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Background: The Duality of Nature

(Good references for this chapter on the historical background are the article by Niels Bohr, entitled "Discussions with Einstein. Epistemological Problems in Atomic Physics." In *Albert Einstein. Philosopher-Scientist.* Vol. VII of Library of Living Philosophers, Paul A. Schilpp, ed., Evanston, Illinois, 1949; and the little book by Werner Heisenberg, *The Physical Principles of the Quantum Theory*, Dover Publications, 1949.)

The results of the experimental developments of the late nineteenth and early twentieth century led us to a picture of nature that showed the *duality* of nature on the atomic scale. *Both* material particles and electromagnetic radiation show *both* particle-like and wave-like aspects. However, particles can be localized in space-time. In classical physics, x,y,z,t for a particle can be specified exactly. Particles are also indivisible. Half an electron, or a fractional part of an electron, does not exist. On the other hand, waves *cannot* be localized. They must be somewhat extended in space-time to give a meaning to wavelength, λ, and frequency, ν. Waves are always divisible. Partial reflection and transmission of a wave at an interface between two media can exist.

This duality poses a real dilemma: The particle picture seems incompatible with that of waves, in particular, the interference effects. Yet, it is precisely the interference effects that determine λ and ν, which via the deBroglie relation, $p = h/\lambda$, and the Bohr relation, $E = h\nu$, determine the dynamical attributes of the particle.

K. T. Hecht, *Quantum Mechanics*

A The Young Double Slit Experiment

To illustrate the paradoxical situation, consider the classical interference experiment of the Young double slit. We could think either of light waves, electromagnetic radiation, or of matter waves, electron deBroglie waves, going through the double slit arrangement.

The incident beam can be made so weak that, on average, only one photon (or electron) at a time will pass through the apparatus and be incident on the photographic plate. Because only one photon at a time goes through the apparatus, the possibility of interference between different photons is eliminated. An interference pattern will still be on the photographic plate, however. Clearly, a photon that has reached the photographic plate must have passed through *either* slit 1 *or* slit 2. Imagine it was slit 1; then, if slit 2 had been closed, no interference pattern would have occurred. Hence, the seemingly terrible paradox that the behavior of the photon is influenced by the presence of a slit, through which it cannot have passed.

The resolution of the paradox rests on the fact that the classical causal space-time description of nature which rests on the "clear-cut separability between the phenomena and the means of observing these phenomena," does not apply. On the atomic scale, an "uncontrollable interaction between the object and the measuring instrument" exists. (The words in quotation marks are those of Niels Bohr.) As a result, the above experiment can be set up in either of two "complementary" ways; as above, and as shown in Fig. 1.1, to exhibit the interference fringes but in a setup that makes it impossible to answer experimentally the question: "Through which slit did the photon pass?". Alternatively, we could alter the experimental setup to answer experimentally the question: "Through which slit did the photon pass?" In setting up the experiment in this second way, however, we have lost the possibility of a precise wavelength measurement through the interference pattern. The interference pattern will have been wiped out.

Because our knowledge of phenomena on the atomic scale is restricted, a wave-field must be associated with the particle motion. For any wavefield, an uncertainty relation exists connecting position and wavelength; $\Delta k \Delta x \approx 2\pi$, where $k = 2\pi/\lambda$. This relation follows from straightforward Fourier analysis of a wave packet of finite extent in space. Now, with the deBroglie relation, $p = h/\lambda$, this leads to $\Delta p \Delta x \approx h$, the Heisenberg uncertainty relation.

