Chapter 6
Wave Properties

This chapter describes some properties of waves—properties like interference, beats, reflection, and refraction. Naturally, the focus will be on sound waves, but the principles apply equally to light waves. Because of the unifying principles of wave physics, you get two for one—once you've learned the acoustics, you automatically know the optics.

6.1 Wave Addition

Your date is talking to you across the table, and the band is playing in the background. You hear them both. You may imagine that there are two sound waves, one from your date and the other from the band and that both sound waves are propagated to your ears. That description is not entirely wrong, but a much better description is that the waves from the two sources are added in the air, and your ear is exposed to only a single wave. The single wave is the sum of the waves from the two sources. The addition of waves is the addition of pressure waves. Because pressure waves can be positive or negative there are possibilities for reinforcement and cancellation. Reinforcement occurs when two waves are added and they have the same sign—both are positive or both are negative. Then the summed wave is more positive or more negative than either of its constituents. Cancellation occurs when the constituent waves have different signs. If one wave leads to a positive pressure at a particular place and time and the other wave leads to a negative pressure at that place and time, then the two waves tend to cancel each other when they both occur at the same time.

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6.2 Interference

In the most general terms, interference is the same thing as the addition of several waves as described in the section above. Specifically though, when we talk about interference, we usually are thinking about a situation where the addition is particularly simple because the two sources are similar. Both sources emit the same waveform and have the same frequency. This simple situation is important because dramatic events can occur. As usual in science, simplicity leads to power.

If the waves from the two sources add up to reinforce one another we have *constructive interference.* If the waves from the two sources add up to cancel one another we have *destructive interference.* Constructive interference occurs at a point in space where the two waves have the same phase. When one is positive the other is positive. When the first one is negative the second one is also negative.

Destructive interference occurs when the two waves have opposite signs. When one is positive the other is negative. Two sine waves can be caused to have opposite signs by delaying one of them by half a period. Then the two sine waves become 180° out of phase. You will recall that what you get on the second half cycle of a sine wave is just the negative of what you got on the first half cycle. *Therefore, the key to completely destructive interference is a delay of half a period.*

Figure 6.1 shows two loudspeakers and a microphone. Speaker 1 is 1.06 m away from the microphone; speaker 2 is 1.4 m away from the microphone. Both speakers emit a sine tone with a frequency of 500 Hz so that the period is 2 ms.

This example happens to be a case of interference that is almost completely destructive. The reason is that the two waves arrive at the microphone almost 180° out of phase. That is because the extra time for the sound from speaker 2, compared to speaker 1, is half a period of the tone. Let's see why that is so. The time required for any waveform feature to get from speaker 1 to the microphone is t_1, $t_1 = D_1/v$, where v is the speed of sound, $v = 344$ m/s, as shown in the time-line of Fig. 6.2. The time for the feature to arrive from speaker 2 is t_2, $t_2 = D_2/v$. The extra time is $t_2 - t_1$, and it is easily calculated.

$$t_2 - t_1 = D_2/v - D_1/v = 1.4/344 - 1.06/344 \approx 0.001 \text{ seconds}, \tag{6.1}$$

or 1 ms. Because the period of the tone is 2 ms, the extra time is half a period, just what is needed for cancellation.

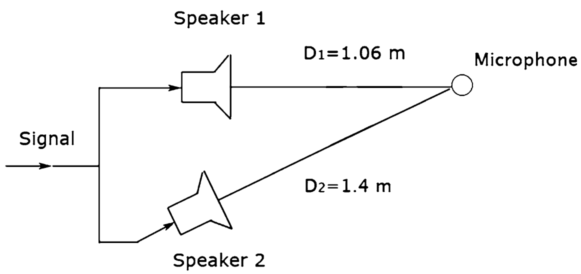

Fig. 6.1 Two loudspeakers and a microphone. The same sine signal is sent to both speakers

