Inventory Management

13

The Learning Objectives for This Chapter

- Understand the trade-off "service level vs. costs" in inventory management
- Understand the role of inventory in the supply chain
- Conduct ABC and XYZ analyses
- Explain and use the EOQ/EPQ models for independent inventory demand
- Compute a reorder point
- Calculate service levels and probabilistic inventory models
- Explain and use the dynamic lot-sizing models
- Understand and compute the effects of inventory aggregation
- Explain ATP/CTP concept

13.1 Introductory Case-Study: Amazon, Volkswagen, and DELL

The trade-off "service level vs. costs" is one of the most important issues in inventory management. Its resolution strongly depends on manufacturing, sourcing, distribution, and SC strategies. Consider three examples.

Amazon

Founded in 1994, Amazon.com, Inc. is the world's largest online retailer. It started as an online bookstore, but has diversified over the last 20 years selling DVDs, CDs,

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MP3, software, games, furniture, food, toys, and more. The basic idea is that everybody with access to the internet can buy a massive range of products at any time and get them delivered wherever they want from (nearly) anywhere in the world. Amazon apps make it possible to buy their products easily with smartphones or tablets. Their inventory management for books is based on the number of sales for each book. Best-selling books are stocked in many regional warehouses to enable fast responsiveness for customers in any region. Books with a smaller number of sales are stocked in only a few regional warehouses to reduce inventory costs. The slowest moving books are not even held in stock, but are obtained directly from the publisher or distributor. Through this strategy, it is possible for Amazon to reduce inventory costs for slow-moving products and to have the best possible service level and reliability for their customers.

Volkswagen

Founded in Germany in 1937, Volkswagen implemented the JIT-based SC strategy in Germany. This means that components and items are delivered to the assembly line when they are needed. Only a small amount of safety stock is held for critical parts. This strategy helps to reduce inventory, transport, labor, and administration costs. Often suppliers locate their plants close to the manufacturer to deliver the required components and items at the right moment and to reduce their transport costs. This choice of location reduces the risk of delivery delays and supply shortages. Examples of these JIT components are dashboards with all the electronic devices or seats. JIT is normally used for A-, B-, X-, and Y-items. For C-items, Volkswagen uses a Kanban replenishment system so that they are in stock at any time. These items normally have low inventory holding costs. Based on the JIT strategy and Kanban replenishment system, Volkswagen is able to minimize inventory costs.

DELL

Founded in US in 1983, DELL runs a MTO production strategy. The assembly starts and materials are replenished only after a customer orders. In such a business model, DELL is able to drastically reduce its inventory. For some components, DELL has 90 inventory turns a year which means that inventory is only on hand 4 days on average. Such inventory dynamics allows Dell to respond to a wide range of demand quantities, meet short lead times, handle a large variety of products, build innovative products, and handle supply uncertainty.

13.2 Role, Functions, and Types of Inventory

The *role of inventory management* is to strike a balance between inventory investment and customer service. Inventory is one of the most expensive assets of many companies, representing as much as 50% of total invested capital. In SCOM we must therefore balance inventory investment and customer service level.

Inventory has different *functions*, e.g.:

- to decouple the company and the SC from fluctuations in demand and hold a stock of goods that will provide a selection for customers;
- to increase SC flexibility by placing inventory in the right places;
- to hedge against facility disruptions in the event of natural catastrophes;
- to decouple or separate various parts of the production process;
- to take advantage of quantity discounts and hedge against inflation.

Typically, inventory is classified according to the following *types*:

- *Raw material* (items which are purchased but not processed);
- *Work-in-process (WIP)* (items which underwent some changes, but are not completed);
- *Maintenance/repair/operating (MRO)* (items which are necessary to keep machinery and processes productive);
- *Finished goods* (completed product awaiting shipment).

In making decisions in the scope of inventory management, the following two *basic questions* are put to the forefront for consideration:

- How much should I replenish?
- When should I replenish?

In calculating inventory amounts, the following *costs* are typically considered:

- Holding costs (variable)—the costs of holding inventory over time;
- Ordering costs (fixed)—the costs of placing an order and receiving goods;
- Setup costs (fixed)—the costs pf preparing a machine or process for manufacturing an order;
- Stockout costs (variable)—the costs of lost customer orders resulting from product shortage, loss-of-goodwill costs.

Holding costs include operating costs, labor costs, material handling costs, etc. and vary considerably depending on the business, location, and interest rates. Generally these costs are greater than 15% of the item price, but some items have holding costs greater than 40%. Holding costs depend on the order quantity.

Setup costs (in manufacturing) and ordering costs (in procurement) are fixed and do not depend on the order quantity. These costs tends less for e-business developments. At the same time, setup/ordering costs can be increased in the event there is a high variety of assortment and market dynamics.

According to inventory functions and types, inventory can be used to *manage:*

- Economy of scale—this is *cycle inventory*;
- Uncertainty—this is *safety inventory*.

Cycle inventory exists as a result of producing or purchasing in large *lots* or *batches*. A lot or *batch size* is the quantity that a stage in the SC either produces or purchases at a time. The SC can exploit economy of scale and order in large lots to reduce fixed costs. With the increase in lot size, however, also comes an increase in carrying costs. As an example of a cycle stock decisions, consider an online book retailer. This retailer's sales average around ten truckloads of books per month. The cycle inventory decisions the retailer must make include how much to order for replenishment and how often to place these orders. We will consider cycle inventory optimization in the "Deterministic models" section (for one period) and "Dynamic lot-sizing models" section for many periods.

Safety inventory is carried to satisfy demand subject to unpredictable demand fluctuations and to reduce product shortages. Safety inventory can help the SC manager improve product availability in the presence of uncertainty. In the presence of safety inventory, shortage costs or overage costs can occur. The calculation of safety inventory is based on a predetermined *service level*. Choosing safety inventory involves making a *trade-off* between the costs of having too much inventory and the costs of losing sales due to inventory shortage. We will consider methods to support decisions on safety inventory in the "Stochastic models" section.

For many industries, some products are sold in high quantities in the summer, and lower quantities in the winter (e.g., mineral water). *Seasonal inventory* is built up to counter predictable variability in demand. Companies using seasonal inventory build up inventory in periods of low demand and store it for periods of high demand when they will not have the capacity to produce all that is demanded. Managers face key decisions in determining whether to build up seasonal inventory, and, if they do build it up, in deciding how much. If a company can rapidly change the rate of its production system at a very low cost, then it may not need seasonal inventory, because the production system can adjust to a period of high demand without incurring large costs. The basic trade-off SCOM managers face in determining how much seasonal inventory to build up is the cost of carrying the additional seasonal inventory versus the cost of having a flexible production rate.

► **Practical Insights** It is intuitively clear that car tires and pins have different inventory management policies. However, in a distribution center, which runs over two million items, it can be quite a complicated task to find the right policy for each item. Many companies that ask for inventory optimization want to start immediately with software and mathematical models. But before starting any calculations, items should be properly analyzed and classified. False organization of inventory management is in many cases, the key point of problems in optimization.

13.3 Material Analysis

13.3.1 ABC Analysis

The first step in item classification is the *ABC analysis*. ABC analysis divides inventory into three classes based on annual dollar volume:

- Class A—high annual dollar volume
- Class B—medium annual dollar volume
- Class C—low annual dollar volume

ABC analysis is used to establish policies that focus on the few critical parts and not the many trivial ones. This method implements the Pareto Principle which states that "there are few critical and many trivial." Consider a simple example. Imagine that you have ten Lego blocks: two of them are red, three green, and five blue (see Fig. 13.1). To perform the ABC analysis, we measure the annual demand of each inventory item times the cost per unit. Assume that one red block costs \$1, one green block costs \$0.1, and one blue block costs \$0.01.

You can easily calculate that total inventory costs are \$2.35. Two red blocks take only 20% of the total inventory amount, but they create 85% of inventory costs. These are critical items in the A group. A-items have to be managed especially carefully. First, the relationships with suppliers of A-items must be managed, i.e., supplier development. Second, tighter physical inventory control for A-items is necessary. Third, we need more care in forecasting demand for A-items.

Consider a numerical example for ABC analysis.

Fig. 13.1 ABC analysis

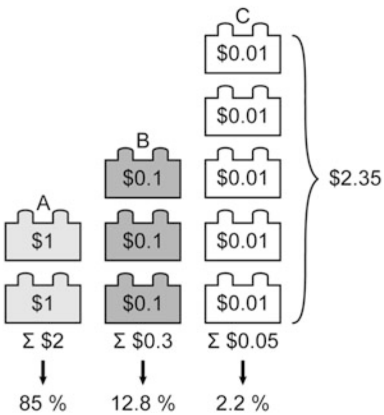


Table 13.1 Initial data for ABC analysis

Table lamp	Annual demand	Cost per unit	Annual expenditure
X1	100	0.5	50
X2	200	0.05	10
X3	50	1.65	82.5
Y1	40	10.75	430
Y2	200	0.11	22
Y3	200	0.19	38
Y4	50	2.4	120
T1	90	0.6	54
T2	10	13.6	136
T3	60	1.35	81

Table 13.2 Results of ABC analysis

Table lamp	Annual demand	Cost per unit	Annual expenditure	Cumulative expenditure	Percentage expenditure	Category
Y1	40	10.75	430	430	42.0	A
T2	10	13.6	136	566	55.3	A
Y4	50	2.4	120	686	67.0	A
X3	50	1.65	82.5	768.5	75.1	A
T3	60	1.35	81	849.5	83.0	B
T1	90	0.6	54	903.5	88.3	B
X1	100	0.5	50	953.5	93.2	B
Y3	200	0.19	38	991.5	96.9	C
Y2	200	0.11	22	1013.5	99.0	C
X2	200	0.05	10	1023.5	100.0	C

Task 13.1 ABC Analysis

Quarted Ltd. is a company which sells table lamps.

In Tables 13.1 and 13.2, a numerical example is provided.

We are given annual demand and the costs. To determine which table lamps are in the A, B, or C-category, we need to calculate the annual expenditure:

$$\text{annual expenditure} = \text{annual demand} \cdot \text{costs per lamp} \tag{13.1}$$

The next step is to arrange the different types of table lamps according to their annual expenditure. After calculating cumulative expenditure, we have to compute the percentage of cumulative expenditure. Then we can classify the A, B, and C items. Assuming that the classification is 80:15:5 we can label the different table lamps as follows (see Table 13.2).

