

Chapter 25

String Instruments

It is hard to overestimate the importance of the stretched string in music—so many instruments, ancient and modern, use the stretched string as the initial source of sound. The string is used in two different ways, as a percussive string and as a bowed string. A percussive string may be plucked, as in a guitar or harpsichord, or it may be struck, as in a piano. A bowed string is continuously excited by the action of the bow and it produces a sustained sound. This chapter treats the percussive and bowed strings in turn.

25.1 Percussive Strings

A percussive string is a free vibrator. After it is once struck or plucked, it vibrates in its natural modes like the vibrators of other percussion instruments such as bells or drums. Chapter 7 described the modes of a freely vibrating string and noted that ideally their frequencies are in a harmonic relationship. Chapter 7 concluded by noting that real strings are not completely ideal and that deviations from ideal behavior would be presented at a later time. That time has now arrived.

Like every oscillator, a stretched string must experience a restoring force that tends to move it back in the direction of equilibrium when it has been displaced. The restoring force comes about in two ways. First, there is the tension in the string, second there is the stiffness of the string. To the extent that the restoring force comes from tension alone, the string is ideal and its modal frequencies are perfect harmonics. To the extent that the restoring force comes from stiffness the string resembles a solid rod. As will be seen in Chap. 26 on percussion instruments, the solid rod is a very inharmonic system. Every real musical string is partly ideal, but partly it is a rod.

The stiffness of a string is not normally important for low-numbered modes of vibration that don't bend the string sharply. However, high-numbered modes of vibration, with many bends, are affected by the stiffness, and their frequencies

are progressively increased. For the highest modes, the deviation from perfect harmonicity is proportional to the cube of the mode number, i.e., proportional to n^3 . A formula that describes this behavior gives the frequency of mode number n as

$$f_n = nf_1[1 + \alpha(n^2 - 1)]. \quad (25.1)$$

The ideal behavior, where f_n is just n times the frequency of the fundamental (f_1), is outside the square brackets. The deviation is within the square brackets.

The size of the deviation from ideal behavior is determined by α , which depends on the properties of the string,

$$\alpha = \frac{\pi^3 r^4 E}{8FL^2}. \quad (25.2)$$

Here r is the radius of the string, and E is a property of the string material. It is a measure of the resistance to being stretched known as Young's modulus. In the denominator, F is the tension in the string, and L is the length of the string. The interpretation of Eq. (25.2), with quantities r , E , L , and F , is that if you want to get ideal behavior then you need a *thin* (r) and *flexible* (E) string. It should be *long* (L) and should be stretched to a *high tension* (F). The opposite conditions tend to lead to a large value of α and more inharmonic behavior.

At this point it is good to reconsider the guitar player's equation, number (7.7), which says that the playing frequency, or fundamental frequency, of a string is

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}}, \quad (25.3)$$

where μ is the linear mass density of the string. This equation suggests that if you want to see ideal behavior then it is the low-frequency tones that are going to give you trouble. The reasoning is this: first, there is a practical limit on how long the string can be. (However much you might enjoy a 20-ft piano, you couldn't get it into the house.) Therefore, to get low frequencies you need to reduce the tension (F) and increase the mass per unit length (μ). But reducing the tension and making the string thicker (r) to increase the mass both tend to increase parameter α and lead to inharmonic components in the tone.

One way to help solve the low-frequency problem is to use a wound string. A wound string consists of a thin nylon or steel string core, with windings of brass wire. The windings add mass to the string to help increase μ but they are not nearly as stiff as a solid wire with comparable mass because the windings are in flexible coils. Guitars, pianos, and violins all use wound strings for the low-frequency strings.