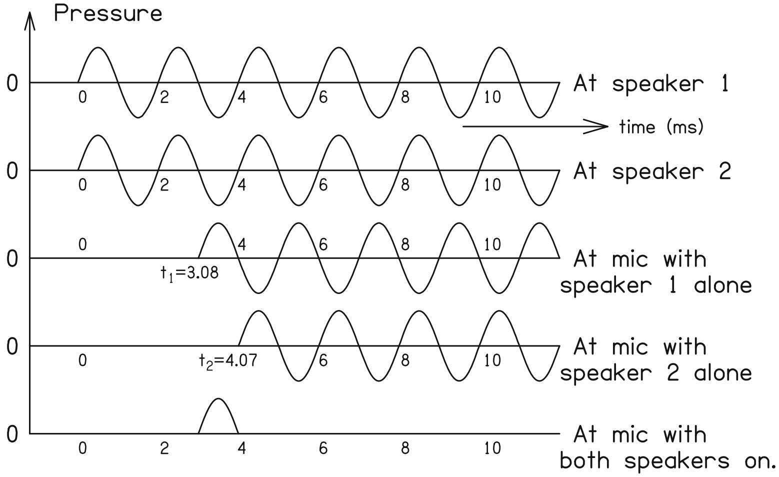

Fig. 6.2 Delay $t_1 = 1.06/344$. Delay $t_2 = 1.4/344$. The two waves cancel each other at the microphone position

Note that if the extra travel time were a full period, then the waves from the two speakers would be in phase, and there would be completely constructive interference.

Note that if the extra travel time were one-and-a-half periods, then the waves from the two speakers would be 180° out of phase again and the interference would be destructive.

It should be clear that the condition for destructive interference is that the extra travel time should be an odd number of half periods, i.e., $T/2$, $3T/2$, $5T/2$, $7T/2$, etc. The formula for destructive interference is then

$$t_2 - t_1 = (2n + 1)T/2, \tag{6.2}$$

where n is an integer. [Note that $(2n + 1)$ is just a way to describe any odd integer. Try it.]

We can express that result in terms of the extra distance that the wave from speaker 2 must go. Call that extra distance $D_2 - D_1$. Then destructive interference occurs when the extra distance is an odd number of half wavelengths, i.e.

$$D_2 - D_1 = (2n + 1)\lambda/2. \tag{6.3}$$

To prove that Eq. (6.3) is correct, we can start with Eq. (6.2) for cancellation described in terms of delay, then multiply both sides of the equation by the speed of sound to get distances:

$$D_2 - D_1 = v(t_2 - t_1) = (2n + 1)vT/2 = (2n + 1)\lambda/2. \tag{6.4}$$

The proof has used the fact that $vT = \lambda$.

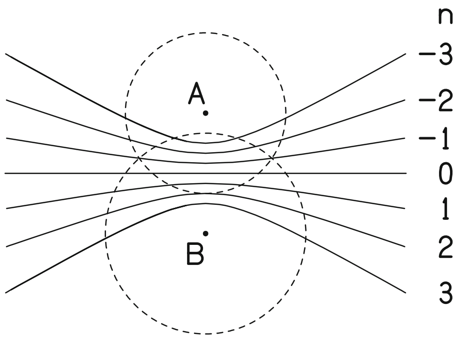

Fig. 6.3 Two sources, A and B, are separated by 3 m. They both emit a tone with wavelength 0.5 m. Hyperbolas where there is perfect constructive interference are labeled with index n. For instance, every point on the hyperbola labeled "−1" is exactly one wavelength closer to A than to B. The hyperbola labeled "−2" is exactly two wavelengths closer, etc. The *dashed circles* show a snapshot taken when a wave crest from A has moved out by 2 m and an earlier wave crest from B has moved out by 2.5 m. The difference is exactly one wavelength and so the circles intersect on the line labeled "−1"

Interference, as described above, is a spatial effect. Two waves having exactly the same frequency may add constructively or destructively at different points in space. At some points in space they may cancel almost entirely so that a dead spot occurs for the particular frequency in question, and the spot remains dead for all time. Interference, as described above, uses spatial differences to create the particular phase relationships between the two waves so that constructive or destructive interference occurs. For instance, a path length difference of 0.34 m led to a 180-degree phase difference in the example.

A two-dimensional view of constructive interference between two sources, A and B, can be seen in Fig. 6.3. Constructive interference occurs at all points where the distance from source A and the distance from source B differ by one wavelength, or two wavelengths, or three, etc., i.e.

$$D_A - D_B = n\lambda \tag{6.5}$$

For a particular value of n we are looking for all the points where the difference between the distance from point A (D_A) and the distance from point B (D_B) differ by a certain amount ($n\lambda$). It is a fact of geometry that all points that fulfil a requirement like that in two dimensions lie on a hyperbola. Such hyperbolas are shown by the solid lines in the figure. At every point on those lines there is perfect constructive interference.

