13 Reflection and Transmission of Plane Waves

God runs electromagnetics on Monday, Wednesday and Friday by the wave theory and the devil runs it on Tuesday, Thursday and Saturday by the Quantum theory.

Sir William Bragg (1862–1942),
physicist, Nobel laureate, 1915, on Electromagnetics

13.1 Introduction

The propagation of waves in free space and in materials was discussed at some length in **Chapter 12**. In this chapter, we discuss properties of waves as they propagate through different materials and changes in their amplitudes and directions as they propagate through the interfaces between materials. This aspect of the propagation of waves is fundamental and many of the properties of waves are defined by materials and their interfaces. As an example, waves are reflected from conducting and dielectric surfaces giving rise to so-called standing waves. The various properties depend on the materials involved, the direction of propagation, and the polarization of the waves. To keep the discussion simple and within the context of plane waves, we will look at a number of simple interface conditions. These include perpendicular and oblique incidence on conducting and dielectric interfaces, conditions often encountered in applications.

The results we obtain here are useful in a variety of applications, ranging from radar operation to fiber optics. For example, reflection from interfaces can be used to detect water levels in underground aquifers as well as oil deposits. The same principle can be used to measure and monitor the thickness of materials on a production line or flaws in plastics. The design of radomes for radar and communication equipment requires that no reflections at interfaces exist, whereas radar evading (stealth) aircraft absorb all incoming radar energy or reflect it in directions other than the transmitting antenna. All these can be accomplished by the proper choice of materials and conditions at the interfaces between materials.

The basic principle involved in describing the behavior of a wave at the interface between materials is to write the waves on both sides of the interface and to match the components of the electric and magnetic fields at the interface. In general, this means applying the interface conditions of the fields, from which the fields on both sides of the interface are found. The general conditions are shown in **Figure 13.1**. The wave in material (1) propagates at an angle to the normal to the interface. This angle is called the ***incidence angle***, θ_i. The tangential components of the electric field intensity are continuous at the interface, as we have seen in **Chapter 11**. The normal components are discontinuous. In general, we will assume that there are no charge densities or current densities at the interface (except for conducting interfaces). The wave is partly transmitted into material (2) and partly reflected at an angle θ_r called the ***reflection angle***. We will also show that the incidence and reflection angle are equal. The ***transmission angle*** (or ***refraction angle***) θ_t is different than the incidence angle θ_i, as one would expect from two different materials. Note however that the discussion of waves at interfaces goes beyond simple interface conditions. Although interface conditions as discussed in **Chapter 11** must be satisfied, the propagation properties of the wave must also be taken into account. This means that we must consider such properties as polarization of the wave and its speed of propagation.

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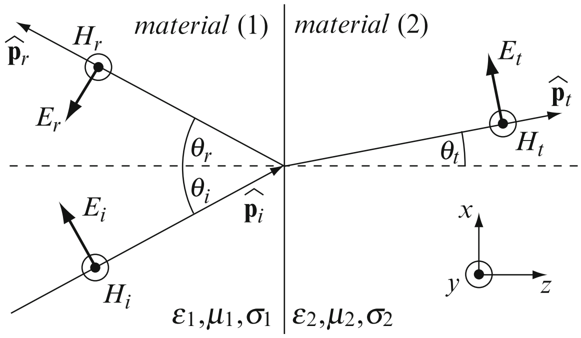


Figure 13.1 The general relationships between fields at an interface between two materials. Directions of the reflected fields are assumed

As we discuss the behavior of waves at interfaces, it is useful to recall the behavior of light waves. We expect similar behavior, including reflection, transmission, and refraction of waves at the interface. For generality, we start with a general, lossy dielectric interface and then proceed to discuss lossless and low-loss dielectrics and conductors. Normal incidence of the wave on the interface is considered first, followed by oblique incidence with polarized waves at conducting and dielectric interfaces.

13.2 Reflection and Transmission at a General Dielectric Interface: Normal Incidence

Figure 13.2 shows an incident wave $\mathbf{E}_i$ propagating in a medium with permittivity ε_1, conductivity σ_1, and permeability μ_1. The wave encounters the interface between material (1) and material (2), which is a general lossy dielectric with permittivity ε_2, conductivity σ_2, and permeability μ_2. Based on the definition of angle of incidence in the previous section, the incident wave hits the interface at a zero-degree angle (perpendicular to the interface) and is reflected at the same angle. The transmitted wave also propagates perpendicular to the interface.

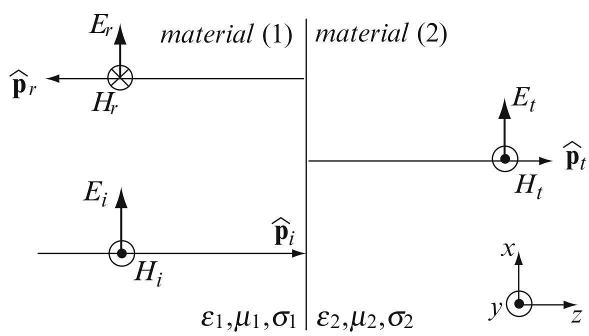


Figure 13.2 Reflection and transmission at a general interface: normal incidence. The directions of the reflected field vectors are assumed

To simplify the discussion, we assume a wave propagating in the positive z direction, as in **Eq. (12.89)**. The incident electric field intensity is

$$\mathbf{E}_i = \hat{\mathbf{x}} E_{i1} e^{-\gamma_1 z} \quad [\mathrm{V/m}] \tag{13.1}$$

where E_{i1} is the amplitude of the incident wave and the propagation constant is

$$\gamma_1 = \alpha_1 + j\beta_1 \tag{13.2}$$

The wave reflected from the interface propagates backward in material (1) and is assumed to be

$$\mathbf{E}_r = \hat{\mathbf{x}} E_{r1} e^{+\gamma_1 z} \quad [\mathrm{V/m}] \tag{13.3}$$

The backward propagation is indicated by the positive sign in the exponent (the wave propagates in the negative z direction). The amplitude E_{r1} is, in general, different than E_{i1} and may be zero (no reflection at the interface). The reflected electric field intensity is assumed, arbitrarily, to be in the same direction as the electric field intensity of the incident wave. The actual direction will be found by imposing the interface conditions at the interface. The total wave in material (1) has two components: one propagating in the positive z direction and one in the negative z direction:

$$\mathbf{E}_1 = \hat{\mathbf{x}} E_{i1} e^{-\gamma_1 z} + \hat{\mathbf{x}} E_{r1} e^{+\gamma_1 z} \quad [\mathrm{V/m}] \tag{13.4}$$

The magnetic field intensity also has two components and can be calculated from **Eq. (12.40)** or directly from Faraday's law as

$$\mathbf{H}_1 = \hat{\mathbf{y}} \frac{E_{i1}}{\eta_1} e^{-\gamma_1 z} - \hat{\mathbf{y}} \frac{E_{r1}}{\eta_1} e^{+\gamma_1 z} \quad [\mathrm{V/m}] \tag{13.5}$$

Note that the backward propagating magnetic field intensity is negative if the backward electric field intensity is positive. This sign can be found directly from Maxwell's first equation (Faraday's law), but, here, we simply point out that the sign must be negative since the propagation of power due to this term is in the negative z direction (direction of the Poynting vector). In practice, we may find that E_{r1} itself may be negative, but, regardless of the sign of E_{r1}, H_{r1} must be such that the Poynting vector $\mathbf{E}_r \times \mathbf{H}_r$ points away from the interface. Thus, from the Poynting vector, we have

$$\boldsymbol{\mathcal{P}}_1^+ = \left(\hat{\mathbf{x}} E_{i1} e^{-\gamma_1 z}\right) \times \left(\hat{\mathbf{y}} \frac{E_{i1}}{\eta_1} e^{-\gamma_1 z}\right) = \hat{\mathbf{x}} \times \hat{\mathbf{y}} \frac{E_{i1}^2}{\eta_1} e^{-2\gamma_1 z} = \hat{\mathbf{z}} \frac{E_{i1}^2}{\eta_1} e^{-2\gamma_1 z} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \tag{13.6}$$

which propagates in the positive z direction. Similarly, the Poynting vector for the backward propagating wave is

$$\boldsymbol{\mathcal{P}}_1^- = \left(\hat{\mathbf{x}} E_{r1} e^{\gamma_1 z}\right) \times \left(-\hat{\mathbf{y}} \frac{E_{r1}}{\eta_1} e^{\gamma_1 z}\right) = -\hat{\mathbf{x}} \times \hat{\mathbf{y}} \frac{E_{r1}^2}{\eta_1} e^{2\gamma_1 z} = -\hat{\mathbf{z}} \frac{E_{r1}^2}{\eta_1} e^{2\gamma_1 z} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \tag{13.7}$$

In these relations, the intrinsic impedance η_1 and the propagation constant γ_1 are completely general and given as **[**see **Eq. (12.102)** and **Eq. (12.85)]**

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}} \quad [\Omega], \quad \gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)} \tag{13.8}$$

To define the transmitted waves, we may assume that **E** and **H** in material (2) are of the same form as in material (1) except that in this case, the backward propagating wave in material (2) cannot exist. This is because material (2) extends to infinity and there is no mechanism for a reflected wave in material (2) to exist. Thus, we get

$$\mathbf{E}_t = \mathbf{E}_2 = \hat{\mathbf{x}} E_2 e^{-\gamma_2 z} \quad [\mathrm{V/m}] \tag{13.9}$$

$$\mathbf{H}_t = \mathbf{H}_2 = \hat{\mathbf{y}} \frac{E_2}{\eta_2} e^{-\gamma_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.10}$$

and, as required, this wave propagates in the positive z direction. The intrinsic impedance and the propagation constant in material (2) are

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} \quad [\Omega], \quad \gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\varepsilon_2)} \tag{13.11}$$

Now that we wrote the fields in each material, we can define a reflection coefficient as the ratio between the amplitudes of the reflected and incident waves:

$$\boxed{\Gamma = \frac{E_{r1}}{E_{i1}} \quad [\text{dimensionless}]} \tag{13.12}$$

The reflection coefficient is a dimensionless quantity which gives the fraction of the incident wave amplitude reflected back from the interface. It can vary from zero (no reflection) to 1 (total reflection) and can be either positive or negative. Since both amplitudes are, in general, complex numbers, the reflection coefficient may also be a complex number.

Similarly, a transmission coefficient is defined as the ratio between the amplitudes of the transmitted and incident waves:

$$\boxed{T = \frac{E_t}{E_{i1}} \quad [\text{dimensionless}]} \tag{13.13}$$

The transmission coefficient is also a dimensionless quantity and gives the fraction of the incident wave amplitude transmitted across the interface. The transmission coefficient is, in general, complex and can vary in magnitude between zero and 2, as we shall see shortly.

Using the definition of the reflection coefficient, the reflected wave can be written in terms of the incident wave as

$$\mathbf{E}_r = \hat{\mathbf{x}} E_{r1} e^{+\gamma_1 z} = \hat{\mathbf{x}} \Gamma E_{i1} e^{+\gamma_1 z} \quad [\text{V/m}] \tag{13.14}$$

Using the transmission coefficient T, we can write

$$\mathbf{E}_2 = \hat{\mathbf{x}} E_2 e^{-\gamma_2 z} = \hat{\mathbf{x}} T E_{i1} e^{-\gamma_2 z} \quad [\text{V/m}] \tag{13.15}$$

Similarly, the incident, reflected, and transmitted magnetic field intensities are

$$\mathbf{H}_i = \hat{\mathbf{y}} \frac{E_{i1}}{\eta_1} e^{-\gamma_1 z} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.16}$$

$$\mathbf{H}_r = -\hat{\mathbf{y}} \Gamma \frac{E_{i1}}{\eta_1} e^{+\gamma_1 z} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.17}$$

$$\mathbf{H}_2 = \hat{\mathbf{y}} T \frac{E_{i1}}{\eta_2} e^{-\gamma_2 z} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.18}$$

The transmission and reflection coefficients can now be evaluated from the relations at the interface. To do so, we place the interface at $z = 0$, write the total electric and magnetic fields on each side of the interface, and equate the tangential components (which are continuous across the interface):

$$\mathbf{E}_i + \mathbf{E}_r = \mathbf{E}_2 \quad \to \quad E_{i1} + \Gamma E_{i1} = T E_{i1} \quad [\text{V/m}] \tag{13.19}$$

Similarly, from the continuity of the tangential components of the magnetic field intensity,

$$\mathbf{H}_i + \mathbf{H}_r = \mathbf{H}_2 \quad \to \quad \frac{E_{i1}}{\eta_1} - \frac{\Gamma E_{i1}}{\eta_1} = \frac{T E_{i1}}{\eta_2} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.20}$$

From these, we can write

$$\boxed{1 + \Gamma = T} \tag{13.21}$$

$$\frac{1}{\eta_1} - \frac{\Gamma}{\eta_1} = \frac{T}{\eta_2} \tag{13.22}$$

or, solving for Γ and T

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \quad [\text{dimensionless}]} \tag{13.23}$$

and

$$\boxed{T = \frac{2\eta_2}{\eta_1 + \eta_2} \quad [\text{dimensionless}]} \tag{13.24}$$

Γ can be negative (depending on the relative values of the intrinsic impedances), but T is always positive. Both coefficients are ratios of impedances and, therefore, are dimensionless.

The reflection and transmission coefficients are important properties in wave propagation. We will meet them in many diverse situations. It is, therefore, important to familiarize ourselves with their general properties. It is often possible to draw simple conclusions on behavior of electromagnetic waves from calculation or estimation of these coefficients. As a simple example, if you were to try to design a stealth aircraft, the immediate conclusion from **Eqs. (13.23)** and **(13.24)** is that the reflected wave should be as small as possible since the lower the reflection, the smaller the amplitude of the wave returning from the aircraft. Thus, the requirement for an aircraft "invisible to radar" is that the reflection coefficient is zero, which, in turn, means that $\eta_2 = \eta_1$. This conclusion was drawn without actually calculating the coefficient or, for that matter, without even commenting on the practicability of the solution. Similarly, when using a microwave oven to heat a material, the lower the reflection coefficient, the more energy couples into the material and the more efficient the heating process. Another way to look at it is in the generator-load context. A perfectly matched load causes no reflection of energy into the generator. All energy on the line is transferred from the line to the load (zero reflection coefficient). An unmatched load means that part of the energy is transferred into the load and part of it is reflected back into the generator. On the other hand, there are situations in which we may wish to maximize reflections. One example is in the design of reflector antennas (most "dish" antennas are parabolic reflectors; the antenna itself is a small "feed" at the focal point of the parabola). In this case, the transmission coefficient should be as close as possible to zero so that power does not penetrate into the reflector and the reflection coefficient as close as possible to -1 **[see Eq. (13.21)]**.

The following properties of the reflection and transmission coefficients will be useful in this and the future sections:

(1) Both coefficients are in general complex numbers.
(2) If the conductivities of both materials are zero, both Γ and T are real numbers. This happens whenever the materials on the two sides of the interface are perfect dielectrics.
(3) If material (2) is a perfect conductor, its intrinsic impedance is zero (i.e., $\sigma_2 \to \infty$ and, therefore, $\eta_2 \to 0$). This means that the reflection coefficient is -1 and the transmission coefficient is zero. The wave does not penetrate into a perfect conductor and the reflected wave is in the opposite direction in space; that is, the reflected wave is 180° out of phase in space with the incident wave.
(4) Both constants are frequency dependent in most cases, as can be seen from **Eqs. (13.8)** and **(13.11)**. In this sense, the constants are not really constant. In other situations, including those of perfect dielectrics and perfect conductors, the constants may be regarded as true constants since the intrinsic impedance of perfect dielectrics is real and independent of frequency, and for perfect conductors it is zero.

The main utility of the reflection and transmission coefficients at this point is to describe the transmitted and reflected waves in terms of the incident wave. This approach is sensible since the incident wave is normally known, whereas the reflection and transmission coefficients are only dependent on materials and possibly on frequency.

Returning now to the fields in material (1), the electric field intensity can be expressed as the sum of the incident and reflected waves as in **Eq. (13.4)**:

$$\mathbf{E}_1(z) = \mathbf{E}_i(z) + \mathbf{E}_r(z) = \hat{\mathbf{x}} E_{i1}\left(e^{-\gamma_1 z} + \Gamma e^{+\gamma_1 z}\right) \quad [\text{V/m}] \tag{13.25}$$

By adding and subtracting the term $\Gamma e^{-\gamma_1 z}$ in the brackets on the right-hand side, we can write

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left((1+\Gamma)e^{-\gamma_1 z} + \Gamma\left(e^{+\gamma_1 z} - e^{-\gamma_1 z}\right)\right) \quad [\text{V/m}] \tag{13.26}$$

or using the identity $e^{+\gamma_1 z} - e^{-\gamma_1 z} = 2\sinh(\gamma_1 z) = -j2\sin(j\gamma_1 z)$ and the fact that $1 + \Gamma = T$,

$$\boxed{\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left(Te^{-\gamma_1 z} - \Gamma j2\sin(j\gamma_1 z)\right) \quad [\mathrm{V/m}]} \tag{13.27}$$

Similarly, the magnetic field intensity in material (1) is the sum of the incident and reflected fields:

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\,\frac{E_{i1}}{\eta_1}\left(e^{-\gamma_1 z} - \Gamma e^{+\gamma_1 z}\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.28}$$

Again adding and subtracting $\Gamma e^{-\gamma_1 z}$ in the brackets, we get

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\,\frac{E_{i1}}{\eta_1}\left((1+\Gamma)e^{-\gamma_1 z} - \Gamma\left(e^{+\gamma_1 z} + e^{-\gamma_1 z}\right)\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.29}$$

With $T = 1 + \Gamma$ and $e^{+\gamma_1 z} + e^{-\gamma_1 z} = 2\cosh(\gamma_1 z) = 2\cos(j\gamma_1 z)$, this gives

$$\boxed{\mathbf{H}_1(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_1}\left(Te^{-\gamma_1 z} - \Gamma 2\cos(j\gamma_1 z)\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]} \tag{13.30}$$

The fields in material (2) are

$$\boxed{\mathbf{E}_2(z) = \hat{\mathbf{x}} TE_{i1}e^{-\gamma_2 z} \quad [\mathrm{V/m}]} \tag{13.31}$$

and

$$\boxed{\mathbf{H}_2(z) = \hat{\mathrm{y}}\, T\frac{E_{i1}}{\eta_2}e^{-\gamma_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]} \tag{13.32}$$

Equations (13.27) and **(13.30)** are useful because they indicate that the sum of the incident and reflected waves consists of a wave component propagating in the positive z direction and a non-propagating wave component. The latter [second term in **Eq.** (13.27) or **Eq.** (13.30)] is called a ***standing wave***. We also observe that if the transmission coefficient is zero, only a standing wave exists, whereas if the reflection coefficient is zero, there are no standing waves. Standing waves will be discussed separately in conjunction with reflection from conducting surfaces, but we point out here that standing waves exist any time there is reflection of a wave, and it is caused by interference between the forward and backward propagating waves.

The above discussion is quite general and assumes nothing about material properties. In practice, there are a number of combinations of materials which are important. For example, the interface between free space (air) and conductors or dielectrics are commonly encountered. Some of the more important material interfaces are described in the following sections.

Example 13.1 Application: Optical Fiber Connectors Two optical fibers are connected through a connector to form an interface as shown in **Figure** 13.3. In optical fibers, the attenuation is indicated in dB/km. Fiber (1) is rated as 1 dB/km (a good fiber) and the second as 10 dB/km. The source of light is in free space (not shown), at a wavelength of 700 nm (a red laser or light emitting diode). Assume both fibers are low-loss dielectrics (which in practice they are; conductivity of glass is about 10^{-12} S/m) and that propagation is from fiber (1) into fiber (2). Calculate:

(a) The reflection and transmission coefficients at the interface.
(b) The amplitude of the electric and magnetic field intensities at $d = 10$ km from the interface, in material (2), assuming the amplitude of the incident electric field intensity in material (1) is known at the interface as E_{i1}.
(c) Show that power is conserved across the interface, that is, that the transmitted time-averaged power density must equal the incident time-averaged power density minus the reflected time-averaged power density.

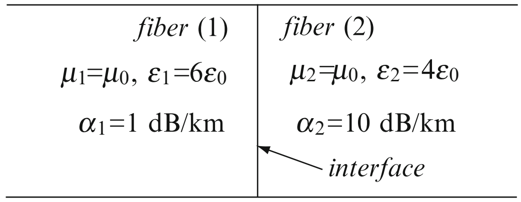


Figure 13.3 Conditions at the interface between two optical fibers

Solution: From the given attenuation, we calculate the attenuation constant, and from **Eqs. (13.8)** and **(13.11)**, we calculate the intrinsic impedance of fibers (1) and (2). Then, the reflection and transmission coefficients are calculated from **Eqs. (13.23)** and **(13.24)**. The electric and magnetic field intensities in material (2) are given in **Eqs. (13.31)** and **(13.32)**.

The attenuation and phase constants for the two fibers are calculated first. Since the attenuation constant is given in dB/km and 1 neper/meter = 8.69 dB/m (see **Section 12.7.1**), the attenuation constants are

$$\alpha_1 = \frac{1}{1000 \times 8.69} = 1.15 \times 10^{-4}, \quad \alpha_2 = \frac{10}{1000 \times 8.69} = 1.15 \times 10^{-3} \quad \left[\frac{\text{Np}}{\text{m}}\right]$$

The phase constant is calculated from the relation $\beta = 2\pi/\lambda$ (low-loss dielectric), where λ is the wavelength in each fiber. The wavelength is given in free space as 700 nm. In the fibers, the speed of propagation is reduced by a factor of $\sqrt{\varepsilon_r}$ and the wavelength is also reduced by this factor since $\lambda = v_p/f$. Thus, the phase constants in the two fibers are

$$\beta_1 = \frac{2\pi \times \sqrt{6}}{700 \times 10^{-9}} = 21.986 \times 10^6, \quad \beta_2 = \frac{2\pi \times \sqrt{4}}{700 \times 10^{-9}} = 17.952 \times 10^6 \quad \left[\frac{\text{rad}}{\text{m}}\right]$$

To calculate the intrinsic impedance, we write [see **Eq. (13.8)**]:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\alpha + j\beta} \approx \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} \quad [\Omega]$$

where the fact that the phase constant (β_1 or β_2) is much larger numerically than the attenuation constant (α_1 or α_2) was used. The intrinsic impedances in the two fibers are

$$\eta_1 = \frac{\omega\mu_1}{\beta_1} = \sqrt{\frac{\mu_0}{6\varepsilon_0}} = \frac{377}{\sqrt{6}} = 153.91, \quad \eta_2 = \frac{\omega\mu_2}{\beta_2} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{377}{2} = 188.5 \quad [\Omega]$$

The transmission and reflection coefficients are calculated in terms of the intrinsic impedances. Using $\mu_1 = \mu_2 = \mu_0$, we get from **Eqs. (13.23)** and **(13.24)**

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = \frac{\omega\mu_0/\beta_2 - \omega\mu_0/\beta_1}{\omega\mu_0/\beta_1 + \omega\mu_0/\beta_2} = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} = \frac{21.986 \times 10^6 - 17.952 \times 10^6}{21.986 \times 10^6 + 17.952 \times 10^6} = 0.101$$

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\beta_1}{\beta_2 + \beta_1} = \frac{43.972 \times 10^6}{21.986 \times 10^6 + 17.952 \times 10^6} = 1.101$$

Note that since the reflection coefficient is positive, the transmission coefficient must be larger than 1, as can be seen from **Eq. (13.21)**.

The incident electric field intensity in fiber (1) is known. We will take this as E_{i1}. First, we calculate the electric field intensity across the interface and then, using the attenuation in material (2), calculate the amplitude of the electric field intensity at a distance of 10 km. The amplitude of the electric field intensity in material (2) is

$$E_2(0) = TE_{i1} = 1.101E_{i1} \quad [\text{V/m}]$$

where (0) indicates that this is at the interface. At $d = 10$ km, the amplitude is

$$E_2(d) = E_2(0)e^{-\alpha_2 d} = 1.101E_{i1}e^{-1.15\times10^{-3}\times10^4} = 1.115\times10^{-5}E_{i1} \quad [\text{V/m}]$$

The electric field intensity has been reduced by about a factor of 10^5 in 10 km. The magnetic field intensity at the same location is

$$H_2 = \frac{E_2}{\eta_2} = \frac{2E_2}{\eta_0} = \frac{2\times1.115\times10^{-5}}{377}E_{i1} = 5.915\times10^{-8}E_{i1} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The time-averaged power across the interface may be written from the Poynting theorem (see **Example 12.9**):

$$\frac{E_i^2}{2\eta_1} - \frac{E_r^2}{2\eta_1} = \frac{E_t^2}{2\eta_2} \quad \rightarrow \quad \frac{E_i^2}{2\eta_1} - \frac{(\Gamma E_i)^2}{2\eta_1} = \frac{(TE_i)^2}{2\eta_2}$$

or

$$\frac{1}{\eta_1} - \frac{\Gamma^2}{\eta_1} = \frac{T^2}{\eta_2} \quad \rightarrow \quad \eta_2 - \eta_2\Gamma^2 = \eta_1 T^2$$

Using the reflection and transmission coefficients and the intrinsic impedances in **(b)** we get

$$188.5 - 188.5\times0.101^2 = 153.91\times1.101^2 \quad \rightarrow \quad 186.57 = 186.57$$

The equality shows that power is conserved across the interface.

Exercise 13.1 In **Example 13.1**, assume the propagation is from material (2) into material (1) and calculate:

(a) The reflection and transmission coefficients at the interface.
(b) The amplitude of the electric and magnetic field intensities at a distance 1 km from the interface in material (1) assuming the amplitude of the incident electric field intensity in material (2) at the interface equals E_2. Use the same properties and assumptions as in **Example 13.1**.

Answer **(a)** $\Gamma = -0.101$, $T = 0.899$. **(b)** $E_1(1000\text{ m}) = 0.8E_2$ [V/m], $H_1(1000\text{ m}) = 5.21\times10^{-3}E_2$ [A/m].

Exercise 13.2 In **Example 13.1**, calculate the exact complex reflection and transmission coefficients.

Answer $\Gamma = 0.101 + j2.9114\cdot10^{-11}$, $T = 1.101 + j2.9114\cdot10^{-11}$.