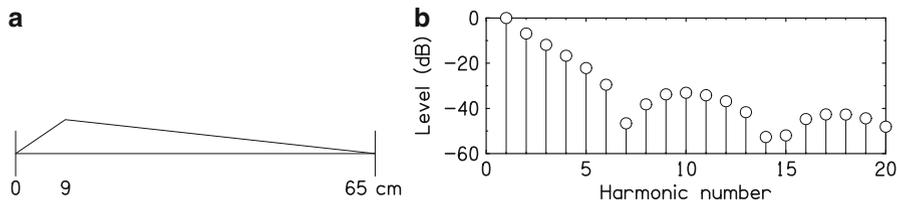


Fig. 25.1 Part (a) shows a guitar string, 65 cm long, plucked with a sharp bend 9 cm from the bridge. Because the initial displacement pattern is sharp the pattern includes high-frequency modes of vibration. The initial spectrum in (b) includes those high frequencies as harmonics. Because the ratio 9/65 is approximately 1/7, it happens that the 7th and 14th harmonics are weak in the spectrum

25.1.1 The Guitar

The guitar has six strings, tuned to frequencies 82, 110, 147, 196, 247, and 330 Hz. The strings are played by plucking, which means that a string is pulled away from equilibrium with some initial displacement pattern and then released.

The tone color of the guitar depends on the initial displacement pattern. If the guitar is plucked in such a way that the initial displacement is rounded, the tone color will be dull. If the string is plucked with a pick so that the initial displacement has a sharp bend, or kink, the tone color will be bright. It is not hard to figure out why the tone color behaves this way. The initial displacement determines the initial amplitudes of the different modes of the string. Imagine that a string is plucked right in the middle using the broad part of the thumb. This initial displacement looks a lot like the first mode of vibration, with little contribution from high-frequency modes. With little amplitude in high-frequency modes, this sound is dull. If the string is plucked with a fingernail or pick near the bridge, the initial pattern has a sharp kink. To form that pattern by adding up the shapes of the modes of a stretched string requires modes with short wavelengths. Therefore, high frequencies will be included in the tone and the tone will be bright, as indicated in Fig. 25.1.

The waveshape and spectrum in Fig. 25.1 look like an unsymmetrical triangle wave, the kind of wave that could be produced by a function generator. But the guitar does not sound at all like a function generator. The difference lies in what happens after the initial sound. As the guitar tone decays away it evolves in several ways.

1. The harmonics of the tone do not decay at equal rates. The higher-frequency modes are damped more than the lower-frequency modes causing the high harmonics to decay more rapidly than the low. The waveform on the string becomes smoother as the decay progresses.
2. The harmonics are not truly harmonic. Because the plucked string is a free vibrator, stiffness in the string causes the high-frequency components to be slightly higher in frequency than true harmonics of the fundamental. Because the frequency shift is not large, we still speak of harmonics, but the shift causes the relative phases

among the harmonics to change with time, and this change is another reason that the shape of the wave evolves as the tone sounds.

3. There may be audible effects because the vertical and horizontal polarizations of the string vibration do not have identical frequencies. Recall that the string displacements are transverse to the string length, but these may be either vertical (perpendicular to the guitar top) or horizontal (parallel to the guitar top). Because of the connection of the string at the bridge, the string appears to have slightly different lengths for these two polarizations and the difference may produce slow beats, a further kind of evolution.

4. Most important, the string vibration is not heard directly. The string itself moves very little air. Instead, in an acoustic guitar, the string is coupled to the top plate through the bridge, and the top plate is coupled to the air cavity within the guitar. The top plate radiates because it has a large surface, and the cavity radiates through the sound hole. The top plate and cavity together have resonances that filter the sound of the string as they radiate it, in much the same way as the vocal tract filters the glottal pulse from the vocal folds.

25.1.2 The Electric Guitar

The electric guitar is different from the acoustic guitar in that the sound is not radiated by the top plate and sound hole. In fact, a top plate and sound hole don't normally exist, and if they do they do not matter acoustically. Instead, the string vibrations are picked up by electromagnetic transducers to create an electrical signal which can then be amplified (often a lot) and reproduced by loudspeakers (often very large loudspeakers).