6.3 Beats

The phenomenon called "beats" is also a form of interference. There are maxima and minima from constructive and destructive addition of tones, but beats take place in time and not in space. At some times there is constructive interference and at

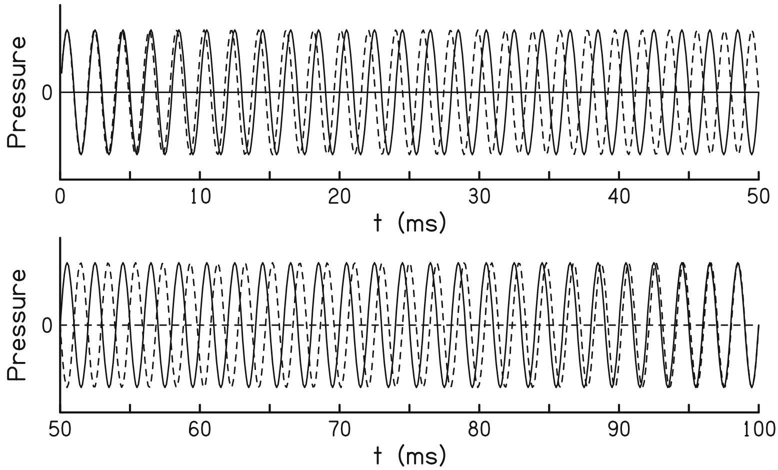

Fig. 6.4 Two waves, 500 and 510 Hz. The time axis begins at the *upper left* and continues on the second graph below. Beats occur when the solid wave and the dashed wave are added together. The pattern shown here repeats for as long as the two tones persist

other times there is destructive interference. When the interference is constructive it is constructive at all points in space, and when it is destructive it is destructive at all points in space.

Beats occur between two sine tones that have *almost* the same frequency but *not exactly* the same frequency. The beat rate (the number of beats per second) is equal to the difference between the two frequencies. If the two frequencies are f_1 and f_2, then the beat rate is

$$f_{beat} = f_2 - f_1 \tag{6.6}$$

For instance, suppose you strike two tuning forks, and one has a frequency of 500 Hz while the other has a frequency of 501 Hz. The difference is $501 - 500 = 1$, or one beat per second. That means that there will be one maximum, when the sum is loud, and one minimum, when the sum is quiet, every second. It is not hard to see why this is so.

Suppose that we measure the two waves from the two tuning forks when each is at a positive maximum. They are "in phase." Then the sum will be as loud as it can be. As time progresses the two waves will become out of phase because the forks have different frequencies. After half a second the 500-Hz wave will have executed 250 complete cycles and it will be at a positive maximum again. However, the 251-Hz wave will have executed 250.5 cycles, and the extra half a cycle will mean that this wave is at a negative maximum. The positive maximum and negative maximum will cancel and the sum will be quiet. After a full second, the 500-Hz wave will have gone through 500 cycles and be at a positive peak; the 501-Hz wave

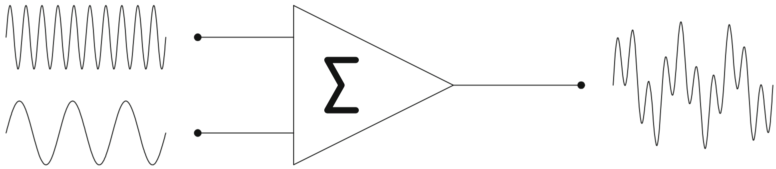

Fig. 6.5 A two-input mixer. As usual, signals go from left to right. Two signals come into the mixer and one signal comes out on the right. The signal that comes out is the sum of the two input signals. The *sigma symbol* indicates summation. To create a musical recording or the sound track of a film, audio engineers use mixers that add dozens of signals together. A 48-input mixer is not uncommon. The principles of interference among audio signals still apply

will have gone through 501 complete cycles and also be at a positive peak. As a result, the waves will once again be perfectly in phase and there will be a maximum again. This example shows that there is one maximum and one minimum per second when the frequency difference is 1 Hz.

