Chapter 1
Agency and Interaction What We Are and What We Do in Formal Epistemology

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Introduction

Formal epistemology is a recent field of study in formal philosophy dating back only a couple of decades or so (Helzner and Hendricks 2011; Hendricks 2006). The point of departure of this essay is rooted in two philosophically fundamental and interrelated notions central to formal epistemology;

- *agency* – what agents are, and
- *interaction* – what agents do.

Agents may be individuals, or they may be groups of individuals working together. In each of the sections that follow, assumptions are made concerning the relevant features of the agents at issue. For example, such relevant features may include the agent's beliefs about its environment, its desires concerning various possibilities, the methods it employs in learning about its environment, and the strategies it adopts in its interactions with other agents in its environment. Fixing these features serves to bound investigations concerning interactions between the agent and its environment. The agent's beliefs and desires are assumed to inform its decisions. Methods employed by the agent for the purposes of learning are assumed to track or approximate or converge upon the facts of the agent's environment. Strategies adopted by the agent are assumed to be effective in some sense.

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In what follows is an attempt to locate predominant paradigms in formal epistemology – e.g., probability theory, belief revision theory, decision theory, logics of knowledge and belief and finally interactive epistemology – within the framework of agency and interaction.

Probability

Probabilities are very useful in formal epistemology. They are used to measure key epistemic components like belief and degrees thereof, the strength of expectations and predictions, may be used to describe actual occurrent frequencies in nature or in the agent's environment and of course probabilities play a paramount in accounting for notions of (Bayesian) confirmation (Earman 1992). Current cognitive models apply probabilities to represent aggregated experience in tasks involving language acquisition, problem solving and inductive learning, conditionalization and updating beliefs and scientific hypotheses.

What sorts of internal states are essential to the agent's representation of its environment? Various doxastic notions e.g., according to which the agent simply believes or is certain of propositions, in contrast to believing the proposition to some degree, are a traditional interest within mainstream epistemology. Some philosophers, e.g. Jeffrey (1992), have argued in favor of a doctrine known as *radical probabilism*. A central tenet of this doctrine is that various propositional attitudes of epistemic interest, especially full belief, are reducible to credal judgments. There are several ways that one might attempt such a reduction. Perhaps the most obvious is to identify full belief with maximal partial belief. For example, if it is assumed that the agent's credal state can be represented by a probability measure, then such a reduction would identify those propositions that are fully believed by the agent with those propositions that have maximal probability according to this representing measure. Following this proposal, it would seem that a proposition counts as a serious possibility for the agent just in case its negation is not assigned maximal probability according to the probability measure representing the agent's credal judgments. Hence, by the probability axioms, a proposition counts as seriously possible for the agent just in case it has nonzero probability under the representing measure. This leads to certain difficulties. For example, if the agent is concerned to estimate the height of an object that is sufficiently distant, then the agent might regard a continuum of values as possible – e.g., the height of the object is judged to be between three and four feet. According to the suggested reduction, such a continuum of possible values for the height of the object could not serve as a set of serious possibilities, since it is a mathematical fact that no probability measure can distribute positive probability to each element of such a continuum. The interested reader is urged to consult van Fraassen (1995) and Arlo-Costa (2001) for more sophisticated versions of probabilism.

Following Levi (1980), one may assume that the agent is, at each point in time, committed to full belief in some set of propositions concerning its environment.

Where the agent is not committed to full belief in a given proposition, the negation of that proposition is a serious possibility for the agent. The agent may judge some serious possibilities to be more probable than others. What can be said about these judgments? The received view, following a tradition that goes back to the work of Ramsey (1931), maintains that such credal judgments ought to be representable by a probability measure. This view has been criticized as being too weak and as being too strong. As for being too weak, the simple requirement that such judgments be representable by a probability measure says little about the extent to which these subjective probabilities should approximate objective probabilities, e.g., limiting frequencies in the sense of von Mises (1957) or perhaps even propensities in the sense of Popper (1959). Various principles have been offered in order to require that the subjective probabilities of a rational agent are informed by that agent's knowledge of objective probabilities (Kyburg 1974; Levi 1978; Lewis 1980). As for being too strong, requiring credal judgments to be representable by a probability measure implies, among other things, that such credal judgments are complete. A consequence of such a requirement is that, for any given pair of serious possibilities, the agent either judges one of the possibilities to be more probable than the other or the agent regards the possibilities as being equally probable. Thus, the requirement bars situations in which the agent, because of a lack of information, is unable to supply such a judgment. Such considerations, which to some extent echo earlier, related concerns of Keynes (1921) and Knight (1921), have motivated some – e.g., Kyburg (1968), Levi (1974) and Walley (1990) – to consider indeterminate probabilities, either in the form of interval-valued measures or sets of traditional measures, in representing rational credences.