A transducer is made by wrapping coils of wire onto a permanent magnet beneath a string. The string is steel, which is a magnetic material. Thus when the string vibrates it alters the field from the permanent magnet and creates a voltage in the coil by means of the generator principle described in Chap. 16. The effect generates an appreciable voltage only if the coil is close to the vibrating string, and there is normally a separate transducer for each string.

The signal that comes out of the electric guitar represents the string motion at the position of the transducer. If the transducer is placed near the bridge, it efficiently picks up high-frequency modes and the tone is bright. If the transducer is farther from the bridge it picks up the large vibrations of low-frequency modes and the tone is mellow. Electric guitars often have two or three sets of transducers at different distances from the bridge to provide options.

Except for this very different sound radiation process, the electric guitar is like an acoustic guitar. Similar processes cause the string vibration to evolve in similar ways as the tone decays, although the decay is of longer duration in the electric guitar (see Exercise 5).

25.1.3 *The Piano*

The piano is like the guitar in that the strings of a piano are free vibrators. However, while the guitar strings are plucked, the piano strings are struck. A modern piano has 88 keys. When a key is depressed, a hammer with a felt surface is thrown against the strings tuned to a single note. The striking of the strings leads to a brief noise at the very start of the piano attack. As in the guitar, the strings themselves radiate very little sound. The strings of the piano pass over a bridge connected to a soundboard, and the soundboard is responsible for the radiation.

As for the guitar, the frequencies of the modes of vibration of the string are “stretched” compared to harmonic frequencies. There are similar questions of scaling. The string length, string tension, and string mass must be chosen to play the right frequency, to obtain good coupling to the radiator, and to satisfy practical considerations such as the size of the instrument and the physical limits on the maximum tension. The piano maker has some advantages. The musician does not hold the piano in his lap, and the piano can be large. The frame can be made of heavy steel to support the tension in the strings—a total of 20 tons of tension in a concert grand piano! Of course, there are correspondingly great demands on the piano maker. The piano needs to cover a wide frequency range—notes tuned from 28 to 4,196 Hz, and the piano is required to produce a big sound.

The three effects of a non-ideal string that lead to an evolving vibration pattern, and an interesting tone, for the guitar also apply to the piano. In addition, the piano tone has another avenue of evolution. Piano tones are played on multiple strings. Corresponding to the 88 different notes on the piano are more than 225 strings. Most of the notes of the piano have three strings. The mid-bass region has two strings, and only the low bass notes have a single string.

The three strings assigned to a single piano tone are not all tuned to identical frequencies. First, it would be physically impossible to make them absolutely identical. Second, there is evidence that piano tuners adjust the mistuning among the three strings to produce the best sound. Mistuning leads to beats, of course, but the different harmonics of the tone beat at different rates and the perceived effect of the beats is subtle. The mistuning among the multiple strings leads to an important characteristic of piano tone, a double decay pattern.

When the hammer strikes the three strings, it first causes them all to move up and down together. All the strings force the bridge in the same way. The bridge moves a lot and extracts energy from the vibrating strings with great efficiency. This leads to an intense attack sound. It is a sound that decays rapidly because the strings are losing energy at such a high rate. But, because the strings have slightly different frequencies, it takes only a few seconds before the vibrations of the individual strings are no longer in phase. Then the strings force the bridge independently. This causes the coupling to the bridge to be less efficient, and the rate of decay becomes slower. The slowly decaying part of the tone is called the “aftersound.” It can easily continue for a minute (Fig. 25.2).

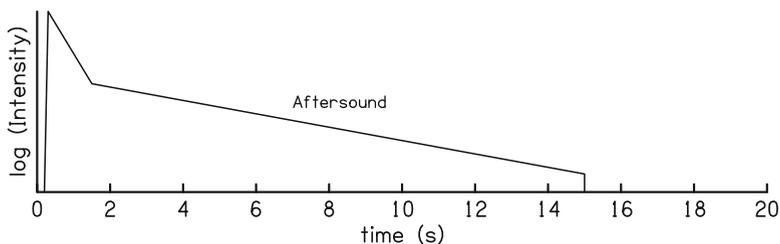


Fig. 25.2 The piano tone has a two-part decay pattern. At first, the strings vibrate in phase, creating an intense rapidly-decaying tone. After the string vibrations become out of phase, the aftersound decays more slowly. The tone shown here stops abruptly after 15 s because the player released the key. When the key is released a damper makes contact with the string and stops the tone