Figure 6.4 shows what happens when the frequencies are 500 and 510 Hz. Now there are ten beats per second. Ten beats per second is rather fast but one can still hear them as a rapidly varying loudness. Again, we choose to start the clock when both waves are together. After 50 ms (0.05 s) the 500-Hz wave will have gone through 25 cycles but the 510-Hz wave will have gone through 25.5 cycles. The waves will be out of phase and will cancel. After 100 ms the two waves will have gone through 50 and 51 complete cycles and will be in phase again.

6.4 Audio Analogies

The previous sections have described interference and beats in acoustical terms. The outputs of two loudspeakers were added in the air to make a single wave that appeared at the microphone. The outputs of two tuning forks were added in the air to create a single beating wave at a listener's ear.

Because we often deal with signals in electronic (audio) form, it is important to note that interference and beats also occur between *audio* signals that are added together. The electronic device that adds audio signals together is called a "mixer." Figure 6.5 shows a mixer with two inputs. If these two inputs contain sine tones that happen to have the same frequency and the same amplitude but are 180° out of phase, then the sine tone in the output will cancel just as surely as it did in the two-loudspeaker experiment. Alternatively if the two inputs contain sine tones with frequencies of 500 and 510 Hz, then the output will contain a sine tone that beats ten times per second. One could see that beating tone by connecting the output to an oscilloscope.

6.5 Generalization

The concepts of interference and beats have been presented above using sine tones as examples. It was natural to use sine tones because the interference and beating effects are determined by frequency and a sine tone has only a single frequency. That makes it simple. It is important to realize that these effects also take place for the more complicated signals of speech and music because sine tones are present as components in complicated signals. An interference effect or a beating effect that occurs for a sine tone in isolation also occurs in the same way when that sine tone is a component of a complex tone.

Telephone Dial Tone To illustrate the generalization above about complex signals, consider the dial tone in an American telephone. The dial tone is a sum of two sine waves, 350 and 440 Hz. This sum makes a complex wave that sounds like a musical chord. Suppose you did not know what the frequencies were and you wanted to find out. An oscilloscope tracing would be a mess because the signal is not particularly periodic. For the same reason, a frequency counter would do you no good either.

What you *could* do is to add another sine tone with adjustable frequency and listen for the beats. You can choose to get beats from either the 350-Hz component or the 440-Hz component. When you adjust the frequency to 345 Hz you hear 5 beats per second because of the 350-Hz component. When you adjust the frequency to 443 Hz you hear 3 beats per second because of the 440-Hz component. By counting the beats for different frequencies of the adjustable tone near 350 and 440 Hz you could learn the exact frequencies of the two components in the dial tone.

Electronic Inversion The present chapter opened with a discussion of wave addition and interference. It showed that a 500-Hz tone can be cancelled by adding another 500-Hz tone if one of these tones is delayed by 1 ms compared to the other. That is because 1 ms is one-half of the period—just what is needed to make the two tones 180° out of phase. Similarly, a 400-Hz tone can be cancelled by adding another 400-Hz tone that has been delayed by 1.25 ms. This kind of cancellation works perfectly for sine tones.

But what happens for a complex tone? Suppose you want to place a single speaker so as to cancel a complex tone that is the sum of a 400-Hz tone and a 500-Hz tone, using a set up like Fig. 6.1. You could use a delay of 1.25 ms to cancel the 400-Hz part of the tone, or you could use a delay of 1 ms to cancel the 500-Hz part of the tone, but there is no delay that would cancel both components of the tone.

Within the domain of acoustics there is no solution to this complex cancellation problem. But electronically there is a solution because electronics allows for waveform inversion by a device called (appropriately) an "inverter." An inverter has an input and an output. Whenever the input voltage is positive, the output voltage is negative—and with the same magnitude. Thus, at an instant of time when the input is 3.45 V, the output is −3.45 V. Similarly, when the input is negative, the output is positive, again with the same magnitude. Therefore, the inverter happens to delay

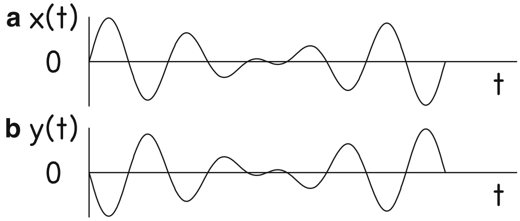

Fig. 6.6 (**a**) Waveform $x(t)$ is the sum of a 400-Hz sine and a 500-Hz sine. (**b**) Waveform $y(t)$ is the result of passing waveform x through an inverter. At any instant of time y is the negative of x. If x and y are added together in a mixer the output of the mixer is zero because x and y cancel completely

each frequency component by exactly the right amount to turn the waveform upside down. The inverted waveform $y(t)$ could be used to cancel the input waveform $x(t)$, as described in Fig. 6.6.

