not normally play melodies, they are used to reinforce selected notes played by the rest of the orchestra, and it is important that the tuning be right. Here we have another case where a tuned percussive sound is needed and yet the basic vibrator, in this case the stretched membrane, has modal frequencies (as given by the ratios in Fig. 26.5) that do not look promising. The solution to this problem for the timpani is to add the kettle below the membrane. The kettle is filled with ordinary air, and this volume of air loads the membrane and reduces all the modal frequencies. However, some modes have their frequencies more reduced than others and that's the key.

The air loading changes the frequency of the [2,1] mode so that the ratio f_{21}/f_{11} changes from 1.34 to 1.51, and it changes the frequency of the [3,1] mode so that the ratio f_{31}/f_{11} changes from 1.67 to 1.99. The ratios 1.51 and 1.99 are close to 1.5 and 2.0, which are related to the fundamental pitch f_{11} by the interval of a musical fifth and an octave. In this way the kettle of air makes the timpani sound more tonal.

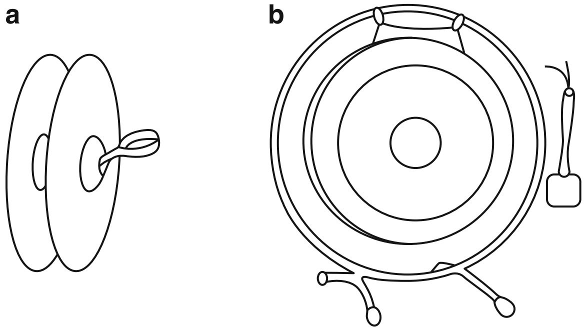

Fig. 26.6 Cymbals (**a**) and gongs (**b**) begin with a free circular plate, but are formed with a dome in the center

26.3 Plates: Cymbals, Gongs, and Bells

The vibrational modes of a flat, uniform, circular plate are like the modes of a circular membrane in that they can be defined by nodal lines and nodal circles. The same notation, [m,n], can be used. There is a useful analogy that can be drawn between these two-dimensional systems and one-dimensional systems, namely that a plate is to a stretched membrane as a bar is to a stretched string. While the membrane and string have fixed boundaries, the plate and bar (as in cymbal and xylophone) have free boundaries. While the membrane and string get most of their restoring force from the applied tension, the plate and the bar get their restoring force from stiffness. Stiff vibrators have modes with frequencies that are widely separated. This is true of both bars and plates. For comparison, note that the lowest six modal frequencies for a membrane, given in Fig. 26.5, have ratios 1, 1.59, 2.14, 2.30, 2.65, and 2.92. The lowest six modal frequencies for a circular plate with a free boundary have ratios 1, 1.73, 2.33, 3.91, 4.11, and 6.30 (Fig. 26.6).

The circular plate with a free boundary is a reasonable starting model for a cymbal or gong, but both instruments differ in important ways from this prototype. Both cymbal and gong have a raised dome in the center which means that the modes of vibration higher than [0,6] or [2,1] are not like those of a plate but are like combinations of different plate modes. The gong has a curved rim that can be expected to change the free boundary condition. The instrument known as a “tamtam” is a large gong without the dome.

26.3.1 Nonlinear Mode Coupling

The description of the vibration of cymbals, gongs, and tamtams in terms of their natural modes of vibration is a linear analysis and does not really do justice to the complicated character of either the vibration or the sound made by these instruments. The modes are coupled in ways that depend on how hard the plate is struck. For a large gong or tamtam, the vibration pattern develops slowly with time