B More Detailed Analysis of the Double Slit Experiment

In region I, to the left of the single slit (see Fig. 1.1), we assume we have a plane wave effectively of infinite extent in the y direction and proceeding in the x direction. Then, in region I, $p_y = 0$; i. e., p_y is known precisely, so $\Delta p_y = 0$;

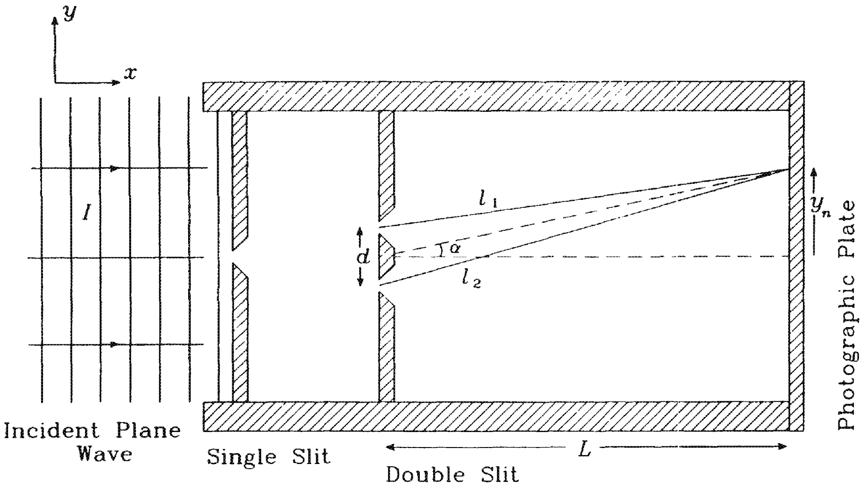

FIGURE 1.1. Conventional double slit experimental setup. Rigidly fixed slits.

now, we have no knowledge of where the photon is located in space, $\Delta y = \infty$. The y position of the particle is completely uncertain.

At the position of the double slit (with massive, rigidly fixed slits bolted to massive apparatus; see the figures in the article by Bohr), no experimental means of determining through which slit the photon is passing exist. It must go through either of the two slits. Hence, at this x-position, $\Delta y \approx d$. In its passage through one of the slits, the photon will interact with the slit jaw, which is massive, bolted firmly to a huge apparatus, and it can absorb recoil momentum without moving. Most of the photons end up within the first few bright fringes near the central maximum. We cannot predict which fringe, however. Let us assume the photon ends up in the n^{th} bright fringe, where n is a small integer. Then, the change in p_y at the double slit position is

$$\Delta p_y \approx p\alpha = \frac{h}{\lambda}\alpha \approx \frac{h}{\lambda}\frac{y_n}{L}. \tag{1}$$

Note

$$\begin{aligned} l_1^2 &= (\tfrac{1}{2}d - y_n)^2 + L^2 \\ l_2^2 &= (\tfrac{1}{2}d + y_n)^2 + L^2, \end{aligned} \tag{2}$$

so

$$l_2^2 - l_1^2 = 2y_n d \approx (l_2 - l_1)2L, \tag{3}$$

and

$$\frac{y_n}{L} = \frac{(l_2 - l_1)}{d} = \frac{n\lambda}{d}. \tag{4}$$

Therefore, at the x-position of the double slit, with $\Delta y \approx d$,

$$\Delta y \Delta p_y \approx d\frac{h}{\lambda}\frac{y_n}{L} = d\frac{h}{\lambda}\frac{n\lambda}{d} = nh, \quad (5)$$

where the uncertainty in the y-component of the momentum at the position of the double slit must be of the same order of magnitude as the change of y-component of momentum of a photon that ends up at the interference maximum given by a relatively small integer, n. Thus, at the double slit,

$$\Delta p_y \Delta y \approx nh, \quad (6)$$

where n is a relatively small number.