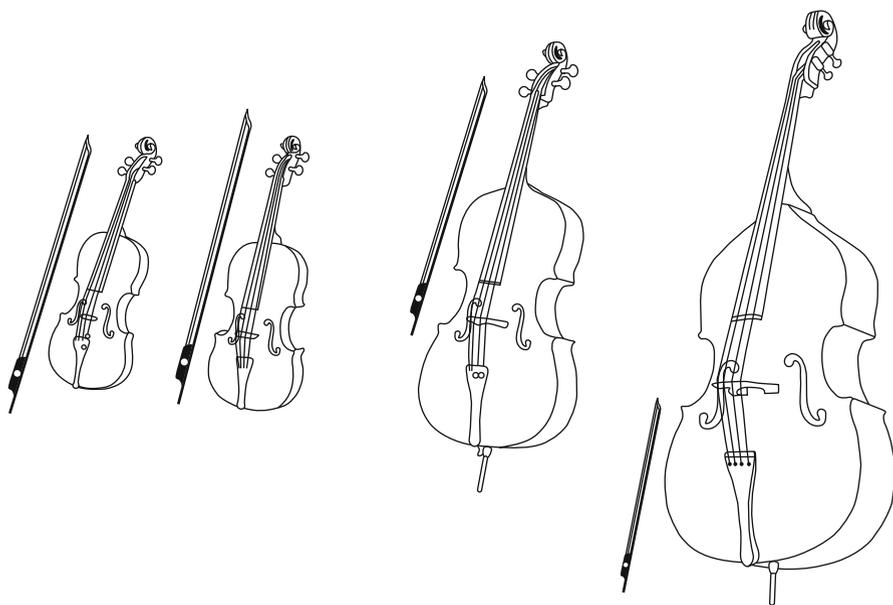


Fig. 25.3 Bowed string instruments, from *left to right*: the violin, the viola, the cello, the doublebass or bass viol

25.2 Bowed Strings

The class of orchestra instruments known as the strings includes the violin, viola, cello, and double bass (Fig. 25.3). These instruments are sometimes called the violin family. The ranges of these important instruments are given in Appendix C. The basic sound producing element is the vibrating string. Sometimes the string is plucked, a style of playing known as “pizzicato.” When plucked, a violin string is a free vibrator, just like a guitar string. Normally, however, the string is bowed.

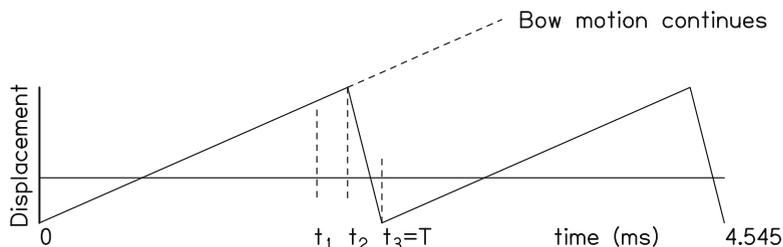


Fig. 25.4 Two cycles of the stick–slip process. The y axis shows the displacement of the string at the location of the bow as time progresses. The slipping takes place between times t_2 and t_3 . Because it takes 4.545 ms for two cycles, you know that the playing frequency is 440 Hz. Time t_1 is a marker for Fig. 25.5

Bowed strings are not free vibrators. They are driven vibrating systems like the brasses and woodwinds. Like the brasses and woodwinds, the bowed strings are sustained-tone instruments. Their tones have exact harmonics and are stable when viewed on an oscilloscope. As in all the other sustained-tone instruments, feedback and a nonlinear tone generation process are responsible for creating a tone with harmonic partials from resonances that are only approximately in a harmonic relationship.

