Chapter 10
Sound Intensity

10.1 Pressure, Power, and Intensity

Sound waves have pressure, power, intensity, and energy. These facts became evident at the walls of Jericho some years ago. If you weren't there at the time to experience the event, then you may anyhow have experienced chest wall vibrations at rock concerts—or when listening to music in a booming car. Like light waves from a laser, sound waves have pressure, power, intensity, and energy.

At this point we mainly know about the pressure in sound waves. But pressure looks like a strange way to describe the flow of sound power because a pressure wave is both positive and negative. Normally it is negative just as much as it is positive, and its average value is zero. That seems quite different from the concepts of power, intensity, and energy. In contrast to pressure, sound power is strictly positive. It is closely related to the amplitude of the pressure (a positive number). We will refer to the pressure amplitude by the symbol A.

Focus on Intensity: I A sound wave travels outward from the source of sound, and the power in the wave gets spread out over space. If a certain amount of power is spread out over a large area, then that is a weak sound. If the same amount of power is concentrated in a small area, then the sound is strong. This concept of weak and strong is embodied in the measure called "intensity." The physical dimensions of intensity are power per unit area. Because power is expressed in metric units of watts, and area is in square meters, intensity is expressed in "watts per square meter." We refer to power and intensity by the symbols P and I respectively.

There is an important relationship between pressure amplitude and intensity: The intensity is proportional to the square of the pressure amplitude. We write

$$I \propto A^2. \tag{10.1}$$

Note that this means that if the pressure is doubled, then the intensity is increased four times. If the pressure becomes ten times larger, then the intensity becomes 100 times larger, i.e., it is increased by a factor of 100.

Proportionality

The concept of proportionality, indicated by the symbol $\propto$, is not difficult to understand. A statement of proportionality, such as Eq. (10.1) simply means that there is some constant number k that would allow us to write a true equation,

$$I = kA^2 \qquad\qquad (10.2)$$

where k does not depend on either A or I. The concept of proportionality is used in mathematics to emphasize a particular functional dependence and to disregard others. Here, proportionality statement Eq. (10.1) emphasizes that intensity depends on the square of the pressure amplitude. What is disregarded in the proportionality statement is all the complexity that might be hidden in Eq. (10.2) because of constant k. For instance, k might depend on the temperature or the density of the gas where the wave is moving. The proportionality statement makes the assumption that all those quantities are unchanging (that's why k is called a "constant") as one changes the pressure amplitude and watches the change in intensity.


10.2   The Inverse Square Law

When you are farther away from a sound source, the sound tends to be less loud. That is such a familiar experience that you would imagine that there ought to be a good physical explanation for it. The most basic attempt to find such an explanation is the inverse square law.

The inverse square law refers to the *intensity* of the sound wave. It says that the intensity at a receiver depends inversely on the square of the distance from the source. If I is the intensity and d is the distance between the source and the receiver, then

$$I \propto 1/d^2. \qquad\qquad (10.3)$$

The statement of proportionality in Eq. (10.3) is an abstract rule about intensity and distance. We apply this rule by making comparisons between several particular situations. For instance, let's think about a trumpet playing 1.5 m (1.5 m) away from your ear, and a trumpet playing 15 m away from your ear. If we say that I_1 is the intensity when the source is at $d_1 = 1.5$ m and I_2 is the intensity when the source is
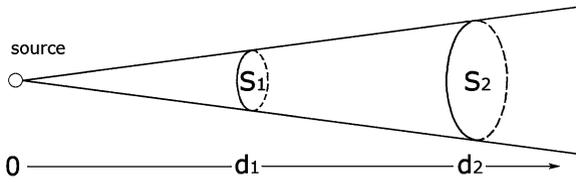
Fig. 10.1 The sound source at the left radiates power in a conical pattern. At distance d_1, the power is spread over area S_1. At distance d_2 the power is spread over area S_2. Because d_2 is twice as large as d_1, S_2 is four times larger than S_1. The sound intensity depends on how the power is spread over the area and so the intensity at distance d_2 is one-quarter of the intensity at distance d_1

at $d_2 = 15\,\text{m}$, then the square of the distance ratio is

$$(d_2/d_1)^2 = (15/1.5)^2 = 10^2 = 100. \tag{10.4}$$

The inverse square law says

$$I_2/I_1 = (d_2/d_1)^{-2} = (15/1.5)^{-2} = 10^{-2} = 1/100. \tag{10.5}$$

That means that the intensity is 100 times less when you are 15 m away from the trumpet compared to when you were 1.5 m away.