Belief Change

As already hinted, some probability theorists tend to think that belief, as opposed to knowledge, may be good enough for action, deliberation and decision. Thus beliefs may suffice as they can serve important epistemic purposes while holding the information, expectations and conjectures that agents act on. Beliefs may be used for making creative leaps beyond what is logically implied by available information, evidence or knowledge and are crucial in agent interaction models representing what agents think about moves, strategies, payoffs and beliefs of other agents and what agent rationality amounts to Finally, beliefs and belief revision are prime vehicles for understanding the mechanism of learning by trial-and-error, one of the main motors of scientific inquiry in general.

Initially, an agent has beliefs about the environment with which it interacts. Sometimes these interactions are such that the agent, on pain of irrationality, must revise its beliefs. The classic example is that of a scientific agent who has beliefs about the world that might need to be revised in light of new data. The study of this sort of example has a long history in the philosophy of science, where it is often discussed at a relatively informal level in connection with topics such

as underdetermination. In the context of formal epistemology, the study of belief revision has been generalized to include various sorts of epistemic agents. Questions such as the following suggest the range of theoretical options that are available in connection with such investigations:

How are the potential belief states to be interpreted? One might take the belief states to represent partial beliefs; e.g., the agent has a certain degree of belief in proposition *P*. Alternatively, one might be interested in states of full belief, expectation or plain belief; e.g., the agent fully believes *P*, expects *P*, etc. Further refinements have been considered. For example, one might consider those full beliefs with respect to which the agent manifests some level of awareness; e.g., I am aware of my belief that I am presently writing the words of this sentence. In contrast to a focus on conscious beliefs, one might consider those propositions that the agent is committed to fully believing; e.g., all of those propositions that are deducible from my conscious beliefs.

How are the potential belief states to be represented? The answers to this question depend, at least to some extent, on how the previous question is answered. For example, if partial beliefs are the issue, then probability distributions might be taken as the basis for the representation so that a potential belief state is represented as a probability measure over the possible states of nature. On the other hand, if the problem is the representation of commitments to full belief (expectation, plain belief), then one might specify a suitably formalized language and represent each potential belief state as a theory formulated over the given language so that membership in the theory indicates full belief.

How are revisions interpreted? If credal states are the concern, then modifications of the credal state might be understood in terms of something like conditionalization. The interested reader is urged to consult (Halpern 2003) for a survey of various proposals concerning the representation and modification of credal states. What about revising or contracting states of full belief? When an instance of belief contraction concerning full beliefs is the result of the agent selecting from a set of (full) belief states that the agent recognizes as potential alternatives, then such an instance may be regarded as the resolution of a decision problem. Isaac Levi has developed a decision-theoretic approach to belief change; important discussions of Levi's approach include (1980), which considers belief change in the context Levi's general approach to epistemology, and Arlo-Costa and Levi (2006), Arlo-Costa and Liu (2011) which gives greater emphasis to the formal details concerning Levi's approach. Different connections between choice and belief revision are emphasized in Rott (1993). Rott demonstrates an important correspondence between the "AGM" account of belief revision offered in Alchourron et al. (1985) and the economists' study of rational choice functions. Finally, it is worth noting that where both partial and full beliefs are considered, there may be significant dependencies between the modification of these two sorts of belief states. For example, if the credal judgments of rational agents are a function of their judgments of full belief, as some philosophers assume, then changes to the latter may result in changes to the former.

There are other alternative interpretations of doxastic change. Spohn have considered change from the point of view of entire epistemic states rather than mere

beliefs in terms of ranking functions and plausibility representations (Spohn 2009) while Hansson have considered change from the point of belief bases as finite sets of sentences that likewise are possible axiomatic bases for a given theory (Hansson 1993).

Decision Theory

An agent interacts with its environment through the choices it makes. Choice presupposes alternatives, and a theory of rational choice should, at least, distinguish some of the available alternatives as admissible. As an example, consider those accounts of rational choice that are built on the concept of preference. One such account assumes that the agent has complete and transitive preferences over the set of available alternatives. Those alternatives that are optimal with respect to the given preference ranking are taken as admissible. This abstract preference-based account says nothing about the way in which preferences are informed by the agent's beliefs about its environment. Subjective expected utility theory [SEU], which is at the center of modern-day decision theory, provides significantly more detail than the abstract theory of preference optimization. SEU assumes that alternatives are acts, which, following Savage's classic formulation of SEU in Savage (1972), are functions from states to consequences. Drawing upon the earlier work of Ramsey (1931) on subjective probability and the work of von Neumann and Morgenstern (1947) on utility, Savage provides conditions on the agent's preferences over acts that guarantee the existence of a probability measure p and a utility function u such that the agent's preferences can be regarded as if they were the result of maximizing utility u with respect to probability p. According to the intended interpretation, the probability measure p represents the agent's degrees of belief concerning the possible states and the utility function u represents the extent to which the agent values the possible consequences.