13.2.1 Reflection and Transmission at an Air-Lossy Dielectric Interface: Normal Incidence

In many practical applications, material (1) is free space; that is, an incident wave propagates in free space and encounters a lossy dielectric. A situation of this type is in a wave transmitted in free space (say from a radar or communication antenna) and encountering a concrete wall. The difference with respect to the previous section is that η_1 becomes η_0 and in the expressions for $\mathbf{E}_i$ and $\mathbf{E}_r$, γ_1 is replaced by $j\beta_0$. This situation is shown in **Figure 13.2** for $\sigma_1 = 0$, $\varepsilon_1 = \varepsilon_0$, and $\mu_1 = \mu_0$. Thus, if material (1) is free space and material (2) is a lossy dielectric, the reflection and transmission coefficients are

$$\boxed{\Gamma = \frac{\eta_2 - \eta_0}{\eta_0 + \eta_2}, \quad T = \frac{2\eta_2}{\eta_0 + \eta_2}} \tag{13.33}$$

The electric and magnetic fields are [from **Eqs. (13.27), (13.30), (13.31),** and **(13.32)**]

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left(Te^{-j\beta_0 z} + \Gamma j2\sin(\beta_0 z)\right) \quad [\mathrm{V/m}] \tag{13.34}$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_0}\left(Te^{-j\beta_0 z} - \Gamma 2\cos(\beta_0 z)\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.35}$$

$$\mathbf{E}_2(z) = \hat{\mathbf{x}} E_{i1} Te^{-j\gamma_2 z} \quad [\mathrm{V/m}] \tag{13.36}$$

$$\mathbf{H}_2(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_2} Te^{-j\gamma_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.37}$$

The phase constant and intrinsic impedance in material (1) are

$$\beta_0 = \omega\sqrt{\mu_0\varepsilon_0} \quad \left[\frac{\mathrm{rad}}{\mathrm{m}}\right], \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad [\Omega] \tag{13.38}$$

and γ_2 and η_2 remain as in **Eq. (13.11)**. Note that even though the expressions for $\mathbf{E}_2$ and $\mathbf{H}_2$ are the same as in **Eqs. (13.31)** and **(13.32)**, the transmission coefficient and the propagation constant are different, and, therefore, the fields in material (2) are also different.

These expressions can be easily adapted to any other lossless dielectric in place of material (1). To do so, the permittivity and permeability of free space in **Eq. (13.38)** are replaced by those of the dielectric.

Example 13.2 Application: Radar Sensing of the Environment In an attempt to map the thickness of the polar ice caps, a special airplane is equipped with a downward-looking radar operating at 10 GHz. The idea is to fly over at a known height and measure the time it takes for narrow pulses to reach the bottom of the ice and return to the receiver. From the knowledge of speed of propagation in air and in ice, it is possible to calculate the thickness of the ice. The properties of ice at 10 GHz are $\mu = \mu_0$ [H/m], $\varepsilon = 3.5\varepsilon_0$ [F/m], $\sigma = 10^{-6}$ S/m and those of free space are $\mu = \mu_0$ [H/m] and $\varepsilon = \varepsilon_0$ [F/m]. The antenna transmits a uniform beam 1 m in diameter and the time-averaged power is 1 kW. Assume plane waves and that the beam remains of constant diameter:

(a) What is the electric field intensity immediately below the surface of the ice?
(b) If the ice is 10 km deep at a measurement point and the surface below the ice is perfectly reflecting, calculate the amplitude of the electric field intensity that reaches back to the aircraft antenna.
(c) Is this measurement feasible?

Solution: To calculate the electric field intensity below the surface of the ice, we need the transmission coefficient from air to ice. To calculate the field at the aircraft, we also need the transmission coefficient from ice into air (for the returning wave). The electric field intensity at the antenna is calculated from the Poynting vector. This electric field intensity is then allowed to propagate through air, transmit into the ice, propagate to the bottom and back to the surface, transmit through the surface, and propagate back to the airplane. The propagation path and coefficients are shown in **Figure 13.4**:

(a) Before calculating the electric field intensity below the ice surface, we calculate the electric field intensity at the antenna. The power density at the antenna is 1 kW/S, where S is the area of the beam:

$$\mathcal{P}_{av} = \frac{P}{S} = \frac{1000}{\pi \times (0.5)^2} = 1273.2 \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

From the discussion in **Chapter 12** (see **Example 12.9**), the amplitude of the time-averaged Poynting vector is

$$\mathcal{P}_{av} = \frac{E^2}{2\eta_0} = 1273.2 \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

In free space, $\eta_0 = 377\ \Omega$ and, therefore, the amplitude of the electric field intensity is

$$E = \sqrt{2\eta_0 \mathcal{P}_{av}} = \sqrt{2 \times 377 \times 1273.2} = 979.8 \quad [\text{V/m}]$$

This is E_0 in **Figure 13.4**. The electric field propagates in air without attenuation. Thus, $E_1 = E_0 = 979.8$ V/m in air, at the surface of the ice.

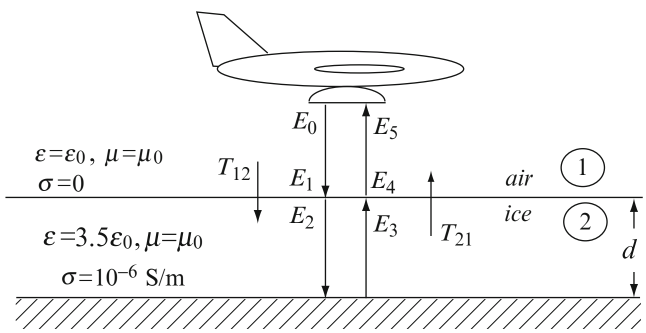


Figure 13.4 Remote sensing. Measurement of thickness of the polar ice caps

To calculate the amplitude of the electric field intensity below the ice, E_2, we need the transmission coefficient between air and ice. Because ice is a low-loss dielectric, we use the formula for intrinsic impedance for low-loss dielectrics given in **Eq. (12.109)**:

$$\eta = \eta_n\left(1 + \frac{j\sigma}{2\omega\varepsilon}\right) = \frac{\eta_0}{\sqrt{\varepsilon_r}}\left(1 + \frac{j10^{-6}}{2 \times 2 \times \pi \times 10^{10} \times 8.854 \times 3.5 \times 10^{-12}}\right) = \frac{377}{\sqrt{3.5}}\left(1 + j2.57 \times 10^{-7}\right) = 201.51 \quad [\Omega]$$

where η_n is the no-loss intrinsic impedance of ice. Thus, the transmission coefficient from air to ice is

$$T_{12} = \frac{2\eta}{\eta + \eta_0} = \frac{2 \times 201.51}{201.51 + 377} = 0.6967$$

The electric field intensity below the surface of the ice is therefore

$$E_2 = T_{12}E_1 = 0.6967 \times 979.8 = 682.63 \quad [\text{V/m}].$$

(b) The electric field intensity in ice propagates with attenuation until it reaches the bottom. Then, it is reflected and propagated with the same attenuation until it reaches the surface again. From the relations for low-loss dielectrics **[Eq. (12.104)]**, the attenuation constant is

$$\alpha \approx \frac{\alpha}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu_0}{3.5\varepsilon_0}} = \frac{10^{-6}\times 377}{2\times\sqrt{3.5}} = 1.0076\times 10^{-4} \quad \left[\frac{\text{Np}}{\text{m}}\right]$$

The total distance traveled is 20 km. The electric field intensity below the surface of the ice after it has propagated to the bottom and back is

$$E_3 = E_2 e^{-2\alpha d} = 682.63 \times e^{-2\times 1.0076\times 10^{-4}\times 10^4} = 91 \quad [\text{V/m}]$$

This is now transmitted across the interface with the transmission coefficient between ice and air, which is

$$T_{21} = \frac{2\eta_0}{\eta+\eta_0} = \frac{2\times 377}{201.51+377} = 1.303$$

The electric field intensity in air, just above the surface of the ice, is

$$E_4 = E_3 T_{21} = 91\times 1.303 = 118.6 \quad [\text{V/m}]$$

This wave propagates in air and reaches the antenna without further change. The electric field intensity returning to the antenna is 118.6 V/m.

(c) The measurement is feasible because the attenuation is rather low. This type of measurement is also used to measure thickness of snow. On the other hand, it is not feasible to use this to map the bottom of oceans because of the high attenuation in seawater.

13.2.2 Reflection and Transmission at an Air-Lossless Dielectric Interface: Normal Incidence

A common combination of materials at an interface is free space and a lossless dielectric. An example is a radome over a radar antenna (a radome is a dielectric cover designed to protect the antenna and, at the same time, should be transparent to the waves). Using **Figure 13.2** again, material (1) is characterized by $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$, and $\sigma_1 = 0$ and material (2) is characterized by ε_2, μ_2, and $\sigma_2 = 0$. The transmission and reflection coefficients can be written in terms of the relative permeability and relative permittivity of material (2) alone:

$$\Gamma = \frac{\eta_2-\eta_0}{\eta_0+\eta_2} = \frac{\sqrt{\mu_{r2}\mu_0/\varepsilon_{r2}\varepsilon_0}-\sqrt{\mu_0/\varepsilon_0}}{\sqrt{\mu_{r2}\mu_0/\varepsilon_{r2}\varepsilon_0}+\sqrt{\mu_0/\varepsilon_0}} = \frac{\sqrt{\mu_{r2}/\varepsilon_{r2}}-1}{\sqrt{\mu_{r2}/\varepsilon_{r2}}+1} = \frac{\sqrt{\mu_{r2}}-\sqrt{\varepsilon_{r2}}}{\sqrt{\mu_{r2}}+\sqrt{\varepsilon_{r2}}} \tag{13.39}$$

$$T = \frac{2\sqrt{\mu_{r2}/\varepsilon_{r2}}}{\sqrt{\mu_{r2}/\varepsilon_{r2}}+1} = \frac{2\sqrt{\mu_{r2}}}{\sqrt{\mu_{r2}}+\sqrt{\varepsilon_{r2}}} \tag{13.40}$$

Both Γ and T are real numbers. The electric and magnetic field intensities are

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left(Te^{-j\beta_0 z} + \Gamma j2\sin(\beta_0 z)\right) \quad [\text{V/m}] \tag{13.41}$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_0}\left(Te^{-j\beta_0 z} - \Gamma 2\cos(\beta_0 z)\right) \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.42}$$

$$\mathbf{E}_2(z) = \hat{\mathbf{x}} E_{i1} T e^{-j\beta_2 z} \quad [\text{V/m}] \tag{13.43}$$

$$\mathbf{H}_2(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_2}Te^{-j\beta_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.44}$$

where

$$\beta_0 = \omega\sqrt{\mu_0\varepsilon_0}, \quad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} \quad [\mathrm{rad/s}], \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \quad [\Omega] \tag{13.45}$$

If material (1) is not free space but a general lossless dielectric, the permittivity and permeability of this material are substituted for those of free space.

We note from these expressions that if the ratio $\mu_{r2}/\varepsilon_{r2}$ is equal to that of free space (i.e., $\mu_{r2}/\varepsilon_{r2} = 1$), there is no reflection at the interface and the whole wave is transmitted across the boundary. This is the case of perfect impedance matching since both materials have the same intrinsic impedance. While this situation is certainly not common in dielectrics, some materials, such as ferrites, can be made to closely resemble this condition by adjusting their permeability.

Similarly, if we are interested in large reflections, the permittivity of material (2) must be made as large as possible. Some materials have relatively high permittivity and are, therefore, reflective. One such material is water ($\varepsilon_r = 81$ at low frequencies). The permittivity of materials is also frequency dependent. For example, the permittivity of water decreases with frequency until, in the optical range, it is only about $1.75\varepsilon_0$.

Example 13.3 Application: Transparent Materials Suppose you are required to design a material which is completely transparent to electromagnetic waves at 1 GHz. For the purpose of this example, assume the material is very thick so that you can consider only the interface between it and free space. You are free to choose any relative permittivity between 2 and 9 and any permeability (it is possible to change the permeability of a dielectric by adding to it ferromagnetic particles). Assume the material remains lossless:

(a) Find the combinations of materials properties that will accomplish this design requirement.
(b) What happens at a different frequency, say 2 GHz?

Solution: For an interface to be transparent, the intrinsic impedance on both sides of the interface must be the same:

(a) Equating the intrinsic impedance in free space and a general material, we get

$$\frac{\mu_0}{\varepsilon_0} = \frac{\mu_2}{\varepsilon_2} = \frac{\mu_0}{\varepsilon_0}\frac{\mu_{r2}}{\varepsilon_{r2}} \quad \rightarrow \quad \frac{\mu_{r2}}{\varepsilon_{r2}} = 1$$

Thus, for $\varepsilon_{r2} = 2$, $\mu_{r2} = 2$, and for $\varepsilon_{r2} = 9$, $\mu_{r2} = 9$. The range of possible relative permabilities is between 2 and 9.

(b) The reflection coefficient in lossless materials is independent of frequency. The material is transparent at all frequencies. In lossy materials, this is not true.

13.2.3 Reflection and Transmission at an Air-Conductor Interface: Normal Incidence

We consider next the propagation of a wave from free space (or any lossless dielectric), impinging on a high-conductivity conductor, as in **Figure 13.5**. This condition is often encountered either accidentally, such as a wave impinging on a metal structure, or purposely, in reflecting a wave from a parabolic reflector or off the body of an airplane. For perfect conductors, the transmission coefficient must be zero and we can write directly from **Eq. (13.21)**

$$T = 0 \quad \text{and} \quad \Gamma = -1 \tag{13.46}$$

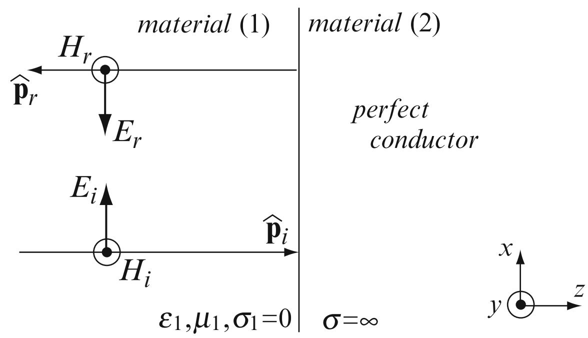


Figure 13.5 Reflection and transmission at a dielectric–conductor interface: normal incidence. The directions of the reflected waves are assumed

Substituting these in **Eqs. (13.27)** and **(13.30)**, the electric and magnetic fields to the left of the conductor in **Figure 13.5** are

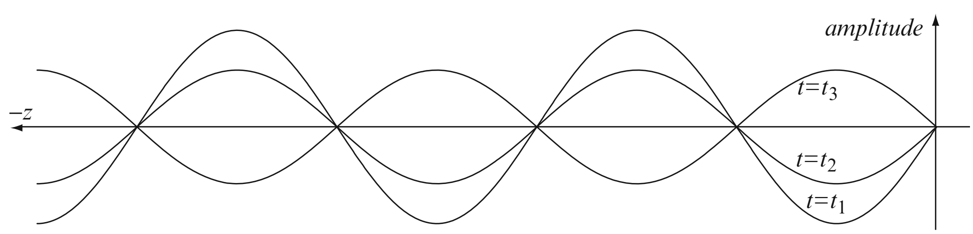
$$\mathbf{E}_1(z) = \hat{\mathbf{x}}\, j2E_{i1}\sin(\beta_1 z) \quad [\mathrm{V/m}] \tag{13.47}$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\, 2\frac{E_{il}}{\eta_1}\cos(\beta_1 z) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.48}$$

The more general notation β_1 rather than β_0 was used here to indicate that the same results apply to any lossless dielectric. The wave formed by these fields is unique in that it does not propagate; that is, as z changes, the amplitudes change, but there is no propagation in space. This is a standing wave. The principle of a standing wave is shown in **Figure 13.6**, where the amplitude of the wave changes with time, but the location of the peaks and zeros (nodes) is constant in space. To see that this is the case, it is most convenient to calculate the time-averaged Poynting vector for the wave:

$$\mathcal{P}_{av1} = \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}_1(z) \times \mathbf{H}_1^*(z)\right\} = \hat{\mathbf{z}}\,\mathrm{Re}\left\{j\frac{2}{\eta_1}|E_{i1}|^2\sin\beta_1 z\cos\beta_1 z\right\} = 0 \tag{13.49}$$

Figure 13.6 Standing wave due to reflection at a conducting interface. Total reflection (complete standing wave) without attenuation is shown



Because $\mathcal{P}_{av}$ is purely imaginary in material (1), no real power is transferred in the direction of propagation of the incident wave and, therefore, into the conductor. This, of course, is exactly what the reflection and transmission coefficients in **Eq. (13.46)** show, but it also means that no power is propagated anywhere to the left of the interface either. The energy in the system is "standing" or, we may say, it propagates back and forth with the same net effect.

To better understand the importance and behavior of standing waves, we first write the waves in the time domain and also calculate the parameters characterizing the standing waves. We start with the electric field intensity in **Eq. (13.47)**. In the time domain, this becomes

$$\begin{aligned}\mathbf{E}_1(z,t) &= \mathrm{Re}\{\mathbf{E}_1(z)e^{j\omega t}\} = \mathrm{Re}\{\hat{\mathbf{x}}\, j2E_{i1}\sin(\beta_{1z})e^{j\omega t}\} = \mathrm{Re}\left\{\hat{\mathbf{x}}\, 2E_{i1}\sin(\beta_1 z)e^{j\omega t}e^{j\pi/2}\right\} \\ &= \hat{\mathbf{x}}\, 2E_{i1}\sin(\beta_1 z)\cos\left(\omega t + \frac{\pi}{2}\right) = -\hat{\mathbf{x}}\, 2E_{i1}\sin(\beta_1 z)\sin\omega t \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]\end{aligned} \tag{13.50}$$

where $j = e^{j\pi/2}$ was used. Now, we can analyze this wave by inspection.

(1) First, we note that the amplitude of the wave varies from zero to a maximum of $2E_{i1}$, depending on the position z:

$$(E_1)_{\min} = 0, \quad (E_1)_{\max} = 2E_{i1} \tag{13.51}$$

(2) The wave varies as sin(ωt) in time.

(3) The wave varies as $\sin(\beta_1 z)$ in space. If we take the interface as the reference point (i.e., $z = 0$), the wave amplitude is zero for any value that makes $\beta_1 z$ a multiple of π. For z negative (i.e., to the left of the conducting surface in **Figure 13.5**), we have

$$E_1 = 0 \quad \text{for} \quad \beta_1 z = n\pi, \quad n = 0, 1, 2, \ldots \tag{13.52}$$

β_1 may be written in terms of the wavelength as $\beta_1 = 2\pi/\lambda_1$. With this, **Eq. (13.52)** becomes

$$E_1 = 0 \quad \text{at} \quad z = -\frac{n\lambda_1}{2}, \qquad n = 0, 1, 2, \ldots \tag{13.53}$$

The amplitude of the wave is zero at $z = 0$, $z = -\lambda_1/2$, $z = -\lambda_1$, $z = -3\lambda_1/2$, etc. These points (also called nodes) are shown in **Figure 13.7a**. If there is only a standing wave, we call this a ***complete standing wave***. An incomplete standing wave means that in addition to the standing wave, there is also a propagating wave. In a complete standing wave, the amplitude varies between zero and twice the amplitude of the incident wave, whereas in an incomplete standing wave, the amplitude varies between a minimum and a maximum value, which depend on the amplitudes of the incident and reflected waves. The magnitude of the ratio between the maximum and minimum amplitude of the standing wave is called the ***standing wave ratio*** (SWR) and will be discussed at length in **Chapter 14**. **Equation (13.50)** is also called the ***standing wave pattern*** for the electric field intensity (a similar standing wave pattern may be obtained for the magnetic field intensity). Either pattern may be plotted as in **Figure 13.6**.

(4) The standing wave is maximum (positive or negative) for any value of $\beta_{1z} = -(n\pi + \pi/2)$:

$$E_1 = 2E_{i1} \quad \text{for} \quad \beta_1 z = -\left(n\pi + \frac{\pi}{2}\right), \quad n = 0, 1, 2, \ldots \tag{13.54}$$

or using the wavelength

$$E_1 = 2E_{i1} \quad \text{at} \quad z = -\frac{n\lambda_1}{2} - \frac{\lambda_1}{4}, \quad n = 0, 1, 2, \ldots \tag{13.55}$$

Thus, the maxima of the standing wave are at $z = -\lambda_1/4$, $z = -3\lambda_1/4$, $z = -5\lambda_1/4$, etc., as shown in **Figure 13.7a**.

(5) Because of the sinusoidal behavior of the wave in space, we can place another conducting surface at any node of the standing wave without affecting the wave behavior. For example, we could place a conducting surface at $z = -\lambda_1/2$, $-\lambda_1$, etc., as shown by dashed lines in **Figure 13.7a**. We will take up this aspect of propagation again in **Chapter 17**. At this point, we simply mention that introducing the plate does not alter the field between the plate and the conducting surface at $z = 0$.

(6) The above analysis was based on the electric field intensity in **Eq. (13.47)**. A similar analysis for the magnetic field intensity in **Eq. (13.48)** may be performed. The magnetic field intensity in the time domain is

$$\mathbf{H}_1(z,t) = \hat{\mathbf{y}}\, 2\frac{E_{i1}}{\eta_1}\cos(\beta_1 z)\cos(\omega t) \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.56}$$

Comparison of **Eqs. (13.56)** and **(13.50)** shows that the electric and magnetic fields are in time quadrature as well as shifted in space by 90° (one-quarter wavelength). Therefore, the magnetic field intensity is maximum wherever the electric field intensity is minimum. The relations are therefore

$$H_1 = 0 \quad \text{at} \quad z = -\frac{n\lambda_1}{2} - \frac{\lambda_1}{4}, \quad n = 0, 1, 2, \ldots \tag{13.57}$$

$$H_1 = 2\frac{E_{i1}}{\eta_1} \quad \text{at} \quad z = -\frac{n\lambda_1}{2}, \quad n = 0, 1, 2, \ldots \tag{13.58}$$

These relations are shown in **Figure 13.7b**.

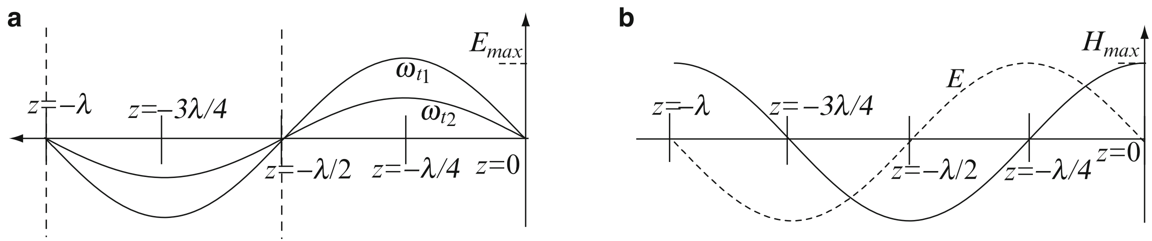


Figure 13.7 Standing waves. (**a**) Location of minima and maxima for two values of ωt. (**b**) Standing wave for the magnetic field intensity and its relation to the standing wave for the electric field intensity

In the above discussion we assumed that the conductor is a perfect conductor. What happens if the conductor is not a perfect conductor? Intuitively, we can say that there should be a transmitted wave, but this should be small. In other words, only a small fraction of the incident wave should propagate into the conductor. The reflection coefficient is large and close to -1. To see how the waves behave at the interface between air and a good conductor, we make use of the propagation properties of good conductors as defined in **Section 12.7.3**, for which $\sigma/\omega\varepsilon \gg 1$. For a good but not perfect conductor we obtained in **Eqs. (12.111)** and **(12.113)**

$$\alpha = \sqrt{\pi f \mu \sigma} \quad \left[\frac{\text{Np}}{\text{m}}\right], \quad \beta = \sqrt{\pi f \mu \sigma} \quad \left[\frac{\text{rad}}{\text{m}}\right], \quad \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad [\text{m}] \tag{13.59}$$

where α is the attenuation constant, β the phase constant, and δ the skin depth in the conductor.

Consider now the general electric and magnetic fields in material (1) as given in **Eqs. (13.27)** and **(13.30)**:

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left(Te^{-\gamma_1 z} - \Gamma j2\sin(j\gamma_1 z)\right) \quad [\text{V/m}], \quad \mathbf{H}_1(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_0}\left(Te^{-\gamma_1 z} - \Gamma 2\sin(j\gamma_1 z)\right) \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.60}$$

and in material (2) as given in **Eqs. (13.31)** and **(13.32)**:

$$\mathbf{E}_2(z) = \hat{\mathbf{x}} TE_{i1}e^{-\gamma_2 z} \quad [\text{V/m}], \quad \mathbf{H}_2(z) = \hat{\mathbf{y}} T\frac{E_{i1}}{\eta_0}e^{-\gamma_2 z} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.61}$$

In this particular case, material (1) is air (or a perfect dielectric) and we can write $\gamma_1 = j\beta_1$. For material (2), $\gamma_2 = \alpha_2 + j\beta_2$, where α and β are given in **Eq. (13.59)**.

The reflection and transmission coefficients in **Eqs. (13.23)** and **(13.24)** still apply, but because the intrinsic impedance of a good conductor is rather small, we write the reflection and transmission coefficients in terms of the skin depth in the conductor. The intrinsic impedance in a good conductor is $\eta_2 = (1 + j)/\sigma_2\delta_2$ **[from Eq. (12.116)]**. Substituting this in **Eqs. (13.23)** and **(13.24)**, we obtain

$$\Gamma = \frac{1 + j - \sigma_2\delta_2\eta_0}{1 + j + \sigma_2\delta_2\eta_0}, \quad T = \frac{2(1 + j)}{\sigma_2\delta_2\eta_0 + (1 + j)} \tag{13.62}$$

These expressions are both complex, and according to the discussion on propagation in conductors **[Eq. (12.116)]**, these are only approximations since the intrinsic impedance itself is an approximation. It is interesting to note that if the skin depth decreases, the term $\sigma_2\delta_2\eta_0$ becomes smaller, until, in the limit, it can be neglected with respect to $1 + j$. In this limit, the reflection coefficient approaches +1 and the transmission coefficient approaches 2. This may seem a contradiction at first since this would imply that there is transmission into the conductor when, in fact, we stated in **Chapter 12** that the lower the skin depth, the lower the penetration into the conductor. However, it is worth restating that the whole idea of skin depth is based on $\sigma/\omega\varepsilon \gg 1$. For a given material, the skin depth can be decreased by increasing the frequency but this also reduces the magnitude of $\sigma/\omega\varepsilon$. Therefore, at some frequency, the conductor ceases to behave as a conductor. This happens when displacement currents dominate, and under these conditions, the skin depth has no meaning. **Equations (13.59)** and **(13.62)** are only valid if the condition for a conductor is satisfied; that is, only if $\sigma/\omega\varepsilon \gg 1$.