It's not hard to see why the inverse square law is true. Sound intensity is like a power density in that it is the number of watts per square meter. A wavefront from the trumpet spreads out in a beam as it leaves the horn. When the ear is only 1.5 m away from the horn, the ear intercepts the wavefront at a distance where it is spread over a small area. When the ear is 15 m from the horn, it intercepts the wavefront where it is spread over an area that is 100 times larger. That is because the surface area of the end of a beam is proportional to the square of the length of the beam.

Figure 10.1 gives another example of the operation of the inverse square law. There, the far distance is only twice as far as the near distance. Therefore, $I_2/I_1 = (1/2)^2 = 1/4$.

Spherical Source The inverse square law does not depend on having a conical radiation pattern like Fig. 10.1. The law works for any pattern. As another example, think about a pulsating sphere that radiates a sound wave equally in all directions. Such a radiator is called *isotropic*. It radiates a spherical wave. You might think of a wavefront of this wave, e.g., a pressure maximum, as the surface of a spherical balloon. The expansion of the wavefront as it moves out from the source is like the expansion of the balloon as it fills with air. As the wave expands, the power is spread out over an ever larger surface area. Similarly, as the balloon expands, the rubber is stretched over a larger area. The surface area of a sphere, S, is proportional to the square of the radius, d. The formula is $S = 4\pi d^2$. Therefore, the intensity of the sound wave measured a distance d away from the source is $I = P/(4\pi d^2)$, where

P represents the power of the source. If the power is in watts and the distance is in meters, then the intensity is in units of watts per square meter, as expected.

Application of the Inverse Square Law The inverse square law comes from a model in which there is a source, a receiver, and nothing else but air. There is nothing in this environment that would lead to reflected sound waves. Obviously, this model is an idealization because whenever a sound occurs in a room there are many reflections from the walls of the room, contrary to the model.

In fact, in many room environments the model is a very bad approximation to reality. For instance, in a classroom with a talker near the chalk board and a listener anywhere else in the room the model fails badly, and the inverse square law does not apply. It fails to describe the dependence of intensity on distance from the source.

There are some other contexts in which the inverse square law does apply. First, if one could imagine a special room where all the walls and other surfaces absorb sound completely and do not reflect any of it, then the model would apply, and the inverse square law would be valid. Such a room is called "anechoic." All surfaces in this room are covered with foam wedges about a meter long. Second, the inverse square law might apply approximately, even in a normal room if the distance between the source and receiver is small compared to the distance to any walls. Then the reflections from the walls, violating the assumptions of the model, might be small compared to the sound that travels directly from source to receiver. Third, if the source and receiver are out of doors, away from all walls, then the inverse square law applies. It applies especially well if the ground is covered with fresh snow, a good sound absorber.

Finally, the inverse square law applies in theoretical calculations in a room where the goal is to follow each sound reflection from one surface to another. This context is different from the others because it deals with sound on a microscopic time scale. Instead of thinking about the propagation of sound from a source to a receiver, this context considers the propagation from the source to the first reflection, and second reflection, and third, and so on, and finally to the receiver. The inverse square law applies to the propagation as it occurs on each step of this complicated path.

Recapitulation

Pressure: Positive and negative, measured in pascals (or micropascals).

Power: Positive only, measured in watts.

Intensity: Positive only, Power/Area, measured in watts/(meter^2).

10.3   Decibels

It's a strange fact that the pressure, power, intensity, and energy in a sound wave are not normally specified in physical units such as watts per square meter. What we do instead is to have standard reference values for all of those physical quantities and

to measure a real sound wave in terms of those reference values. That's the basis of the decibel scale.

To explain how the decibel scale works, this chapter tells a long story about you and your neighbor's music. The story gives a specific example, but you should be able to generalize the concepts to other circumstances.

The story begins with a definition of a reference intensity. The standard reference for intensity is 10^{-12} W/m^2. This intensity is called the "threshold of hearing" because it is the weakest sound that humans can hear. Making measurements at your place of the music coming from your neighbor, we might find that the intensity is one million times greater than the reference intensity. That is how we refer the intensity of a sound, like music coming from your neighbor, to the threshold of hearing.

To put this in the form of an equation, if the threshold of hearing is an intensity called I_o and the music measured at your place has an intensity called I, then

$$I/I_o = 1,000,000 \text{ or } 10^6. \tag{10.6}$$

Because the power of ten that is needed to make one million is 6, we could also say that the music is six orders of magnitude more intense than the threshold of hearing. The number 6 is an exponent here. It is a power of 10. It is the power of 10 that is needed to make a million. Therefore, the number 6 is the logarithm of one million. In equation form we say,

$$6 = \log(1,000,000). \tag{10.7}$$

There is only one step remaining to describe the music in terms of decibels. We multiply by 10. In the end we say that the music *has a level of 60 decibels* or 60 dB.