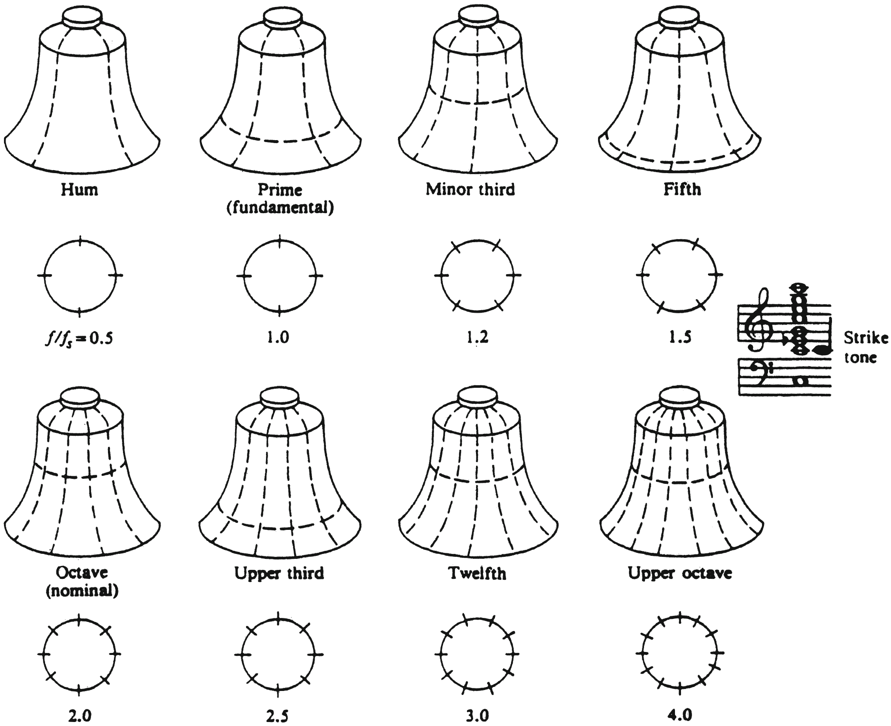

Fig. 26.7 The modes of vibration of a bell have been given names. Their nodal meridians and nodal circles are shown by *dashed lines*. Their frequencies are given relative to the prime frequency which corresponds to the nominal pitch of the bell. This figure was borrowed from *The Science of Sound* by Thomas D. Rossing, Addison Wesley, 1990

as the plate is repeatedly struck. Beating softly on a tamtam with a soft beater leads to a low-frequency rumble. Continued and more intense beating appears to cause a transfer of energy from low-frequency modes into high-frequency modes producing a loud shimmering sound. Psychologically, the effect is dramatic. Intellectually, the scientist also finds this unexpected mode coupling to be dramatic because it is not readily understandable. Whoever figures out how this effect works is going to make scientific history—big time.

26.3.2 Bells

Many civilizations, in the east and in the west, have used bells throughout history. The church bell, or carillon bell, that we know in the west was developed in the low countries of Europe in the seventeenth century.

Figure 26.7 shows that the modes of vibration have nodal meridians and nodal circles that recall the nodal lines and nodal circles of a circular plate. From left to right, the top line of the figure shows modes [2,0], [2,1], [3,1], and [3,1a]. What is different about the bell is that the thickness of the walls is not uniform. Instead, the thickness is caused to vary so that the modes are tuned, with relative frequencies corresponding to notes of the scale, such as the minor third and fifth. Of course, when the bell is struck, all the modes are excited at once and a listener does not hear the individual notes of the scale. The pitch of the bell, or fundamental, is established by the modes with frequencies 2.0, 3.0, and 4.0 times the fundamental. The prime tone acts to reinforce this pitch, and the "hum tone" is consonant, an octave lower. But the modes of the bell decay at different rates, and eventually a listener can hear several different notes of the scale. That ambiguity is part of the charm of the bell sound.

Exercises

Exercise 1, Modes of a uniform bar

Use Eq. (26.1) to find the frequencies of the first four modes of vibration of a uniform bar. Express these frequencies as multiples of the frequency of the first mode. In other words, find f_2/f_1, f_3/f_1, and f_4/f_1.

Exercise 2, The chime tone

It is said that the modes of a chime tube are proportional to 9^2, 11^2 and 13^2, which are in the ratio of 2, 2.99, and 4.17. Show that this is true.

Exercise 3, Drums with two heads

Bass drums and snare drums often have two heads. One is beaten and the other is forced to vibrate by the compression of air inside the drum. The two heads are normally stretched to different tensions. Do you think that the addition of the second head makes the drum more tonal or less tonal?

Exercise 4, Plates you have known

Some dinner plates are made of china, others are made of paper. They sound very different when struck. What is responsible for the difference?

Exercise 5, Bell mode notation

What are the values of $[m, n]$ for the bottom row of Fig. 26.7?

Exercise 6, Bell modes

On opposite sides of a nodal line the vibration is in opposite directions. For instance the nodal meridians in Fig. 26.7 divide the bell into segments. When one segment moves outward, the adjacent segment moves inward. Show how this applies to the circles in Fig. 26.7 that indicate the nodes on the rim of the bell. Use solid and dashed lines—as usual for standing waves—to indicate vibrations as separated in time by half a period.

♠