Using these expressions in **Eq. (13.60)**, we obtain the electric and magnetic field intensities in material (1) as

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}\left(\frac{2(i+j)}{\sigma_2\delta_2\eta_0 + (i+j)}e^{-j\beta_1 z} + \frac{1+j-\sigma_2\delta_2\eta_0}{1+j+\sigma_2\delta_2\eta_0}(2j\sin(\beta_1 z))\right) \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right] \tag{13.63}$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\,\frac{E_{i1}}{\eta_0}\left(\frac{2(i+j)}{\sigma_2\delta_2\eta_0 + (i+j)}e^{-j\beta_1 z} + \frac{1+j-\sigma_2\delta_2\eta_0}{1+j+\sigma_2\delta_2\eta_0}(2\cos(\beta_1 z))\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.64}$$

where $\gamma_1 = j\beta_1$ was used. The fields in material (2) are

$$\mathbf{E}_2 = \hat{\mathbf{x}}\,TE_{i1}e^{-\gamma_2 z} = \hat{\mathbf{x}}\,E_{i1}\frac{2(i+j)}{\sigma_2\delta_2\eta_0 + (i+j)}e^{-\gamma_2 z} \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right] \tag{13.65}$$

$$\mathbf{H}_2 = \hat{\mathbf{y}}\,\mathrm{T}\frac{E_{i1}}{(1+j)+(1/\sigma_2\delta_2)}e^{-j\gamma_2 z} = \hat{\mathbf{y}}\,E_{i1}\frac{2\sigma_2\delta_2}{\sigma_2\delta_2\eta_0 + (i+j)}e^{-\gamma_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.66}$$

The fields in material (2) are rather small and decay fast, as expected for a good conductor, but they are not zero.

The expressions in **Eqs. (13.63)** through **(13.66)** are not different from those in **Eqs. (13.25)** through **(13.32)**, except for the fact that the standing wave component is large, as it should be for a good conductor. A useful calculation at this point is the power flow into the conductor. This is found by calculating the Poynting vector for the transmitted wave:

$$\begin{aligned}\mathbf{E}\times\mathbf{H}^* &= \hat{\mathbf{z}}\left(TE_{i1}e^{-\gamma_2 z}\right)\left(\frac{TE_{i1}e^{-\gamma_2 z}}{\eta^2}\right)^* = \hat{\mathbf{z}}\left(TE_{i1}e^{-(\alpha_2+j\beta_2)z}\right)\left(\frac{\sigma_2\delta_2 TE_{i1}e^{-(\alpha_2+j\beta_2)z}}{1+j}\right)^* \\ &= \hat{\mathbf{z}}E_{i1}^2\sigma_2\delta_2 T\left(\frac{T}{1+j}\right)^* e^{-2\alpha_2 z} = \hat{\mathbf{z}}\,4E_{i1}^2\sigma_2\delta_2\left(\frac{1+j}{(\sigma_2\delta_2\eta_0+1)^2}\right)^* e^{-2\alpha_2 z} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]\end{aligned} \tag{13.67}$$

where $e^{-\gamma z}(e^{-\gamma z})^* = e^{-(\alpha+j\beta)z}e^{-(\alpha-j\beta)} = e^{-2\alpha}$ was used. Assuming that $|\eta_2| \ll |\eta_0|$ (this is a good approximation in a good conductor), the total power flow into the conductor is minimal. The real part of the Poynting vector represents the dissipated power in the conductors and this decays as $e^{-2\alpha z}$, indicating very rapid attenuation. For a perfect conductor $\eta_2 = 0$ $(T = 0)$ and the power density in the conductor is zero, as required. This aspect of power propagation shows why a microwave oven, while heating lossy materials such as food, will not dissipate much power in its own walls, which are made of steel (see **Example 13.6**).

Example 13.4 Application: Reflectometry It is required to measure the distance from an antenna to a reflecting surface (such as a wall, planet, etc.). A plane wave is transmitted to the wall and a zero (minimum reception) in the standing wave pattern is recorded using a second antenna at a distance d_1 [m] from the sending antenna, as shown in **Figure 13.8**. The frequency of the wave is f_1 [Hz]. Now, the frequency is decreased until the receiving antenna reads a maximum in the electric field at the same location. If the frequency for the maximum reading is f_2 [Hz], calculate the distance between the transmitting antenna to the conducting surface. The values are $f_1 = 100$ MHz, $f_2 = 99.9$ MHz, and $d_1 = 10$ m. Use the properties of free space without attenuation.

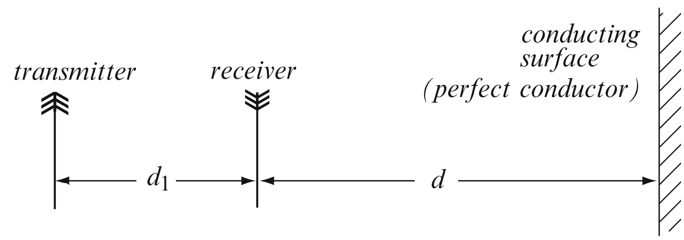


Figure 13.8 Reflectometry: measurement of distance to a target by identifying the nodes in the standing wave pattern

Solution: To calculate the distance between the receiving antenna and wall, we first calculate the number of minima in the standing wave pattern. From this, the distance is obtained.

For frequency f_1, assuming there are n minima between the receiving antenna and wall,

$$d = n\frac{\lambda_1}{2} = n\frac{c}{2f_1} \quad \text{since} \quad \lambda_1 f_1 = c$$

As the frequency is decreased, the wavelength increases and the distance between minima increases. The maxima (as well as minima) move to the left until, at frequency f_2, the first maximum to the right of the receiving antenna moves to the location of the receiving antenna. There are now $n - 1$ minima in the standing wave pattern between the wall and the receiver (because the minimum at the receiver has now moved to the left) plus the distance between a minimum and a maximum. Thus, the distance between the wall and the receiver is

$$d = (n-1)\frac{\lambda_2}{2} + \frac{\lambda_2}{4} = (n-1)\frac{c}{2f_2} + \frac{c}{4f_2} \quad \text{since} \quad \lambda_2 f_2 = c$$

Equating the two expressions for d,

$$n\frac{c}{2f_1} = (n-1)\frac{c}{2f_2} + \frac{c}{4f_2}$$

From this,

$$n = \frac{f_1}{2(f_1 - f_2)}$$

For the values given,

$$n = \frac{100 \times 10^6}{2 \times \left(100 \times 10^6 - 99.9 \times 10^6\right)} = \frac{100}{2 \times (0.1)} = 500$$

The distance d is therefore

$$d = 500\frac{3 \times 10^8}{2 \times 100 \times 10^6} = 750 \text{ m}$$

and adding the 10 m between the two antennas, we get 760 m.

Note: This type of measurement is quite sensitive and is routinely performed in many applications such as measurement of thickness of conductors or even measurement of coatings such as paint. In practical applications, the standing wave pattern is not complete and, therefore, only minima and maxima are detected (not zeros of the pattern). In most cases, minima are easier to detect than maxima because they are sharper. At still higher frequencies, the distance between minima and maxima is very small and very sensitive measurements are possible even over short distances.

Example 13.5 A plane wave propagates in free space and encounters a perfect conductor. The frequency of the wave is 1 GHz and its amplitude is 1 V/m. The electric field intensity is directed in the x direction and the wave propagates in the z direction, as shown in **Figure 13.5**. Calculate:

(a) The electric and magnetic field intensities everywhere to the left of the conductor's surface.
(b) The power relations everywhere to the left of the interface.

Solution: The conducting interface reflects the wave and, since there is no attenuation in free space, the amplitude of the reflected wave equals that of the incident wave but opposite in sign. The electric field intensity to the left of the interface is given by **Eq. (13.50)**.

(a) The instantaneous electric field intensity is

$$\mathbf{E}_1(z,t) = -\hat{\mathbf{x}}\,2E_{i1}\sin(\beta_1 z)\sin(\omega t) \quad [\mathrm{V/m}]$$

The phase constant at the given frequency in free space is

$$\beta_1 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c} = \frac{2\pi\times 10^9}{3\times 10^8} = \frac{20\pi}{3} \quad \left[\frac{\mathrm{rad}}{\mathrm{m}}\right]$$

Thus, the electric field intensity is

$$\mathbf{E}_1(z,t) = -\hat{\mathbf{x}}\,2\sin\left(\frac{20\pi z}{3}\right)\sin\left(2\pi\times 10^9 t\right) \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

From **Eq. (13.56)**, the instantaneous magnetic field intensity is

$$\mathbf{H}_1(z,t) = \hat{\mathbf{y}}\frac{2}{377}\cos\left(\frac{20\pi z}{3}\right)\cos\left(2\pi\times 10^9 t\right) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]$$

The electric and magnetic fields are in time quadrature (90° phase difference) and shifted in space by $\lambda/4$, as well as being orthogonal in their direction in space. The electric field intensity is zero at

$$\frac{20\pi z}{3} = -n\pi \quad \rightarrow \quad z = -\frac{3n}{20}, \quad n = 0, 1, 2, 3, \ldots$$

that is, the electric field intensity is zero at $z = 0$, -0.15 m, -0.3 m, -0.45 m, etc.

The magnetic field intensity is zero at

$$\frac{20\pi z}{3} = -\frac{n\pi}{2}, \quad z = -\frac{3n}{40}, \quad n = 0, 1, 3, 5, 7, \ldots$$

that is, the nodes of the magnetic field are at $z = -0.075$ m, -0.225 m, -0.375 m, -0.525 m, etc.

Note that the distance between two nodes of either **E** or **H** is one-half wavelength and the distance between a node of the electric field intensity and the nearest node of the magnetic field intensity is one-quarter wavelength.
The electric and magnetic fields also vary with time, but the nodes remain fixed in space, thus, again, the meaning of standing waves.

The solution for the electric and magnetic field intensities is shown in **Figure 13.9**, where the field at three time instances is shown. Note the locations of the zeros and the variation in time.

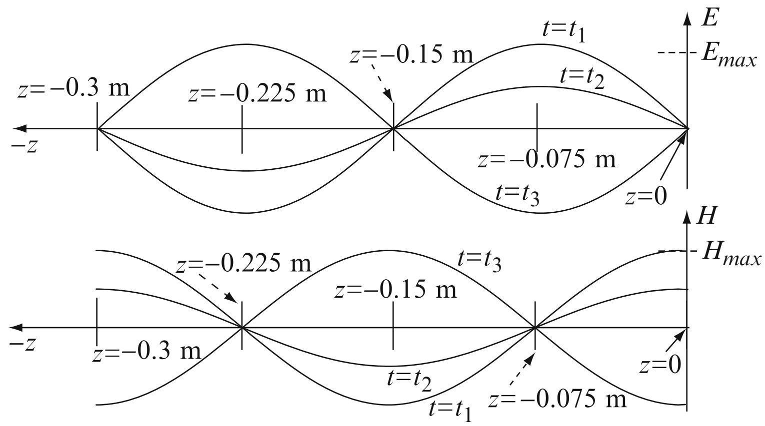


Figure 13.9 Electric and magnetic field intensities to the left of an air–conductor interface for different times. Note the relative location of the nodes

(b) We look first at the instantaneous power density in space using the instantaneous Poynting vector is

$$\mathcal{P}_1(z,t) = \mathbf{E}_1(z,t) \times \mathbf{H}_1(z,t) = -\hat{\mathbf{z}}\frac{4}{377}|E_{i1}|^2 \sin\left(\frac{20\pi z}{3}\right)\cos\left(\frac{20\pi z}{3}\right)$$
$$\times \sin\left(2\pi \times 10^9 t\right)\cos\left(2\pi \times 10^9 t\right) \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

This exists anywhere to the left of the conducting interface. The time-averaged power density may be calculated by integrating the instantaneous power density over one cycle of the wave, but it is easier to evaluate it from the time-harmonic forms of the electric and magnetic fields given in **Eqs. (13.47)** and **(13.48)**:

$$\mathbf{E}_1(z) = \hat{\mathbf{x}}j2E_{i1}\sin(\beta_1 z) \quad [\text{V/m}], \quad \mathbf{H}_1(z) = \hat{\mathbf{y}}\,2\frac{E_{i1}}{\eta 1}\cos(\beta_1 z) \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The time-averaged power density is therefore

$$\mathcal{P}_{av1}(z) = \frac{1}{2}\text{Re}\{\mathbf{E}(z) \times \mathbf{H}^*(z)\} = \frac{1}{2}\text{Re}\left\{\hat{\mathbf{x}}j2E_{i1}\sin(\beta_1 z) \times \hat{\mathbf{y}}\,2\frac{E_{i1}}{\eta 1}\cos(\beta 1z)\right\} = 0 \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

that is, there is no propagation of real power.

Example 13.6 Application: Microwave Cooking—Why the Oven Itself Does Not Get Hot? The microwave oven is a closed cavity so the waves that exist inside are not plane waves. However, to approximate the conditions in the walls of the oven, we take an equivalent sheet of metal, equal in thickness and area to that of the oven, and expose it to the same power density it would be subject to in the oven. We assume here that the total area of the walls is 1 m^2 (a medium-sized domestic oven), operating at 2.45 GHz and the walls are made of steel.

A very large sheet of steel is illuminated by a plane wave at 2.45 GHz. The steel is 1 mm thick, has conductivity of 10^7 S/m, and has a relative permeability of 200. The time-averaged power density impinging on the sheet is 1,000 W/m^2 and the wave impinges on the walls perpendicularly. Calculate the power dissipated in the steel per unit area.

Solution: Steel is a good conductor; therefore, we expect little penetration. From the given data, we first calculate the incident electric and magnetic field intensities at the interface. From material data, we calculate the skin depth δ and then calculate the transmission coefficient in **Eq. (13.62)** or use **Eqs. (13.65)** and **(13.66)** directly to calculate the electric and magnetic field intensities just below the surface of the walls. Using the Poynting vector with these fields will give the power density at the walls. Since all transmitted power is dissipated in the walls, the product of the power density in the wall and the area of the walls gives the total power dissipated in the walls.

The amplitude of the incident electric field intensity E_{i1} is calculated from the time-averaged power density (see **Example 12.9**):

$$\mathcal{P}_{av} = \frac{E_1^2}{2\eta_0} \quad \rightarrow \quad E_{i1} = \sqrt{2\eta_0 \mathcal{P}_{av}} = \sqrt{2 \times 377 \times 1000} = 868 \quad \left[\frac{\text{V}}{\text{m}}\right],$$

$$H_{i1} = \frac{E_{i1}}{\eta_0} = \frac{868}{377} = 2.3 \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The skin depth is

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 2.45 \times 10^9 \times 200 \times 4 \times \pi \times 10^{-7} \times 10^7}} = 2.27 \times 10^{-7} \quad [\text{m}]$$

The skin depth is only 0.227 μm. Therefore, we can assume the sheet to be infinitely thick for practical purposes.

The electric field intensity immediately inside the iron sheet [from **Eq. (13.65)**, with $z = 0$] is

$$E_2 = \frac{2(1+j)E_{i1}}{\eta_0 \sigma_2 \delta_2 + (1+j)} = \frac{2(1+j)E_{i1}}{377 \times 10^7 \times 2.27 \times 10^{-7} + (1+j)} \approx \frac{2(1+j)E_{i1}}{855.8} = 2.028(1+j) \quad \left[\frac{\text{V}}{\text{m}}\right]$$

where j in the denominator was neglected in comparison to the large real part. The magnetic field intensity is calculated (using $\eta_2 = (1 + j)/\sigma_2\delta_2$) as

$$H_2 = \frac{E_2}{\eta_2} \approx \frac{2(1+j)\sigma_2\delta_2 E_{i1}}{855.8(1+j)} = \frac{2\sigma_2\delta_2 E_{i1}}{855.8} = \frac{2 \times 10^{-7} \times 2.27 \times 10^{-7} \times 868}{855.8} = 4.6 \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The time-averaged Poynting vector gives the power per unit area entering the steel sheet:

$$\mathcal{P}_{av} = \left|\frac{1}{2}\text{Re}\{\mathbf{E}_2 \times \mathbf{H}_2^*\}\right| = \frac{1}{2}\text{Re}\{2.028 \times (1+j) \times 4.6\} = 4.66 \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

For a 1 m square of the conductor only 4.66 W enters the steel sheet. This is dissipated in the conductor. Thus, less than 0.5 % of the power generated by the oven enters its walls and contributes to heating. Things are more complicated than this in a real oven but this example gives a sense of the quantities involved.

Exercise 13.3 Derive **Eq. (13.67)** from **Eqs. (13.65)** and **(13.66)**.

Exercise 13.4 A plane wave propagates from free space into copper. Calculate the time-averaged power density entering the copper and show that the amount of power per unit area of the interface, going into the copper, is very small. Assume frequency is 100 MHz and the magnitude of the incident electric field intensity at the surface, in air, is 100 V/m. The conductivity of copper is 5.7×10^7 S/m.

Answer The time-averaged power density into copper is 3.7×10^{-4} W/m^2. The time-averaged incident power density in air is 13.26 W/m^2.

13.3 Reflection and Transmission at an Interface: Oblique Incidence on a Conductor Waves.m

A plane wave, obliquely incident on an interface between two materials, undergoes changes similar to those for normal incidence. Part of the wave is transmitted and part of it is reflected. In some cases, there is either only transmission or only reflection, depending on the values of the reflection and transmission coefficients. In oblique incidence, the continuity of the field components is taken into account as interface conditions at the interface between the two materials. The behavior of the wave at the interface depends on the polarization of the wave. To assist us in describing the waves at the interface, a ***plane of incidence*** is defined as the plane described by the direction of propagation of the incident wave (i.e., the Poynting vector) and the normal to the surface at the interface, as shown in **Figure 13.10a**. The direction of the electric or magnetic field intensities for uniform plane waves is always normal to the direction of propagation. If the electric field intensity is parallel to the plane of incidence, we refer to this as ***parallel polarization*** (sometimes called H polarization). ***Perpendicular polarization*** refers to the case of the electric field intensity perpendicular to the plane of incidence (sometimes called E polarization). The general case of arbitrary polarization can always be treated by separating the electric field intensity into its normal and parallel components as a combination of parallel and perpendicular polarizations.

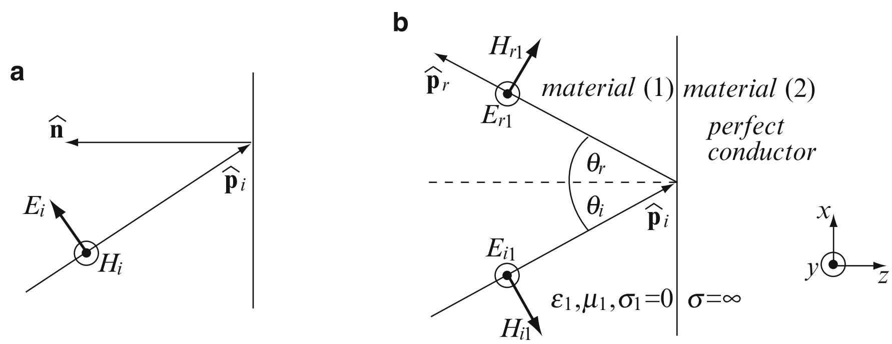


Figure 13.10 (**a**) Definition of the plane of incidence. The electric field intensity is parallel to the plane of incidence. (**b**) Oblique incidence at a dielectric–conductor interface. The electric field is polarized perpendicular to the plane of incidence and the direction of the reflected wave is assumed

We treat the problem of oblique incidence by looking at the propagation of waves obliquely incident at an interface for parallel and perpendicular polarization separately for dielectrics and conductors. The case of incidence on conducting interfaces is given first, followed by incidence on lossless dielectrics. The more general case of incidence on lossy dielectrics is not treated here because it results in nonuniform plane waves and this is a subject for more advanced study. However, the principles used here are directly applicable to any interface.

13.3.1 Oblique Incidence on a Conducting Interface: Perpendicular Polarization

In this configuration, the electric field intensity is perpendicular to the plane of incidence, as shown in **Figure 13.10b**. Incidence is from a dielectric with permittivity ε_1 on a perfect conductor surface. The direction of propagation can be written directly from **Figure 13.10b**. For the incident wave, the unit vector in the direction of propagation is

$$\hat{\mathbf{p}}_i = \hat{\mathbf{x}}\sin\theta_i + \hat{\mathbf{z}}\cos\theta_i \tag{13.68}$$

For the reflected wave, the direction of propagation is

$$\hat{\mathbf{p}}_r = \hat{\mathbf{x}}\sin\theta_r - \hat{\mathbf{z}}\cos\theta_r \tag{13.69}$$

where the unit vectors $\hat{\mathbf{p}}_i$ and $\hat{\mathbf{p}}_r$ refer to the direction of the Poynting vector for the incident and reflected waves, respectively.

Inspection of the electric field intensity in **Eq. (13.1)** shows that the variable z in the exponent is a distance the wave propagates from some reference point, whereas β is the phase constant of the wave. The product βz is the phase of the wave after it has propagated a distance z from some reference point. Since the phase constant β_1 is given for the wave propagating in the $\hat{\mathbf{p}}_i$ direction, we can view the wave in **Figure 13.10b** as two components, one propagating in the positive x direction with phase constant β_{1x} and one in the positive z direction with phase constant β_{1z} (see **Figure 13.11a**), where

$$\beta_{1x} = \beta_1 \sin\theta_i, \quad \beta_{1z} = \beta_1 \cos\theta_i \quad [\text{rad/m}] \tag{13.70}$$

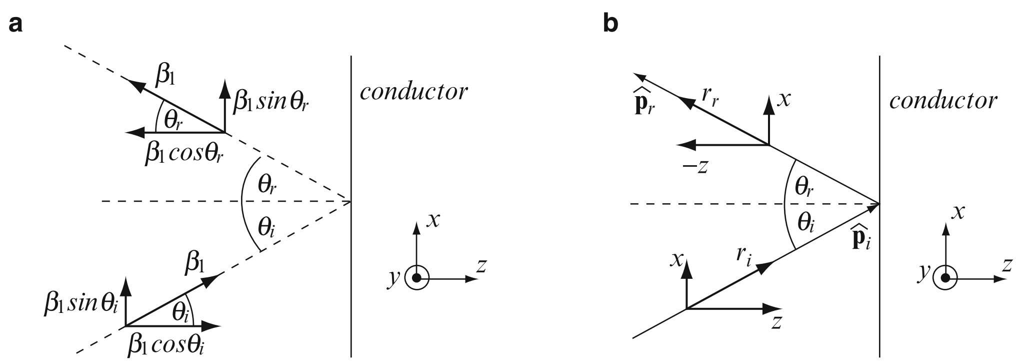


Figure 13.11 Direction of propagation and phase constants at an air-conductor interface. (**a**) Relation between phase constants. (**b**) Distances traveled by the incident and reflected waves

When the wave propagates a distance r_i along $\hat{\mathbf{p}}_i$ (see **Figure 13.11b**), the vertical component of the wave propagates a distance x with phase constant $\beta_{1x,}$ and the horizontal component propagates a distance z with a phase constant β_{1z}, where x and z are the vertical and horizontal distances traveled, respectively. The phase of the incident wave is therefore

$$\beta_1 r_i = \beta_1 x \sin\theta_i + \beta_1 z \cos\theta_i \quad [\text{rad}] \tag{13.71}$$

Similarly, for the backward propagating wave, the components of β are as in **Eq. (13.70)**, but now the wave travels a distance x in the vertical direction and a distance $-z$ in the horizontal direction and the angle is θ_r. The phase of the reflected wave is

$$\beta_1 r_r = \beta_1 x \sin\theta_r - \beta_1 z \cos\theta_r \quad [\text{rad}] \tag{13.72}$$

Using the directions of the incident electric and magnetic field intensities shown in **Figure 13.10b**, and the phase of the incident wave from **Eq. (13.71)**, the incident electric field intensity is in the positive y direction and the magnetic field intensity has both a negative x and a positive z component. These are

$$\mathbf{E}_i(x,z) = \hat{\mathbf{y}} E_{i1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad [\text{V/m}] \tag{13.73}$$

$$\mathbf{H}_i(x,z) = \frac{E_{i1}}{\eta_1}(-\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.74}$$

The reflected fields are obtained using **Eq. (13.72)** for the phase of the wave:

$$\mathbf{E}_r(x,z) = \hat{\mathbf{y}} E_{r1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad [\text{V/m}] \tag{13.75}$$

$$\mathbf{H}_r(x,y) = \frac{E_{r1}}{\eta_1}(\hat{\mathbf{x}}\cos\theta_r + \hat{\mathbf{z}}\sin\theta_r) e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.76}$$

In a perfect conductor, there is no transmitted wave; therefore, at the interface ($z = 0$), the total tangential electric field intensity (in this case the electric field intensity has only a tangential component) must be zero:

$$\mathbf{E}_i(x,0) + \mathbf{E}_r(x,0) = \hat{\mathbf{y}}\left[E_{i1} e^{-j\beta_1 \sin\theta_i} + E_{r1} e^{-j\beta_1 x \sin\theta_r}\right] = 0 \tag{13.77}$$

For this to be satisfied, the following must hold:

$$E_{r1} = -E_{i1} \quad \text{and} \quad \theta_r = \theta_i \tag{13.78}$$

that is, the sum of the amplitudes must be zero whereas the phases at the interface remain constant ($\beta_1 x \sin\theta_i = \beta_1 x \sin\theta_r$). The latter, relation, namely, $\theta_r = \theta_i$ is called Snell's law of reflection. The amplitudes of the reflected and incident waves are the same in absolute values and the angle of incidence and reflection are the same. The reflected electric and magnetic fields can be written in terms of the incident field as

$$\mathbf{E}_r(x,z) = -\hat{\mathbf{y}}\,E_{i1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad [\mathrm{V/m}] \tag{13.79}$$

$$\mathbf{H}_r(x,z) = -\frac{E_{i1}}{\eta_1}(\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.80}$$

The reflection coefficient in this case is equal to -1. This indicates that the reflected electric and magnetic field intensity that we assumed in **Figure 13.10b** are in the directions opposite those shown. The total electric and magnetic field intensities are (after rearranging terms)

$$\mathbf{E}_1(x,z) = \hat{\mathbf{y}}\,E_{i1}\left[e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i}\right]e^{-j\beta_1 z\sin\theta_i} = -\hat{\mathbf{y}}\,j2E_{i1}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \quad [\mathrm{V/m}] \tag{13.81}$$

$$\mathbf{H}_1(x,z) = -2\frac{E_{i1}}{\eta_1}[\hat{\mathbf{x}}\cos\theta_i\cos(\beta_1 z\cos\theta_i) + \hat{\mathbf{z}}\,j\sin\theta_i\sin(\beta_1 z\cos\theta_i)]e^{-j\beta_1 z\sin\theta_i} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.82}$$

To obtain these relations, **Eqs. (13.73)** and **(13.75)** were summed together and **Eqs. (13.74)** and **(13.76)** were summed together, after substituting the relations in **Eq. (13.78)**. In addition, the relations $e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i} = -j2\sin(\beta_1 z\cos\theta_i)$ and $e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i} = 2\cos(\beta_1 z\cos\theta_i)$ were used to simplify the expressions. The term $\beta_{1x} = \beta_1 \sin\theta_i$ may be viewed as the modified phase constant in the x direction due to the presence of the conductor; that is, the conducting surface causes the wave to propagate parallel to the surface with a phase constant $\beta_{1x} = \beta_1\sin\theta_i$. As a consequence, the phase velocity of the wave propagating parallel to the conducting surface is different than the phase velocity in material (1) since $\beta = \omega/v_p$. The phase velocity in the x direction is now $v_{px} = \omega/\beta_{1x} = \omega/(\beta_1\sin\theta_i) = v_p/\sin\theta_i$. A similar discussion shows that the phase velocity in the z direction is $v_{pz} = v_p/\cos\theta_i$. In fact, if material (1) is free space, the phase velocities in the x and y directions are greater than the speed of light. This, of course, is admissible and does not imply that energy propagates faster than the speed of light (see **Section 12.7.4**).