The assumptions of SEU may be questioned in various ways. We focus on two ways that have generated significant interest among philosophers. First, why should it be that the rational agent's degrees of belief can be represented by a probability distribution p? As already noted, it is not clear why such an assumption should obtain in cases where the agent has very little information concerning the possible states. Second, in SEU it is assumed that the agent's subjective probability concerning the states is independent of the act that is chosen. Some question this assumption and offer examples in which a modification of SEU that provides for such dependencies, through the use of conditional probabilities, is supposed to give an irrational recommendation. The first line of questioning has led some – e.g., Ellsberg (1961), Levi (1974, 1977), and Gardenfors and Sahlin (1982) – to use indeterminate probabilities in their normative accounts of decision making under uncertainty. The second line of questioning has led some – e.g., Gibbard and Harper (1978), Lewis (1981), and Joyce (1999) – to investigate causal decision theory.

Logics of Knowledge and Belief

What is now known as epistemic logic started with the study of proper axiomatizations for knowledge, belief, certainty and other epistemic attitudes. Hintikka inaugurated the field with his seminal book Hintikka (1962) which focuses on axiomatizing knowledge and belief in mainly mono-agent systems. Agents are syntactically represented as indices on epistemic operators in a formal logical language. From the semantic perspective, to be an agent is to be an index on an accessibility relation between possible worlds representing the epistemic alternatives over which the agent has to succeed in order to know some proposition (interesting alternative semantics to Kripke semantics have been developed by Arlo-Costa and Pacuit 2006, Baltag and Moss 2004 and others). Like many other philosophical logics in their infancy, interesting axiomatizations governing the logics of knowledge and belief took center stage in the beginning together with nailing down important logical properties for these new logics. The field was living a somewhat isolated life remote from the general concerns of mainstream epistemology. Hintikka himself (and a few others like Lenzen 1978) was a notable exception and insisted on telling a better story, not about what agents are in the logical language, but about what they do and the meaning of epistemic axioms for epistemology (Stalnaker 2006). Accordingly, Hintikka took axioms of epistemic logic to describe a certain kind of strong rationality much in sync with the autoepistemological tradition of G.E. Moore and especially Norman Malcolm. Axioms of epistemic logic are really prescriptions of rationality in mono-agent systems. Epistemic logic has since been used address a number of important philosophical problems including for instance the Fitch Paradox (Brogaard and Salerno 2009), the problem of logical omniscience (Duc 1997; Parikh 2005), and various conceptual characterizations of knowledge and other epistemic attitudes (Kraus and Lehmann 1988).[1]

But rationality considerations are not only central to the singular agent acting in some environment, call it nature, but likewise, and perhaps especially, central to agents when in presence of other agents and interacting with these. Thus mono-agent systems had to be extended to multi-modal systems in order to get both agency and interaction off the epistemological ground for real. A sea-change took place in epistemic logic in the late 1980s and the beginning of the 1990s especially due to the work of Joseph Halpern and his collaborators (Fagin et al. 1995) and others (Meyer and Hoek 1995). Multiple agents were introduced into the logical language which, along with multiple epistemic accessibility relations on the semantic level, gave rise to a precise and adequate representation of the flow of information through an agent system, together with the nature of various protocols governing such systems. In this setting, possible worlds are to be understood as the states of the system taken as a whole, or sometimes the possible histories or consecutive runs of the system as a

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[1]For solid overviews refer to De Bruin (2008) and Gochet and Gribomont (2006).

whole, that are compatible with the state transition directives which rule the system. Stalnaker has recently summarized the consequences of this sea-change precisely:

> The general lesson I drew from this work was that it was useful for epistemology to think of communities of knowers, exchanging information and interacting with the world, as (analogous to) distributed computer systems. (Hendricks and Roy 2010: 78)