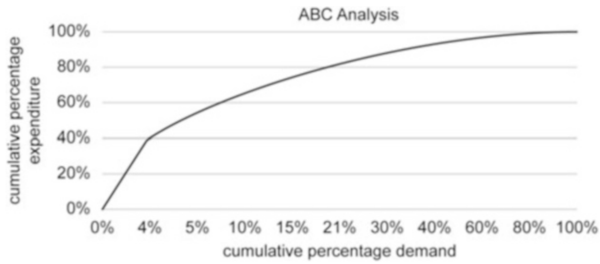


Fig. 13.2 ABC analysis in graphical form

We can observe that 150 table lamps generate around 80% of inventory costs (A), whereas 600 table lamps generate only 5% of inventory costs (C). This relationship can be presented in graphic form as shown in Fig. 13.2.

From Fig. 13.2 it can be observed that 80% of expenditure is created by only 20% of demand.

13.3.2 XYZ Analysis

The monetary value of capital commitment is not the only criterion for classifying materials. Other criteria include the physical volume of items, demand patterns, or delivery times. For example, in terms of the physical volume of items, it is possible to classify unwieldy items in group X and small and handy items in groups Y and Z. The principle is the same as for the ABC analysis. Another option for XYZ analysis is to divide inventory into three classes based on different demand patterns:

- Class X: constant, non-changing demand;
- Class Y: neither constant nor sporadic demand (fluctuating demand);
- Class Z: sporadic or strongly fluctuating demand.

Changes in demand make it possible to determine the prediction accuracy of each inventory class. Information on XYZ analysis provides an opportunity to develop strategies concerning alternative stocking arrangements, reorder calculations, and intervals of inventory control. Consider Fig. 13.3 to understand the connection between the ABC and XYZ analyses.

XYZ analysis can be used to enhance ABC analysis. A combined ABC/XYZ analysis helps to define the purchasing method for different inventory classes and to determine which parts must be in stock, when they should be ordered JIT, and when it is only viable for them to be ordered on a forecast basis.

A Hollywood star is constantly complaining of the lack of space in her wardrobe, represented as Tables 13.3 and 13.4.

Perform a combined ABC/XYZ analysis to help the Hollywood star to gain at least 50% of space and to reduce the expenditure by 70%!

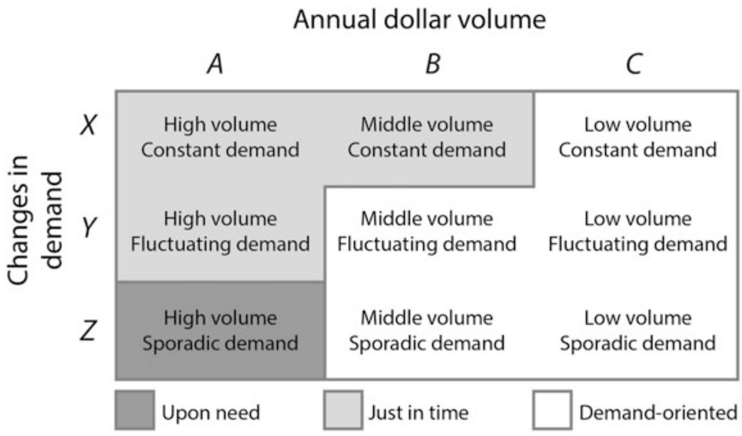


Fig. 13.3 ABC/XYZ analysis

Table 13.3 Initial data for ABC/XYZ analysis

	Quantity	Cost per unit ($)	Annual expenditure	Volume per unit (dm^3)	Total volume (dm^3)
Blouses	120	200	24,000	1.00	120
Pantsuits	420	200	84,000	1.00	420
Jeans	50	200	10,000	2.00	100
Dresses and skirts	450	500	225,000	2.00	900
Costumes	280	1000	280,000	2.00	560
Fur coats	120	10,000	1,200,000	10.00	1200
Sport pants	10	80	800	2.00	20
T-shirts	1200	80	96,000	0.50	600
Scarfs	100	50	5000	0.40	40
Underwear	500	75	37,500	0.02	10
Belts	600	95	57,000	0.05	30
Total	3850		2,019,300		4000

Solution. First, we sort the items according to their volume (see Table 13.5).

Second, we perform ABC analysis (Table 13.6).

Finally, we sort the items according to the XYZ classification in order to see how much potential there is for space reduction. Of course, we will have to identify X-items and the percentage of never used X-items, which are primary candidates for leaving the wardrobe. In parallel, we will have to identify A-items, since our second goal in this task is also to reduce expenditure to 70% (Table 13.7).

Table 13.4 Initial data for ABC/XYZ analysis

	Quantity	Frequent use	Seldom use	No use	% of no use from quantity
Blouses	120	30	30	60	50.00
Pantsuits	420	100	120	200	47.62
Jeans	50	20		30	60.00
Dresses and skirts	450	10	20	420	93.33
Costumes	280	20	30	230	82.14
Fur coats	120	5	15	100	83.33
Sport pants	10	4	2	4	40.00
T-shirts	1200	300	400	500	41.67
Scarfs	100	30	30	40	40.00
Underwear	500	200	100	200	40.00
Belts	600	200	200	200	33.33

Table 13.5 XYZ analysis

	Quantity	Volume per unit (dm^3)	Total volume (dm^3)	Percentage	Cumulative percentage
Fur coats	120	10	1200	30	30
Dresses and skirts	450	2	900	22.5	52.5
T-shirts	1200	0.5	600	15	67.5
Costumes	280	2	560	14	81.5
Pantsuits	420	1	420	10.5	92
Blouses	120	1	120	3	95
Jeans	50	2	100	2.5	97.5
Scarfs	100	0.4	40	1	98.5
Belts	600	0.05	30	0.75	99.25
Sport pants	10	2	20	0.5	99.75
Underwear	500	0.02	10	0.25	1
Total	3850	20.97	4000	100.00	100

It can be observed that due to the high value, high volume, and high percentage of never used items, such as the fur coats, dresses, skirts, and costumes, are key in achieving both the objectives, i.e. value reduction of 70% and space reduction of 50%. Clearing the wardrobe of never used fur coats, dresses, skirts, and costumes will enable the Hollywood star both to reduce the value of the wardrobe items to 70% and space to 50%.

Table 13.6 ABC analysis

	Quantity	Cost per unit ($)	Annual expenditure	Cumulative expenditure	Percentage expenditure	Category
Fur coats	120	10,000	1,200,000	1,200,000	59.4	A
Costumes	280	1000	280,000	1,480,000	73.3	A
Dresses and skirts	450	500	225,000	1,705,000	84.4	B
T-shirts	1200	80	96,000	1,801,000	89.2	B
Pantsuits	420	200	84,000	1,885,000	93.4	B
Belts	600	95	57,000	1,942,000	96.2	C
Underwear	500	75	37,500	1,979,500	98.0	C
Blouses	120	200	24,000	2,003,500	99.2	C
Jeans	50	200	10,000	2,013,500	99.6	C
Scarfs	100	50	5000	2,018500	99.7	C
Sport pants	10	80	800	2,019,300	100	C
Total	3850		2,019,300	2,019,300	100	

Table 13.7 Integrated ABC-XYZ analysis

	Quantity	% of never used items	Cumulative percentage (volume)	Space saving (%)	ABC	Cost saving
Fur coats	120	50.00	30	25.00	A	1000,000
Dresses and skirts	450	47.62	52.5	21.00	B	210,000
T-shirts	1200	60.00	67.5	6.25	B	40,000
Costumes	280	93.33	81.5	11.50	A	230,000
Pantsuits	420	82.14	92	5.00	B	40,000
Blouses	120	83.33	95	1.50	C	12,000
Jeans	50	40.00	97.5	1.50	C	6000
Scarfs	100	41.67	98.5	0.40	C	2000
Belts	600	40.00	99.25	0.25	C	19,000
Sport pants	10	40.00	99.75	0.20	C	3200
Underwear	500	33.33	100	0.10	C	15,000
Total	3850		100	100		1,577,200

We thank Mr. Martin Pruy for preparing this task

13.4 Deterministic Models

After classifying the items, the next step is to determine order quantities. In this section, we consider items with independent deterministic demand in the setting of economy of scale, i.e., the *cycle inventory*. Why do we need to determine optimal order quantities? Theoretically, we could order exactly the quantity that corresponds to the daily demand each day. However, then we would also have ordering costs each day. We would pay each day for transportation.

In most cases, it is reasonable to exploit *economy of scale* and order in large lots to reduce fixed ordering costs. We are going to learn how to determine when and how much to order. We will consider the following methods:

- Basic economic order quantity (EOQ)
- Economic production order quantity (EPQ)
- Quantity discount model
- Reorder point (ROP).

13.4.1 EOQ Model

Let us start with the *EOQ model*. It is simple and helps us to understand the relationship between ordering and holding costs. Consider the system that exhibits the following characteristics (see Fig. 13.4):

- Demand and lead-time are known and constant;
- Receipt of inventory is instantaneous and complete;
- Quantity discounts are not possible;
- The only variable costs are setup and holding;
- Stock-out can be avoided.

We introduce the following notations:

q is the number of units per order;

q^* is optimal number of units per order (EOQ);

b is annual demand in units for the inventory item;

f is set-up or ordering cost for each order;

c is holding or carrying cost per unit per year.

Under the assumption of linear inventory consumption, cycle inventory, and lot-sizes are related as follows in Eq. (13.2):

$$\text{Cycle inventory} = \frac{q}{2}. \tag{13.2}$$

This means that average inventory on hand is equal to 50% of the order quantity.

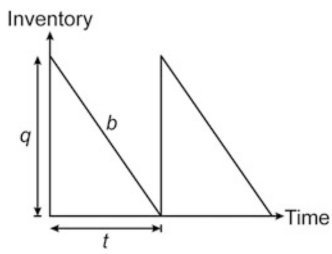


Fig. 13.4 Inventory consumption and replenishment pattern in EOQ model

$$\text{Then annual inventory holding costs is } c \cdot \frac{q}{2}. \tag{13.3}$$

In order to calculate ordering costs we have to know the number of orders per year. This number can be easily calculated as shown in Eq. (13.4):

$$\text{Number of orders per year} = \frac{b}{q} \tag{13.4}$$

$$\text{Then, annual fixed ordering costs is} = f \cdot \frac{b}{q} \tag{13.5}$$

Optimal order quantity is found when annual ordering costs equal annual holding costs [see Eq. (13.6)]:

$$c \cdot \frac{q}{2} = f \cdot \frac{b}{q} \tag{13.6}$$

In solving Eq. (13.6) for q*, we get the EOQ formula as follows (Eq. 13.7):

$$q^* = \sqrt{\frac{2b \cdot f}{c}} \tag{13.7}$$

Consider graphical representation (see Fig. 13.5).