To see how the wave propagates, it is useful to write the time-averaged Poynting vector:

$$\begin{aligned}\boldsymbol{\mathcal{P}}_{av} &= \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}_1(x,z)\times\mathbf{H}_1^*(x,z)\right\} = \frac{1}{2}\mathrm{Re}\left\{\left(-\hat{\mathbf{y}}\,j2E_{i1}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1(x\sin\theta_i)}\right)\right.\\ &\quad\left.\times\left(-2\frac{E_{i1}}{\eta_1}[\hat{\mathbf{x}}\cos\theta_i\cos(\beta_1 z\cos\theta_i) - \hat{\mathbf{z}}\,j\sin\theta_i\sin(\beta_1 z\cos\theta_i)]e^{+j\beta_1(x\sin\theta_i)}\right)\right\}\\ &= \mathrm{Re}\left\{\hat{\mathbf{y}}\times\hat{\mathbf{x}}\,j2\frac{E_{i1}^2}{\eta_1}\sin(\beta_1 z\cos\theta_i)\cos(\beta_1 z\cos\theta_i)\cos\theta_i\right.\\ &\quad\left.+(-\hat{\mathbf{y}}\times\hat{\mathbf{z}})2j^2\frac{E_{i1}^2}{\eta_1}\sin(\beta_1 z\cos\theta_i)\sin(\beta_1 z\cos\theta_i)\sin\theta_i\right\} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]\end{aligned} \tag{13.83}$$

where the conjugate of $\mathbf{H}_1(x,z) = (-\hat{\mathbf{x}}\,\mathbf{H}_{1x} - \hat{\mathbf{z}}\,j\mathbf{H}_{1z})e^{-j\beta_1 x\sin\theta_i}$ is $\mathbf{H}_1^*(x,z) = (-\hat{\mathbf{x}}\,\mathbf{H}_{1x} + \hat{\mathbf{z}}\,j\mathbf{H}_{1z})e^{+j\beta_1 x\cos\theta_i}$. This can be simplified by writing $\hat{\mathbf{y}}\times\hat{\mathbf{x}} = -\hat{\mathbf{z}}$, $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$, $\sin(\beta_1 z\cos\theta_i)\cos(\beta_1 z\cos\theta_i) = (1/2)\sin(2\beta_1 z\cos\theta_i)$ and $j^2 = -1$:

$$\boldsymbol{\mathcal{P}}_{av} = \mathrm{Re}\left\{-\hat{\mathbf{z}}\,j\frac{E_{i1}^2}{\eta_1}\sin(2\beta_1 z\cos\theta_i)\cos\theta_i + \hat{\mathbf{x}}\,\frac{2E_{i1}^2}{\eta_1}\sin^2(\beta_1 z\cos\theta_i)\sin\theta_i\right\} = \hat{\mathbf{x}}\,\frac{2E_{i1}^2}{\eta_1}\sin^2(\beta_1 z\cos\theta_i)\sin\theta_i \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \tag{13.84}$$

Note that the wave in the z direction is a standing wave since the power density in this direction is purely imaginary, but the wave in the x direction is propagating. There are a number of important properties associated with these results, properties which we will use extensively in **Chapter 17**. We note the following:

(1) For any angle $0 < \theta_i < \pi/2$, there is a propagating term and a standing wave term. The standing wave becomes smaller as θ_i increases, whereas the propagating term becomes smaller as θ_i decreases.
(2) For perpendicular incidence ($\theta_i = 0$), the propagating term is zero and the wave is a purely standing wave. The time-averaged power density is zero.
(3) The wave propagates parallel to the surface of the conductor (in this case, in the positive x direction) for any angle $0 < \theta_i < \pi/2$, as can be seen from **Eq. (13.84)**. The conducting surface has the net effect of guiding the waves parallel to its surface. In **Chapter 17**, we will call these guided waves and will use the results of this and the following section to define the properties of guided waves.
(4) The amplitude of the propagating wave depends on the z-coordinate. This means that the amplitude is not constant on the plane perpendicular to the direction of propagation and hence this is not a uniform plane wave. Note in particular that whereas the incident and reflected waves are uniform plane waves, their sum is not.

Example 13.7 Application: Reflection of Waves by the Ionosphere A perpendicularly polarized plane wave propagates in air and impinges on the ionosphere as shown in **Figure 13.12a**. The amplitude of the electric field is 100 V/m, its frequency is 3 GHz, and the angle of incidence is 30°. Assume air has properties of free space and the ionosphere is a perfect conductor at the frequency of the wave.

(a) Calculate the total electric and magnetic field intensity in air.
(b) Calculate the time-averaged power density in air.

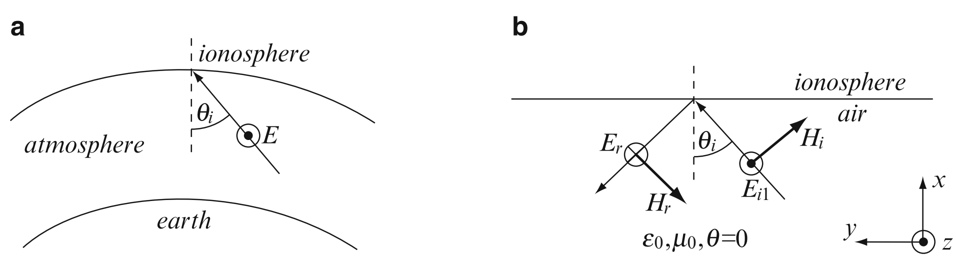


Figure 13.12 (**a**) A wave impinging on the ionosphere which is assumed to be a perfect conductor. (**b**) The system of coordinates used for solution

Solution: The electric and magnetic field intensities in the dielectric are given in **Eqs. (13.81)** and **(13.82)** and the time-averaged power density in **Eq. (13.84)**, but before calculating the fields, we must define a system of coordinates which, of course, is arbitrary. One possible system is shown in **Figure 13.12b**. Also, whereas the electric field intensity is perpendicular to the plane of incidence, it can be either in the negative or positive z direction. We choose the latter.

(a) The incident electric field intensity and magnetic field intensity are

$$\mathbf{E}_i(x,y) = \hat{\mathbf{z}} E_{i1} e^{-j\beta_1(y\sin\theta_i + x\cos\theta_i)} \quad [\text{V/m}]$$

$$\mathbf{H}_i(x,y) = \frac{E_{i1}}{\eta_1}(-\hat{\mathbf{y}}\cos\theta_i + \hat{\mathbf{x}}\sin\theta_i) e^{-j\beta_1(y\sin\theta_i + x\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

From these, because the tangential components of E_i and E_r are in opposite directions (see **Figure 13.12b**) $\Gamma = -1$, $E_{r1} = -E_{i1}$, and the reflected fields are

$$\mathbf{E}_r(x,y) = -\hat{\mathbf{z}} E_{i1} e^{-j\beta_1(y\sin\theta_i - x\cos\theta_i)} \quad [\text{V/m}]$$

$$\mathbf{H}_r(x,y) = -\frac{E_{i1}}{\eta_1}(\hat{\mathbf{y}}\cos\theta_i + \hat{\mathbf{x}}\sin\theta_i) e^{-j\beta_1(y\sin\theta_i - x\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

and the total fields in air are

$$\mathbf{E}_1(x,y) = -\hat{\mathbf{z}}j2E_{i1}\sin(\beta_1 x\cos\theta_i)e^{-j\beta_1(y\sin\theta_i)} \quad [\text{V/m}]$$

$$\mathbf{H}_1(x,y) = -2\frac{E_{i1}}{\eta_1}[\hat{\mathbf{y}}\cos\theta_i\cos(\beta_1 x\cos\theta_i) + \hat{\mathbf{x}}j\sin\theta\sin(\beta_1 x\cos\theta_i)]e^{-j\beta_1(y\sin\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

With the values given ($\beta_1 = \beta_0 = 2\pi f/c = 2\pi \times 3 \times 10^9/3 \times 10^8 = 20\pi$ [rad/m], $\eta_1 = \eta_0 = 377\ \Omega$, $E_{i1} = 100$ V/m), the fields are

$$\mathbf{E}_1(x,y) = -\hat{\mathbf{z}}j200\sin(17.32\pi x)e^{-j10\pi y} \quad [\text{V/m}]$$

$$\mathbf{H}_1(x,y) = -0.265[\hat{\mathbf{y}}\,1.732\cos(17.32\pi x) + \hat{\mathbf{x}}j\sin(17.32\pi x)]e^{-j10\pi y} \quad [\text{A/m}].$$

(b) The time-averaged power density is as in **Eq. (13.84)** except that the propagation is in the y direction:

$$\mathcal{P}_{av} = \frac{1}{2}\text{Re}\{\mathbf{E}\times\mathbf{H}^*\} = \hat{\mathbf{y}}\frac{2E_{i1}^2}{\eta_1}\sin^2(\beta_1 x\cos\theta_i)\sin\theta_i \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

With the given values, we get

$$\mathcal{P}_{av} = \hat{\mathbf{y}}\,26.53\sin^2(17.32\pi x) \quad [\text{W/m}^2]$$

Exercise 13.5 For the oblique incidence described in this section (perpendicular polarization), calculate for the incident wave shown in **Figure 13.10b**:

(a) The phase velocities of the wave in the x direction (along the conductor's surface), z direction (perpendicular to the conductor's surface), and $\hat{\mathbf{p}}_i$ direction (along the direction of propagation of the incident wave).
(b) What are the three velocities if $\theta_i \to 0$?

Answer **(a)** $v_x = c/\sin\theta_i$, $v_z = c/\cos\theta_i$, $v_{pi} = c$ [m/s]. **(b)** $v_x \to \infty$, $v_z \to c$, $v_{pi} = c$ [m/s].

Exercise 13.6 A perpendicularly polarized plane wave with amplitude E_0 [V/m] propagates in a perfect dielectric and impinges on a perfectly conducting surface as in **Figure 13.13**. The intrinsic impedance of material (1) is η_1 [Ω] and the phase constant is β_1 [rad/m].

(a) Calculate the total electric and magnetic fields in the dielectric.
(b) Calculate the time-averaged power density in the dielectric.

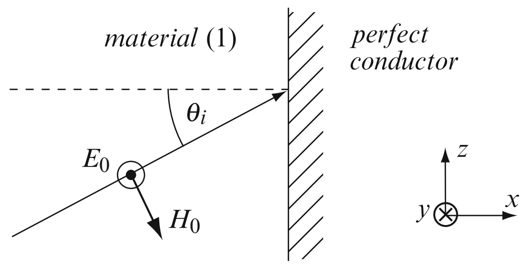


Figure 13.13 A perpendicularly polarized wave impinging on a perfect conductor from a dielectric medium

Answer

(a) $\mathbf{E}_1(x,z) = -\hat{\mathbf{y}}\, j2E_0 \sin(\beta_1 x\cos\theta_i) e^{-j\beta_1(z\sin\theta_i)} \quad [\mathrm{V/m}]$

$$\mathbf{H}_1(x,y) = -2\frac{E_0}{\eta_1}[\hat{\mathbf{z}}\cos\theta_i\cos(\beta_1 x\cos\theta_i) + \hat{\mathbf{x}}\, j\sin\theta_i\sin(\beta_1 x\cos\theta_i)]e^{-j\beta_1(z\sin\theta_i)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right].$$

(b) $\mathcal{P}_{av} = \hat{\mathbf{z}}\,\frac{2E_0^2}{\eta_1}\sin^2(\beta_1 x\cos\theta_i)\sin\theta_i \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$

13.3.2 Oblique Incidence on a Conducting Interface: Parallel Polarization

The discussion of the previous section applies here as well, but the components of the fields are different. The electric field intensity now lies in the incidence plane and, therefore, will have x and z components, whereas the magnetic field intensity is perpendicular to the plane of incidence (y direction in **Figure 13.14**). The direction of propagation of the incident and reflected waves is identical to those in **Eqs. (13.68)** and **(13.69)**. Also, the phases of the incident and reflected waves are as in **Eqs. (13.71)** and **(13.72)**. However, the incident electric field now has two components; one is in the positive x direction, the second in the negative z direction. Note that once we choose the direction of **E**, the direction of **H** must be such that the cross product between **E** and **H** is in the direction of propagation. From these considerations and from **Figure 13.14**, the incident electric and magnetic field intensities are

$$\mathbf{E}_i(x,z) = E_{i1}(\hat{\mathbf{x}}\cos\theta_i - \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad [\mathrm{V/m}] \tag{13.85}$$

$$\mathbf{H}_i(x,z) = \hat{\mathbf{y}}\,\frac{E_{i1}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.86}$$

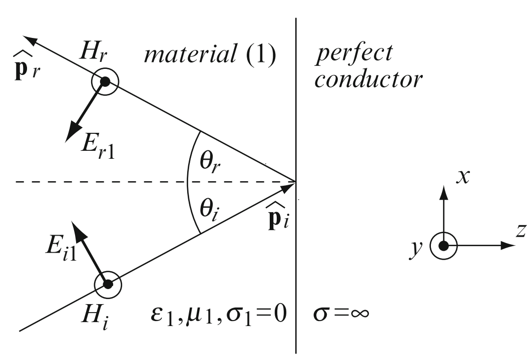


Figure 13.14 Incident and reflected waves for oblique incidence at a dielectric–conductor interface: parallel polarization

Similarly, the reflected electric field intensity has components in the negative x and negative z directions, and the reflected magnetic field intensity is still in the positive y direction. The reflected fields are assumed to be

$$\mathbf{E}_r(x,z) = E_{r1}(-\hat{\mathbf{x}}\cos\theta_i - \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad [\text{V/m}] \tag{13.87}$$

$$\mathbf{H}_r(x,z) = \hat{\mathbf{y}}\,\frac{E_{r1}}{\eta_1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.88}$$

where, again, we used the relation $\theta_r = \theta_i$.

At the conducting interface, the tangential components of the electric field intensity $\mathbf{E}(x,0) = \mathbf{E}_i(x,0) + \mathbf{E}_r(x,0)$ must be zero. Setting $z = 0$ in **Eqs. (13.85)** and **(13.87)** and summing the tangential (x) components of **E**, we get

$$E_{i1}(\hat{\mathbf{x}}\cos\theta_i)e^{-j\beta_1(x\sin\theta_i)} + E_{r1}(-\hat{\mathbf{x}}\cos\theta_i)e^{-j\beta_1(x\sin\theta_i)} = 0 \quad \rightarrow \quad E_{r1} = E_{i1} \tag{13.89}$$

Substituting this in **Eqs. (13.87)** and **(13.88)**, and summing the incident and reflected waves in **Eqs. (13.85)** and **(13.87)** for the total electric field intensity and **Eqs. (13.86)** and **(13.88)** for the total magnetic field intensity, we get the total fields in material (1) as the sum of the incident and reflected fields:

$$\mathbf{E}_1(x,z) = -2E_{i1}[\hat{\mathbf{x}}\,j\cos\theta_i\sin(\beta_1 z\cos\theta_i) + \hat{\mathbf{z}}\,\sin\theta_i\cos(\beta_1 z\cos\theta_i)]e^{-j\beta_1 x\sin\theta_i} \quad [\text{V/m}] \tag{13.90}$$

$$\mathbf{H}_1(x,z) = \hat{\mathbf{y}}\,2\frac{E_{i1}}{\eta_1}\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.91}$$

As with perpendicular polarization, the wave propagating in the z direction consists of E_{1x} and H_{1y} and these are out of phase. Therefore, we have a standing wave, oscillating exactly as for the perpendicular polarization. The wave in the x direction is a propagating wave (see **Exercise 13.7**). Again, as for perpendicular polarization, the propagating wave is not a uniform plane wave since its amplitude depends on z (see **Exercise 13.7**).

Exercise 13.7

(a) Show that **Eq. (13.90)** is the sum of **Eqs. (13.85)** and **(13.87),** and **Eq. (13.91)** is the sum of **Eqs. (13.86)** and **(13.88)**.

(b) Calculate the time-averaged Poynting vector for the total fields and show that waves propagate in the positive x direction and waves in the z direction are standing waves.

Answer **(b)** $\mathcal{P}_{av}(x,z) = \dfrac{\text{Re}\left(\mathbf{E}_1(x,z)\times\mathbf{H}_1^*(x,z)\right)}{2} = \hat{\mathbf{x}}\,\dfrac{2E_{i1}^2}{\eta_1}\cos^2(\beta_1 z\cos\theta_i)\sin\theta_i \quad \left[\dfrac{\text{W}}{\text{m}^2}\right].$

Exercise 13.8 A wave propagates in a perfect dielectric and impinges on a perfectly conducting surface as in **Figure 13.15**. Calculate the total electric and magnetic fields in the dielectric.

Answer

$$\mathbf{E}_1(x,z) = 2E_{i1}[\hat{\mathbf{x}}\,\sin\theta_i\cos(\beta_1 x\cos\theta_i) + \hat{\mathbf{z}}\,j\cos\theta_i\sin(\beta_1 x\cos\theta_i)]e^{-j\beta_1 z\sin\theta_i} \quad [\text{V/m}]$$

$$\mathbf{H}_1(x,z) = \hat{\mathbf{y}}\,2\frac{E_{i1}}{\eta_1}\cos(\beta_1 x\cos\theta_i)e^{-j\beta_1 z\sin\theta_i} \quad \left[\frac{\text{A}}{\text{m}}\right].$$

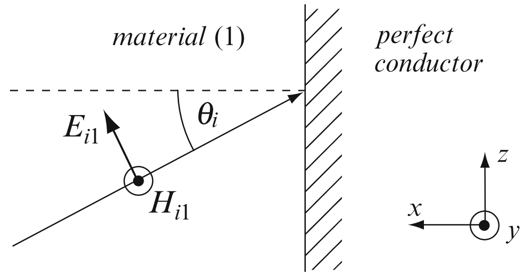


Figure 13.15 Obliquely incident wave on a conducting interface (parallel polarization)

13.4 Oblique Incidence on Dielectric Interfaces

Based on the previous two sections, oblique incidence on a dielectric should be quite similar: an incident wave gives rise to a reflected wave, both propagating in the same material. However, unlike the perfect conductors in the previous two sections, there is also a wave propagating in material (2) in a fashion similar to that of **Section** 13.2, but, since the incident wave is at an angle θ_i to the normal, we expect the wave in material (2) to also propagate at an angle to the normal. These considerations are shown in **Figure** 13.16. The reflection angle θ_r and refraction angle θ_t both depend on the incident angle θ_i. We have already shown in **Section** 13.3.1 that $\theta_r = \theta_i$ [see **Eq. (**13.78**)]** and will use this relation (actually Snell's law of reflection) from now on without comment. To be able to describe all wave properties in terms of the incident wave alone, we must also define a relation between the refraction angle θ_t and the incidence angle θ_i. Also, we must expect that the reflection and transmission coefficient should be different than those obtained for normal incidence. We start with perpendicularly polarized waves.

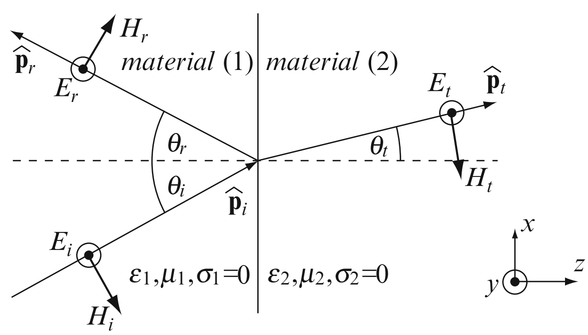


Figure 13.16 Incident, reflected, and refracted waves for oblique incidence at a dielectric–dielectric interface. Perpendicular polarization

13.4.1 Oblique Incidence on a Dielectric Interface: Perpendicular Polarization

To define the conditions for the reflection and transmitted waves, we use **Figure** 13.16, write the electric and magnetic field intensities on both sides and apply the boundary conditions on the interface for the tangential components of the electric field intensity. Now, we can return to the electric and magnetic fields on both sides of the interface in **Figure** 13.16. The incident electric and magnetic field intensities are the same as **Eqs. (**13.73**)** and **(**13.74**)**:

$$\mathbf{E}_i(x,z) = \hat{\mathbf{y}}\, E_{i1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad [\mathrm{V/m}] \tag{13.92}$$

$$\mathbf{H}_i(x,z) = \frac{E_{i1}}{\eta_1}(-\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.93}$$

where, again, the direction of propagation of the incident wave is given by $\hat{\mathbf{p}}_i = \hat{\mathbf{x}} \sin\theta_i + \hat{\mathbf{z}} \cos\theta_i$. The reflected electric and magnetic field intensities are

$$\mathbf{E}_r(x,z) = \hat{\mathbf{y}}\, E_{r1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad [\text{V/m}] \tag{13.94}$$

$$\mathbf{H}_r(x,z) = \frac{E_{r1}}{\eta_1}(\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.95}$$

Similarly, the transmitted electric and magnetic field intensities have the same form as the incident wave but with different amplitudes and propagate at a different angle (see **Figure 13.16**):

$$\mathbf{E}_t(x,z) = \hat{\mathbf{y}}\, E_{t2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad [\text{V/m}] \tag{13.96}$$

$$\mathbf{H}_t(x,z) = \frac{E_{t2}}{\eta_2}(-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.97}$$

To determine the transmission and reflection coefficients, the tangential components of the electric field intensity and those of the magnetic field intensity on both sides of the interface (i.e., at $z = 0$) are equated. From **Figure 13.16** and **Eqs. (13.92)** through **(13.97)**, and taking only the tangential components (y component for **E** and x component for **H**) at $z = 0$, we have

$$(E_{i1} + E_{r1})e^{-j\beta_1 x\sin\theta_i} = E_{t2}e^{-j\beta_2 x\sin\theta_t} \quad \text{and} \quad \left(-\frac{E_{i1}}{\eta_1} + \frac{E_{r1}}{\eta_1}\right)\cos\theta_i e^{-j\beta_1 x\sin\theta_i} = -\frac{E_{t2}}{\eta_2}\cos\theta_t e^{-j\beta_2 x\sin\theta_t} \tag{13.98}$$

There are three relations that must be satisfied:

$$e^{-j\beta_1 x\sin\theta_i} = e^{-j\beta_2 x\sin\theta_t} \quad \text{or} \quad \beta_1\sin\theta_i = \beta_2\sin\theta_t \tag{13.99}$$

and

$$E_{i1} + E_{r1} = E_{t2} \quad \text{and} \quad -\frac{E_{i1}}{\eta_1}\cos\theta_i + \frac{E_{r1}}{\eta_1}\cos\theta_i = -\frac{E_{t2}}{\eta_2}\cos\theta_t \tag{13.100}$$

From **Eq. (13.99)**, we get

$$\omega\sqrt{\varepsilon_1\mu_1}\sin\theta_i = \omega\sqrt{\varepsilon_2\mu_2}\sin\theta_t \tag{13.101}$$

or

$$\boxed{\sin\theta_t = \frac{\sqrt{\varepsilon_1\mu_1}}{\sqrt{\varepsilon_2\mu_2}}\sin\theta_i} \tag{13.102}$$

This relation between the incident and refraction angle is Snell's law of refraction. Since $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$, $\varepsilon_2 = \varepsilon_0\varepsilon_{r2}$, $\mu_1 = \mu_0\mu_{r1}$ and $\mu_2 = \mu_0\mu_{r2}$ (where ε_{r1}, ε_{r2}, μ_{r1}, μ_{r2} are the relative permittivities and relative permeabilities of the two media), and since the phase velocities in medium (1) and (2) are $v_{p1} = 1/\sqrt{\varepsilon_1\mu_1}$ and $v_{p2} = 1/\sqrt{\varepsilon_2\mu_2}$ respectively, we can also write Snell's law of refraction as

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{v_{p2}}{v_{p1}}} \tag{13.103}$$

where $n_1 = \sqrt{\varepsilon_{r1}\mu_{r1}}$ is the (optical) index of refraction in medium (1) and $n_2 = \sqrt{\varepsilon_{r2}\mu_{r2}}$ is the (optical) index of refraction in medium (2).

Now returning to **Eq. (13.100)**, the solution of the two relations for E_{r1} and E_{t2} gives

$$E_{r1} = E_{i1}\frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}, \qquad E_{t2} = E_{i1}\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right] \tag{13.104}$$

Because E_{r1} and E_{i1} are in the same direction, the reflection coefficient may be written as $\Gamma_\perp = E_{r1}/E_{i1}$ and the transmission coefficient as $T_\perp = E_{t1}/E_{i1}$:

$$\boxed{\Gamma_\perp = \frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad [\text{dimensionless}]} \tag{13.105}$$

$$\boxed{T_\perp = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad [\text{dimensionless}]} \tag{13.106}$$

The notation $\perp$ indicates these are the reflection and transmission coefficients for perpendicular polarization, because, as we shall see, the coefficients for parallel polarization differ.

Now, the total fields in each material can be written directly. In material (1), the fields are the sum of the incident and reflected waves [from **Eqs. (13.92)** and **(13.94)** for E_1 and from **Eqs. (13.93)** and **(13.95)** for H_1]:

$$\mathbf{E}_1(x,z) = \hat{\mathbf{y}}E_{i1}\left[e^{-j\beta_1 z\cos\theta_i} + \Gamma_\perp e^{j\beta_1 z\cos\theta_i}\right]e^{-j\beta_1 x\sin\theta_i} \quad [\mathrm{V/m}] \tag{13.107}$$

$$\begin{aligned}\mathbf{H}_1(x,z) = {} & \hat{\mathbf{x}}\frac{E_{i1}\cos\theta_i}{\eta_1}\left[\Gamma_\perp e^{j\beta_1 z\cos\theta_i} - e^{-j\beta_1 z\cos\theta_i}\right]e^{-j\beta_1 x\sin\theta_i} \\ & + \hat{\mathbf{z}}\frac{E_{i1}\sin\theta_i}{\eta_1}\left[e^{-j\beta_1 z\cos\theta_i} + \Gamma_\perp e^{j\beta_1 z\cos\theta_i}\right]e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]\end{aligned} \tag{13.108}$$

In medium (2), where the only wave is the transmitted wave, **Eqs. (13.96)** and **(13.97)** describe the wave. Using the transmission coefficient, we can write

$$\mathbf{E}_t(x,z) = \hat{\mathbf{y}}T_\perp E_{i1}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad [\mathrm{V/m}] \tag{13.109}$$

$$\mathbf{H}_t(x,z) = \frac{T_\perp E_{i1}}{\eta_2}(-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.110}$$

In all these relations, we could also use **Eq. (13.102)** to write the refraction angle θ_t in terms of the incident angle θ_i. However, this would complicate the expressions considerably.