25.2.1 Tone Generation in the Bowed Strings

The tone generation process is similar in all the bowed string instruments. For definiteness, the following discussion will refer to the violin, but it might as well be directed toward any of the others.

The bow of a violin is made of many strands of horse hair, stretched into a thin ribbon. When the bow is drawn across a string, the string is set into vibration. The process by which the bow excites the string can be described as a “stick–slip” process. As the bow moves across a string, it pulls the string with it. This is the “sticking” phase of the stick–slip process. Violin players put rosin on the bow to control the amount of sticking. As the string is pulled aside by the bow it is stretched, and tension builds up. Eventually the tension is so great that the string slips and moves in a direction that is opposite to the bow motion. This is the “slipping” phase of the stick–slip process. Once the string has slipped back, it is ready to be picked up by the bow again and another cycle begins. During the sticking part of the cycle, the speed of the string is the speed of the bow. But when the string slips, it snaps back suddenly, as shown in Fig. 25.4.

This stick–slip description of the bowing process is easy to understand. What is more, it correctly describes the motion of a bowed string as seen at the bowing point. However, there is something dreadfully incomplete about it. It takes no account of the length of the string. But you know that the length of the string is crucial in

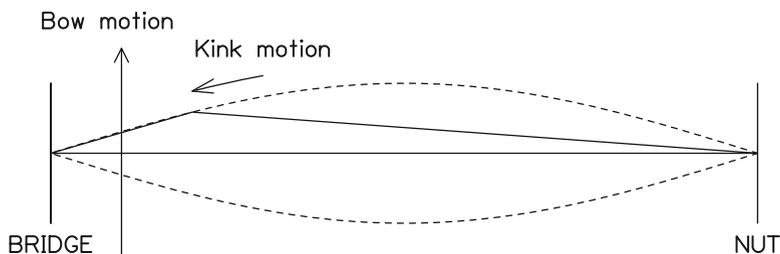


Fig. 25.5 When a string is correctly bowed, the string displacement has a sharp kink which travels around the path shown by *dashed lines*, once per cycle. A snapshot of the string is here taken at time t_1 (see Fig. 25.4), just before the kink hits the bowing point. When the kink hits the bowing point the slipping phase will start

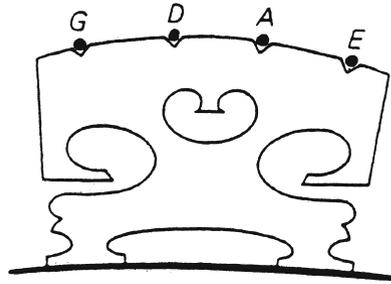
determining the playing frequency. A violin player shortens the string by pressing it down on the fingerboard to make tones with higher frequency. So there is a puzzle here to solve.

Perhaps you are now thinking about how it was with woodwind instruments—how resonant feedback from the body of a woodwind instrument entrained the opening and closing of the reed, which then fed energy into the oscillating system. That feedback was critical in determining the playing frequency of the instrument. Perhaps you are now thinking by analogy and imagining that feedback from the well-known resonances of a stretched string, might entrain the stick–slip process. If that is what you are thinking, you would be right. It is absolutely uncanny how this general physical principle keeps on appearing, in different guises, in the different sustained-tone instruments.

If you look down on a string while it is being bowed, you will see a fuzzy pattern looking like the dashed outline in Fig. 25.5. This outline is actually the path of a sharp bend, or kink, in the string, as shown by the solid line. This kink travels around the path once per cycle. If the playing frequency is 440 Hz, the kink makes 440 round trips per second. Figure 25.5, is a snapshot of the string taken at time t_1 , just before the kink arrives at the bowing point. An instant later, at time t_2 , the kink hits the bowing point and knocks the string free from the bow. That initiates the slipping phase, as shown in Fig. 25.4. While the string is slipping, the kink will travel to the bridge, where it will be reflected onto the bottom dashed path in Fig. 25.5. When the kink hits the bowing point again, the bow will pick up the string and the sticking process will begin again. In that way, the modes of the string make their impact on the stick–slip mechanism.