Noise-Cancelling Headphones

Our high-speed industrialized world is a noisy place. The sounds of machinery and transportation can mask other sounds that we need to hear or want to hear. Noise can be more than just annoying. It can be a hazard to health and safety. Wouldn't it be great if we could use electronics and the concept of interference to cancel the noise around us? The idea is simple in principle. All we need to do is to pick up the annoying sound with a microphone to convert it into an audio (electronic) signal, then invert the signal, and finally play it back through a loudspeaker. The signal from the loudspeaker should then cancel the annoying sound. This is the concept of *active noise cancellation.*

In fact, it is almost impossible with current technology to achieve active noise cancellation over a three-dimensional region of space, except for the very longest wavelengths (lowest frequencies). However, there are several acoustical circumstances where active noise cancellation can be successful. One application is in ducts where the sound is confined, and there is only one important direction of propagation, namely along the length of the duct.

The most common application of active noise cancellation is in headphones. Headphones are another confined environment, and they all contain little drivers with diaphragms that reproduce speech and music. In noise-cancelling headphones, the phone for each ear contains a tiny microphone to pick up the outside noise. The signal from the microphone is inverted and sent back, to be mixed with the desired speech or music. The combined signal is then sent to the headphone diaphragm driver. The result is that the inverted noise cancels the noise from outside and the user can hear the speech and music better. Alternatively, the user can disconnect the source of speech or music and simply listen to quiet.

Noise-cancelling headphones work well for low-frequency sounds where the wavelengths are long compared to the size of the headphone ear cup. Commercially available units are effective below 500 Hz but not at higher frequencies. Because much of the noise in aircraft and other vehicles is actually below 500 Hz, noise-cancelling headphones are useful. However, the sounds of intelligible human speech extend well above 500 Hz. Thus, noise-cancelling headphones can eliminate the low rumble inside an airplane, leaving a high-frequency rushing sound, but they do not prevent you from being disturbed by people using their cell phones while you are waiting in the airport.

6.6 Reflection

Waves can be reflected. A wave is reflected when it abruptly changes its direction of propagation. Suppose that you are in a room at night with a single lamp burning. If you look directly at the lamp you see a source of direct light. If you look at anything else in the room you are seeing reflected light. Similarly, sound waves are reflected from surfaces in a room. A wave is reflected when it encounters a change in medium for propagation. Both light and sound are reflected from a wall because the material of the wall is different from ordinary air.

There are two kinds of reflection depending on the regularity of the reflected wave. A mirror leads to *specular* reflection. You can see your face in a mirror. A typical wall leads to *diffuse* reflection. You cannot normally see your face in a wall. The difference can be seen in Fig. 6.7.

The specular reflection preserves the order of the rays and therefore preserves the image on reflection like a mirror. The diffuse reflection scrambles the order.

Whether a reflection is specular or diffuse depends on the size of the bumps on the surface compared to the wavelength. If the bumps are small compared to

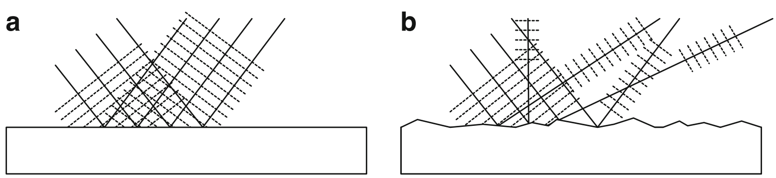

Fig. 6.7 (**a**) A wave comes in from the upper left. Four long, parallel lines show the direction of propagation of the rays, and the spaces between the wavefronts (shown *dashed*) indicate the wavelength. The wave is specularly reflected because the reflecting surface has irregularities that are much smaller than the wavelength. (**b**) The same wave comes in from the upper left, but it is diffusely reflected because the bumps on the surface are comparable to the wavelength or larger than the wavelength