The electric field intensity $\mathbf{E}_1$ is in the y direction, but $\mathbf{H}_1$ has a component in the x and z directions. As was the case for conducting interfaces, we have a propagating wave in the x direction and a standing wave in the z direction (see **Exercise 13.9**).

Exercise 13.9 Show that the wave in material (1) in **Figure 13.16** propagates in the positive x direction, whereas in the z direction, there are both a standing wave and a propagating wave, by calculating the time-averaged Poynting vector in material (1).

Exercise 13.10 Show that the wave in material (2) in **Figure 13.16** propagates in the x and z directions and that there are no standing waves in this region, by calculating the Poynting vector.

Exercise 13.11 Show that for perpendicular polarization the relation ($1 + \Gamma_\perp = T_\perp$) holds.

Example 13.8 A perpendicularly polarized plane wave impinges on a very thick sheet of plastic at an angle $\theta_i = 30^\circ$ from free space, as shown in **Figure 13.16**. The relative permittivity of the plastic is $\varepsilon_r = 4$ and its relative permeability is $\mu_r = 1$. If the amplitude of the incident electric field intensity is $E_{i1} = 100$ V/m, calculate the time-averaged power density transmitted into the plastic.

Solution: First, we express the refraction angle and transmission coefficient in terms of the incident angle θ_i. Then, using **Eqs. (13.109)** and **(13.110)**, we calculate the time-averaged power density in material (2), which should have both x and z components.

The reflection and refraction angles are evaluated from the following relations:

$$\theta_r = \theta_i, \qquad \sin\theta_t = \sin\theta_i \frac{n_1}{n_2} = \sin\theta_i \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \sin\theta_i \sqrt{\frac{1}{\varepsilon_{r2}}} = 0.5\sqrt{\frac{1}{4}} = 0.25$$

The transmission coefficient for perpendicular polarization **[Eq. (13.106)]** with $\mu_1 = \mu_2 = \mu_0$ is

$$T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\sqrt{\mu_0/\varepsilon_2}\cos\theta_i}{\sqrt{\mu_0/\varepsilon_2}\cos\theta_i + \sqrt{\mu_0/\varepsilon_0}\cos\theta_t} = \frac{2\sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_0}\cos\theta_i + \sqrt{\varepsilon_0\varepsilon_{r2}}\cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + 2\cos\theta_t}$$

Rewriting $\cos\theta_t$ in terms of $\sin\theta_t$ as $\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\sin^2\theta_i/\varepsilon_{r2}}$, and then substituting the values for θ_i and $\sin\theta_t$, we get

$$T_\perp = \frac{2\cos\theta_i}{\cos\theta_i + 2\sqrt{1-\sin^2\theta_i/\varepsilon_{r2}}} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{4-\sin^2\theta_i}} = \frac{1.732}{0.866+\sqrt{4-0.25}} = 0.618$$

Now, the transmitted electric and magnetic field intensity may be calculated using **Eqs. (13.109)** and **(13.110)**. However, we are interested in the time-averaged
Poynting vector:

$$\begin{aligned}\mathcal{P}_{av} &= \frac{1}{2}\text{Re}\{\mathbf{E}\times\mathbf{H}^*\} = \hat{\mathbf{y}}\,\frac{T_\perp E_{i1}}{2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \times \frac{T_\perp E_{i1}}{2}(-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)e^{j\beta_2(x\sin\theta_t + z\cos\theta_t)} \\ &= \frac{T_\perp^2 E_{i1}^2}{2\eta_2}\hat{\mathbf{y}} \times (-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t) = \frac{T_\perp^2 E_{i1}^2}{2\eta_2}(-\hat{\mathbf{z}}\cos\theta_t + \hat{\mathbf{x}}\sin\theta_t) \quad \left[\frac{\text{W}}{\text{m}^2}\right]\end{aligned}$$

where $\eta_2 = \eta_0/2 = 377/2\ \Omega$. With $\sin\theta_t = 0.25$, $\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-(0.25)^2} = 0.968$, the time-averaged power density is

$$\mathcal{P}_{av} = \frac{0.618^2 \times 100^2}{377}(\hat{\mathbf{z}}\,0.968 + \hat{\mathbf{x}}\,0.25) = \hat{\mathbf{x}}\,2.533 + \hat{\mathbf{z}}\,9.8 \qquad \left[\frac{\text{W}}{\text{m}^2}\right]$$

13.4.2 Oblique Incidence on a Dielectric Interface: Parallel Polarization

The situation considered here is shown in **Figure 13.17a**. The incident electric field intensity is parallel to the plane of incidence and the magnetic field intensity is perpendicular in the y direction so that the incident wave propagates toward the interface. The directions of the reflected fields in **Figure 13.17a** are assumed. The correct directions are found from the interface conditions that must be satisfied.

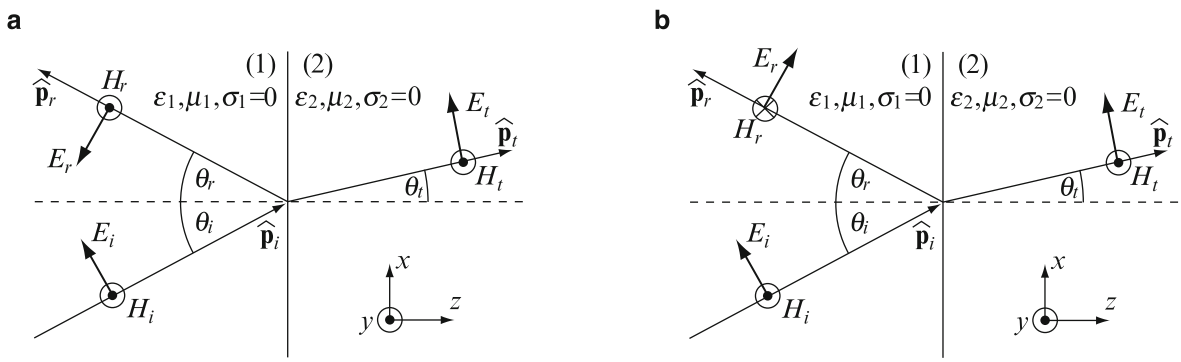


Figure 13.17 (**a**) Assumed incident, reflected, and refracted waves for oblique incidence at a dielectric–dielectric interface. The electric field is polarized parallel to the plane of incidence. (**b**) Correct, calculated waves (for $\Gamma_{\|}$ positive)

The incident and reflected electric and magnetic field intensities for the configuration in **Figure 13.17a** are as given in **Eqs. (13.85)** through **(13.88)**:

$$\mathbf{E}_i(x,z) = E_{i1}(\hat{\mathbf{x}}\cos\theta_i - \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad [\mathrm{V/m}] \tag{13.111}$$

$$\mathbf{H}_i(x,z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.112}$$

$$\mathbf{E}_r(x,z) = E_{r1}(-\hat{\mathbf{x}}\cos\theta_i - \hat{\mathbf{z}}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad [\mathrm{V/m}] \tag{13.113}$$

$$\mathbf{H}_r(x,z) = \hat{\mathbf{y}}\frac{E_{r1}}{\eta_1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.114}$$

The transmitted wave into material (2) can be written directly from **Figure 13.17a**:

$$\mathbf{E}_t(x,z) = E_{t1}(\hat{\mathbf{x}}\cos\theta_t - \hat{\mathbf{z}}\sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad [\mathrm{V/m}] \tag{13.115}$$

$$\mathbf{H}_t(x,z) = \hat{\mathbf{y}}\frac{E_{t2}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.116}$$

At the interface between the two media (at $z = 0$), the continuity conditions on the tangential components of the electric and magnetic field intensities are

$$E_{i1}\cos\theta_i - E_{r1}\cos\theta_i = E_{t2}\cos\theta_t \quad \text{and} \quad \frac{E_{i1}}{\eta_1} + \frac{E_{r1}}{\eta_1} = \frac{E_{t2}}{\eta_2} \tag{13.117}$$

Solving for E_{r1} and E_{t2}, we get

$$E_{r1} = E_{i1}\frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}, \qquad E_{t2} = E_{i1}\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \tag{13.118}$$

To define the reflection coefficient for parallel polarization, we note from **Figure 13.17a** and from **Eq. (13.117)** that E_{i1} and E_{r1} are in opposite directions. Therefore, the reflection coefficient for parallel polarization is defined as

$$\boxed{\Gamma_{\parallel} = -\frac{E_{r1}}{E_{i1}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \quad [\text{dimensionless}]} \tag{13.119}$$

On the other hand, E_{t2} and E_{i1} are in the same direction and therefore the transmission coefficient is

$$\boxed{T_{\parallel} = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \quad [\text{dimensionless}]} \tag{13.120}$$

The total fields in medium (1) are calculated by summing the incident and reflected waves. With the use of the reflection coefficient (i.e., using $E_{r1} = -\Gamma_{\parallel} E_{i1}$), these become

$$\mathbf{E}_1(x,z) = \hat{\mathbf{x}} E_{i1} \cos\theta_i \left(\Gamma_{\parallel} e^{j\beta_1 z\cos\theta_i} + e^{-j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i} + \hat{\mathbf{z}} E_{i1} \sin\theta_i \left(\Gamma_{\parallel} e^{j\beta_1 z\cos\theta_i} - e^{-j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i} \quad [\text{V/m}] \tag{13.121}$$

$$\mathbf{H}_1(x,z) = -\hat{\mathbf{y}} \frac{E_{i1}}{\eta_1} \left(\Gamma_{\parallel} e^{j\beta_1 z\cos\theta_i} - e^{-j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i} \quad [\text{V/m}] \tag{13.122}$$

Using $E_{t2} = T_{\parallel} E_{i1}$ in **Eqs. (13.115)** and **(13.116)**, we get the fields in medium (2):

$$\mathbf{E}_t(x,z) = T_{\parallel} E_{i1} (\hat{\mathbf{x}} \cos\theta_t - \hat{\mathbf{z}} \sin\theta_t) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \quad [\text{V/m}] \tag{13.123}$$

$$\mathbf{H}_t(x,z) = \hat{\mathbf{y}} \frac{T_{\parallel} E_{i1}}{\eta_2} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \quad \left[\frac{\text{A}}{\text{m}}\right] \tag{13.124}$$

Because of the relation between the incident and reflected electric fields, the direction of the reflected electric field intensity in **Figure 13.17a** must be reversed if $\Gamma_{\parallel}$ is positive. The correct electric and magnetic field intensities are shown in **Figure 13.17b** for $\Gamma_{\parallel}$ positive.

Example 13.9 A plane wave is generated underwater and propagates toward the surface at an angle α as shown in **Figure 13.18a**. Assume lossless conditions (distilled water) with relative permittivity $\varepsilon_r = 25$, relative permeability $\mu_r = 1$ and an amplitude of 1 V/m. The electric field intensity is in the plane as shown. Calculate:

(a) The electric and magnetic field intensities in air for an incidence angle $\alpha = 5^{\circ}$.
(b) The incident instantaneous and time-averaged power densities just below the water surface.
(c) The transmitted instantaneous and time-averaged power densities in air immediately above the water's surface.

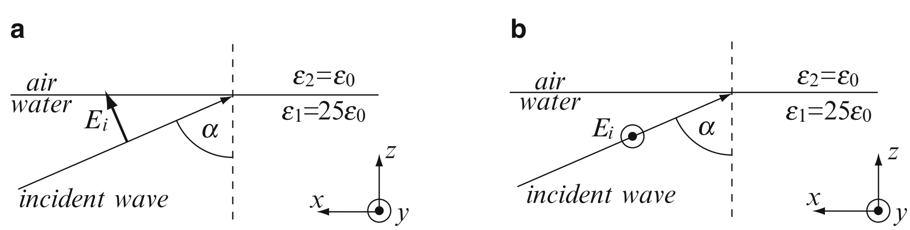


Figure 13.18 A wave propagating from water into air. (**a**) Parallel polarization. (**b**) Perpendicular polarization

Solution: The electric field intensity is parallel to the plane of incidence as shown in **Figure 13.18a**. The polarization is therefore parallel. At the interface, we use the transmission coefficient to find the electric and magnetic field intensities in air, using the relations for parallel polarization.

(a) To define the electric and magnetic fields, we need to calculate the intrinsic impedances in air and water, the transmission coefficient, and the transmission angle θ_t. We know the incident angle: $\theta_i = \alpha = 5^{\mathrm{o}}$. The intrinsic impedances are

$$\eta_2 = \eta_0 = 377\ \Omega\ \text{in air}, \quad \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{25}} = 75.4\ \Omega\ \text{in water}$$

The relation between the incidence and refraction angle is given in **Eq. (13.102)** for any polarization:

$$\sin\theta_t = \frac{n_{water}}{n_{air}}\sin\alpha = \frac{\sqrt{25}}{1}\sin\alpha = 5\sin\alpha$$

where the refraction angle is larger than the incident angle as expected for propagation from a high- to a low-permittivity dielectric. For the given incidence angle, the transmission angle is $\theta_t = \sin^{-1}(5\sin\alpha) = 25.83^{\mathrm{o}}$ $(25°50')$. The transmission coefficient is

$$T_{\|} = \frac{2\eta_2\cos\alpha}{\eta_2\cos\theta_t + \eta_1\cos\alpha} = \frac{2\eta_0\cos\alpha}{\eta_0\sqrt{1-25\sin^2\alpha} + (\eta_0/5)\cos\alpha} = \frac{10\cos\alpha}{5\sqrt{1-25\sin^2\alpha}+\cos\alpha} = \frac{10\cos 5°}{5\sqrt{1-25\sin^2 5°}+\cos 5°} = 1.812$$

where the relations $\cos\theta_t = \sqrt{1-\sin^2\theta_t}$ and $\eta_1 = \eta_0/5$ were used.

The electric and magnetic field intensities generated underwater are known and are of the same general form as in **Eqs. (13.111)** and **(13.112)**, although directions of individual components are different. Substituting the intrinsic impedance of water ($\eta_0/5$) and using the coordinate system in **Figure 13.18a**, the incident electric and magnetic field intensities are

$$\mathbf{E}_i(x,z) = 1(\hat{\mathbf{x}}\cos\alpha + \hat{\mathbf{z}}\sin\alpha)e^{-j\beta_1(-x\sin\alpha+z\cos\alpha)} \quad [\text{V/m}]$$

$$\mathbf{H}_i(x,z) = \hat{\mathbf{y}}\frac{5}{\eta_0}e^{-j\beta_1(-x\sin\alpha+z\cos\alpha)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The transmitted electric and magnetic fields have the same form as the incident fields, but the propagation is at an angle θ_t and the intrinsic impedance is that of free space [see **Eqs. (13.123)** and **(13.124)**]:

$$\mathbf{E}_t(x,z) = T_{\|}(\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)e^{-j\beta_2(-x\sin\theta_t+z\cos\theta_t)} \quad [\text{V/m}]$$

$$\mathbf{H}_t(x,z) = \hat{\mathbf{y}}\frac{T_{\|}}{\eta_0}e^{-j\beta_2(-x\sin\theta_t+z\cos\theta_t)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

With $\sin\theta_t = 5\sin\alpha$, $T_{\|} = 1.812$, $E_{i1} = 1$ V/m, $\cos\theta_t = \sqrt{1-25\sin^2\alpha}$, we get

$$\mathbf{E}_t(x,z) = 1.812\left(\hat{\mathbf{x}}\sqrt{1-25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right)e^{-j\beta_2\left(-x5\sin\alpha+z\sqrt{1-25\sin^2\alpha}\right)} \quad [\text{V/m}]$$

$$\mathbf{H}_t(x,z) = \hat{\mathbf{y}}\frac{1.812}{377}e^{-j\beta_2\left(-x5\sin\alpha+z\sqrt{1-25\sin^2\alpha}\right)} \quad \left[\frac{\text{A}}{\text{m}}\right].$$

(b) To calculate the incident, instantaneous Poynting vector, we must first write the time-dependent fields:

$$\mathbf{E}_i(x,z,t) = \text{Re}\{\mathbf{E}_i(x,z)e^{j\omega t}\} = (\hat{\mathbf{x}}\cos\alpha + \hat{\mathbf{z}}\sin\alpha)\cos(\omega t - \beta_1(-x\sin\alpha + z\cos\alpha)) \quad [\text{V/m}]$$

$$\mathbf{H}_i(x,z,t) = \text{Re}\{\mathbf{H}_i(x,z)e^{j\omega t}\} = \hat{\mathbf{y}}\frac{5}{\eta_0}\cos(\omega t - \beta_1(-x\sin\alpha + z\cos\alpha)) \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The instantaneous power density at the surface is given by the Poynting vector at $z = 0$:

$$\begin{aligned}\boldsymbol{\mathcal{P}}_i(x,0,t) &= \mathbf{E}_i(x,0,t) \times \mathbf{H}_i(x,0,t) \\ &= (\hat{\mathbf{x}}\cos\alpha + \hat{\mathbf{z}}\sin\alpha)\cos(\omega t + \beta_1 x\sin\alpha) \times \hat{\mathbf{y}}\frac{5}{\eta_0}\cos(\omega t + \beta_1 x\sin\alpha) \\ &= \hat{\mathbf{z}}\frac{5\cos\alpha}{\eta_0}\cos^2(\omega t + \beta_1 x\sin\alpha) - \hat{\mathbf{x}}\frac{5\sin\alpha}{\eta_0}\cos^2(\omega t + \beta_1 x\sin\alpha) \\ &= \left(\hat{\mathbf{z}}\,1.32 \times 10^{-2} - \hat{\mathbf{x}}\,1.16 \times 10^{-3}\right)\cos^2(\omega t + \beta_1 x\sin\alpha) \quad \left[\frac{\text{W}}{\text{m}^2}\right]\end{aligned}$$

Note that the phase of the instantaneous power varies with x as the wave propagates in this direction. It also varies with z, but because we calculated the power density at the surface ($z = 0$), this variation is not shown.

The time-averaged power density may be calculated by integration over one cycle of the wave or using the complex Poynting vector and the fields in **(a)**. The latter is simpler:

$$\begin{aligned}\boldsymbol{\mathcal{P}}_{iav}(x,0) &= \frac{1}{2}\text{Re}\{\mathbf{E}_i(x,0) \times \mathbf{H}_i^*(x,0)\} = \frac{1}{2}\text{Re}\left\{(\hat{\mathbf{x}}\cos\alpha + \hat{\mathbf{z}}\sin\alpha)e^{-j\beta_1(-x\sin\alpha)} \times \hat{\mathbf{y}}\frac{5}{\eta_0}e^{j\beta_1(-x\sin\alpha)}\right\} \\ &= \hat{\mathbf{z}}\frac{5\cos\alpha}{2\eta_0} - \hat{\mathbf{x}}\frac{5\sin\alpha}{2\eta_0} = \hat{\mathbf{z}}\,0.66 \times 10^{-2} - \hat{\mathbf{x}}\,0.52 \times 10^{-3} \quad \left[\frac{\text{W}}{\text{m}^2}\right].\end{aligned}$$

(c) To calculate the transmitted power (instantaneous or time averaged) at the surface, we use the same steps as in **(b)**, but now we must use the equations for the transmitted wave given in **(a)**. The time-dependent forms (at $z = 0$) are

$$\mathbf{E}_i(x,0,t) = 1.812\left(\hat{\mathbf{x}}\sqrt{1 - 25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right)\cos(\omega t - \beta_2(-5x\sin\alpha)) \quad [\text{V/m}]$$

$$\mathbf{H}_t(x,0,t) = \hat{\mathbf{y}}\frac{1.812}{\eta_0}\cos(\omega t - \beta_2(-5x\sin\alpha)) \quad \left[\frac{\text{A}}{\text{m}}\right]$$

The transmitted instantaneous power density is

$$\begin{aligned}\boldsymbol{\mathcal{P}}_t(x,0,t) &= \mathbf{E}_t(x,0,t) \times \mathbf{H}_t(x,0,t) \\ &= 1.812\left(\hat{\mathbf{x}}\sqrt{1 - 25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right)\cos(\omega t - \beta_2(-5x\sin\alpha)) \times \hat{\mathbf{y}}\frac{1.812}{\eta_0}\cos(\omega t - \beta_2(-5x\sin\alpha)) \\ &= \hat{\mathbf{z}}\frac{1.812^2 \times \sqrt{1 - 25\sin^2\alpha}}{\eta_0}\cos^2(\omega t + 5\beta_2 x\sin\alpha) - \hat{\mathbf{x}}\frac{1.812^2 \times 5\sin\alpha}{\eta_0}\cos^2(\omega t + 5\beta_2 x\sin\alpha) \\ &= \left(\hat{\mathbf{z}}\,7.84 \times 10^{-3} - \hat{\mathbf{x}}\,3.79 \times 10^{-2}\right)\cos^2(\omega t + 5\beta_2 x\sin\alpha) \quad \left[\frac{\text{W}}{\text{m}^2}\right]\end{aligned}$$

The transmitted time-averaged power density (calculated at $z = 0$) is

$$\mathcal{P}_{tav}(x,0) = \frac{1}{2}\text{Re}\{\mathbf{E}_t(x,0) \times \mathbf{H}_t^*(x,0)\}$$
$$= \frac{1}{2}\text{Re}\left\{1.812\left(\hat{\mathbf{x}}\sqrt{1-25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right)e^{-j\beta_2(-x9\sin x)} \times \hat{\mathbf{y}}\frac{1.812}{\eta_0}e^{j\beta_2(-x9\sin x)}\right\}$$
$$= \frac{1.812}{2}\left(\hat{\mathbf{x}}\sqrt{1-25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right) \times \hat{\mathbf{y}}\frac{1.812}{\eta_0} = \frac{1.812^2}{2\times\eta_0}\left(\hat{\mathbf{z}}\sqrt{1-25\sin^2\alpha} - \hat{\mathbf{x}}\,5\sin\alpha\right)$$
$$= \hat{\mathbf{z}}\,3.92\times10^{-3} - \hat{\mathbf{x}}\,1.895\times10^{-3} \quad \left[\frac{\text{W}}{\text{m}^2}\right].$$

Exercise 13.12 Show that the following relations hold:

$$1 + \Gamma_{\|} = T_{\|}\left(\frac{\cos\theta_t}{\cos\theta_i}\right), 0 \le \theta_i \le \pi/2$$
$$|\Gamma_{\|}|^2 < |\Gamma_{\perp}|^2,\ 0 < \theta_i < \pi/2$$
$$\Gamma_{\|} = \Gamma_{\perp} = \Gamma,\ \theta_i = 0$$

Exercise 13.13 Repeat **Example 13.9** for the configuration in **Figure 13.18b**.

Answer

$$\mathbf{E}_t(x,z) = \hat{\mathbf{y}}\,1.67e^{-j\beta_2\left(-x5\sin\alpha + z\sqrt{1-25\sin^2\alpha}\right)} \quad \left[\frac{\text{V}}{\text{m}}\right]$$

$$\mathbf{H}_t(x,z) = \frac{1.67}{\eta_0}\left(-\hat{\mathbf{x}}\sqrt{1-25\sin^2\alpha} + \hat{\mathbf{z}}\,5\sin\alpha\right)e^{-j\beta_2\left(-x5\sin\alpha + z\sqrt{1-25\sin^2\alpha}\right)} \quad \left[\frac{\text{A}}{\text{m}}\right]$$

$$\mathcal{P}_i(x,0,t) = \left(-\hat{\mathbf{x}}\,1.156\times10^{-3} + \hat{\mathbf{z}}\,1.32\times10^{-2}\right)\cos^2(\omega t + \beta_1 x\sin\alpha) \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

$$\mathcal{P}_{iav}(x,0) = -\hat{\mathbf{x}}\,0.578\times10^{-3} + \hat{\mathbf{z}}\,0.66\times10^{-2} \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

$$\mathcal{P}_t(x,0,t) = \left(\hat{\mathbf{z}}\,8.348\times10^{-3} - \hat{\mathbf{x}}\,4.042\times10^{-3}\right)\cos^2(\omega t + 5\beta_2 x\sin\alpha) \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

$$\mathcal{P}_{tav}(x,0) = \hat{\mathbf{z}}\,4.174\times10^{-3} - \hat{\mathbf{x}}\,2.021\times10^{-3} \quad \left[\frac{\text{W}}{\text{m}^2}\right].$$

13.4.3 Brewster's Angle

The reflection and transmission coefficients we obtained in the previous sections depended on the incidence and transmission angle and on the intrinsic impedances of the two materials. A closer inspection of the reflection coefficients and their behavior is now in order. The reflection coefficients for perpendicular and parallel polarizations [from **Eqs. (13.105)** and (**13.119**)] are

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}, \qquad \Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \tag{13.125}$$

Either reflection coefficient is zero if the numerator is zero; that is, $\Gamma_{\perp} = 0$ if $\eta_2\cos\theta_i = \eta_1\cos\theta_t$ and $\Gamma_{\parallel} = 0$ if $\eta_2\cos\theta_t = \eta_1\cos\theta_i$. The angle at which either condition is satisfied is called the ***Brewster angle***. We will now explore these possibilities starting with the reflection coefficient for parallel polarization in lossless dielectrics.