This statement, that the sound has a level of 60 dB, is a little curious. It looks as though we are describing the level in absolute terms. We know, however, that this level has really been referred to the threshold of hearing. So we might have said instead, "The sound has a level of 60 dB with respect to the threshold of hearing." The point is that the threshold of hearing has been agreed upon as an international standard. Therefore, it is OK to say simply that the level is 60 dB. We must bear in mind though that levels measured in dB always are *relative* measures, even though levels with respect to the threshold of hearing are expressed as though they were *absolute.* That distinction becomes clarified in the next section.

To summarize what has gone above, we can calculate the level, L, of a sound that has intensity I by the equation

$$L = 10 \log(I/I_o), \tag{10.8}$$

where $I_o = 10^{-12}$ W/m^2.

10.4   Absolute vs Relative dB

You go to your neighbor's place and walk in. There, the music is 100 times more intense than it was when you heard it back in your own place. Therefore, its intensity is 10^8 times the threshold of hearing. The intensity is actually 10^{-4} W/m². To calculate the level in dB, we use the formula above,

$$L = 10 \log(10^{-4}/10^{-12}) = 10 \log(10^8) \tag{10.9}$$

$$L = 10 \times 8 = 80 \text{ dB.} \tag{10.10}$$

At your neighbor's place the level is 80 dB. That is 20 dB more than back at your place. We arrive at this difference of 20 dB by remembering that at your place the level was 60 dB, and $80 - 60 = 20$. We calculated two values on the "absolute" dB scale and subtracted one from the other.

Another Way   There is another way to do this calculation that emphasizes the dB measure as a relative measure. We return to the fact that the sound intensity was 100 times greater at your neighbor's place.

We define the level at your own place as L_1 and the level at your neighbor's place as L_2. The difference is

$$L_2 - L_1 = 10 \log(I_2/I_1). \tag{10.11}$$

Because $I_2/I_1 = 100$, we have

$$L_2 - L_1 = 10 \log(100) = 10 \times 2 = 20 \text{ dB} \tag{10.12}$$

So we get the same answer, 20 dB, whichever way we do it. We say, "Relative to the sound at your place, the level at your neighbor's place is up by 20 dB."

Another way to say it is, "Relative to the sound at your neighbor's place, the level at your place is down by 20 dB." Still another way to say it is, "The barriers between you and your neighbor attenuate the sound by 20 dB." To *attenuate* means to reduce the level.

It is instructive to think about the attenuation picture algebraically. The reference sound is the sound at the source (your neighbor). Its level is L_2. The sound of interest is the sound at your place, and its level is L_1. Therefore,

$$L_1 - L_2 = 10 \log(I_1/I_2) \tag{10.13}$$

We know that the ratio I_1/I_2 is 1/100 or 10^{-2}. We calculate

$$L_1 - L_2 = 10 \log(10^{-2}) = 10 \times (-2) = -20 \text{ dB} \tag{10.14}$$

We say that relative to the source, the level at your place is -20 dB.

Examples

By studying the following examples, you ought to be able to figure out how to do decibel problems. The first example uses powers of ten that are integers and don't require a calculator to calculate logs. Examples 2–5 use powers of ten that are not integers. You should use the *log* button on your calculator to check the numbers.

Example 1(a), An easy dB problem
    If sound A is 1,000 times more intense than sound B, then $I_A = I_B \times 10^3$. The exponent is 3 and the level of A is said to be 30 dB greater than the level of B $(3 \times 10 = 30)$. *viz*

$$L_A - L_B = 10 \log(I_A/I_B) = 10 \log(10^3) = 30 \text{ dB}.$$

Example 1(b), A negative dB problem
    If sound C has one tenth the intensity of sound D, then $I_C = I_D \times (1/10)$ or $I_C/I_D = 10^{-1}$. The exponent is -1 and the level of C is said to be 10 dB *less* than the level of D.

viz

$$L_C - L_D = 10 \log(I_C/I_D) = 10 \log(\frac{1}{10}) = 10 \log(10^{-1}) = -10 \text{ dB}.$$

Example 2(a), A dB problem
    Noise Y is five times more intense than noise X. What is the level difference?

Solution
    With the levels of X and Y defined as L_Y and L_X, the level difference is given by

$$L_Y - L_X = 10 \log(I_Y/I_X)$$
$$L_Y - L_X = 10 \log(5) = 10 \times 0.7 = 7 \text{ dB}.$$

We conclude that the level of Y is 7 dB greater than the level of X.

Example 2(b), The same dB problem
    Another way to compare noises X and Y is to say that noise X is one fifth as intense as noise Y. What is the level difference?

Solution
    The level difference is given by

$$L_X - L_Y = 10 \log(I_X/I_Y)$$
$$L_X - L_Y = 10 \log(1/5) = 10 \times (-0.7) = -7 \text{ dB}.$$

We again conclude that the level of Y is 7 dB greater than the level of X.