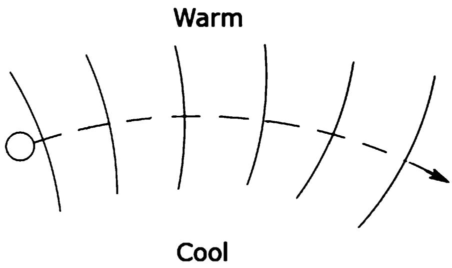

Fig. 6.8 The figure shows six *lines* indicating the pressure peaks in a traveling sound wave. The *lines* are separated by a wavelength. The wavelength is longer in warm air where the speed of sound is greater. The direction of propagation is perpendicular to the *lines* that show the peaks. Therefore, the direction of propagation is bent downwards toward the cool region

the wavelength the reflection is specular. If the bumps are large compared to the wavelength the reflection is diffuse. A specular reflection of sound waves in a room can lead to focusing of sound—often leading to an uneven sound distribution in the room.

6.7 Refraction

Refraction of a wave is different from reflection because the change of direction is gradual and not abrupt. A lens refracts a light wave. For instance, eyeglasses consist of lenses that make slight changes in the direction of the rays of light in order to correct for defects in the optics of the user's eyes. Lenses work because the conditions for propagation of light in glass are slightly different from the conditions in air or vacuum.

Sound waves can also be refracted when the conditions for propagation change. For instance, when sound travels from a region of cool air to a region of warm air, the wavefront moves faster in the warm air and bends toward the direction of the cool air. This effect occurs on a sunny winter day when the air immediately above the ground is cooled by snow but the air well above the ground is warm (Fig. 6.8).

6.8 Diffraction

You can't see around a corner, but you can hear around a corner. That essential difference between vision and hearing is the result of the very different wavelengths of light and sound. While visible light has a wavelength of about 500 nm (500×10^{-9} m, or 5×10^{-7} m), audible sound has a wavelength as long as 7 m, about ten million times longer than light. Sound waves bend around corners. Light waves bend around corners too—just ten million times less.

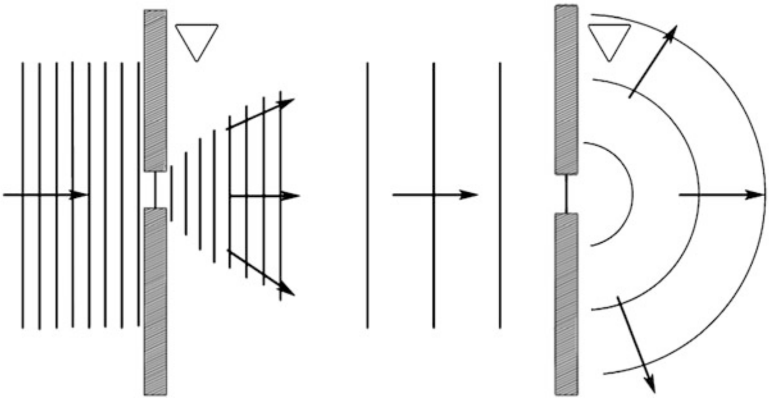

Fig. 6.9 A sound wave travels toward a door opening. In the figure on the *left* the sound wavelength is smaller than the door opening. In the figure on the *right* the wavelength is larger than the door opening. The large-wavelength sound spreads out beyond the door opening because of diffraction. Thus, a listener located at the position of the triangle can hear the large-wavelength sound

The ability of a wave to bend around a solid object is known as diffraction. The amount that a wave can bend depends on the ratio of the wavelength to the length of the bend. Diffraction also is involved when a sound wave passes through an opening such as a door. Then the amount of bending depends on the ratio of the wavelength of the sound to the width of the door opening. If the wavelength is small compared to the opening, then the wave goes straight through and does not bend much. Thus short-wavelength (high-frequency) sound behaves somewhat like light would behave. By contrast, long-wavelength (low-frequency) sound bends around the door opening, as shown in Fig. 6.9.

6.9 Segregation

We now return briefly to the example that began this chapter—your date and the band. This example illustrates the truly remarkable human capacity to selectively attend to individual sound sources. The sounds you want to hear are normally acoustically mixed in the air together with many other sounds. Moreover, the sounds that come directly to your ears are mixed with reflections from the surfaces in a room, creating a still more complicated sound field. As a listener, you sample this sound field with your two ears, and from this mixed up mess you extract the images of individual sources, e.g., your date and the band. This human ability is called "source segregation." It is a very impressive bit of signal processing. Scientists who study this aspect of human hearing for a living have yet to create a computer program that can segregate sound sources as well as the human brain. The source segregation problem is one of the greatest challenges in the field of artificial intelligence, but you routinely solve the problem without even thinking.