13.4.3.1 Brewster's Angle for Parallel Polarization

For parallel polarization, the reflection coefficient is zero if [from **Eq. (13.125)**]

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_i \tag{13.126}$$

To find the angle θ_i at which this is satisfied, we rewrite sin θ_t in terms of θ_i. From **Eq. (13.102)**, we have

$$\sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i \tag{13.127}$$

Using $\cos\theta_{\rm i} = \sqrt{1-\sin^2\theta_i}$ and $\cos\theta_t = \sqrt{1-\sin^2\theta_t}$, we can write **Eq. (13.126)** as

$$\eta_2\sqrt{1-\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_i} = \eta_1\sqrt{1-\sin^2\theta_i} \tag{13.128}$$

Now, using $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ and $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$, squaring **Eq. (13.128)**, separating $\sin^2\theta_i$, and then taking the square root, we get

$$\sin\theta_b = \sqrt{\frac{\varepsilon_2(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\mu_1({\varepsilon_1}^2 - {\varepsilon_2}^2)}} \tag{13.129}$$

The index b indicates θ_b as the Brewster's angle. This may also be written as

$$\theta_b = \sin^{-1}\sqrt{\frac{\varepsilon_2(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\mu_1({\varepsilon_1}^2 - {\varepsilon_2}^2)}} \tag{13.130}$$

Thus, for any two materials except two materials of identical permittivity ($\varepsilon_1 = \varepsilon_2$), there is a specific angle at which there is no reflected wave. In the particular but very common case in which both materials have the permeability of free space ($\mu_1 = \mu_2 = \mu_0$), the expression for Brewster's angle is greatly simplified:

$$\boxed{\sin\theta_b = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}} \quad \text{or} \quad \theta_b = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}} \quad \text{if} \quad \mu_1 = \mu_2} \tag{13.131}$$

The importance of Brewster's angle is twofold. First, it shows that by proper choice of the angle of incidence, the reflection from a material for a parallel polarized wave can be canceled. Second, if a wave has an electric field intensity which has components parallel and perpendicular to the plane of incidence and if the wave impinges on a material interface at the Brewster's angle, the reflection of the parallel polarized component is canceled but not that of the perpendicularly polarized component. The reflected wave consists of the perpendicularly polarized component of the wave alone. Thus, for any general wave (polarized or unpolarized), the reflected wave at the Brewster angle of incidence is linearly polarized perpendicular to the plane of incidence. For this reason, Brewster's angle is also called a ***polarizing angle***. Because the wave at the Brewster angle is not reflected, it follows that it must be transmitted across the interface. Thus, the Brewster angle may also be called the angle of total transmission.

13.4.3.2 Brewster's Angle for Perpendicular Polarization

An angle of no reflection may also be defined for perpendicular polarization by starting with $\Gamma_\perp$ in **Eq. (13.125)**. The condition for zero reflection is now

$$\eta_2 \cos\theta_i = \eta_1 \cos\theta_t \tag{13.132}$$

Following steps identical to those for parallel polarization, the Brewster angle is

$$\boxed{\theta_b = \sin^{-1}\sqrt{\frac{\mu_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\varepsilon_1\left(\mu_2^2 - \mu_1^2\right)}}} \tag{13.133}$$

However, for two dielectrics with identical permeabilities, the condition for no reflection cannot be satisfied. (When $\mu_1 = \mu_2$, the denominator in **Eq. (13.133)** is zero.) If μ_1 and μ_2 are not the same, the condition can be satisfied and a Brewster angle exists. For materials with identical permittivities but different permeabilities, the Brewster angle for perpendicular polarization is

$$\boxed{\theta_b = \sin^{-1}\sqrt{\frac{\mu_2}{\mu_2 + \mu_1}}} \quad \textit{if} \quad \varepsilon_1 = \varepsilon_2 \tag{13.134}$$

While both **Eqs. (13.133)** and **(13.134)** describe correct relationships, most materials do not fall in this category; that is, very few dielectrics have different permeabilities and almost none have different permeabilities and the same permittivity. For this reason, the Brewster angle is most often associated with parallel polarization rather than perpendicular polarization.

Example 13.10 A plane wave is generated underwater ($\varepsilon = 81\varepsilon_0$ [F/m], $\mu = \mu_0$ [H/m]). The wave is parallel polarized, propagates in water, and reflects and/or transmits through the interface between water and air as shown in **Figure 13.19**. Calculate the angle α for which there is no reflection at the interface

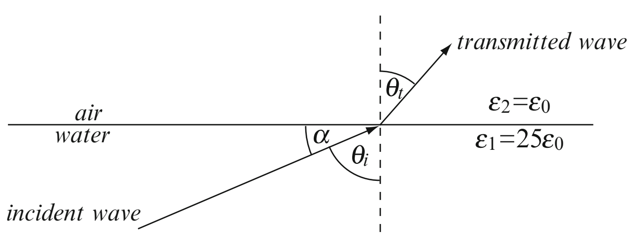


Figure 13.19 A wave incident on the water surface from below

Solution: The incident angle is $\theta_i = 90 - \alpha$. The angle of no reflection is the Brewster angle. In this case, both dielectrics (air and water) have the same permeability; therefore, the Brewster angle is given in **Eq. (13.131)**.

The Brewster angle for a parallel polarized wave propagating from water (ε_1) into air (ε_2) is

$$\theta_b = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} = \sin^{-1}\sqrt{\frac{\varepsilon_0}{25\varepsilon_0 + \varepsilon_0}} = 11.31°$$

The angle α at which there is no reflection is

$$\alpha = 90 - \theta_b = 78.69^\circ$$

13.4.4 Total Reflection

If the wave propagates across an interface such that the angle of refraction is larger than the angle of incidence, an increase in the angle of incidence leads to an angle at which the refracted wave propagates at 90° to the normal (see **Figure 13.20**). This angle is called a ***critical angle***. Any increase in the angle of incidence results in total reflection of the incident wave since what would have been the transmitted wave in material (2) is transmitted into material (1). This condition occurs in lossless dielectrics if $\varepsilon_1 > \varepsilon_2$. For example, waves incident on the surface of water from below satisfy this condition. If we view the water surface at a low angle to the normal (i.e., almost perpendicular to the surface of the water), we can see from under the surface. At high angles of incidence, the surface looks as a mirror because of total reflection. The phenomenon exists in either perpendicular or parallel polarization. In terms of the reflection coefficient, total reflection occurs when the reflection coefficient is equal to unity. Substitution of this angle ($\theta_t = 90^\circ$) into the relations for the reflection coefficient in **Eqs. (13.105)** and **(13.119)** gives

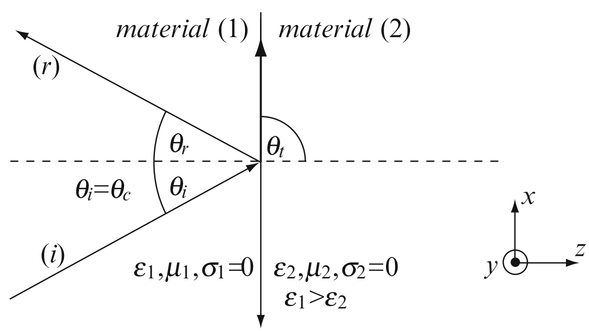


Figure 13.20 Total reflection at the interface between two dielectrics occurs when $\theta_t \geq 90^\circ$

$$\Gamma_\perp = 1, \qquad \Gamma_\parallel = -1, \qquad \text{at } \theta_t = 90^\circ \tag{13.135}$$

To define the critical angle, we again use Snell's law in **Eq. (13.127)**:

$$\sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i \tag{13.136}$$

Substituting $\theta_t = 90^\circ$ gives the critical angle:

$$\boxed{\sin\theta_c = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} \quad \text{for} \quad \mu_2\varepsilon_2 \leq \mu_1\varepsilon_1} \tag{13.137}$$

The condition $\mu_2\varepsilon_2 \leq \mu_1\varepsilon_1$ is also necessary otherwise, sin θ_c would be larger than 1.

Now, suppose we increase the angle of incidence above θ_c. This leads to sin $\theta_t > 1$; that is, sin $\theta_t = 1$ for $\theta_i = \theta_c$. Because $\theta_i = \theta_c < 90^\circ$, an increase in θ_i increases the right-hand side of **Eq. (13.136)** above 1. When substituting this condition in the reflection coefficients in **Eqs. (13.105)** and **(13.119)**, it leads to complex values for the reflection coefficients. The magnitude of the reflection coefficients remains equal to 1, but they are no longer real values.

Therefore, total reflection occurs for $\theta_i \geq \theta_c$

$$\boxed{\theta_i \geq \sin^{-1}\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} \quad \text{for} \quad \mu_2\varepsilon_2 \leq \mu_1\varepsilon_1} \tag{13.138}$$

In dielectric media for which the permeability is equal in both materials, the condition for total reflection is

$$\boxed{\theta_i \geq \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \quad \text{for} \quad \varepsilon_2 \leq \varepsilon_1,\ \mu_1 = \mu_2} \tag{13.139}$$

The relations in **Eqs.** (13.138) and (13.139) are independent of polarization.

Example 13.11 Application: Propagation Within Dielectric Layers A wave propagates inside a glass pane of thickness d [m] at angle of incidence θ_i. Material properties are shown in **Figure** 13.21. What must be the minimum angle of incidence θ_i so that the wave inside the pane does not escape (total reflection at the interior surfaces)? Assume that the wave has entered the glass pane from an edge (not shown). Use the following properties: $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_2 = 3\varepsilon_0$ [F/m], and $d = 10$ mm.

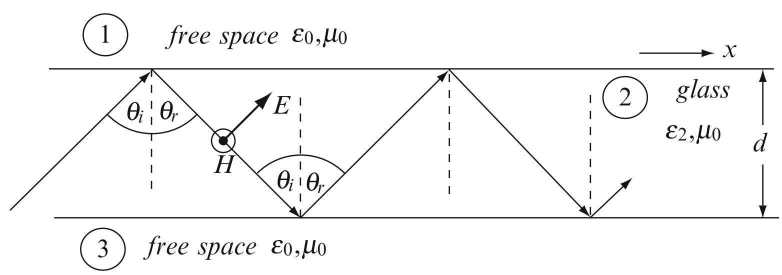


Figure 13.21 Total internal reflection at the interface between glass and air: the glass is said to guide the wave

Solution: For total internal reflection, there are two conditions that must be satisfied: (1) the material outside must be less dense (lower permittivity) than inside and (2) the angle must be above the critical angle. This is calculated from **Eq.** (13.136):

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\varepsilon_2}{\varepsilon_0}}$$

since for total reflection $\theta_t = \pi/2$, we have

$$\sin\theta_c = \sqrt{\frac{\varepsilon_0}{\varepsilon_2}} = \sqrt{\frac{1}{3}} = 0.57753 \quad \text{or} \quad \theta_c = 35°16'$$

For any angle $\theta_i \geq \theta_c$, the wave is totally reflected from either interface. We say that the wave is now guided by the glass pane. This is the basic principle of all wave guiding structures but, in particular, that of optical fibers. The requirements are quite simple: a lossless (or low loss) material with dielectric constant higher than the surrounding medium and a means of coupling electromagnetic waves into the material. Much more will be said about this in **Chapter** 17, but see also the following example.

Example 13.12 Application: Integrated Optical Waveguides In integrated optical devices, it is often necessary to propagate waves in particular materials or to "guide" them from one point to another. The idea of channeling a wave is not different in principle than that of guiding a sound wave between two points by means of a pipe. The main requirement here is that the wave should not penetrate through the boundaries of the dielectric. This is easily accomplished by means of a dielectric layer, provided that the materials with which the layer interfaces have lower permittivities.

In the optical device shown in **Figure 13.22**, it is required that light does not escape through either interfaces. The device is made of a low-permittivity layer (silicon dioxide) on which a high-permittivity layer is grown or deposited (guiding layer, made of silicon nitride), and on top of this, there is a cladding of a low-permittivity layer. In practical devices, the substrate is silicon, followed by a silicon dioxide layer to produce a low-permittivity material. The guiding layer may be silicon or some other high-permittivity material such as silicon nitrate. In this case, there is no cladding, but in practice, there will be some cladding to protect the guiding layer. The light is incident from the left at an angle θ as shown in **Figure 13.22**.

(a) What must be the angle θ for light not to escape through the upper surface?
(b) What must be the angle θ for light not to escape through the lower surface?
(c) What must be the angle θ for light not to escape the guiding layer?

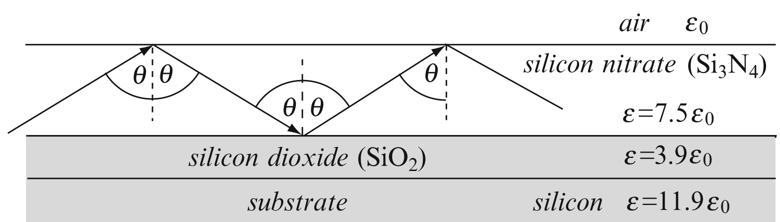


Figure 13.22 An integrated optical waveguide. The device relies on total internal reflection at angles above the critical angle to confine the wave

Solution: The critical angles for the upper and lower surface are different. Thus, the condition for each surface is different. For light not to escape either surface, the incidence angle must be larger than the larger of the two angles.

(a) The critical angle for a wave propagating from the guiding layer into free space at the upper surface is

$$\sin\theta_{us} = \sqrt{\frac{\varepsilon_0}{7.5\varepsilon_0}} = 0.365, \quad \theta_{us} = 21.42°$$

For angles equal to or larger than 21.42°, there is total reflection on the upper surface and light cannot escape into air.

(b) The critical angle at the lower surface (for a wave propagating from the guiding layer into the substrate) is

$$\sin\theta_{ls} = \sqrt{\frac{3.9\varepsilon_0}{7.5\varepsilon_0}} = 0.721, \quad \theta_{ls} = 46.15°$$

For any angle smaller than this, there will be transmission through the lower surface.

(c) Since for angles above 21.42°, there is no transmission through the upper surface but there is transmission through the lower surface up to 46.15°, the angle of incidence must be above 46.15°.

Note: This device is an optical waveguide similar to optical fibers. One important aspect of waveguides has been shown here: that of confining the wave between surfaces which do not allow transmission (total internal reflection). In these types of devices, any transmission through interfaces means losses. It is therefore important to ensure that total internal reflection occurs at all allowable angles of incidence.

13.5 Reflection and Transmission for Layered Materials at Normal Incidence

Waves.m

At multiple interfaces, such as in dielectric slabs and layered dielectrics, we expect both reflection and transmission at each interface. The methods used in **Sections** 13.2.1 through 13.2.3 are difficult to apply directly to layered media. It is much easier to calculate the general fields on each side of each layer and then apply the interface conditions at each interface. From these conditions, the fields on each side of each interface are calculated, completely specifying the problem. To outline the method, we treat here a lossy dielectric slab between two general, lossy dielectrics as shown in **Figure** 13.23, where the electric and magnetic field intensities are also shown and are chosen so that power propagates to the right, incident from material (1). The field intensities in each medium are as follows.

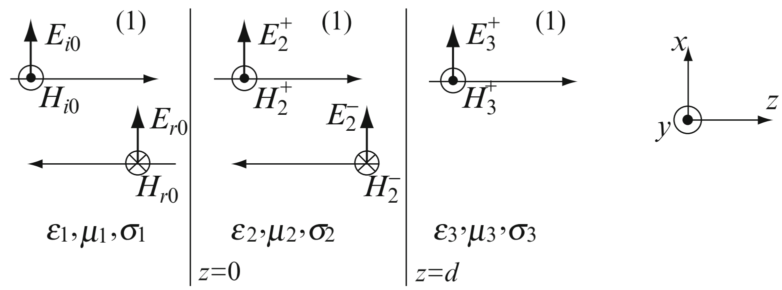


Figure 13.23 Reflection and transmission for a lossy dielectric slab in free space: normal incidence

In material (1) the propagation constant is $\gamma_1 = \alpha_1 + j\beta_1$, and we can write

$$\mathbf{E}_1 = \hat{\mathbf{x}}\left[E_{i0}e^{-\gamma_1 z} + E_{r0}e^{\gamma_1 z}\right] \quad [\mathrm{V/m}] \tag{13.140}$$

$$\mathbf{H}_1 = \hat{\mathbf{y}}\left[\frac{E_{i0}}{\eta_1}e^{-\gamma_1 z} - \frac{E_{r0}}{\eta_1}e^{\gamma_1 z}\right] \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.141}$$

In material (2), $\gamma_2 = \alpha_2 + j\beta_2$ and we can write

$$\mathbf{E}_2 = \hat{\mathbf{x}}\left[E_2^+e^{-\gamma_2 z} + E_2^-e^{\gamma_2 z}\right] \quad [\mathrm{V/m}] \tag{13.142}$$

$$\mathbf{H}_2 = \hat{\mathbf{y}}\left[\frac{E_2^+}{\eta_2}e^{-\gamma_2 z} - \frac{E_2^-}{\eta_2}e^{\gamma_2 z}\right] \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.143}$$

In material (3), $\gamma_3 = \alpha_3 + j\beta_3$ and there is only forward propagation:

$$\mathbf{E}_3 = \hat{\mathbf{x}}E_3^+e^{-\gamma_3 z} \quad [\mathrm{V/m}] \tag{13.144}$$

$$\mathbf{H}_3 = \frac{\hat{\mathbf{y}}}{\eta_3}E_3^+e^{-\gamma_3 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{13.145}$$

Because both **E** and **H** are tangential to the various interfaces, the interface conditions are as follows:
At $z = 0$,

$$E_1(0) = E_2(0) \quad \rightarrow \quad E_{i0} + E_{r0} = E_2^+ + E_2^- \tag{13.146}$$

$$H_1(0) = H_2(0) \quad \rightarrow \quad \frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_2^+}{\eta_2} - \frac{E_2^-}{\eta_2} \tag{13.147}$$

At $z = d$,

$$E_2(d) = E_3(d) \quad \rightarrow \quad E_2^+ e^{-\gamma_2 d} + E_2^- e^{\gamma_2 d} = E_3^+ e^{-\gamma_3 d} \tag{13.148}$$

$$H_2(d) = H_3(d) \quad \rightarrow \quad \frac{E_2^+}{\eta_2} e^{-\gamma_2 d} - \frac{E_2^-}{\eta_2} e^{\gamma_2 d} = \frac{E_3^+}{\eta_3} e^{-\gamma_3 d} \tag{13.149}$$

For convenience, we write **Eqs. (13.146)** through **(13.149)** as a matrix:

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ \dfrac{1}{\eta_1} & \dfrac{1}{\eta_2} & -\dfrac{1}{\eta_2} & 0 \\ 0 & e^{-\gamma_2 d} & e^{-\gamma_2 d} & -e^{-\gamma_3 d} \\ 0 & \dfrac{e^{-\gamma_2 d}}{\eta_2} & -\dfrac{e^{-\gamma_2 d}}{\eta_2} & -\dfrac{e^{-\gamma_3 d}}{\eta_3} \end{bmatrix} \begin{Bmatrix} E_{r0} \\ E_2^+ \\ E_2^- \\ E_3^+ \end{Bmatrix} = \begin{Bmatrix} E_{i0} \\ \dfrac{E_{i0}}{\eta_1} \\ 0 \\ 0 \end{Bmatrix} \tag{13.150}$$

The system in **Eq. (13.150)** may be extended to any number of layers by following the method above (see **Problem 13.41**). This system of equations may be solved numerically once the various constants (η_1, η_2, η_3, γ_1, γ_2, γ_3, and d) and the incident electric field intensity E_{i0} are specified. It may also be solved in general terms to obtain the general forms of E_{r0}, $E_2{}^+$, $E_2{}^-$, and $E_3{}^+$, and then, by substitution in **Eqs. (13.140)** through **(13.145)**, the total fields in the various media. A slab reflection coefficient as well as a slab transmission coefficient may also be calculated. These calculations are discussed next.

By direct solution of **Eq. (13.150)**, we obtain the following relations:

$$E_{r0} = \frac{\Gamma_{12} + \Gamma_{23} e^{-2\gamma_2 d}}{1 + \Gamma_{12}\Gamma_{23} e^{-2\gamma_2 d}} E_{i0} \quad \left[\frac{\text{V}}{\text{m}}\right] \tag{13.151}$$

$$E_2^- = \frac{T_{12}\Gamma_{23} e^{-2\gamma_2 d}}{1 + \Gamma_{12}\Gamma_{23} e^{-2\gamma_2 d}} E_{i0} \quad \left[\frac{\text{V}}{\text{m}}\right] \tag{13.152}$$

$$E_2^+ = \frac{T_{12}}{1 + \Gamma_{12}\Gamma_{23} e^{-2\gamma_2 d}} E_{i0} \quad \left[\frac{\text{V}}{\text{m}}\right] \tag{13.153}$$

$$E_3^+ = \frac{T_{12} T_{23} e^{-\gamma_2 d} e^{\gamma_3 d}}{1 + \Gamma_{12}\Gamma_{23} e^{-2\gamma_2 d}} E_{i0} \quad \left[\frac{\text{V}}{\text{m}}\right] \tag{13.154}$$

where Γ_{12} and T_{12} are the reflection and transmission coefficients at the interface between materials (1) and (2) and Γ_{23} and T_{23} are the reflection and transmission coefficients at the interface between materials (2) and (3). These are given as

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}, \quad T_{12} = \frac{2\eta_2}{\eta_2 + \eta_1}, \quad T_{23} = \frac{2\eta_3}{\eta_3 + \eta_2} \tag{13.155}$$

In turn, the intrinsic impedances are

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}}, \quad \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}, \quad \eta_3 = \sqrt{\frac{j\omega\mu_3}{\sigma_3 + j\omega\varepsilon_3}} \quad [\Omega] \tag{13.156}$$

From the definition of the reflection coefficient as the ratio between the reflected and incident waves, we define the ***slab reflection coefficient*** from **Eq. (13.151)** as

$$\boxed{\Gamma_{slab} = \frac{E_{r0}}{E_{i0}} = \frac{\Gamma_{12} + \Gamma_{23} e^{-2\gamma_2 d}}{1 + \Gamma_{12} + \Gamma_{23} e^{-2\gamma_2 d}} \quad [\text{dimensionless}]} \tag{13.157}$$

Similarly, the ***slab transmission coefficient*** [from **Eq. (13.154)**] is

$$T_{slab} = \frac{E_3^+}{E_{i0}} = \frac{T_{12}T_{23}e^{-\gamma_2 d}e^{\gamma_3 d}}{1+\Gamma_{12}+\Gamma_{23}e^{-2\gamma_2 d}} \qquad \text{[dimensionless]} \tag{13.158}$$

The slab reflection and transmission coefficients indicate the degree of transparency of the slab. A low transmission coefficient indicates a material opaque to propagation of electromagnetic waves, whereas a high transmission coefficient indicates a more transparent material. However, it should be noted that the transmission coefficient may be small even if the reflection coefficient is small since the lossy dielectric slab attenuates the field in addition to internal reflections in the slab.

The results in **Eqs. (13.151)** through **(13.158)** were obtained assuming general lossy dielectrics. Lossless dielectrics as well as perfect conductors may be treated by simply replacing the appropriate properties. For example, a common application is a lossless dielectric in free space. Under these conditions, $\gamma_1 = \gamma_3 = j\beta_0$, $\gamma_2 = j\beta_2$, $-\Gamma_{23} = \Gamma_{12}$, $T_{12} = 1 + \Gamma_{12}$, $T_{23} = 1 - \Gamma_{12}$ where $\beta_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the phase constant in free space and $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$ the phase constant in the dielectric slab. With these the slab reflection and transmission coefficients are (see also **Example 13.13**)

$$\Gamma_{slab} = \frac{\Gamma_{12}\left(1-e^{-j2\beta_2 d}\right)}{1-\Gamma_{12}^2 e^{-j2\beta_2 d}}, \qquad T_{slab} = \left[\frac{\left(1-\Gamma_{12}^2\right)e^{-j2\beta_2 d}e^{j2\beta_0 d}}{1-\Gamma_{12}^2 e^{-j2\beta_2 d}}\right] \tag{13.159}$$

These properties may be designed for minimum or maximum transparency of the slab either through specification of the thickness of the layer or through specification of its permittivity. Two particular methods of reducing reflections are widely used as follows.

Half-Wavelength Impedance Matching Section Consider again **Figure 13.23** but with all three dielectrics assumed to be lossless ($\sigma_1 = \sigma_2 = \sigma_3 = 0$). If $\varepsilon_{r1} = \varepsilon_{r3}$, such as for a lossless dielectric layer in free space, and assuming the thickness of the layer to be $d = \lambda/2$, we get, from **Eq. (13.159)**,

$$\Gamma_{slab} = \frac{\Gamma_{12}\left(1-e^{-j2\pi}\right)}{1-\Gamma_{12}^2 e^{-j2\pi}} = 0 \tag{13.160}$$

where $\beta_2 d = (2\pi/\lambda)(\lambda/2) = \pi$ and $e^{-j2\pi} = \cos 2\pi - j\sin 2\pi = 1$ were used.

Thus, a $\lambda/2$ layer guarantees no reflection into medium (1). One common application of this method is in radomes designed to both protect equipment such as antennas and to allow transmission without reflections through the radome.

Quarter-Wavelength Impedance Matching Section If $\varepsilon_{r1} \neq \varepsilon_{r3}$ and $d = \lambda/4$, **Eq. (13.157)** becomes

$$\Gamma_{slab} = \frac{\Gamma_{12}+\Gamma_{23}e^{-2\beta_2 d}}{1+\Gamma_{12}\Gamma_{23}e^{-2\beta_2 d}} = \frac{\Gamma_{12}+\Gamma_{23}e^{-j\pi}}{1+\Gamma_{12}\Gamma_{23}e^{j\pi}} = \frac{\Gamma_{12}-\Gamma_{23}}{1-\Gamma_{12}\Gamma_{23}} \tag{13.161}$$

where $\beta_2 d = (2\pi/\lambda)(\lambda/4) = \pi/2$ and $e^{-j2\pi/2} = \cos\pi - j\sin\pi = -1$ were used.

For this to be zero, we must have

$$\Gamma_{12} = \Gamma_{23} \quad \rightarrow \quad \eta_2 = \sqrt{\eta_1\eta_3} \tag{13.162}$$

or, if $\mu_1 = \mu_2 = \mu_3 = \mu_0$ [H/m],

$$\varepsilon_2 = \sqrt{\varepsilon_1\varepsilon_3} \qquad \text{[F/m]} \tag{13.163}$$

This method is widely used to reduce reflections in optical devices such as lenses. Proper choice of the coating's permittivity guarantees reduction in reflections.

Another practical application is a lossy or lossless dielectric slab backed by a perfect conductor (see **Figure 13.24**). In this case, the reflection coefficient at the slab-conductor interface is $\Gamma_{23} = -1$, and again we can obtain simple expressions by substituting this in **Eqs. (13.157)** and **(13.158)** (see **Example 13.14**). Finally, it should be noted that if the slab itself is a perfect conductor, then $\Gamma_{12} = -1$, $\Gamma_{23} = +1$ and substitution of these in **Eqs. (13.157)** and **(13.158)** results in $\Gamma_{slab} = -1$, $T_{slab} = 0$ as expected.

Example 13.13 Application: Antenna Radomes A radome is a protective dielectric cover placed over antennas to protect them from the environment. These can be a "dielectric window" in the skin of an airplane to allow its radar to transmit while still maintaining the required smooth surface or may be a dome over a large antenna on the ground or on a ship. In either case, one of the main requirements is that the radome be transparent to transmitted and received waves at the frequency or frequencies at which the antenna operates:

(a) What must be the relative permittivity of a lossless radome material 0.05 m thick for the slab reflection coefficient to be zero at 1 GHz? The permeability of the material is $\mu = \mu_0$ [H/m].