C Complementary Experimental Setup

The question now arises: Could we have modified the experimental setup at the x-position of the double slit to narrow the uncertainty in y at this x-position? In particular, could we have modified the experimental setup to answer experimentally the question: Through which slit did the photon pass? We could do this by making the slit jaws movable, so the momentum exchange between photon and slit jaw could be detected. Bohr imagines the very light slit jaws being suspended from springs, so the photon will jiggle the slit as it goes through the slit opening. (See Fig. 1.2, drawn in the style of the Bohr article.) Imagine the photon ends up at the position where the central interference maximum would have occured in the conventional double slit experiment, (with the apparatus of Fig. 1.1, that is, at the most likely final position of the photon for the experimental setup of Fig. 1.1). Then,

$$p_{y_{\text{final}}} - p_{y_{\text{initial}}} \approx p \times \theta = \frac{h}{\lambda} \times \frac{d}{2L} = \frac{h}{\lambda} \times \frac{\lambda}{2(y_1 - y_0)} = \frac{h}{2(y_1 - y_0)}, \quad (7)$$

where $(y_1 - y_0)$ is the distance between the first and zeroth (central) bright fringe in the interference pattern of the conventional experimental setup. This actual change in the photon's momentum at the position of the slits would now lead to a recoil in the slit jaws, which can be detected. An uncertainty will still exist in the y-position of the photon as it passes through the slit, because of this jiggling of the slit; even though our jiggle detectors can tell us through which slit the photon has passed (so $\Delta y \ll d$). Now, let us use the uncertainty relation to determine the best possible Δy caused by to the jiggling of the slit,

$$\Delta y \approx \frac{h}{\Delta p_y} \approx \frac{h \times 2(y_1 - y_0)}{h} = 2(y_1 - y_0). \quad (8)$$

That is, Δy is of the order of the distance between bright fringes in the conventional double slit setup. This Δy, however, is now due to the jiggling of the slit. If the slit jiggles on average by an amount equal to the distance between interference fringes, the interference pattern on the photographic plate will surely be washed out

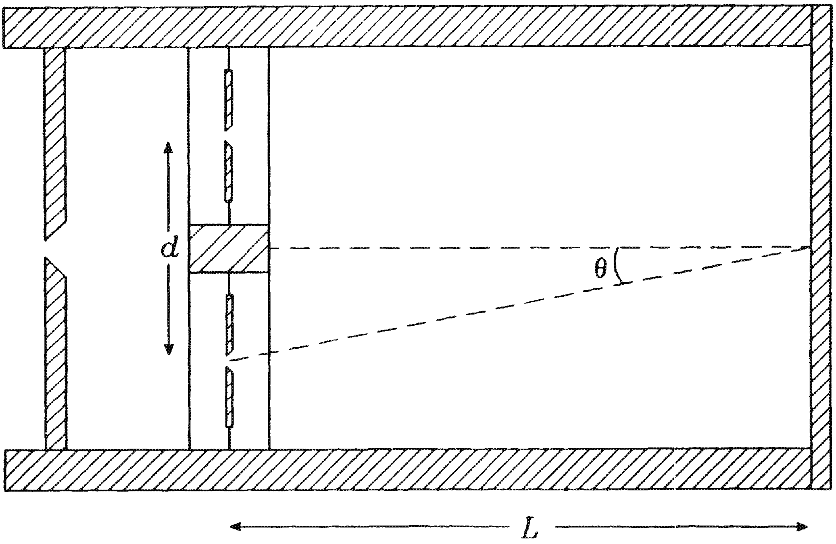

FIGURE 1.2. Complementary double slit experimental setup. Movable slits.

completely. In answering experimentally the question through which slit did the photon pass, we have by altering the experimental setup destroyed those features of the setup that previously made the precise wavelength measurement possible. This illustrates Bohr's complementarity principle. We can set up the double slit experiment to get very precise wavelength (hence, momentum) information about the photon. In this case, we cannot make a position measurement of the photon precise enough to tell us through which slit the photon passed. Alternatively, if we use the complementary experimental setup, which can answer this question about the position of the photon experimentally, we cannot determine the wavelength (hence, momentum) of the photon with sufficient accuracy.

Because we can have only partial position and partial momentum information about a photon or a material particle on the atomic scale, it becomes natural to associate a wave packet with the motion of the particle (either photon or material particle). A wave packet can give us partial position and wavelength (or wave-number k) information through the wave packet relation, $\Delta k_y \Delta y \approx 2\pi$, which follows from the Fourier analysis of the wave packet, the subject of the next chapter.