It can be observed from the graph that the smallest total cost (the top curve) is the sum of the two curves below it. Minimal total costs are achieved at the intersection point of the fixed and variable costs curves. This corresponds to the EOQ point q*. It can also be observed that total cost function is quite flat in the minimal region. This

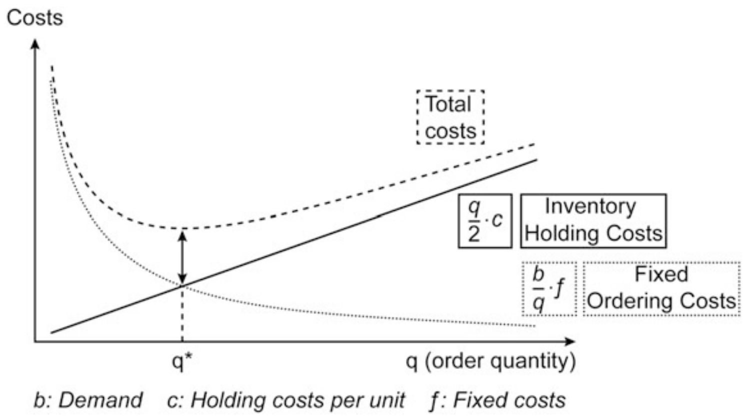


Fig. 13.5 Graphical representation of EOQ model

means that moderate EOQ changes will not influence a significant increase in total costs by tendency. This exemplifies the useful *robustness property* of this method.

We can also determine the expected number of orders per year (Eq. 13.8) and expected time between orders (Eq. 13.9):

$$N = \frac{b}{q*} \tag{13.8}$$

$$T = \frac{365}{N} \tag{13.9}$$

Total annual cost is calculated as shown in Eq. (13.10):

$$TC = c \cdot \frac{q^*}{2} + f \cdot \frac{b}{q*}. \tag{13.10}$$

Consider an example.

Task 13.2 EOQ

Demand for the Sakri2 LED TV at Amillos is 3200 units per quarter. Amillos charges fixed costs of \$2500 per order. Annual holding costs per LED TV are \$80. Calculate the number of LED TVs that the store manager should order per refill.

Solution

Annual demand: b = 3200 × 4 = 12,800 units; Ordering cost/order: f = \$2500

Holding cost per unit per year: c = \$80

Using the EOQ formula, the optimal order quantity is

$$q^* = \sqrt{\frac{2 \cdot 12,800 \cdot 2500}{80}} = 895$$

To minimize total costs at Amillos, the store manager orders 895 LED TVs per refill. The cycle inventory is the average resulting inventory and is calculated as follows: cycle inventory = 895/2 ≈ 448 units.

For an order size of q* = 895, the store manager evaluates:

Number of orders per year: N = 12,800/895 = 14.3

Expected time between orders: T = 365/14.3 = 25.5 days

Total cost: TC = 80 · 895/2 + 2500 · 12,800/895 = \$71,554

Total cost for 1 year is \$71,554 for 14.3 orders with 895 LED TVs in each order.

▶ **Practical Insights** The EOQ model was developed around the beginning of the twentieth century in the era of mass production and economy of scale. By that time, procurement processes were being performed manually and were quite costly and time-consuming. Today, economy of flexibility and small lot-sizes exists in many industries and services. Transportation costs and SC coordination has become more and more important. Procurement processes have been automated, many of them performed via the internet. A significant part of fixed ordering costs (telephone, fax, etc.) has been cut.

▶ **Practical Insights** In applying EOQ computation results and inventory control policies, the practical reality must also be considered. For example, in some cases suppliers fix the dates for new orders (e.g., each Friday) or allow orders at fixed quantities (e.g., 50, 150 or 300 units) only.

Deliberate the following questions:

- Can we include transportation costs in the EOQ model?
- Is the EOQ of each firm really an optimal solution for the whole SC?
- For which department is EOQ really "optimal"? (Hint: what happens to transportation costs if EOQ decreases?)

Consider an example. EOQ is 40 units. This corresponds to 25 deliveries a year subject to annual demand of 1000 modules. Consider the following procedure for determining transportation costs: 400 € per delivery + 4 € per module. It is possible to transport up to 100 units at a time.

Costs Analysis for 100 Units

- Transportation costs = 10 deliveries $\cdot$ (400 + 4 $\cdot$ 100) = 8 € per unit
- Cycle inventory = (100/2) $\cdot$ 29 + (1000/100) $\cdot$ 23.2 = 1682 €
- Safety inventory = 1.65 $\cdot$ 4 $\cdot$ 10 = 66 items $\cdot$ 29 = 1914 €
- Total costs per module: 11.6 €

Costs Analysis for 40 Units

- Transportation costs = 25 deliveries $\cdot$ (400 + 4 $\cdot$ 40) = 14 € per unit
- Cycle inventory = (40/2) $\cdot$ 29 + (1000/40) $\cdot$ 23.2 = 1160 €
- Safety inventory = 1.65 $\cdot$ 4 $\cdot$ 10 = 66 units $\cdot$ 29 = 1914 €
- Total costs per module: 17.1 €

Conclusion: EOQ is not optimal for the integrated inventory-transportation setting. At lot-sizes of 100 units, total costs are reduced from 17.1 € to 11.6 € per unit. For an annual demand of 1000 modules, the cost savings is 5500 €.

13.4.2 EOQ Model with Discounts

Inventory costs can also be calculated on the basis of the unit prices p (i.e., the actual costs of the material purchased) as shown in Eq. (13.11):

$$Costs(q) = p_1(1 - r_1)b + \frac{q}{2}p_1(1 - r_1)I + \frac{b}{q}f \tag{13.11}$$

In this case for calculating EOQ, Eq. (13.12) is used:

$$q^* = \sqrt{\frac{2 \cdot b \cdot f}{p \cdot I}}, \tag{13.12}$$

where I is the *interest rate* (*capital commitment*) and p is the unit price.

As such, different prices can be included in the analysis. This allows us to apply the EOQ model for situations with quantity discounts. Reduced prices are often available when larger quantities are purchased. In this case, the *trade-off* is between reduced item costs and increased holding costs.

The *algorithm of calculating EOQ with discounts* involves the following steps:

- For each discount, calculate q*;
- If q* does not qualify for a discount, choose the smallest possible order size to get the discount;
- Compute the total cost for each q* or adjusted value from Step 2;
- Select the q* that gives the lowest total cost.

Task 13.3 EOQ with Discounts
Carlo Inc. operates a chocolate shop in New York. The chocolate is ordered from a supplier in Switzerland. Normally, cost for one unit of chocolate is $5.00, but a quantity discount is provided by the manufacturer (see Table 13.8).

Carlo Inc.'s annual demand for chocolate is 10,000 units and the setup cost per order is $50. Interest rate is 20%.

Solution:

$$\text{For discount } 0\ \%, q^* = \sqrt{\frac{2 \cdot 10,000 \cdot 50}{5 \cdot 0.2}} = 1000.$$

Table 13.8 Initial data for EOQ calculation with discounts

Discount quantity in units	Discount (%)	Discount price p ($)
0–999	0	5.00
1000–1999	4	4.80
2000–10,000	10	4.50

$$Costs(1000) = 5 \cdot (1 - 0) \cdot 10,000 + \frac{1000}{2} \cdot 5(1 - 0) \cdot 0.2 + \frac{10,000}{1000} \cdot 50 = \$51,000$$

$$\text{For discount } 4\ \%, q^* = \sqrt{\frac{2 \cdot 10,000 \cdot 50}{4.8 \cdot 0.2}} = 1021$$

q* = 1021 units qualifies for the interval [1000–1999] units and therefore can be used for costs calculation:

$$Costs(1021) = 5 \cdot (1 - 0.04) \cdot 10,000 + \frac{1021}{2} \cdot 5(1 - 0.04) \cdot 0.2 + \frac{10,000}{1021} \cdot 50 = \$48,980$$

$$\text{For discount } 10\ \%, q^* = \sqrt{\frac{2 \cdot 10,000 \cdot 50}{4.5 \cdot 0.2}} = 1054$$

q* = 1054 units does not qualify for the interval [2000–10,000] units and therefore the smallest possible order size to get the discount of 10% should be used for cost calculations:

$$Costs(2000) = 5 \cdot (1 - 0.1) \cdot 10,000 + \frac{2000}{2} \cdot 5(1 - 0.1) \cdot 0.2 + \frac{10,000}{2000} \cdot 50 = \$46,150$$

It can be observed that we have the lowest total costs with 2000 units of chocolate per order. We can also determine the expected number of orders per year $N = 10,000/2000 = 5$ and the expected time between orders: $T = 365/5 = 73$.

13.4.3 EPQ Model

The EPQ model is fairly similar to the EOQ model, but it is applied to manufacturing. This model is used when:

- inventory builds up over a period of time after an order is placed;
- units are produced and sold simultaneously.

In the EOQ model, we assume that the receipt of inventory is instantaneous and complete. Now, we allow the receipt of inventory *over a period of time* and introduce three new parameters:

- r is daily production rate;
- d is daily demand;
- t is the length of the production run in days.

Setup costs remain unchanged; holding costs are now calculated subject to the relation of production and demand as follows (Eq. 13.13):

$$c \cdot \frac{q}{2}[1-(d/r)] \tag{13.13}$$

For example, if we produce the quantity that corresponds exactly to the daily demand each day, we will not have any holding costs (the expression in brackets will always equal zero), but we will have very high set-up costs.

Then, the following EPQ formula (13.14) can be stated as follows:

$$q^* = \sqrt{\frac{2b \cdot f}{(1-d/r) \cdot c}} \tag{13.14}$$

Maximal inventory level in the system can be calculated as per Eq. (13.15):

$$\mathrm{I}_{max} = q\left(1 - \frac{b}{r}\right) \tag{13.15}$$

Note: In EPQ, we consider daily demand. If we are given annual demand, it should be divided into the corresponding number of working days.

Consider an example.

Task 13.4 EPQ

Natural Inc. produces high-quality food processors. It sells 18,000 processors per year and is able to produce 125 machines per day. Natural Inc. works 250 days per year. Annual holding costs per food processor is $18 and setup costs are $800. Calculate the economic production quantity for Natural Inc. and the maximal inventory level.