The stick–slip process is periodic, and the period is equal to the time it takes for the kink to make a round trip. In one cycle the kink travels a total distance of twice the string length ($2L$), and it travels with the speed of sound on the string (v). Consequently, the period is $2L/v$, and the fundamental frequency is $v/(2L)$. It should not surprise you that this frequency is exactly what we found in Chap. 7 for the fundamental frequency of a stretched string.

Fig. 25.6 A violin bridge with the pitch-names of the strings indicated



This account of the vibrations of a bowed string describes what happens when the string is correctly bowed and a good tone is produced. It is worth noting that there is no guarantee that just any application of a bow to a string will be correct or that a good tone will be produced automatically. Getting a string to vibrate so that there is useful cooperation between the energy fed in from the bow and the feedback from the string itself is the art of bowing a string, and every violin player learns to do that. In fact, if a string is bowed with too much bow force, the feedback process fails. Then the stick–slip process is uncontrolled, and the string produces a noisy squawk with no clear frequency.

25.2.2 *The Violin Body*

In a wind instrument, the air column plays a double role. It provides a resonant feedback mechanism responsible for playing a periodic tone with a stable frequency, and it is responsible for radiating the tone, either through a bell or through tone holes. The violin is not like that. As described above, the resonant feedback mechanism is the stretched string and does not involve the body, but a string makes a very inefficient radiator. It is so thin that it cannot move much air. What happens instead is that the violin string causes the bridge to vibrate, and the bridge causes the top plate of the body to vibrate in turn. The coupling of the vibration through the bridge is a very delicate operation. The bridge is carved in a special shape that allows it to provide a good match between the string and the top plate. The *mute* on a violin, for especially soft playing, adds a little mass to the bridge and greatly reduces the coupling efficiency for high-frequency vibrations. The fact that a small change in mass in the bridge dramatically reduces the transfer of high-frequency power indicates that the function of the bridge is highly sensitive to its physical details (Fig. 25.6).

The radiation from the violin comes mainly from the top plate and from the vibration of air inside the body. Holes, called “f-holes,” are cut into the top plate and allow sound waves inside the body to radiate to the outside. The resonances of the violin body and air volume are independent of the modes of the strings, and they are of the utmost importance for the sound quality of a violin. They mark the difference between a poor sounding student instrument and a great sounding masterpiece.

It is thought that the art of violin making reached its peak, in Italy, at the time of Niccolò Amati (1596–1684) and Antonio Stradivari (1644–1737). In the centuries since then countless violin makers (and scientists too) have tried to make instruments of comparable quality. It has been supposed that these Italian craftsmen held a secret—perhaps it was a special varnish—and that one only had to discover the secret to achieve the playing characteristics and sound of these splendid instruments. Possibly all this is true, and yet there is more than just a little mystique about these instruments. Most of the fine old Italian violins that are so prized have been rather thoroughly worked over in more recent times to keep them in playing condition. One wonders how much of the original is left.

Whether or not contemporary violin makers can make violins that compare with the old masters can be debated. What can be said for sure is that contemporary makers know enough about the art and science of violin making to make fine playing instruments and can do so reliably.

Vibrato A violinist does not press the string down on the fingerboard and hold it there motionless. Instead, the violin player oscillates the hand (it's always the left hand) so as to make the string slightly shorter and longer with time. This leads to a playing frequency that rises and falls periodically. The effect is well characterized as frequency modulation (FM) of the violin tone. The parameters of this FM are different for different playing styles and are subject to fashions of the times. In renaissance music there was no vibrato at all. Over the course of the twentieth century preferred vibrato rates decreased from a range of 6–8 cycles per second to a range of 5–7 cycles per second. For a contemporary violinist typical parameters are: a rate of 5.5 cycles per second, a sinusoidal modulation waveform, and an overall frequency excursion of 1 semitone (equivalent to +3% and –3% of the playing frequency).