(b) Suppose you cannot find the material required in (**a**) but have plenty of Perspex, which has relative permittivity of 6 and may be assumed to be lossless. Calculate the required thickness to avoid any reflection by the radome at 1 GHz.

Solution:

(a) From the slab reflection coefficient in **Eq. (13.159)**, the condition for zero reflection is

$$\Gamma_{slab} = \frac{\Gamma_{12}\left(1 - e^{-j2\beta_2 d}\right)}{1 - \Gamma_{12}^2 e^{-j\beta_2 d}} = 0$$

This reduces to

$$1 - e^{-j2\beta_2 d} = 0$$

since, for any dielectric, $\Gamma_{12} \neq 0$. Expanding this, we have

$$1 - e^{-j2\beta_2 d} = 1 - \cos(2\beta_2 d) + j\sin(2\beta_2 d) \quad \rightarrow \quad \cos(2\beta_2 d) - j\sin(2\beta_2 d) = 1$$

Since the right-hand side is real, we have

$$\cos(2\beta_2 d) = 1 \quad \rightarrow \quad 2\beta_2 d = 2n\pi \quad \rightarrow \quad \beta_2 d = n\pi, \quad n = 0, 1, 2 \ldots$$

Although any value of n will do, n cannot be zero; otherwise the thickness must be zero. For the given thickness and taking $n = 1$, we get

$$\beta_2 = \frac{\pi}{d} = \frac{\pi}{0.05} = 62.832 \quad \left[\frac{\text{rad}}{\text{m}}\right]$$

For lossless dielectrics, the phase constant is $\beta = \omega\sqrt{\mu\varepsilon}$ [rad/m] and we have

$$\beta_2 = \omega\sqrt{\mu\varepsilon} \quad \rightarrow \quad \varepsilon = \frac{\beta_2^2}{\omega^2\mu} \quad \left[\frac{\text{F}}{\text{m}}\right]$$

Thus,

$$\varepsilon = \frac{3947.84}{4 \times \pi^2 \times 10^{18} \times 4 \times \pi \times 10^{-7}} = 7.958 \times 10^{-11} \quad \rightarrow \quad \varepsilon_r = \frac{7.958 \times 10^{-11}}{8.854 \times 10^{-12}} = 9.0.$$

(b) Now, we go in reverse. We start with known permeability, permittivity, and frequency. This gives the phase constant in the radome. From this, we calculate

$$\beta_2 = \omega\sqrt{\mu_0 6\varepsilon_0} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{6} = \frac{2\times\pi\times 10^9\sqrt{6}}{3\times 10^8} = 51.3 \quad \left[\frac{\text{rad}}{\text{m}}\right]$$

The thickness is (again using the thinnest possible radome: $n = 1$)

$$d = \frac{\pi}{\beta_2} = \frac{\pi}{51.3} = 0.06124 \quad [\text{m}]$$

Note that the reflection coefficient is frequency dependent. This means that at other frequencies the reflection coefficient is not zero (the radome is not completely transparent).

Example 13.14 Application: Measurement of Dielectric Constant of Dielectric Coatings A simple method for measurement of the permittivity of dielectric coatings such as paints is shown in **Figure 13.24**. The measurement consists of a source producing a wave that impinges on the dielectric. The reflection coefficient is then measured. The frequency of the source is varied until the reflection coefficient is either maximum or minimum. The dielectric constant is then calculated from the formula for the slab reflection coefficient.

A perfect dielectric 10 mm thick has permeability of free space and is placed next to a perfect conductor as shown in **Figure 13.24**. A wave impinges on the dielectric and the reflected wave is measured. As the frequency is varied, it is found that the reflected wave's magnitude is maximum at $f = 10$ GHz. Calculate the dielectric constant of the dielectric.

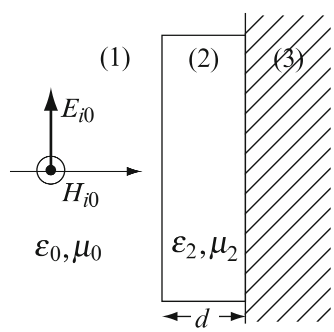


Figure 13.24 A conductor-backed dielectric

Solution: Using **Eq. (13.157)** for the reflection coefficient (with $\gamma_2 = j\beta_2$) we get

$$\Gamma_{slab} = \frac{\Gamma_{12} + \Gamma_{23}e^{-j2\beta 2d}}{1 + \Gamma_{12}\Gamma_{23e^{-j2\beta 2d}}}$$

The reflection coefficient at the dielectric–conductor interface is

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} = \frac{0 - \eta_2}{0 + \eta_2} = -1$$

Thus,

$$\Gamma_{slab} = \frac{\Gamma_{12} - e^{-j2\beta_2 d}}{1 - \Gamma_{12}e^{-j2\beta_2 d}} = -1$$

The reason the reflection coefficient should be -1 is that Γ_{12} itself is also negative for any dielectric since $\varepsilon_2 > \varepsilon_0$. We have

$$\Gamma_{12} - e^{-j2\beta_2 d} = -1 + \Gamma_{12}e^{-j2\beta_2 d} \quad \rightarrow \quad (\Gamma_{12}+1) = (\Gamma_{12}+1)e^{-j2\beta_2 d}$$

or

$$e^{-j2\beta_2 d} = 1 \quad \rightarrow \quad \cos 2\beta_2 d = 1 \quad \rightarrow \quad 2\beta_2 d = 2n\pi \ \rightarrow \ \beta_2 \frac{n\pi}{d}, \quad n = 1, 2, 3, \ldots$$

Thus, for the first maximum, $n = 1$,

$$\beta_2 = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{\pi}{d} \quad \rightarrow \quad \sqrt{\varepsilon_r} = \frac{\pi c}{\omega d}$$

The relative permittivity is therefore

$$\varepsilon_r = \left(\frac{\pi c}{\omega d}\right)^2 = \left(\frac{\pi \times 3 \times 10^8}{0.01 \times 2 \times \pi \times 10^{10}}\right)^2 = 2.25.$$

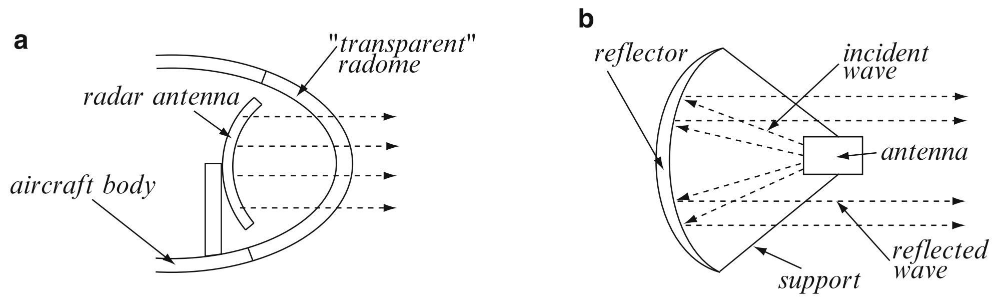
13.6 Applications

Application: Microwave Cooking We mentioned microwave cooking earlier. Its utility comes from the fact that at certain frequencies, water absorbs energy from electromagnetic waves. Although water itself has a relatively low conductivity, it has dielectric losses which are high at certain frequencies. A particularly useful frequency is 2.45 GHz, which is used almost universally in microwave ovens. Although some foods are, in fact, lossy dielectrics in the true sense (animal tissue, for example, has a relatively large conductivity), the main function of microwave ovens is to act on water present in the substance being cooked. A moist substance will cook well but a dry substance will not. Although microwave ovens of moderate size (around 1 kW or less) are common in the kitchen, they also find considerable utility in industrial applications where quick drying of wet substances is needed. Examples are drying of grain before storage and shipment, drying and curing of polymers, large-scale cooking, dielectric welding of plastics, and many others.

Application: Freeze-Drying One application of wave propagation in lossy dielectrics is freeze-drying of foods. It consists of freezing the substance to be dried and then placing it in a vacuum chamber. At that point, heat is applied to evaporate the ice (by sublimation) and thus extract the moisture from the substance. This is different than cooking the substance at high temperature in that the water in the material is evaporated and extracted without boiling and, therefore, with minimum damage to the substance itself. The result is a dehydrated substance (food, plasma, etc.) without damage to tissue. The structure remains essentially unaltered, as do color and texture. Rehydration restores it to the original condition. The idea of using microwaves for this purpose is almost natural, especially since heat conduction through vacuum is very poor, as is the conduction through dried-up tissue. With a microwave source, the waves are absorbed in the ice itself, and if the energy is high enough, ice is evaporated as required.

Application: Radomes and Dielectric Windows It is often necessary to transmit electromagnetic waves from one area into another through a physical barrier. One example was mentioned earlier: the radome. Whenever an antenna must be physically separated from the environment, its energy can only be transmitted and received through this barrier. For example, in an airplane, the radar antenna must be located within the body of the airplane for aerodynamic purposes. A window is then provided to allow transmission and reception. On ships, the antenna must be protected from the environment by a cover. Another example is the magnetron (a microwave tube). The waves generated in a vacuum chamber must be coupled to the outside such as into a microwave oven with as little reflected energy as possible. These covers and windows operate like radomes (see **Figure 13.25a**) and are designed to transfer energy without any reflection (see **Example 13.13**). A radome may be designed in two ways. One is to choose permittivities and permeabilities such that the intrinsic impedance of the radome material equals that of the surrounding domain (air or free space). For this purpose, the material must be a lossless dielectric with material properties such that $\mu/\varepsilon = \mu_0/\varepsilon_0$. This method has the distinct advantage that the design is independent of frequency. The second method is to choose a perfect dielectric and design its thickness such that there are no reflections at the required frequency. In practical applications, it is often required to switch frequencies, and the design of the radome must be such that it is transparent at all required frequencies, a design which is often difficult to achieve.

Figure 13.25 (**a**) An aircraft antenna radome. (**b**) The "dish" or parabolic antenna is made of a small feed and a large reflector



Application: Microwave Reflectors Many antennas rely on reflectors to guide the beam in specific directions. A typical parabolic (dish) antenna is shown in **Figure 13.25b**. It consists of a parabolic dish very much like the surface in a car's headlights. The antenna itself (also called a feed) is a small horn located in the focal point of the reflector (similar to the bulb in the headlight). The feed radiates toward the reflector and the reflector then reflects the beam into the direction required. These antennas are highly useful because they transmit energy in narrow beams in the required direction. They are common in satellites and other communication systems. Reflectors may also be used in a passive form: their purpose is to reflect waves or perhaps to change their direction. For example, the actual antenna, which may include a reflector, may be placed at the bottom of a transmission tower, whereas a reflector may be placed on top of the tower to reflect the waves. The antenna transmits upward and the reflector changes the direction of propagation horizontally. This has the advantage that the antenna may be serviced on the ground, and the reflector may be made much lighter than the antenna itself. One particularly interesting aspect of reflectors dates to the early time of satellites. Back then, it was seriously considered using reflectors in space to bounce transmissions. Experiments with aluminized balloons in space were performed and these, while not considered for permanent service, provided the first experimental data for satellite communication. Other ideas included the use of the moon's surface as a reflector. Needless to say, these had minimal success. However, the reflection off surfaces is the common method of radar and microwave mapping of the Earth and the planets. The common thread in these applications is the need for good reflectivity. In antenna reflectors, this is obtained by use of highly conductive, polished materials, whereas in mapping surfaces, the natural reflectivity of surfaces (or any other reflecting materials or regions such as clouds) is used. The variations in reflectivity of materials and surfaces are then used for remote sensing and monitoring.

Application: Scattering of Waves The reflection of waves by any material, including perfect dielectrics, is due to the fact that the reflection coefficient is almost always nonzero. For example, the atmosphere has a permittivity different than free space and the permittivity differs from place to place depending on atmospheric pressure and weather conditions. Similarly, any substance in the atmosphere such as a dust cloud, an airplane, rain or snow, or a pressure front will have permittivities that differ from that of air. These variations may be detrimental to communication in that some energy is reflected in various directions (scattered) rather than serving a useful purpose, whereas in some cases, this scattering is quite useful. Because of scattering, many of the effects mentioned above can be detected by measuring the reflectivity of the materials or conditions present. This is extremely important in weather prediction and remote sensing of the environment. Other applications include communication such as the tropospheric scattering method shown in **Figure 13.26a**. In this method, the transmitter sends a rather narrow beam upward into the troposphere. The waves are scattered and some of the scattered waves are then reflected back into the receiver. With this method there is no need for a reflector to reflect the waves back into the receiver; use is made of the natural reflections that occur in the troposphere. Another simple use of scattering is in microwave testing of lossless dielectrics, shown in **Figure 13.26b**. A microwave beam illuminates the test sample. Some of the waves propagate through the material and some are reflected back into the transmitter. However, none will be coupled to the upper receiver. If, however, there are inclusions, defects, etc., in the material, these will scatter waves in many directions, some of which will be received in the upper receiver. This reception is then an indication of the defects or foreign materials in the test sample.

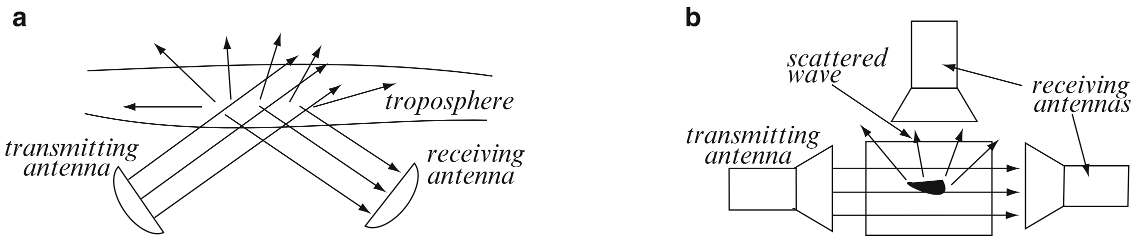


Figure 13.26 (**a**) Tropospheric scattering method of communication. (**b**) Microwave method of testing dielectrics for defects and inclusions

Application: Stealth Aircraft There are two methods of avoiding detection by radar. One is to ensure that the reflection coefficient of the aircraft, as a whole, is as nearly as possible close to zero. If there is no energy reflected back from the aircraft, there is no energy reaching the antenna of the radar and the aircraft is "transparent" to the incoming wave. To do so, the aircraft is coated with materials which have the same intrinsic impedance as air but which also absorb (dissipate) energy. The latter is required because if it were not for this, the wave would propagate through the coating and reflect off the metallic surfaces of the aircraft. Materials appropriate for this purpose are those for which $\mu/\varepsilon = \mu_0/\varepsilon_0$. The required ratio is usually obtained by varying the permeability by adding ferromagnetic powders. In addition, these materials must have some loss to attenuate the wave. Alternatively, a number of layers of different materials are used so that the general reflection coefficient is zero. The absorption of energy must be over a wide enough spectrum to avoid detection by shifting frequencies of the radar system (shifting frequencies is the simplest way to detect "undetectable" aircraft). Absorbing paints and coatings such as rubber and polymers exist that will absorb certain frequencies or range of frequencies. In most cases, radar-absorbing materials are used only where necessary (such as engine intakes, wing tips and edges, etc.) to reduce rather than eliminate the aircraft radar visibility.

A second method of avoiding detection is to reflect the incoming waves but to deflect these in directions away from the antenna. In this method, no energy is absorbed, but little is reflected back to the antenna. Aircraft of this type will have sharp angles, as shown in **Figure** 13.27. The sharper the corners, the less energy will be reflected. Note, however, that the flat surfaces employed are quite visible if viewed from a steep enough angle. In the example in **Figure** 13.27, the aircraft is visible from underneath or even from above, but these are not normal angles of observation. Typically a radar installation will try to detect aircraft at low angles, possibly from the front or side. For these angles, the bottom flat surface in **Figure** 13.27 is not detectable.

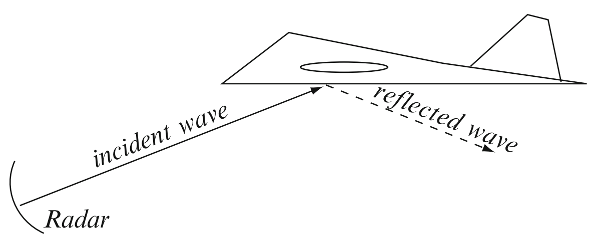


Figure 13.27 A "stealth aircraft" based on sharp corners and flat surfaces that reflect waves away from the radar antenna

Wave-absorbing materials have many applications that are not related to the military. For instance, in evaluation of antennas, it is important to avoid reflections from structures around the antenna so that a proper evaluation can take place. These structures are routinely coated with absorbing materials, usually in the form of narrow foam pyramids, impregnated with conductive materials to attenuate the incoming waves. These form the so-called anechoic chambers because they do not reflect waves (no echoes).

13.7 Experiments

Experiment 1 (Demonstrates: Total Reflection) Total reflection may be most easily demonstrated with light waves and water. Using an aquarium, shine a narrow beam flashlight from inside as shown in **Figure 13.28**. Vary the angle α until you get no transmission through the surface of the water. Calculate this angle based on the properties of water. Alternatively, suppose you measure the angle. If permeability of water equals μ_0 [H/m] and in air $\mu = \mu_0$ [H/m] and $\varepsilon = \varepsilon_0$ [F/m], calculate the relative permittivity of water in the visible range.

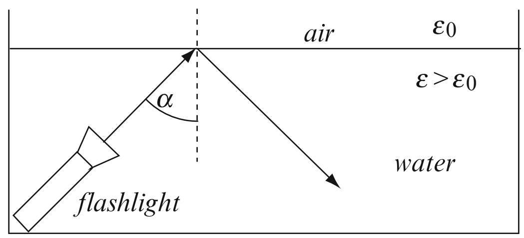


Figure 13.28 Method of demonstrating total reflection

Experiment 2 (Demonstrates: Heating of Lossy Dielectrics—Cooking with Microwaves) Place some popcorn in a small dish in a microwave oven. For most convincing results, take the kernels from a microwaveable package. Try to pop the corn by running the microwave oven for the amount of time specified on the package. Few if any kernels will pop. Now, try again by placing some oil in the dish. The corn should pop properly. Now, take the open package and inspect the bottom of the package. A small piece of material is embedded in the package. If you were to remove the material, the popcorn will not pop. With the material, it will. The package normally specifies which side should be down. Why is this important? Try inverting the package. Why doesn't the popcorn pop? The material is a microwave-absorbing material (low-reflection, lossy material) which gets hot enough to pop the corn. The popcorn is normally quite dry and will absorb little energy. The addition of oil or absorbing material allows proper heating of the popcorn.

Experiment 3 (Demonstrates: Heating of Lossy Dielectrics, Industrial Uses in Drying Of Materials) An instructional experiment is the following: Place a small dry foam sponge in a microwave oven. Turn on the oven for about 20 s. Check the temperature of the sponge. Now, soak the sponge in water and squeeze it as dry as you can. Repeat the heating. The temperature now should be considerably higher. Be careful: it may get real hot! Microwave heating is due to losses. The sponge is a good dielectric and will not heat up. Water is lossy and, therefore, the heating.

13.8 Summary

This chapter takes up the issues of transmission, reflection, and refraction of plane waves at the interface between two different media. The dominant quantities are the reflection and transmission coefficients at interfaces between media.

Definitions

Plane of incidence: the plane formed by the direction of propagation of the incident wave and the normal to the interface (**Figures 13.1** and **13.10**).

Incidence angle: the angle between the direction of propagation of the incident wave and the normal to the interface (**Figure 13.1**).

Reflection angle: the angle between the direction of propagation of the reflected wave and the normal to the interface (**Figure 13.1**).

Transmission angle: the angle between the direction of propagation of the transmitted wave and the normal to the interface (**Figure 13.1**).

Perpendicular (***normal***) ***incidence***: the wave impinges on an interface perpendicularly (**Figure 13.2**).

Oblique incidence: the wave impinges on an interface at an angle (**Figures 13.10** and **13.1**).

Perpendicular polarization: the electric field intensity is perpendicular to the plane of incidence (see **Figures 13.13** and **13.16**).

Parallel polarization: the electric field intensity is parallel to the plane of incidence (see **Figure 13.15**).

Perpendicular Incidence on general media For a wave propagating from medium (1) into medium (2), the reflection and transmission coefficients are (see **Figure 13.2**)

$$\Gamma = \frac{E_{r1}}{E_{i1}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (13.23) \qquad T = \frac{E_t}{E_{i1}} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad (13.24) \qquad 1 + \Gamma = T \quad (13.21)$$

where η_1 and η_2 are given in **Eqs. (13.8)** and **(3.11)**. η_1 and η_2 and therefore Γ and T can be complex.

The total fields (sum of incident and reflected waves) in medium (1) with E_{i1} known (**Figure 13.2**) are

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} E_{i1}(Te^{-\gamma_1 z} - \Gamma j2\sin(j\gamma_1 z)) \quad [\mathrm{V/m}] \quad (13.27) \qquad \mathbf{H}_1(z) = \hat{\mathbf{y}} \frac{E_{i1}}{\eta_1}(Te^{-\gamma_1 z} - \Gamma 2\cos(j\gamma_1 z)) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \quad (13.30)$$

where $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)}$ [see **Eq. (13.8)**]. The total fields in medium (2) are

$$\mathbf{E}_2(z) = \hat{\mathbf{x}} T E_{i1} e^{-\gamma_2 z} \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right] \quad (13.31) \qquad \mathbf{H}_2(z) = \hat{\mathbf{y}} T \frac{E_{i1}}{\eta_2} e^{-\gamma_2 z} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \quad (13.32)$$

where $\gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\varepsilon_2)}$ [see **Eq. (13.11)**].

At the interface between a perfect dielectric and a perfect conductor: $\Gamma = -1$, $T = 0$ and $\gamma_1 = j\beta_1$. Only ***standing waves***, that is, waves that oscillate but do not propagate, can exist in the dielectric:

$$\mathbf{E}_1(z) = \hat{\mathbf{x}} j2E_{i1}\sin(\beta_1 z) \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right] \quad (13.47) \qquad \mathbf{H}_1(z) = \hat{\mathbf{y}}\, 2\frac{E_{i1}}{\eta_1}\cos(\beta_1 z) \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \quad (13.48)$$

Nodes of the standing wave (zero electric field intensity, maximum magnetic field intensity) are at $z = -n\lambda_1/2$, with λ_1 the wavelength in the dielectric ($z = 0$ is assumed at the conducting interface). Maxima in **E** or minima in **H** are $\lambda_1/4$ on either side of the minima in **E**.

Perpendicular Polarization, Oblique Incidence on a Conductor (Figure 13.10b)

$$\Gamma = -1,\ T = 0 \quad \rightarrow \quad E_{r1} = -E_{i1} \quad \text{and} \quad \theta_r = \theta_i \qquad (13.78)$$

Total fields in the dielectric [medium (1)]:

$$\mathbf{E}_1(x,z) = -\hat{\mathbf{y}} j2E_{i1}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \quad [\mathrm{V/m}] \qquad (13.81)$$

$$\mathbf{H}_1(x,z) = -2\frac{E_{i1}}{\eta_1}[\hat{\mathbf{x}}\cos\theta_i\cos(\beta_1 z\cos\theta_i) + \hat{\mathbf{z}} j\sin\theta_i\sin(\beta_1 z\cos\theta_i)]e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \qquad (13.82)$$

Parallel Polarization, Oblique Incidence on a Conductor (Figure 13.14) Reflections and transmission coefficients are the same as for perpendicular polarization. The total fields are

$$\mathbf{E}_1(x,z) = -2E_{i1}[\hat{\mathbf{x}} j\cos\theta_i\sin(\beta_1 z\cos\theta_i) + \hat{\mathbf{z}}\sin\theta_i\cos(\beta_1 z\cos\theta_i)]e^{-j\beta_1 x\sin\theta_i} \quad [\mathrm{V/m}] \qquad (13.90)$$

$$\mathbf{H}_1(x,z) = \hat{\mathbf{y}}\, 2\frac{E_{i1}}{\eta_1}\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \qquad (13.91)$$

Conclusions for Both Types of Polarizations

(1) The wave propagates parallel to the conducting surface.
(2) Only standing waves exist perpendicular to the surface.
(3) Power is guided along the conducing surface.

Oblique Incidence on Dielectrics (Figure 13.16) Snell's law (lossless dielectrics):

$$\theta_r = \theta_i \quad (13.78) \qquad \frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\varepsilon_1\mu_1}}{\sqrt{\varepsilon_2\mu_2}} = \frac{n_1}{n_2} = \frac{v_{p2}}{v_{p1}} \qquad (13.102) \text{ and } (13.103)$$

where $n_i = \sqrt{\varepsilon_{ri}\mu_{ri}}$ is the index of refraction of medium i and ε_{ri}, μ_{ri} are the relative permittivity and relative permeability of the medium.

Perpendicular Polarization, Oblique Incidence on a Dielectric Reflection and transmission coefficients:

$$\Gamma_\perp = \frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \text{ [dimensionless]} \quad (13.105) \qquad T_\perp = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \text{ [dimensionless]} \quad (13.106)$$

and $1 + \Gamma_\perp = T_\perp$

The electric and magnetic field intensities in both media are given in **Eqs. (13.107)** through **(13.110)**.

Parallel polarization, Oblique Incidence on a Dielectric Reflection and transmission coefficients

$$\Gamma_{||} = -\frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \text{ [dimensionless]} \quad (13.119) \qquad T_{||} = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \text{ [dimensionless]} \quad (13.120)$$

and $1 + \Gamma_{||} = T_{||}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$ (see **Exercise 13.12**).

The electric and magnetic field intensities in both media are given in **Eqs. (13.121)** through **(13.124)**.

Brewster's angle is the angle of no reflection (also called polarizing angle) for waves propagating from medium (1) into medium (2). For parallel polarization, provided $\varepsilon_1 \neq \varepsilon_2$,

$$\theta_b = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\mu_1}\frac{(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\varepsilon_1^2 - \varepsilon_2^2}} \quad (13.130) \qquad \text{or} \qquad \theta_b = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \quad \text{if} \quad \mu_1 = \mu_2 \quad (13.131)$$

For perpendicular polarization, provided $\mu_1 \neq \mu_2$

$$\theta_b = \sin^{-1}\sqrt{\frac{\mu_2}{\varepsilon_1}\frac{(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\mu_2^2 - \mu_1^2}} \quad (13.133) \qquad \text{or} \qquad \theta_b = \sin^{-1}\sqrt{\frac{\mu_2}{\mu_2 + \mu_1}} \quad \text{if} \quad \varepsilon_1 = \varepsilon_2 \quad (13.134)$$

Critical Angle and ***Total Reflection*** A wave propagating in medium (1) is reflected back into medium (1) without transmission if

$$\theta_i \geq \sin^{-1}\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}, \quad \text{for} \quad \mu_2\varepsilon_2 \leq \mu_1\varepsilon_1 \qquad (13.138)$$

That is, total reflection can only occur when propagating from a higher to a lower permittivity dielectric (most dielectrics have the permeability of free space) at and above the critical angle.