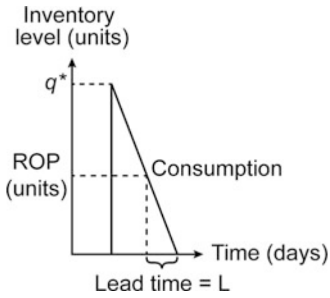
Solution

d = 18,000/250 = 72 units per day

$$q^* = \sqrt{\frac{2 \cdot 18{,}000 \cdot 800}{(1-72/125) \cdot 18}} = 1943 \ \mathrm{I}_{max} = 1943\left(1 - \frac{72}{125}\right) = 824$$

It can be observed that the optimal lot-size is 1943 units. At this quantity, minimal setup and holding costs can be achieved. Taking into account the actual consumption rate of 72 units a day and the production rate of 125 units a day, maximal inventory level in the system is 824 units.

Fig. 13.6 Re-order point



13.4.4 Re-order Point

The EOQ model answers the "how much" question. The *re-order point* (ROP) tells "when" to order. ROP is introduced to take into account the so called *lead time*, i.e. the time between placement and receipt of an order (see Fig. 13.6).

With the assumption of constant demand and a set lead time, ROP is calculated as in Eq. (13.16):

$$ROP = d \cdot L, \tag{13.16}$$

where d is daily demand and L is lead time.

Note: In ROP, we consider daily demand. If we are given annual demand it should be divided into the corresponding number of working days in a year. Consider an example.

Task 13.5 Re-order Point

A company experiences an annual demand of 8500 cheese knives per year (250 working days). Lead time for an order is five working days. Calculate the ROP and explain its meaning.

Solution

1. Daily demand = 8500/250 = 34 units
2. ROP = 34 · 5 = 170 units

If our inventory reaches 170 cheese knives, then we have to place a new order.

13.5 Stochastic Models

We already know how to determine order quantities and ROPs for situations where demand and lead time are deterministic. However, in many practical cases, both demand and lead time fluctuate. We do not know their values, but can only estimate them on the basis of probability. For such cases, *stochastic* (*probabilistic*) models are needed.

13.5.1 Service Level and Safety Stock

Imagine that one Saturday evening you are sitting at home and notice that you have only 16 nappies left for your baby. You are in a province city in Germany where all stores are closed on Sunday. You recall that normally you need eight nappies a day. Normally, you would need three nappies for the rest of Saturday, eight for Sunday, and three for Monday morning until the stores open again. So normally 16 nappies would be enough and you would not need to go out now to buy a new pack of nappies (the stores close in 1 h and it is rainy). But what will happen if the demand for nappies increases and deviates from the mean value? So here the trade-off in your decision is to balance the risk of being out of stock and the additional costs for driving the car to the store and spending your time buying nappies.

Uncertainty in demand makes it necessary in SCOM to maintain a certain customer service level or *level of product availability* to avoid stock-outs. The level of product availability is the fraction of demand that is served on time from a product held in inventory. A high level of product availability provides a high level of *responsiveness,* but increases costs because much inventory is held, but rarely used. In contrast, a low level of product availability lowers inventory holding cost, but results in a higher fraction of customers who are not served on time. The basic trade-off when determining the level of product availability is between the cost of inventory to increase product availability and the loss from not serving customers on time.

For example, a 0.05 probability of stock-out corresponds to a 95% service level (see Fig. 13.7).

> **Practical Insights** In practice, different service level estimations are used. For example, anyLogistix software uses three service level indicators (Ivanov 2017):
>
> - The Alpha service level measures the probability that all customer orders that arrive within a given time interval will be completely delivered from stock on hand. In other words, a lack of stock will not delay the deliveries.
> - The Beta service level is a quantity-oriented service level with consideration of backordering.
> - The Lead Time service level is the ratio of orders delivered to the customers within the expected lead time to total orders.

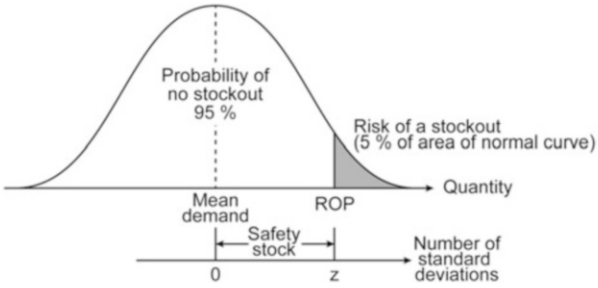


Fig. 13.7 Interrelation of demand distribution, ROP, service level and safety stock. Adapted from Heizer and Render (2013)

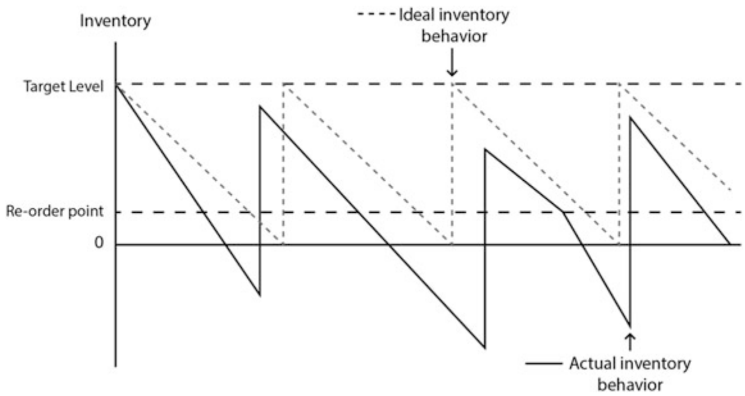


Fig. 13.8 Actual and ideal inventory behavior

In the further course of this Chapter, we will focus on the Alpha service level. In a situation of demand uncertainty, *safety inventory* is introduced with the objective to ensure product availability even in the case of demand fluctuations. Consider an example in Figs. 13.8–13.10 based on practice of inventory control in software anyLogistix.

Assume that we use Eqs. (13.12) and (13.16) to compute EOQ and ROP, respectively. The dashed line in Fig. 13.8 reflects the inventory dynamics in the case of using optimal EOQ and ROP and can be named as an ideal inventory behavior. The ideal inventory behavior means in this case that all assumptions of EOQ and ROP models subject to Eqs. (13.12) and (13.16) are met, i.e., demand and lead-time are constant. In reality, this is not the case. Both demand and lead-time fluctuate resulting in actual inventory behavior which is different as the ideal one.

In order to cope with this situation, the ROP should be increased by the safety stock. Consider Figs. 13.9 and 13.10.

In Fig. 13.9, the ROP from Fig. 13.8 is increased by safety stock. It considers an example where safety stock allows to cope with demand fluctuations in some cases. However, in other cases there exists a backlog. Figure 13.10 shows an example

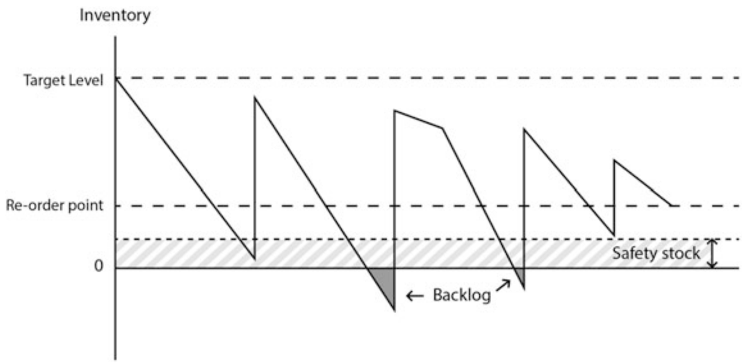


Fig. 13.9 ROP with safety stock and backlogs

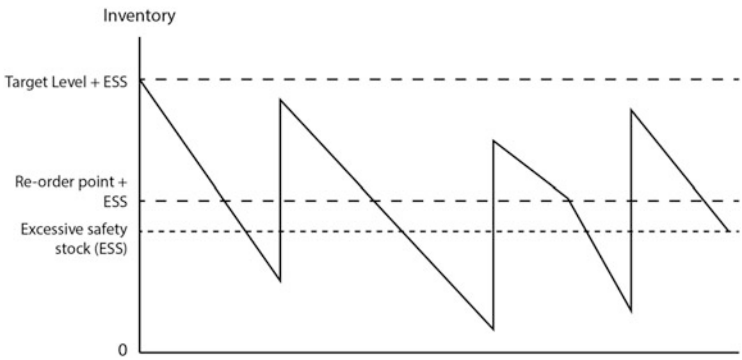


Fig. 13.10 ROP with excessive safety stock and without backlogs

where ROP from Fig. 13.9 is increased by an excessive safety stock (ESS). The ESS is so high that demand fluctuations would never result in a backlog which means a 100% product availability on stock resulting in a 100% service level. However, the inventory level in Fig. 13.10 is much higher as compared to Figs. 13.8 and 13.9 resulting in higher inventory costs.

The question is *how much safety stock should we plan to find a right balance between the inventory investment and customer satisfaction?* Technically, the safety stock computation is based on the desired service level and demand volatility (see Fig. 13.7 and Eq. 13.17).

It can be observed from Fig. 13.9 that ROP is enlarged by the safety stock subject to a 95% service level.

In order to compute safety stock subject to a desired service level, Eq. (13.17) is used:

Table 13.9 Table of normal distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

$$ss = z \cdot \sigma_{dLT}, \tag{13.17}$$

where ss is safety stock, σ_{dLT} is standard deviation of demand during lead-time and z is the number of standard deviations.

Demand deviation can be gleaned, e.g., from analysis of demand forecasts and actual sales in the past. For example, $\sigma = 1.25MAD$ is a typical value. The Z-value can easily be determined (see Table 13.9).

For example, z = 1.65 for a service level of 95%, z = 2.33 for service level of 99%, and z = 1.28 for a service level of 90%. If standard deviation of demand during lead time is 10, then safety stocks equals $1.65 \cdot 10 = 16.5$; $2.33 \cdot 10 = 23.3$; $1.28 \cdot 10 = 12.8$. We can observe that the increase of service level from 90% to 99% results in a doubling of inventory.

Another important issue is that stock-outs can be objective and subjective. For example, the absence of a certain sort of milk in a supermarket does not automatically mean that all milk is missing since other kinds are available. In retail, 3–8% of stock-out is typical and results not only from false inventory planning, but could for many other reasons, e.g., false shelf placement.