Example 3, A reverse dB problem

The level of the Chevrolet horn (C) is 5 dB greater than the level of the Ford horn (F). Compare the intensities.

Solution

$$L_C - L_F = 5 = 10 \log(I_C/I_F)$$

$$0.5 = \log(I_C/I_F)$$

$$I_C/I_F = 10^{0.5} = 3.16.$$

We conclude that the Chevrolet horn is about three times more intense than the Ford horn.

Example 4, Absolute dB

The intensity of the trombone at your ear is 4×10^{-3} W/m^2. What is the level?

Solution

$$L = 10 \log(4 \times 10^{-3}/10^{-12})$$

$$L = 10 \log(4 \times 10^{9})$$

$$L = 10 \log(4) + 10 \log(10^{9})$$

$$L = 10 \times 0.6 + 10 \times 9 = 96 \text{ dB}.$$

That's loud!

Example 5, Reverse absolute dB problem

A sound level meter at the back of the hall measures 103 dB during a rock concert. What is the intensity of sound there?

Solution

Let I be the intensity of interest. Then

$$103 = 10 \log(I/10^{-12})$$

$$I/(10^{-12}) = 10^{10.3}$$

$$I = 10^{10.3-12} = 10^{-1.7} = 0.02$$

i.e., 0.02 W/m^2. That's dangerously loud!!

Falling off a log

1. Prove that the log of 10 is 1, in other words, $\log(10) = 1$.
2. Prove that the log of 1/10 is -1, in other words, $\log(1/10) = -1$.

3. A logarithm is an exponent. It is a power of 10. For instance, the log of 17 [$\log(17)$] is the power of 10 needed to make the number 17.

   3a. Use a calculator to show that $\log(17)$ is approximately 1.23
   3b. Use a calculator to show that $10^{1.23} \approx 17$.

4. Show that because exponents add, the log of a product is the sum of the logs. To be more precise, if A and B are any two numbers, then the log of the product of A times B is the log of A plus the log of B. In other words,

$$\log(AB) = \log(A) + \log(B).$$

5. The number 170 is 10×17. If the log of 17 is 1.23, then prove that the log of 170 is 2.23.
6. The reciprocal of 17 (1/17) is about 0.0588. If the log of 17 is 1.23, how do you know that $\log(0.0588) = -1.23$?

Exercises

Exercise 1, Outdoors
   The inverse square law for sound intensity applies pretty well outdoors where the only reflection is from the ground. (a) Show that if you go three times farther away from a source of sound, the intensity goes down by a factor of 9. (b) Show that this decrease corresponds to a level decrease of 9.5 dB.

Exercise 2, The inverse first-power law

(a) Use the inverse square law and the relationship between intensity and amplitude to show that the amplitude A of a sound wave depends inversely on the first power of the distance d to the source, i.e.,

$$A \propto 1/d.$$

(b) To solve part (a) you need to take the square root of both sides of a proportionality. Show that this is a legitimate step writing the proportionality as an equation and then taking the square root of both sides of the equation (a legitimate step in this context).

Exercise 3, A spherical radiator

A spherical source radiates 10 W of acoustical power uniformly in all directions. (a) What is the sound intensity at a distance of 1 km, assuming no obstructions. (b) What is the absolute level in dB? (c) Where would one find such conditions?

Exercise 4, Life in different dimensions

You know that the inverse square law for intensity arises because an expanding wave spreads over the surface of a sphere, and the surface area is proportional to the *square* of the distance from the source. The spherical surface itself is appropriate in this case because the world is three dimensional.

In a two-dimensional world, the wave is confined to two dimensions. An expanding wave spreads over a circle, and the circumference of the circle is proportional to the *first power* of the distance from the source. In a two-dimensional world the inverse square law for sound intensity is replaced by an inverse first-power law.

How about a one-dimensional world? A tin-can telephone or the speaking tube on a ship are examples of sound waves confined to one dimension. What then is the appropriate law for sound intensity?

Exercise 5, Orchestra vs solo

Thirty violins lead to an intensity that is 30 times greater than one violin. Show that the difference is 14.8 dB.

Exercise 6, A factor of two

Show that doubling the intensity of sound leads to a level increase of 3 dB.

Exercise 7, Unequal duet

The trombone is playing at a level that is 20 dB higher than the flute. (a) Compare the intensities of trombone and flute. (b) Compare the sound pressure amplitudes of trombone and flute.

Exercise 8, Converting the equation to amplitude

Show that

$$L_2 - L_1 = 20 \log(A_2/A_1), \qquad (10.15)$$

where A refers to the pressure amplitude.

Exercise 9, Loud!

Not far from the space shuttle main engines the sound level is measured to be 130 dB. Show that the intensity is 10 W/m^2.

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