Reflection from Layered Structures, Normal Incidence The slab reflection and transmission coefficients (lossy slab of thickness d between lossy dielectrics) (see **Figure 13.23**) are

$$\Gamma_{slab} = \frac{E_{r0}}{E_{i0}} = \frac{\Gamma_{12} + \Gamma_{23}e^{-2\gamma_2 d}}{1 + \Gamma_{12}\Gamma_{23}e^{-2\gamma_2 d}} \quad (13.157) \qquad T_{slab} = \frac{E_3^-}{E_{i0}} = \frac{T_{12}T_{23}e^{-2\gamma_2 d}e^{\gamma_3 d}}{1 + \Gamma_{12}\Gamma_{23}e^{-2\gamma_2 d}} \quad (13.158)$$

where Γ_{ij} and T_{ij} are the reflection and transmission coefficients at the interface as the wave propagates from material i into material j.

Notes:

(1) The reflection and transmission coefficients are defined only for the electric field intensity based on the continuity of the tangential components.
(2) The reflected/transmitted magnetic field intensity components are calculated from the electric field intensity by dividing by the appropriate intrinsic impedance.

Problems

Reflection and Transmission at a General Dielectric Interface: Normal Incidence

13.1 Reflection and Transmission at Air-Lossy Dielectric Interface. A plane wave impinges perpendicularly on a half-space made of a lossy dielectric. Calculate the reflected and transmitted waves. Use **Figure 13.29** for reference. Assume the material to the left is free space and to the right it is water ($\sigma_2 = 10^{-9}$ S/m, $\varepsilon_2 = 72\varepsilon_0$ [F/m] , $\mu_2 = \mu_0$ [H/m]), and the frequency is 100 MHz.

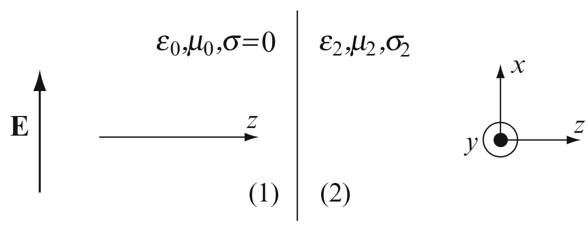


Figure 13.29

13.2 Incident and Reflected Waves at a Lossless Dielectric Interface. A plane wave is given as $E = E_0 e^{-j\beta z}$ [V/m] and propagates in free space. The wave hits a dielectric wall ($\varepsilon = 2\varepsilon_0$ [F/m]) at normal incidence. With $E_0 = 10$ V/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $f = 1$ GHz, calculate:

(a) The peak electric field intensity, left of the wall.
(b) The peak magnetic field intensity, left of the wall.

13.3 Incident and Reflected Waves at a Lossy Dielectric Interface. The configuration in **Figure 13.29** is given. A wave propagates in the direction perpendicular to the interface between free space and a general lossy material (z direction) and has an electric field intensity directed as shown. Calculate the ratio between the maximum and minimum electric field amplitudes in material 1.

13.4 Application: **Transmission of Power into Solar Cells**. Consider the question of generating electricity with silicon solar cells. The relative permittivity of silicon at optical wavelengths is 1.75 m and it may be considered to be lossless. Assume uniform plane waves, perpendicular incidence, and that 25 % of the power entering the cells is converted into electric power. The Sun power density at the location of the cells is 1,400 W/m^2.

(a) Calculate the power per unit area of the cell it can generate and its overall efficiency.
(b) Suppose a new type of material is designed which has properties identical to those of silicon except that its permittivity equals that of free space. How much larger is the power that solar cells made of this material can generate and its efficiency?

13.5 Power Transmitted into Glass at Normal Incidence. A laser beam is incident on a glass surface from free space. The beam is narrow, 0.1 mm in diameter, with a power density in the beam of 0.1 W/m^2. Assume normal incidence on the surface and plane wave behavior. Glass is lossless and has a relative permittivity of 1.8 at the frequency used:

(a) Calculate the amplitude of the incident electric and magnetic field intensities in space and the transmitted electric and magnetic field intensities in the glass.
(b) Calculate the total power transmitted into the glass.

13.6 Show that time-averaged power is conserved across an interface between two media for:

(a) Lossless media, perpendicular incidence.
(b) Lossy media, perpendicular incidence.

13.7 **Application**: **The Sun at the Beach or**: **Why Do We Get Sunburns**? The Sun impinges on the ground at 1,300 W/m^2 (time-averaged power density). If the properties of the skin are known as $\sigma = 0.01$ S/m, $\mu = \mu_0$ [H/m] and $\varepsilon = 24\varepsilon_0$ [F/m], calculate the amount of power dissipated in the skin of a person. Assume the area exposed is 1 m^2, the Sun radiates at an average frequency of 5×10^{14} Hz and is perpendicular to the surface of the skin.

13.8 **Application**: **Radiation Exposure**. One of the main concerns in exposure to microwave radiation is heating effects in the body. The US radiation safety code specifies that the total amount of radiation should not exceed 10 mW/cm^2 of skin for 6 hours. Suppose an average person is exposed to this radiation at a frequency of 10 GHz. The effective area of the skin is 1.5 m^2, and the body properties are $\sigma = 0.01$ S/m, $\mu = \mu_0$ [H/m], and $\varepsilon = 24\varepsilon_0$ [F/m], at the given frequency. Calculate the total power absorbed by the body and the total energy absorbed during maximum exposure.

Reflection and Transmission at a Dielectric Conductor Interface: Normal Incidence

13.9 **Application**: **Standing Waves and Reflectometry**. An antenna generates an electric field intensity in the positive y direction. The amplitude of the generated wave is $E_0 = 100$ V/m, at a wavelength of 12 m:

(a) Calculate the location of the antenna in relation to a perfectly conducting wall such that a standing wave is generated with three positive maxima in the electric field between the wall and antenna, and the antenna is at the location of the fourth positive peak. Assume propagation in free space.
(b) If propagation occurs in a low-loss medium, $\varepsilon_1 = 4\varepsilon_0$ [F/m], $\mu_1 = \mu_0$ [H/m], $\sigma_1 = 10^{-5}$ S/m, calculate the amplitude of the electric field intensity at the location of the first positive maximum to the right of the antenna.

13.10 **Reflection of Waves from Conducting Surfaces**. A wave propagates in free space and impinges perpendicularly on a perfectly conducting surface. Show that the ratio between the electric field intensity and the magnetic field intensity anywhere to the left of the conducting surface is purely imaginary or that the electric and magnetic field intensities are out of phase. **Hint:** Use relations $e^{j\beta z} - e^{-j\beta z} = j2\sin\beta z$ and $e^{j\beta z} + e^{-j\beta z} = 2\cos\beta z$.

13.11 **Surface Current Generated by Incident Waves**. A wave impinges perpendicularly on a perfectly conducting surface. The amplitude of the incident electric field intensity is 10 V/m and the wave propagates in free space. For orientation purposes, assume the wave propagates in the positive z direction and the electric field intensity is in the negative y direction:

(a) Calculate the surface current density (A/m) produced by the incident field.
(b) Show that the total field in free space is the sum of the incident field and the field produced by the surface current density.

Oblique incidence on a Conducting Interface: Perpendicular Polarization

13.12 **Interface Conditions at a Conductor Interface**. A uniform plane wave impinges on a good conductor at an arbitrary angle. The wave is polarized perpendicular to the plane of incidence.

(a) What are the interface conditions that exist at the interface between the conductor and air?
(b) What happens to the wave inside the conductor (i.e., describe the relations for phase velocity, depth of penetration, intrinsic impedance, and propagation constant)?

13.13 **Application**: **Condition of No Reflection—Stealth Principles**. A plane wave propagates from the left in free space. It hits a corner of a conducting material and is reflected **(Figure 13.30)**. Calculate the angle α for which no power is reflected in the negative z direction. The conductor may be considered to be a perfect conductor. Properties for free space are $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m. You can also assume a unit magnitude for the electric field intensity.

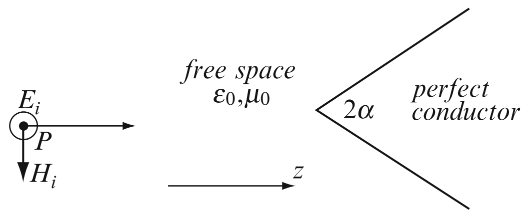


Figure 13.30

13.14 Oblique Incidence on a Conducting Surface: **Perpendicular Polarization.** A perpendicularly polarized plane wave at 100 GHz impinges on a flat metallic reflector at an angle of incidence α. The incident electric field intensity is in the positive x direction, has amplitude 100 V/m, and propagates in free space. Assume the incident magnetic field intensity has components in the positive y and negative z directions and that the interface coincides with the x-y plane. Calculate:

(a) The incident magnetic field intensity.
(b) The reflected electric and magnetic field intensities.
(c) The surface current density as a function of the incidence angle α on the surface of the conductor. Plot its magnitude and show for what values of the incidence angle the current density is maximum and for what values it is minimum.

Oblique Incidence on a Conducting Interface, Parallel Polarization

13.15 Oblique Incidence on a Conductor: **Parallel Polarization**. A uniform plane wave impinges on a good conductor at an arbitrary angle. The wave is polarized parallel to the plane of incidence.

(a) What are the interface conditions that exist at the interface between the conductor and air?
(b) Compare the results obtained here with those in **Problem 13.12**.

13.16 Oblique Incidence on a Conductor: **Parallel Polarization**. A parallel polarized plane wave impinges on a flat metallic reflector at an angle of incidence α. The incident magnetic field intensity is in the positive x direction, has amplitude 100 A/m, and propagates in free space at a frequency of 100 GHz. Assume the incident electric field intensity has components in the negative y and positive z directions and the interface is on the x–y plane.

(a) Calculate the incident electric field intensity.
(b) Calculate the reflected electric and reflected magnetic field intensities.
(c) Calculate the surface current density as a function of the incidence angle α on the surface of the conductor. Plot and show for what values of the incidence angle the current density is maximum and for what values it is minimum.
(d) Compare the results obtained here with those in **Problem 13.14**.

13.17 Show that time-averaged power is conserved across an interface between two media for:

(a) Lossless media, incidence at an angle, perpendicular polarization.
(b) Lossless media, incidence at an angle, parallel polarization.
Hint: Recall that the transmission and reflection coefficients are defined for the tangential components of the electric field intensity.

13.18 Standing Waves for Oblique Incidence on a Conductor. A plane wave is polarized parallel to the plane of incidence, its magnetic field intensity is directed in the positive x direction, and it has an amplitude of 15 A/m. Assume the incident electric field intensity has components in the negative y and positive z directions. The phase constant of the wave is 200 rad/m. The wave impinges on a conducting surface on the x–y plane at 30°.

(a) Calculate the standing wave pattern.
(b) Find the location and amplitude of the standing wave peaks.
(c) Calculate the total time-averaged power density in space. Show that real power propagates parallel to the surface.

13.19 Propagation of Waves in the Presence of a Conducting Surface. A plane wave is parallel polarized and impinges on the surface of a perfect conductor. For a given amplitude and frequency and assuming the wave propagates in free space before hitting the conductor:

(a) Determine the phase velocity in the direction parallel to the surface of the conductor (in which real power propagates) as a function of the angle of incidence.
(b) What is the phase velocity if the incident wave is parallel to the surface of the conductor?
(c) Compare the results in **(a)** and **(b)** with the phase velocity in free space in the absence of the conductor.

13.20 Surface Currents Induced by an Obliquely Incident Wave. A plane wave impinges on a perfectly conducting surface at 30° to the normal. The amplitude of the incident electric field intensity is 10 V/m and the wave propagates in free space. Assume the surface is in the x–y plane and calculate:

(a) The surface current density (A/m) produced by the field if the polarization is perpendicular and the incident electric field intensity is in the positive y direction.
(b) The surface current density for parallel polarization if the incident magnetic field intensity is in the positive y direction.

Parallel and Perpendicular Polarization in Dielectrics

13.21 Oblique Incidence on a Dielectric: **Perpendicular Polarization**. A perpendicularly polarized plane wave impinges on a perfect dielectric from free space. The electric field intensity is in the positive x direction, has amplitude E_{i1}, and the incident wave propagates so that it makes an angle θ_i to the normal. Assume the magnetic field intensity has components in the positive y and negative z directions and the interface is on the x–y plane. The properties of the dielectric are μ [H/m] and ε [F/m].

(a) Calculate the time-averaged power density in air.
(b) Calculate the time-averaged power density in the dielectric.
(c) What is the most fundamental difference between the two power densities calculated above?

13.22 Oblique Incidence on a Dielectric. A uniform plane wave is incident at an angle on an interface between two perfect dielectrics incoming from dielectric (1). The interface coincides with the y–z plane and the dielectrics have properties $\mu_2 = \mu_1 = \mu_0$ [H/m], $\varepsilon_2 = 3\varepsilon_0$ [F/m] and $\varepsilon_1 = 2\varepsilon_0$ [F/m]. The scalar components of the incident electric field are $E_{ix} = 10$ V/m and $E_{\text{iy}} = 5$ V/m:

(a) Find the angle of incidence and the transmission angle.
(b) Identify the polarization of the wave in relation to the given geometry.
(c) Calculate the reflection and transmission coefficients.
(d) From **(a)** and **(c)**, find the scalar components of the reflected and transmitted waves.

13.23 Phase Shift of Transmitted and Reflected Waves. A plane wave at given amplitude and frequency propagates in free space, is polarized perpendicular to the plane of incidence, and impinges on the surface of a high-loss dielectric at an angle θ:

(a) Find the phase shift of the wave at the interface between air and the lossy dielectric; that is, find the phase shift of the transmission coefficient.
(b) Is there also a phase shift in the reflected wave? If so, calculate this phase shift.

13.24 Phase Velocity and its Dependence on Incidence Angle. A plane wave is parallel polarized and impinges at 30° to the normal, on the surface of a perfect dielectric with relative permittivity ε_r and relative permeability μ_r. For a given amplitude and frequency and and assuming the wave propagates in free space before hitting the dielectric:

(a) Determine the phase velocity in the direction parallel to the surface of the dielectric (in which real power propagates) as a function of the angle of incidence.
(b) What is the phase velocity if the incident wave is parallel to the surface of the dielectric?
(c) Compare the results in **(a)** and **(b)** with the phase velocity in free space in the absence of the dielectric.
(d) Compare the answers in **(a)** and **(b)** with those in **(a)** and **(b)** in **Problem 13.19**.

13.25 Reflection Coefficient and its Dependency on Angle of Incidence. Calculate the reflection coefficient for a planar surface of Teflon versus incidence angle when the electric field intensity remains tangential to the surface and when the electric field intensity has both a normal and a tangential component. Properties of Teflon are $\varepsilon = 2.1\varepsilon_0$ [F/m] and $\mu = \mu_0$ [H/m]. Plot the reflection coefficients for an incidence angle between zero and $\pi/2$.

13.26 Parallel and Perpendicular Incidence: **Reflection and Transmission Coefficients**. After obtaining the general expressions for the reflection and transmission coefficients in **Eqs. (13.119)** and **(13.120)**, find the reflection and transmission coefficients for parallel ($\theta_i = 90°$) and perpendicular incidences ($\theta_i = 0°$) on a perfect dielectric.

13.27 Measurement of Thickness of Dielectrics. To measure the thickness of a dielectric material (or if thickness is known its dielectric constant), a collimated wave is sent at an angle θ_1 **(Figure 13.31)**. By receiving the reflection from the first surface and from the second surface, the distances d_1 and d_2 can be directly related to the thickness or dielectric constant of the material. Find the relation needed to measure d. Assume parallel polarization, all material properties are known, dielectric and free space are lossless, and the beam is a narrow beam. Given ε_0 [F/m] and μ_0 [H/m] for free space, $\varepsilon_1 = 4\varepsilon_0$ [F/m], $\mu_1 = \mu_0$ [H/m] for the dielectric, d_1 and d_2 are known, and $d_3 = 10$ mm.

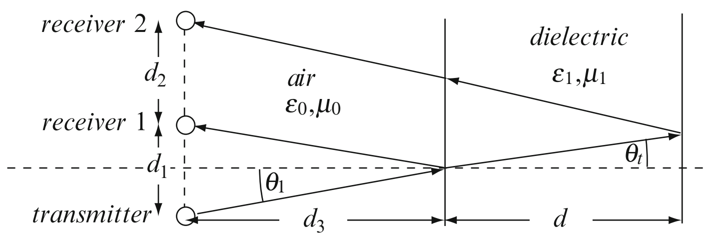


Figure 13.31

Brewster's Angle

13.28 Brewster's Angle in Dielectrics. Calculate the Brewster angle for the following dielectric interfaces for a wave propagating from material (2) into material (1):

(a) Distilled water (1) and air (2): $\varepsilon_1 = 24\varepsilon_0$ [F/m], $\varepsilon_2 = \varepsilon_0$ [F/m], $\mu_2 = \mu_1 = \mu_0$ [H/m].
(b) Plexiglas (1) and air (2): $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 1$, $\mu_2 = \mu_1 = \mu_0$ [H/m].
(c) Teflon (1) and air (2): $\varepsilon_{r1} = 2.25$, $\varepsilon_{r2} = 1$, $\mu_2 = \mu_1 = \mu_0$ [H/m].

13.29 Calculation of Permittivity from the Brewster Angle. A plane electromagnetic wave is incident on the surface of a dielectric at 62° from air (free space). Calculate the permittivity of the dielectric if at this angle there is no reflection from the surface. Assume parallel polarization of the wave.

Total Reflection

13.30 Critical Angles in Dielectrics. What are the critical angles for the following dielectric interfaces? The wave propagates from material (1) into material (2) and all materials have permeability of free space:

(a) Distilled water (1) and air (2): $\varepsilon_1 = 24\varepsilon_0$ [F/m], $\varepsilon_2 = \varepsilon_0$ [F/m].
(b) Plexiglas (1) and glass (2), $\varepsilon_{r1} = 4.0$, $\varepsilon_{r2} = 1.75$.
(c) Teflon (1) and air (2), $\varepsilon_{r1} = 2.25$, $\varepsilon_{r2} = 1$.

13.31 Application: **Use of Critical Angle to Measure Permittivity**. A plane electromagnetic wave is incident on the surface of a dielectric at 36° from within the dielectric, at the interface between the dielectric and free space. Calculate the relative permittivity of the dielectric if at this angle there is total reflection from the surface. Assume the dielectric has permeability of free space.

13.32 Critical Angle in Dielectric. A plane wave with parallel polarization is incident on the interface between a perfect dielectric and free space, at 28° from within the dielectric. The dielectric has permeability of free space and relative permittivity $\varepsilon_r = 4$. Calculate:

(a) The reflection and transmission coefficients at the interface.
(b) The critical angle.

13.33 Application: **Design of Sheathing for Optical Fibers**. An optical fiber is made of glass with a thin plastic coating as shown in **Figure 13.32**. Both materials are transparent at the frequencies of interest. Relative permittivity of glass is 2.25 and that of the plastic material may be chosen as (1) 4.0 or (2) 2.0. The optical fiber operates in free space:

(a) Which coating is a better choice and why?
(b) Calculate the critical angle for propagation inside the glass material, based on your answer in (**a**).
(c) Suppose propagation is also allowed in the coating. What is now the critical angle for propagation?

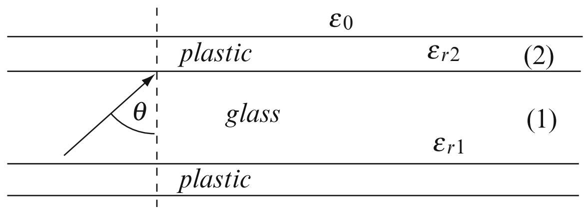


Figure 13.32

Reflection and Transmission for Lossy and Lossless Dielectric Slabs at Normal Incidence

13.34 Propagation through lossless slab. A lossless dielectric layer of thickness d [m] and material constants ε_2 [F/m], μ_2 [H/m], and $\sigma_2 = 0$ are given. The dielectric is in free space **(Figure 13.33)**. Assume a plane wave at frequency f impinges on the dielectric (perpendicular to the surface) from the left. Calculate the total electric field intensity in materials (1), (2), and (3) and the slab reflection and transmission coefficients.

13.35 Propagation Through Lossless Dielectric Slab. A dielectric layer of thickness d [m] and material constants ε_2 [F/m], μ_2 [H/m] is given as shown in **Figure 13.33**. The dielectric is lossless, in free space. Assume a plane wave impinges on the dielectric (perpendicular to the surface) from the left. Given: $\mu_2 = \mu_0$ [H/m], $\varepsilon_2 = 4\varepsilon_0$ [F/m], $\sigma_2 = 0, f = 1$ GHz, $d = 0.01$ m.

(a) Calculate the intrinsic impedances in material (1), (2), and(3).
(b) Assuming the incident electric field intensity is known, find expressions for the electric and magnetic field intensities in each material.
(c) Evaluate the electric and magnetic field intensities in material (1) at the interface, in material (3) at the interface, and in material (2) at $d/2$. Assume the amplitude of the incident electric field intensity is 1 [V/m].

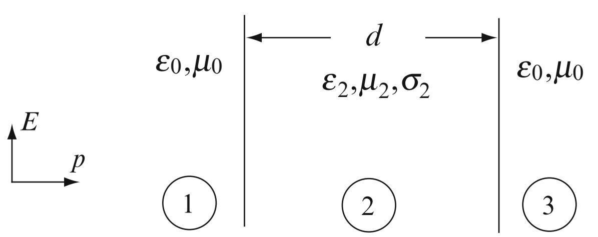


Figure 13.33

13.36 Conditions for Transparency of Dielectrics. Given a dielectric slab in free space, with material properties $\mu = \mu_0$ [H/m], $\varepsilon = 4\varepsilon_0$ [F/m], $\sigma = 0$, what must be the thickness of the slab so that there is no reflection from the material at 1 GHz (i.e., the slab reflection coefficient at the surface is zero)? Is this at all possible with the material properties given?

13.37 Conditions for Transparency. A perfect dielectric of thickness d [m] is placed in front of a perfect conductor. Write an expression for the thickness of the dielectric for which the slab reflection coefficient is zero. Show that this condition cannot be satisfied for any perfect dieletric.

13.38 Application: Design of Radomes. A radome is placed in front of a radar antenna to protect the antenna from the elements. Material properties of the radome are known and the radome is a perfect dielectric. Calculate the minimum thickness of the radome so that the radome is transparent for waves propagating from the antenna and to the antenna. The radome properties are $\mu = \mu_0$ [H/m], $\varepsilon = 4\varepsilon_0$ [F/m], $\sigma = 0, f = 10$ GHz. Use properties of free space for air.

13.39 Application: Design of a Dielectric Window. In a microwave oven it is necessary to place a transparent dielectric window made of a quartz sheet between the magnetron (microwave power generator) and the cavity of the oven, so that the magnetron operates under vacuum. If the oven operates at 2.45 GHz, what must be the thickness of the quartz sheet? Use $\mu = \mu_0$ [H/m], $\sigma = 0$ and $\varepsilon = 3.8\varepsilon_0$ [F/m] for quartz and μ_0, ε_0 everywhere else.

13.40 Propagation Through a Lossy Dielectric Slab. Solve **Problem 13.35** but now the dielectric is a lossy material with conductivity $\sigma_2 = 0.001$ S/m.

13.41 Propagation Through a Two-Layer Slab. A two-layer dielectric slab in free space is given as shown in **Figure 13.34**. An incident wave propagates from material (1) and impinges on the first dielectric interface at $z = 0$. The wave has an electric field intensity of magnitude $E_0 = 1$ V/m directed in the x direction and is at frequency 150 MHz. Hint: Set up the general equations in each section of space and match the fields at the interfaces. Set up a system of equations based on these relations to solve for the forward and backward components in each section and solve for the numerical values of the fields. Find:

(a) The electric and magnetic field intensities everywhere.
(b) The reflection and transmission coefficient of the composite slab.

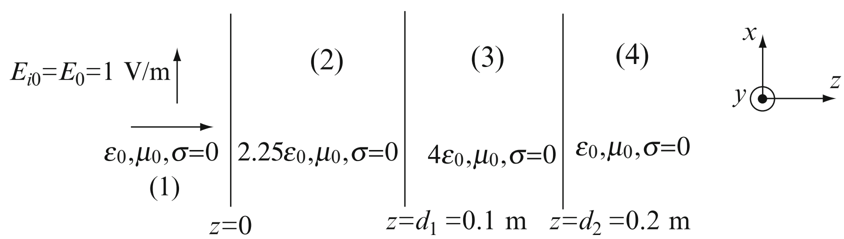


Figure 13.34

13.42 Transmission of Power Through an Interface. A plane wave with an electric field intensity equal to E_0 [V/m] propagates from free space into a material with properties ε_1, μ_1, σ_1, and thickness 1 m. The direction of propagation is perpendicular to the surface of the material. Calculate the time-averaged power dissipated in material (1) per unit area of the material. Given: $\varepsilon_1 = 2\varepsilon_0$ [F/m], $\mu_1 = 50\mu_0$ [H/m], $\sigma_1 = 10$ S/m, frequency = 100 MHz, $E_0 = 100$ V/m.

Reflection and Transmission for a Dielectric Slab Backed by a Perfect Conductor: Normal Incidence

13.43 Reflection from a Conductor-Backed Slab. A perfect dielectric of permittivity ε_2 [F/m], permeability μ_2 [H/m], and thickness d [m] is backed by a perfect conductor as shown in **Figure 13.35**. A plane wave is incident from the left as shown:

(a) Assuming that the incident wave is known, calculate the reflection coefficient at the interface between free space and dielectric.
(b) Calculate the required thickness d for the reflection to be maximum.

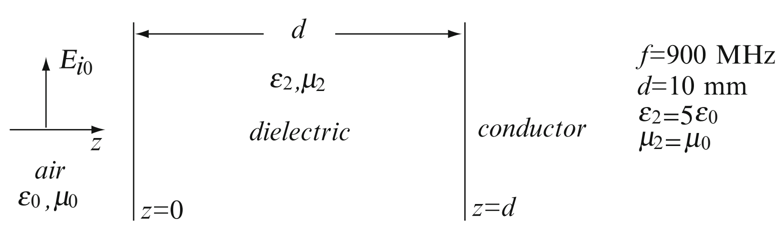


Figure 13.35