▶ **Practical Insights** The service level is determined subject to SC strategy (efficient vs responsive). The higher the service level, the higher SC responsiveness, but also the higher inventory costs. If managers were to set the service level to 100% this would mean that each customer order would be satisfied from the inventory without delay. However, a 100% service level would result in a huge inventory. This cost can scatter the positive effects of high customer satisfaction. That is why service level is typically set at less than 100%.

The inclusion of safety stock changes the calculation of ROP [see Eq. (13.18)]:

$$ROP = \overline{d} \cdot L + ss, \tag{13.18}$$

where $\overline{d}$ is average daily demand.

In order to calculate ROP, four situations are possible:

- demand is assumed to be normally distributed during the lead time;
- daily distribution of demand is given (i.e., demand is variable) and lead time is constant;
- daily demand is constant and lead time is variable;
- both demand and lead time are variable.

In order to calculate ROP if demand is assumed to be normally distributed during the lead time, formula (13.19) can be used:

$$ROP = \overline{d} \cdot L + z \cdot \sigma_{dLT} \tag{13.19}$$

If daily distribution of demand is given (i.e., demand is variable) and lead time is constant, formula (13.20) can be used:

$$ROP = \overline{d} \cdot L + z \cdot \sigma_d \cdot \sqrt{L} \tag{13.20}$$

If daily demand is constant and lead time is variable, formula (13.21) can be used:

$$ROP = \overline{d} \cdot L + z \cdot \overline{d} \cdot \sigma_L \tag{13.21}$$

▶ **Practical Insights** Equation (13.21) nicely provides evidence of the importance of reducing lead time variability. We can observe that lead time variability reduction directly influences safety stock levels. This observation depicts the integration of supplier selection, contracting, and inventory management decisions.

Finally, if both demand and lead time are variable, formula (13.22) can be used:

$$ROP = \overline{d} \cdot L + z\sqrt{L \cdot \sigma_d^2 + \overline{d}^2 \cdot \sigma_L^2} \tag{13.22}$$

Consider an example.

Task 13.6 ROP with Safety Stock
Average demand for toothbrushes is 35 units per day. Standard deviation of normally distributed demand during lead time is ten toothbrushes per day. Lead time is 3 days. Service level is 95%. Calculate ROP and safety stock.

Solution

$$\text{ROP} = 35 \cdot 3 + 1.65 \cdot 10 = 122 \text{ units; Safety stock is } 1.65 \cdot 10 = 16.5 \text{ units}$$

Now we assume that we are given daily distribution of demand instead of standard deviation of normally distributed demand during lead time. Consider ten units as daily standard deviation of demand and calculate ROP:

Solution

$$\text{ROP} = 35 \cdot 3 + 1.65 \cdot 10 \cdot \sqrt{3} = 134 \text{ units}$$

Next consider a situation where demand is constant, but lead time may fluctuate with a standard deviation of 1 day. Calculate ROP.

Solution

$$ROP = 35 \cdot 3 + 1.65 \cdot 35 \cdot 1 = 163$$

Finally, we assume that both demand and lead time are variable. Calculate ROP.

Solution

$$ROP = 35 \cdot 3 + 1.65\sqrt{3 \cdot 10^2 + 35^2 \cdot 1^2} = 170$$

► **Practical Insights** Practical implementation of the statistical methods for inventory management is not easy. In many companies, decisions have been taken manually for many years on the basis of expert knowledge. In this case, it can be reasonable to allow manual re-writing of the calculated results in software to start working with new technology. As practice shows, in a short period of time, inventory managers will see the advantages of the new system and accept 95% of automatically generated orders.

13.5.2 Single Period Systems ("Newsvendor Problem")

The newsvendor problem is a mathematical model for calculating the optimal inventory level for one single period. It is called the newsboy or newsvendor problem. A newspaper vendor who must decide every day how many daily newspapers he wants to stock for the next day is faced by uncertain demand and the knowledge that unsold copies will be almost worthless next day.

The newsvendor problem is characterized by the following conditions:

- fixed price for each unit,
- perishable product,
- uncertain demand,
- no additional delivery in period *t*,
- short purchase time.

Consider the following notation for single period model:
c is purchase price;
r is retail price;
v is salvage price;
c_o is overage cost;
c_u is underage cost;
z is the number of standard deviations;
σ is the standard deviation of demand;
μ is the expectation of demand;
S is order quantity;
$S*$ is optimal order quantity;
Z(S*) is the expected cost for optimal order quantity;
∏(S*) is the expected profit for optimal order quantity.
To calculate the *overage* and *underage costs* we can use Eqs. (13.23) and (13.24):

$$c_o = c - v \tag{13.23}$$

$$c_u = r - c \tag{13.24}$$

Then we use the *critical ratio* (CR) to find the z-value from the table of normal distribution (Eq. 13.25):

$$CR = \frac{c_u}{c_u + c_o}; F(CR) = z \tag{13.25}$$

For example, $CR = \frac{0.75}{0.75+0.25}; F(0.75) = 0.68 = z.$
Next step is the calculation of S* according to Eq. (13.26):

$$S^* = \mu + z \cdot \sigma \tag{13.26}$$

In order to calculate the expected cost and profit for optimal order quantity, Eqs. (13.27) and (13.28) can be used:

$$Z(S^*) = (c_o + c_u) \cdot f_{01}(z) \cdot \sigma \tag{13.27}$$

$$\Pi(S^*) = c_u \cdot (\mu - Z(S^*)) \tag{13.28}$$

Consider an example.

Task 13.7 Newsvendor Problem
Coff&Co., a coffee shop, sells vegan chocolate croissants. They purchase the croissants from a small bakery at the end of the street. The bakery sells chocolate croissants to Coff&Co. for \$0.70 each. Coff&Co. sells them for \$2.40 to their customers. Unsold croissants can be returned to the bakery for \$0.15 each. On the basis of the last few months, Coff&Co. expects a normal distributed demand for chocolate croissants. Expectation of demand is 14 croissants per day with a standard deviation of four per day. Calculate the optimal order quantity, and expected costs and profit for the chocolate croissants.

Solution

$$\begin{aligned}&\text{Overage cost}: c_o = c - v = 0.70 - 0.15 = \$0.55\\&\text{Underage cost } c_u = 2.40 - 0.70 = \$1.70\\&CR = \frac{1.70}{1.70 + 0.55} = 0.75; F(0.7556) = 0.7 = z\end{aligned}$$

$S^* = 14 + 0.7 \cdot 4 = 17$ units; therefore Coff&Co. should order 17 vegan chocolate croissants per day.

$Z(S^*) = (0.55 + 1.70) \cdot f_{01}(0.7) \cdot 4 = \$2.81/day.$; so expected cost is \$2.81 per day.

$\Pi(S^*) = 1.70 \cdot (14 - 2.81) = \$20.99/day.$; so expected profit is \$20.99 per day.

Note: $f_{01}(z)$ value can be taken from the full version of a normal distribution table.

13.5.3 Safety Stock and Transportation Strategy: Case DailyMaersk

This case study focuses on the impact of a global transportation concept on inventory management using the example of the Daily Maersk. It illustrates how the sea-leg part of SCs relates to shippers' inventory management at destination countries. It shows that Daily Maersk can offer substantial benefits to shippers in terms of safety stock reduction.

Maersk Line is part of Denmark's largest corporation, the Copenhagen-based Maersk Group. Within the Maersk Group, Maersk Line makes up roughly half of the revenues, making it the group's largest segment. Maersk Line operates in the container shipping industry, offering liner services between seaports. It is the largest container line worldwide with a fleet of 576 vessels. Normally, customers can be split into either direct customers (producers) or forwarders (e.g. Kuehne&Nagel, DHL, DB Schenker) who book slots for the cargo on container vessels to the respective locations.

Fig. 13.11 Daily Maersk's transportation network

In 2011, Maersk announced the introduction of Daily Maersk, a first-of-its-kind concept where daily instead of weekly departures from key ports in Asia to key ports in Europe were offered. Maersk started by offering daily rather than weekly departures from key Asian ports such as Shanghai, Ningbo, Yantian (all Chinese) and Tanjung Pelepas (Malaysia) to Felixstowe (UK), Rotterdam (Netherlands), and Bremerhaven (Germany) (Fig. 13.11).

Consider an example. The sea journey from Yantian to Felixstowe takes 30 days on average; now, one departure of a Maersk vessel is offered each day instead of once a week. We assume a constant demand of 10 units a day, 98% service level, and a normally distributed lead-time from Yantian to Felixstowe. The safety stock (ss) at destination (Felixstowe) can be then defined as

$$\mathrm{ss} = \mathrm{z} \cdot \overline{d} \cdot \sigma_{\mathrm{LT}}, \tag{13.29}$$

where z is the number of standard deviations, d is daily demand, and σ_{LT} is standard deviation of lead time.

First, an analysis of lead time was performed for weekly departures. It showed that the standard deviation of lead time for weekly departures is 2 days. Subsequently, an analysis of lead time was performed for daily departures: standard deviation for daily departures of 0.3 days.

Assuming a service level of 98% and daily demand of 10 units, safety stock can be calculated as follows:

$$\begin{aligned} &\mathrm{ss}\ \ (\text{weekly departure}) = 2.055 \cdot 10 \cdot 2.0 = 41.1 \ \text{units} \\ &\mathrm{ss}\ \ (\text{daily departure}) = 2.055 \cdot 10 \cdot 0.3 = 6.2 \ \text{units} \end{aligned}.$$

The calculation above shows that the safety stock level falls significantly as a result of the Daily Maersk concept to only 6.2 units required in stock versus 41.1 units in a weekly service. This translates into lower inventory costs for direct customers or intermediaries, such as forwarders, at destination points and offers a measurable benefit to Maersk Line's customers.

Discussion Questions

What impact does daily vessel departure have on inventory?

Daily vessel departures have two impacts: (1) lower average lead time (due to shorter average waiting time at port of origin); and (2) a lower standard deviation of lead time (as the increased sailing frequency leads to a high probability of catching a vessel within 24 h, even if a vessel is missed for 1 day).

Which other factors could be included in this analysis?

We restricted our analysis only to sailing lead time and included no other elements of lead time (thus ignoring e.g., hinterland transportation).

Why is Maersk Line able to provide daily service and what risks might be encountered?

As the largest player in the market, Maersk exploits economy of scale, being among the few able to offer sufficient capacity for daily departures. To offer daily instead of weekly departure, Maersk Line must deploy more vessels instead of only one on respective routes, demanding a much higher investment in container ships. Its environmental footprint can be assumed to be higher, as more vessels travel on the same route. Maersk faces the risk of not being able to fully utilize the increased capacity on Daily Maersk routes. This risk can be reduced by re-routing other services.