Vibrato is said to be an “ornament” in performance. That statement is not false, but it may not do justice to the importance of vibrato in the sound of the violin. It has become an essential element of playing, so much so that when a violinist has the possibility of playing a sustained note on an open string (no finger down on the fingerboard) the violinist will choose to play the note on a different string where vibrato can be used.

A violin tone with vibrato is lively. Its character is more interesting than a violin tone played without vibrato. The difference may well be that vibrato leads to an effect known as “FM-induced AM.” In this effect, the harmonics of the violin tone rise and fall with the frequency excursions in the vibrato. For example, imagine a violin body with a strong resonance at 800 Hz. Imagine now a violin tone with a nominal frequency of 392 Hz, but excursions that make the instantaneous fundamental frequency vary from 380 to 404 Hz. The second harmonic will vary in step with the fundamental from 760 to 808 Hz because the waveform remains periodic throughout the vibrato cycle. When the second harmonic is near 800 Hz its amplitude will be boosted by the resonance, but when the second harmonic is near 760 Hz its amplitude will be weaker. While this second harmonic has *increased* in amplitude with increasing playing frequency, there will no doubt be other harmonics

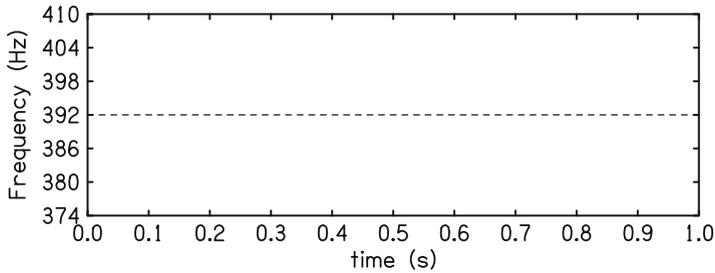


Fig. 25.7 The note being played is a G with a central frequency of 392.0 Hz. One second of the sound is shown here. The *vertical axis* is intended to show the instantaneous frequency to be drawn in Exercise 4

of the tone that line up with different resonances of the body in such a way that their amplitudes *decrease* with increasing playing frequency. Because the violin body has many resonances, there are many opportunities for this effect. The dynamic character of FM induced AM leads to a shimmering quality to the violin tone.

Exercises

Exercise 1, Speed of sound on a violin string

A violin string is 346 mm long, from bridge to nut. If the string is tuned to 440 Hz, what is the speed of sound on the string?

Exercise 2, How to play the violin

An open violin string is 346 mm long and plays 440 Hz. To play higher notes the violinist shortens the string. Each semitone step of the ascending scale corresponds to an increase of about 6 % in frequency. Thus, successive notes of the chromatic scale have frequencies of about 440, 466.2, 493.9, 523.3, 554.4, 587.3, 622.3, and 659.3 Hz, as given in Appendix E. Calculate the string lengths for all of those tones. How close are these string lengths to being equally spaced on the fingerboard?

Exercise 3, The violin and the voice

The violin sound begins with a string and is radiated by the body. The human voice begins with vocal folds and is radiated by the vocal tract. Find the parallels between the violin and the voice.

Exercise 4, Vibrato

On Fig. 25.7 sketch the instantaneous frequency of a violin tone played with vibrato that is acceptable by contemporary standards.

Exercise 5, Guitars

The section on electric guitars says that the string vibration decays more slowly in an electric guitar than in an acoustic guitar. Why is this so?

Exercise 6, Bozo's string theory

Bozo says that the stick-slip process for bowing a string predicts an effect of adding rosin to the bow. He says that adding rosin makes the bow stickier so that the string continues to be dragged by the bow for a longer time. That makes the period longer. The result is that adding rosin to the bow decreases the playing frequency.

Use what you know about the reflected kink on the string and its feedback to the bowing process to explain to Bozo why adding rosin to the string does *not* affect the playing frequency.