Exercises

Exercise 1, Magic wavelengths, magic frequencies

Two loudspeakers are connected so that they radiate the same sine tone. You are 3 m away from one loudspeaker and 3.4 m away from the other. (a) Find two values of the wavelength for which cancellation occurs. (b) What are the two frequencies for which cancellation occurs? (c) Are there more than two different wavelengths for which cancellation occurs?

Exercise 2, Total destruction

You are 1 m away from a source of a 1,000-Hz sine tone. (a) Where would you put a second source, driven by the same 1,000-Hz sine signal, in order to get complete cancellation? (b) Does it matter what the amplitudes of the two tones are?

Exercise 3, Stereo

In a standard stereo audio system, there are two loudspeakers. The loudspeakers can be connected wrongly so that they are out of phase. Why is this error more important for low frequencies than for high?

Exercise 4, Beats

You hear two tones with a beat rate of 5 Hz. One tone has a frequency of 440 Hz. What are possible frequencies for the other tone?

Exercise 5, The dial tone experiment

Fig. 6.10 depicts the dial tone experiment. The added tone at 345 Hz has an amplitude that is somewhat smaller than the amplitude of the dial-tone component at 350 Hz. Explain why the beats would be stronger if the added tone had the same amplitude as the component at 350 Hz.

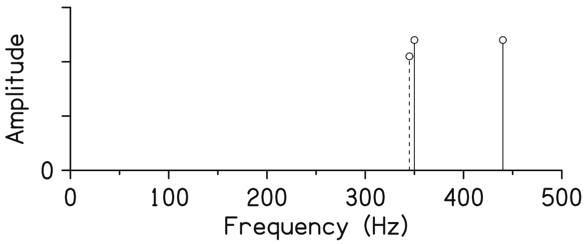

Fig. 6.10 The *solid lines* show the amplitude spectrum of the telephone dial tone with components at 350 and 440 Hz. The *dashed line* shows a sine tone that can be added at 345 Hz to cause beats, five per second, to expose the 350 Hz tone

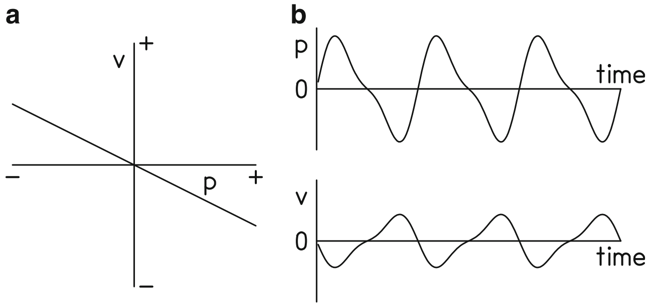

Fig. 6.11 Figure for Exercise 10. The input/output relationship of a linear transducer is a straight line. How would you describe this line?

Exercise 6, How smooth is smooth?

How smooth does a mirror need to be? Answer this question by working out the wavelength of light. Remember that the speed of light is 3×10^8 m/s. Recall that frequency of visible light is about 6×10^{14} Hz.

Exercise 7, How rough is rough?

Imagine a rough stone wall where some stones randomly protrude a few inches. How does a 100-Hz sound wave reflect from this wall? How about a 10,000-Hz wave? The answers to this exercise illustrate the general principle that a surface appears to be rough or smooth depending on the wavelength of the radiation that is used to examine the surface.

Exercise 8, Sound channels by refraction

With a complicated temperature profile, refraction can lead to narrow channels that trap sound waves. Which profile leads to a channel: (1) a layer of warm air trapped between layers of cool air, or (2) a layer of cool air trapped between layers of warm air?

Exercise 9, Elective deafness

Although it seems like blocking out the loud sounds of our busy and noisy world would be a good idea, having perfect noise-cancelling headphones would have its disadvantages. Explain why you would not want to completely knock out the noise of everyday life.

Exercise 10, Another linear transducer

Figure 4.2 shows the input/output relationship for a linear transducer that converts pressure waves into electrical waves. Figure 6.11 is similar but different. Explain how the difference in parts (a) of the figures leads to the difference in the voltage v in parts (b).

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