13.6 Inventory Control Policies

Inventory control policy is a managerial procedure that helps to define how much and when to order. The review may happen periodically (e.g., at the end of a month) or continuously (i.e., tracking each item and updating inventory levels each time an item is removed from inventory). Four parameters are important in the setting up of inventory control policies:

- t is replenishment interval;
- q is order quantity;
- s is re-order point;
- S is target inventory level.

Since order quantity and replenishment intervals may be both fixed and variable, four basic inventory control policies can be classified (see Fig. 13.12).

If the period between two orders is always the same, we talk about *periodic review systems*. If the point of time of the next replenishment depends on the ROP, we talk about the ROP method of stock control or a *continuous review system*. The above-mentioned four parameters can be fixed or changed (adjusted) in dynamics according to changes in demand and supply. Therefore, *static* and *dynamic* views on inventory control policies can be considered.

13.6.1 Fixed Parameters

When the replenishment interval, order quantity, ROP, and target inventory levels are fixed, the following policies can be classified.

Policy 1: t,q

- t: fixed time between two orders
- q: fixed order quantity

In *(t,q)* policy, a fixed amount (q) is ordered for a fixed period of time (t) (see Fig. 13.13).

(*t,q*) is a simple policy for handling the ordering process. This policy opens possibilities to further automatic control, which improves quality and saves resources, such as labor, energy, or materials. However, the (*t,q*)-policy is inflexible and used very seldom in business. Should uncertainty or fluctuation in demand exist, this policy cannot be adjusted. In addition, shortage or overstocking make the (*t,q*)-policy an unattractive tool for many companies. Thus, it is recommended to implement this policy under constant demand.

Policy 2: t,S

Fig. 13.12 Inventory control policies

		Order interval	
		Fixed	Variable
Order quantity	Fixed	(t,q)-policy	(s,q)-policy
	Variable	(t,S)-policy	(s,S)-policy

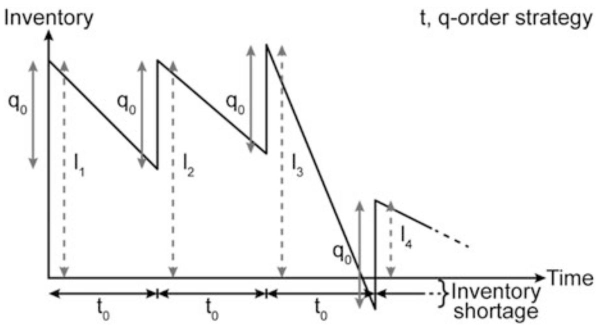


Fig. 13.13 (t,q)-inventory control policy

- T: fixed time between two orders
- Q: variable order quantity to stock up to the target level S

In the (*t,S*)-policy, the order quantity (*q*) is variable, and *q* is placed at a fixed time (*t*). We need to order a certain amount of inventory to reach the desired quantity *S* subject to lead time (*lt*). Order quantity is calculated as the target level S—stock on hand (see Fig. 13.14).

This policy avoids excessive inventory, which cannot be used for any other purpose and thus involves opportunity costs. The model is easy to use for control of orders. However, the physical control of the inventory could be so expensive that the exact count is only performed once a month, for example. In certain cases the (*t, S*)-policy can lead to relatively high capital commitment because of the high average inventory. This policy also implies high ordering costs because we might not place a large order on the fixed day. At the same time, we might need to wait too long to fulfil our target inventory and thus a shortage can occur. The (*t,S*)-policy is recommended for use in companies with cycled replenishment.

Policy 3: s,q

- t: variable time between two orders
- q: fixed order quantity

This model operates when order quantity (*q*) is fixed and the interval (t) between orders can vary. In this case, the order point (*s*) is defined as ROP (see Fig. 13.15). Every order arrives to replenish inventory after a lead time. The lead time is assumed to be known and constant. The only uncertainty is associated with demand. In the following analysis, one should be most concerned with the possibility of shortage during an order cycle, that is, when the inventory level falls below zero. This is also called a stock-out event. Every time we extract inventory, we compare what is left with *s*.

Note: for calculating the ROP, Eq. (13.20) should be slightly modified to take into account the replenishment interval [see Eq. (13.30)]:

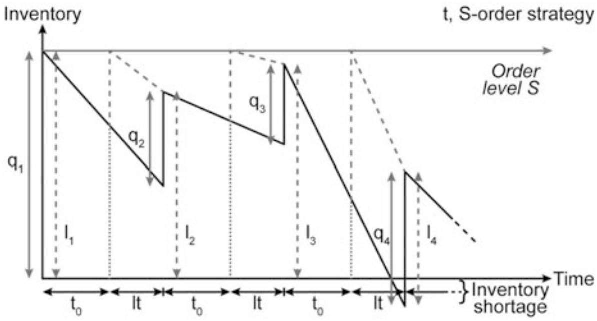
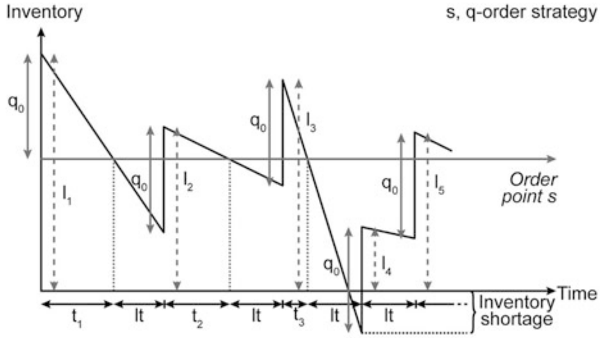


Fig. 13.14 (t,S)-inventory control policy

Fig. 13.15 (s,q)-inventory control policy



$$ROP = d \cdot (T + L) + z \cdot \sigma \cdot \sqrt{(T + L)} \tag{13.30}$$

If the stock level is less than s, then we place an order at the rate of q. Similar to the (t,q)-policy, in the (s,q)-policy, q also refers to the optimal order quantity. The policy (s,q) results in shorter time between orders if there is inventory shortage. Because of its simple operation and full control over results, this policy is widely used in organizations. An advantage of the model is that it considers demand fluctuations. Disadvantages lie in regular inventory control.

Policy 4: s,S

- t: variable time between two orders
- q: variable order quantity between the order level S and ROP s

This strategy is used to define the drop of order quantity s after every inventory usage. Should this be the case, a manager should refill inventory to raise the inventory position to the level S, which is desirable property (see Fig. 13.16).

Therefore, both order quantity and the time interval between orders is variable. This system can handle any level of demand and at any time, and includes demand fluctuations in planning. Target level is calculated as Eq. (13.31):

$$\mathrm{S} = (\mathrm{ROP} + q) \tag{13.31}$$

Order policy (s,S) avoids an excessive level of inventory and ensures that the business has the right goods on hand to avoid stock-outs. However, this policy requires much effort and high control. It is used in industrial and commercial areas of business, given the fact that flexible order quantity is possible and a target quantity can be predetermined.

Task 13.8 Inventory Control Policy

Consider the given data and determine parameters and annual holding costs for (s,S)-policy for 95% and 98% service levels respectively:

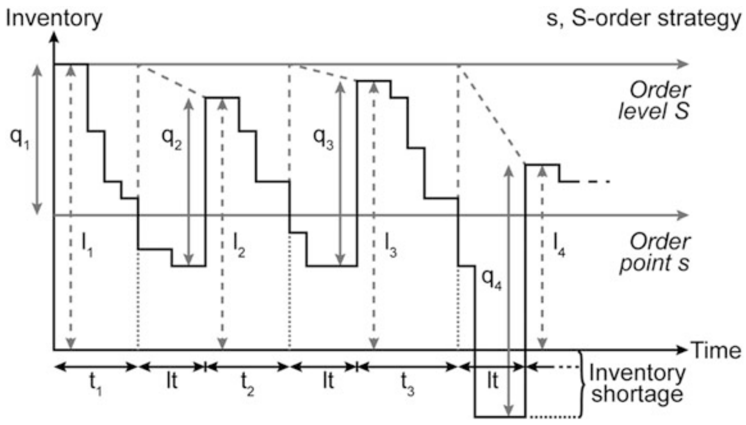


Fig. 13.16 (s,S)-inventory control policy

- demand per day (d)
  100 units;
- standard daily deviation of demand (σ)
  20 units;
- annual holding costs (h)
  \$10 per unit;
- fixed ordering costs (f)
  \$100 per order;
- order interval (T)
  4 weeks;
- lead time (L)
  2 weeks.

Solution

1. Find z-values for 95% and 98% service level; we get 1.65 and 2.05 respectively.

$$ss = z \cdot \sigma \cdot \sqrt{(T + L)} = 1.65 \cdot 20 \cdot \sqrt{4 + 2} = 81 \text{ units}$$

$$ROP = 100 \cdot (4 + 2) + 81 = 681 \text{ units}$$

2. S = ROP + q; $q^* = \sqrt{\frac{2 \cdot 36,500 \cdot 100}{10}} = 855$ units; $S = 681 + 855 = 1536$ units.
3. The policy is (681;1536); Average inventory position is (681 + 1536)/2 = 1108.
4. Costs = 1108 · 10 = \$11,082.
5. $ss = 2.05 \cdot 20 \cdot \sqrt{4 + 2} = 100$; $ROP = 100 \cdot (4 + 2) + 100 = 700$ units
6. S = ROP + q; $q^* = \sqrt{\frac{2 \cdot 36,500 \cdot 100}{10}} = 855$ units.; S = 700 + 855 = 1555 units.

Fig. 13.17 Dynamic view of inventory control (based on SupplyOn VMI solution, used with permission)

7. The policy is (700;1555); Average inventory position is (700 + 1555)/2 = 1128.
8. Costs = 1128 · 10 = \$11,282.

13.6.2 Dynamic View

When the replenishment interval, order quantity, ROP, and target inventory levels are not fixed, but change in dynamics subject to changes in demand, the following changes to the above-mentioned policies must be considered. We have to take into account demand, current and projected inventory, and in-transit quantities as well as planned deliveries (see Fig. 13.17).

It can be observed from Fig. 13.17 that both target inventory level and the re-order point change in dynamics subject to changes in demand. The calculation basis is the planned days of supply.

13.7 Dynamic Lot-Sizing Models

We already know how to manage inventory with both deterministic and stochastic demand. However, our previous discussion addressed one period problems. Now we will consider *multi-period* problems. Assume that you need tickets for the tram, say 20 tickets for a month. You can buy all of them right at the beginning of the month. In this case, you spend time for buying tickets only once, but you invest quite a lot of money at one time. Alternatively, you can buy tickets every day. Your capital commitment will be lower, but you will spend more time buying tickets.