Agent systems can now be thought of as encompassing everything from a group of robots on an assembly line to a group of poker players in Texas Hold 'Em. In turn, there is much more to what agents are nowadays, but also much more to what they do dynamically (as opposed to statically in terms of, say, (van Ditmarsch et al. 2008) epistemic axioms describing the rationality of single agents). Dynamic epistemic logic is a rich blend of studies ranging multi-agent axiomatizations of knowledge, belief, common knowledge and belief (Barwise 1988) certainty, uncertainty, doubt, ignorance and a host of other epistemic attitudes; models of the interplay between knowledge and games (Benthem 2001, 2007), knowledge and justification in mainstream epistemology (Artemov and Nogina 2005), social software (Parikh 2002), knowledge and public announcement of information (Baltag et al. 2002), knowledge intertwined with preferences, actions and decisions (Liu 2011); knowledge acquisition in light of formal learning theory, logical reliability, methods of scientific inquiry and computability studies (Gierasimczuk 2009; Hendricks 2001; Kelly 1996, 2004), belief revision (Baltag and Smets 2008), models of agent interaction in multi-agent systems; combined multi-agent and multi-modal systems in which for instance the development of knowledge over time may be scrutinized (Kraus and Lehmann 1988), relations between knowledge and deontic commitments investigated, divisions of cognitive labor modeled and so forth (for epistemic logic paired up with mainstream epistemological concerns, refer to Williamson (2006), Hendricks (2006) and Hendricks and Pritchard (2007)).

Interactive Epistemology

Theoretical economics is to a significant extent about understanding, anticipating and modeling phenomena like trading, stock speculation, real-estate dealing, hostile company take-overs, shareholding, convention and so forth. Obviously, agency and interaction play a paramount role here and seen from this perspective economics is about multi-individual and collective action balancing information and utility.

Independently, but informed by the developments in epistemic logic, economists have used game theory to scrutinize an extensive spread of the mentioned phenomena. By way of example, in 1976 the later Nobel Prize Laureate Robert Aumann published his famous Agreement Theorem in "Agreeing to Disagree" in which he describes conditions under which two "like minded" agents or players cannot "agree to disagree" in the sense that if the two players' posteriors of some event are common knowledge then they must coincide. In other words, in order to make trade

possible, agents have to agree to disagree (Aumann 1976). That is agency in terms of players, interaction in terms of games.

On the way to this result Aumann made a host of assumptions about the nature knowledge much in tune with what is to be found in epistemic logic like the axiomatic strength of knowledge in order to infer the backwards induction equilibrium and assumptions about what is common knowledge among the players (Halpern 2001). In 1999, Aumann coined a term for these kinds of study in theoretical economics: "Interactive epistemology" (Aumann 1999). It denotes an epistemic program studying shared knowledge and belief given more than one agent or player in an environment and has, as already suggested, strong ties to game theoretic reasoning and questions of common knowledge and belief, backward induction, various forms of game equilibria and strategies in games, (im)perfect information games, (bounded) rationality etc (Aumann and Brandenburger 1995; Stalnaker 1996, 2006).

Given its inauguration with Aumann, the program was in the beginning dominated by scholars drawn from theoretical economics and computer science rather than philosophy and logic, but recently philosophers and logicians have begun to pay close attention to what is going on in this striving program of formal epistemology. And for good reason too; social epistemology focuses on knowledge acquisition and justification in groups or institutions (Goldman 1999) and the extent to which exactly institutions may be viewed as genuine agents (List and Pettit 2011) while the interactive epistemological approach to agency and interaction also have close shaves with the major new focal points in dynamic epistemic logic (Benthem 2011) and much of the technical machinery is a common toolbox for both paradigms (Brandenburger 2007).

Formal Epistemology

Formal epistemology is the study of crucial concepts in mainstream epistemology including knowledge, belief (-change), certainty, rationality, reasoning, decision, justification, learning, agent interaction and information processing using formal tools from three streams; probability, logic and computability. In particular, the tools may come from tool boxes like modal logic, probability calculus, game theory, decision theory, formal learning theory, computability theory and distributed computing. Practitioners of formal epistemology include philosophers, computer scientists, social scientists, cognitive psychologists, theoretical economists, mathematicians, and theoretical linguists but also scholars from the empirical sciences like cognitive science, engineering and biology are onboard. This mixed bag of practitioners is surely a witness to the thoroughly interdisciplinary nature of formal epistemology and its wide range of applications in natural science, social science, humanities and the technical sciences.

Formal epistemology is right in the middle; between mainstream epistemology's fairly abstract theories on the one hand and the more concrete cognitive sciences

devoted to the empirical reality of agency and interaction on the other. In formal epistemology we are walking the fine line between theory and reality. This is as it should be: The hallmark of a progressive research program.

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