In multi-period problems, the parameters (e.g., demand, lead time, costs) can vary in different periods. In addition, inventory from the previous periods can be used to cover demand in the future. This is why the basic *trade-off in multi-period models* is how to balance inventory holding costs and ordering costs over time.

Table 13.10 Demand data

t	1	2	3	4	5	6
b_t	500	200	600	300	200	100

Dynamic lot-sizing models help us to decide in every period if we should order/ produce only for one period or if we should build lot-sizes over several periods together to minimize order and holding costs. These models can be divided into two groups: optimization and heuristics. An example for optimization is the Wagner–Whitin model, which is a generalization of the EOQ model which considers changes in demand over time. The second group contains the heuristics, e.g., the Silver-Meal heuristic, the least unit cost heuristic, and the part-period heuristic.

To explain the different dynamic lot-sizing models and show the different procedures and results, we use the following example.

Task 13.9 Dynamic Lot-Sizing Problem
Sheeran Ltd. has the following periodic demand for a material they need for their production (Table 13.10):

Orders take place at the beginning of each period and the product is immediately available. Every order generates a fixed cost f of \$100 and there are holding costs c per unit per period of \$0.5.

13.7.1 Least Unit Cost Heuristic

The least unit cost heuristic is based on an average unit cost per period K_t^{unit}. For the calculation of the average costs for the first period we can use Eq. (13.32):

$$K_1^{unit} = \frac{f_1}{b_1} \tag{13.32}$$

In the second step, the task is to find out if it is more efficient to order the material for period 1 and period 2 jointly in the first period. To find out the average unit cost jointly for the first and second period we use Eq. (13.33):

$$K_{1,2}^{unit} = \frac{f_1 + b_2 \cdot c}{b_1 + b_2} \tag{13.33}$$

We continue with this extension until we reach the minimum of the average costs. This means that as soon as the cost increases we stop the calculation and start from the beginning with the next period onwards.

For example:

$$K_{1,1}^{unit} = \frac{100}{500} = 0.2 K_{1,2}^{unit} = \frac{100 + 200 \cdot 0.5}{500 + 200} = 0.286$$

We can see the average cost per unit increases, so we decide not to order the demand for period 1 and period 2 together. Instead of ordering them together, we start the calculation with period 2 again and order in period 1 only the material for this single period:

$$K_{2,2}^{unit} = \frac{100}{200} = 0.5 K_{2,3}^{unit} = \frac{100 + 600 \cdot 0.5}{200 + 600} = 0.5$$

Because the average unit cost remains at the same level we continue the calculation:

$$K_{2,4}^{unit} = \frac{100 + 600 \cdot 0.5 + 300 \cdot 2 \cdot 0.5}{200 + 600 + 300} = 0.636$$

Note: since the inventory for period 3 is held over two periods, we multiply $(300 \cdot 0.5)$ by 2.

In this case, we might want to order material in period 2 for periods 3, 4, and 5 jointly and start the calculation for period 4 again.

$$K_{4,4}^{unit} = \frac{100}{300} = 0.33 K_{4,5}^{unit} = \frac{100 + 200 \cdot 0.5}{300 + 200} = 0.4$$

The average cost increases again, so we decide to order only the material for period 4 and not for periods 4 and 5 together.

$$K_{5,5}^{unit} = \frac{100}{200} = 0.5 K_{5,6}^{unit} = \frac{100 + 100 \cdot 0.5}{200 + 100} = 0.5$$

In the result, order quantities for each period are determined as follows:

$$q_1 = 500; \; q_2 = 800; \; q_3 = 0; \; q_4 = 300; \; q_5 = 300; \; q_6 = 0$$

In order to calculate the total cost, we add up the fixed ordering costs for every order and the holding costs for every period in which we generate them:

$$K^{total} = 4 \cdot 100 + (600 \cdot 0.5) + (100 \cdot 0.5) = \$750.$$

We order in four periods and in period 2 and 5 we order material for the next period as well. Total cost is $750.

13.7.2 Silver-Meal Heuristic

The Silver-Meal method uses the average cost per period instead of the average unit cost. As long as the cost decreases, the lot-size is extended similarly to the least unit cost heuristics.

For period 1 Eq. (13.34) can be used:

$$K_1^{period} = \frac{f_1}{1} \tag{13.34}$$

If we want to find out if it is less expensive to order the material for period 2 in period 1, we use Eq. (13.35):

$$K_{1,2}^{period} = \frac{f_1 + b_2 \cdot c}{2} \tag{13.35}$$

Consider an example:

$$K_{1,1}^{period} = \frac{100}{1} = 100 \; K_{1,2}^{period} = \frac{100 + 200 \cdot 0.5}{2} = 100$$

Now we expand the first calculation with the material holding costs for period 2. Because they do not increase we continue with period 3.

$$K_{1,3}^{period} = \frac{100 + 200 \cdot 0.5 + 600 \cdot 2 \cdot 0.5}{3} = 267$$

As you can see, the average period cost increases remarkably. For this reason we order only material for period 1 and 2 together and start our calculation again with period 3.

$$K_{3,3}^{period} = \frac{100}{1} = 100 \; K_{3,4}^{period} = \frac{100 + 300 \cdot 0.5}{2} = 125$$

The cost increases again and so we order the material for period 3 only.

$$K_{4,4}^{period} = \frac{100}{1} = 100 \, K_{4,5}^{period} = \frac{100 + 200 \cdot 0.5}{2} = 100$$

$$K_{4,6}^{period} = \frac{100 + 200 \cdot 0.5 + 100 \cdot 2 \cdot 0.5}{3} = 100$$

As we can see we can order the material for periods 4, 5, and 6 jointly, resulting in the following solution.

$$q_1 = 700 \;\; q_2 = 0 \;\; q_3 = 600 \;\; q_4 = 600 \;\; q_5 = 0 \;\; q_6 = 0$$

$$K^{total} = 3 \cdot 100 + (200 \cdot 0.5) + (200 \cdot 0.5 + 100 \cdot 2 \cdot 0.5) = \$600$$

The total cost is lower with Silver-Meal heuristic than with the least unit cost heuristic. In this case the Silver-Meal order strategy is the preferable method because we can save $150.

► **Practical Insights** Silver-Meal heuristic is most preferable in cases with sporadically fluctuating demand.

13.7.3 Wagner–Whitin Model

The *Wagner–Whitin model* considers deterministic demand that changes over several periods for one product. This model helps us to find the optimal solution that will tend to be (but not necessarily!) better than Silver-Meal and offer the least unit cost heuristic solutions. The Wagner–Whitin method provides optimal results for the periods under consideration, but not beyond them. The basic idea of the model is to minimize holding and order costs. Unlike with the heuristics, the Wagner–Whitin method does not stop when costs are increasing in one period, because the optimization method compares different periods with each other.

For the first period we only have the fixed ordering cost as $K_1^{WW} = f_1$. In the second step, we want to find out the costs for ordering periods 1 and 2 jointly in the first period instead of ordering them separately.

Consider the basic Wagner–Whitin formula for costs calculation (13.36):

$$K_{t,t+1} = Min\left[Min(1 \leq t < j)\left[f + c \cdot \sum_{t=t+1}^{j} b_t \cdot n + K_{t-1}\right]; f + K_{j-1}\right] \qquad (13.36)$$

According to formula (13.36), for each period t, all possible options to order for n-periods in this period until the n-period or to not order at all are considered. The option with minimal costs is selected.

The calculation for the first period is performed as $K_{1,2}^{WW} = f_1 + b_2 \cdot c$. Normally, this calculation is continued for all possible options. This is remarkable for the Wagner–Whitin method and differentiates it from the heuristics. It is safer to calculate one or two more costs rather than build the wrong lot-sizes. In case we want to calculate costs for the next period, we do not start only with the fixed ordering costs in this period. We have to add up the ordering costs from this period and the total costs from the previous ordering period.

Consider the following example to better understand the procedure of the Wagner–Whitin method (Table 13.11).

Table 13.11 Wagner–Whitin calculation

	Demand b_t					
	500	200	600	300	200	100
Period t	1	2	3	4	5	6
1	100	200	800			
2		200	500	800		
3			300	450	600	
4				400	500	600
5					500	**550**
6						600
K_t^{WW}	100	200	300	400	500	550

The bold value indicates minimal total costs

$$K_{1,1}^{WW} = 100;$$
$$K_{1,2}^{WW} = 100 + 200 \cdot 0.5 = 200; K_{1,3}^{WW} = 100 + 200 \cdot 0.5 + 600 \cdot 2 \cdot 0.5 = 800$$
$$K_{2,2}^{WW} = K_{1,1}^{WW} + 100 = 200; K_{2,3}^{WW} = K_{1,1}^{WW} + 100 + 600 \cdot 0.5 = 500$$

In this case, the costs for ordering the materials for periods 1 and 2 jointly (strategy 1) is similar to ordering them separately (strategy 2). We can follow both ways and decide at the end which is the preferred strategy.

$$K_{2,2}^{WW} = K_{1,1}^{WW} + 100 + 600 \cdot 0.5 + 300 \cdot 2 \cdot 0.5 = 800$$

$$K_{3,3}^{WW} = K_{1,2}^{WW} + 100 = 300$$

If we decide to order in period 1 for the first and second periods jointly, the calculation we use for period 3 is the total cost of ${K_{1,2}}^{WW}$. If we decide to order for the first and second periods separately, we use the total cost of ${K_{2,2}}^{WW}$ for the calculation of ${K_{3,3}}^{WW}$. We will continue with strategy 1.

$$K_{3,4}^{WW} = K_{1,2}^{WW} + 100 + 300 \cdot 0.5 = 450$$

$$K_{3,5}^{WW} = K_{1,2}^{WW} + 100 + 300 \cdot 0.5 + 200 \cdot 2 \cdot 0.5 = 4650$$

$$K_{4,4}^{WW} = K_{3,3}^{WW} + 100 = 400; K_{4,5}^{WW} = K_{3,3}^{WW} + 100 + 200 \cdot 0.5 = 500$$

$$K_{4,5}^{WW} = K_{3,3}^{WW} + 100 + 200 \cdot 0.5 + 100 \cdot 2 \cdot 0.5 = 600$$

$$K_{5,5}^{WW} = K_{4,4}^{WW} + 100 = 500; K_{5,6}^{WW} = K_{4,4}^{WW} + 100 + 100 \cdot 0.5 = 550$$

$$K_{6,6}^{WW} = K_{5,5}^{WW} + 100 = 600$$

The costs $K_{\min}^{WW}$ at the end of the table in the last period are the total costs. When trying different strategies, we realize that the total cost can never be below \$550.

Practical Insights The dynamic lot-sizing models considered are basically applied to make-to-stock (MTS) production strategies (see Chap. 6). In small series manufacturing and assemble-to-order (ATO) production strategies, other methods are typically used. These methods are based on bottleneck-based manufacturing control such as *EPEI* (Every Part Every Interval) lot-sizing, *heijunka* or DBR (*drum-buffer-rope*). Lot-size can also be constrained by limited storage capacity within the production processes. In addition, integration of manufacturing lot-sizing and vehicle scheduling belong to practical trends, e.g., as so-called BIB (*batch-in-batch*) lot-size.

13.8 Aggregating Inventory

In a number of cases, *many markets* are replenished from the same warehouse. In this context, interesting *trade-offs* between the number of facilities, inventory, and transportation costs may arise when comparing different options, e.g.:

- building warehouse in each market or
- centralizing inventory.

Some important questions can be discussed regarding these options.

How does the replenishment interval affect the safety stock requirement in warehouses?

A larger replenishment interval results in higher safety stock requirements, because safety stock is often used at the end of replenishment intervals; if replenishment intervals are larger, it is important that safety stock is available to be used.

How does the replenishment interval affect the level of inventory (and thus the size of the warehouse)?

A smaller replenishment interval means a lower level of inventory within the warehouse and thus a smaller warehouse is useful. On the other hand, a higher replenishment interval leads to a higher level of inventory and thus a bigger warehouse must be provided.

How does the replenishment interval affect other warehouse costs (such as labor cost)?

A shorter replenishment interval probably results in higher labor cost because of greater material handling.

Furthermore, it is necessary to evaluate other factors that affect the decision, such as the market in which the firm produces. Perhaps in a smaller market the firm would focus on responsiveness rather than efficiency, and in a larger market the opposite. Also other SC drivers affect the decision. Since sustainability is becoming more important, firms also have to find a trade-off between transportation costs and responsiveness while considering *sustainability*.

Another specific case is a situation where two warehouses have to be merged in order to save warehousing fixed and operating costs. How can we determine the right level of safety stock in the new larger warehouse? Here are two possible situations:

- Two warehouses served the same market previously;
- Two warehouses served different markets previously.

When two warehouses served the same market previously, safety stock should remain unchanged, according to the formula $ss = z \times \sigma_{dLT}$. Indeed, if deviation of demand for tomatoes in Berlin is 100,000 tons, it will not change only because we merge our warehouses. However, if two warehouses served different markets previously, the following holds true: $\sigma_{new} = \sqrt{\sigma_{new}^2}$ where $\sigma_{new}^2 = \sigma_1^2 + \sigma_2^2$.

Note: If average demand and demand deviation differ significantly from each other in two warehouses which previously served the same market, the formula for different markets has to be applied.

Task 13.10 Merging Warehouses
Mr. Tsching has two wholesale flower markets and his most important products during the Christmas period are Christmas stars. Mr. Tsching has to order Christmas stars for the next season from his supplier. His markets are located in Berlin and Cologne and provide Christmas stars for different regions in Germany, Switzerland, and Austria. The warehouses serve different markets. Cologne serves west and south Germany and Switzerland, and Berlin serves north and east Germany and Austria. Based on sales data from the last year, Mr. Tsching assumes demand to be evenly distributed. Demand for his market in Cologne is 12,000 Christmas stars with a standard deviation of 4600 stars. The market in Berlin has demand for 14,300 Christmas stars with a standard deviation of 6200 stars. The purchase price for one Christmas star is 1.12 € and the retail price is 3.65 €. Christmas stars which are not sold can be sold on to the textile industry for 0.31 € per star. The textile industry can use the red pigments for dying. Mr. Tsching thinks about merging his markets. Merging will involve extra transport costs of 22,500 €, but a reduction in fixed costs of 20,000 €.

Calculate optimal order volume, and expected costs and profit for every market, and decide if a merging of Mr. Tsching's markets is a profitable idea.

Solution.

We refer to the newsvendor model considered in Sect. 13.5.2. First, the expected costs and profit for the non-merged case are as follows

$$c_o = 1.12 - 0.31 = 0.81\text{€}\ c_u = 3.65 - 1.12 = 2.53\text{€}$$
$$CR = \frac{2.53}{2.53 + 0.81} \approx 0.76 F(0.76) = z = 0.71$$
$$f(0.71) = 0.31 S^*_{Berlin} = 14,300 + 0.71 \cdot 6200 = 18,702$$
$$S^*_{Co\log ne} = 12,000 + 0.71 \cdot 4600 = 15,266$$

Expected cost for Christmas season:

$$Z\left(S^*_{Berlin}\right) = (0.81 + 2.53) \cdot 0.31 \cdot 6200 = 6420\text{€}$$

$$Z\left(S^*_{Co\log ne}\right) = (0.81 + 2.53) \cdot 0.31 \cdot 4600 = 4763\text{€}$$

Expected profit for Christmas season:

$$\Pi\left(S^*_{Berlin}\right) = 2.53 \cdot 14,300 - 6420 = 29,759\text{€}$$

$$\Pi\left(S^*_{Co\log ne}\right) = 2.53 \cdot 12,000 - 4763 = 25,597\text{€}$$

$$\Pi(total) = 25,597 + 29,759 = 55,356\text{€}$$

Merging the markets:

$$\mu_{new} = \mu_{Berlin} + \mu_{Co\log ne} = 14,300 + 12,000 = 26,300$$

$$\sigma_{new} = \sqrt{\sigma^2_{Berlin} + \sigma^2_{Co\log ne}} = \sqrt{6200^2 + 4600^2} = 7720$$

$$S^*_{new} = 26,300 + 0.71 \cdot 7720 = 31,781$$

Expected cost for the merged the market:

$$Z\left(S^*_{new}\right) = (0.81 + 2.53) \cdot 0.31 \cdot 7720 = 7993\text{€}$$

Compare expected costs with total costs for two markets:

$$Z\left(S^*_{Berlin}\right) + Z\left(S^*_{Co\log ne}\right) - Z\left(S^*_{new}\right) = 6420 + 4763 - 7993 = 3190\text{€}$$

Do not forget to add the costs of merging ($22,\ 500\text{€} = \ -20,\ 000\text{€}$) when calculating the expected profit:

$$\Pi\left(S^*_{new}\right) = 2.53 \cdot 26,300 - (7993 + 2500) = 56.046\text{€}$$

We opt for merging the warehouses because the expected profit of 56,046 € is higher than in initial situation (55,356 €).

13.9 ATP/CTP

John was browsing the internet to find new furniture for his flat. After entering the desired parameters, he had an overview of beds. The overview contained offers from four suppliers. The prices were quite similar, but lead times differed significantly. Two suppliers indicated 3 weeks of lead time, one supplier indicated 2–3 days, and the last indicated 8–10 days since it does not have the item in stock but will be able to produce it and deliver within a week. John selected the offer of the third supplier, since the prices were almost equal and lead time became the competitive advantage. His friend Alex selected the offer of the fourth supplier since the price was a bit lower than that of the third supplier, but lead time was shorter than for suppliers 1 and 2.

Indication of a lead time of “3 weeks” is a signal that the supplier works with standard lead times. This means that it does not take into account actual inventory or production capacity loads. If there is inventory or slack in production capacity, delivery time can be significantly shorter than 3 weeks. Otherwise, it may be longer.

Suppliers 3 and 4 apply so-called ATP/CTP (available-to-promise/capable-to-promise) systems. This means that they have introduced IT [e.g., SAP APO (advanced planning and optimization)] which has the ability to check actual inventory and production capacity along the SC and to define an exact lead time and delivery date (see Fig. 13.18).

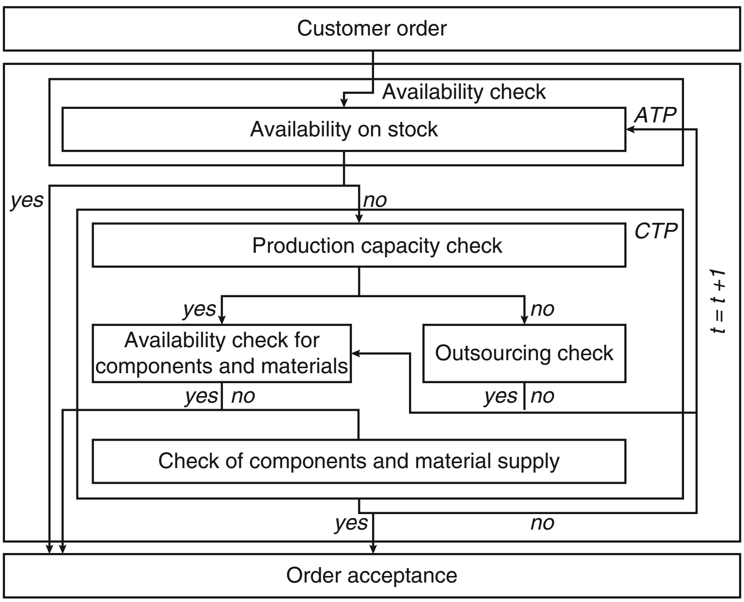


Fig. 13.18 ATP/CTP process. Based on Teich (2003)

At the order fulfilment stage, customer orders are matched with quantities available in stock and from scheduled receipts. In addition, customer requests for delivery of a product with regard to quantity, time, and location have to be answered.

The investigation into whether a delivery can actually be made is called ATP. The standard method of ATP is to search for available stocks, which can be promised for delivery.

CTP can be applied when conventional ATP checks fail, i.e., the requested product is not in stock at the respective locations and not available from scheduled receipts. CTP refers to the planning and scheduling run with regard to work-in-progress (WIP) during an ATP check. CTP checks whether it is possible to modify existing scheduled production orders with regard to time and order size or whether to create a new production order, taking production capacities and component availability into account.

Such concepts are used by many online retail companies such as Amazon. Manufacturing companies apply such systems to manage complexity which frequently involves thousands of orders per day which may be placed for hundreds of thousands of items. Investment in IT for ATP/CTP can be high, but will be paid for through the achievement of competitive advantages.

13.10 Key Points and Outlook

In this chapter, we learned about methods and practical tools for inventory management. Let us summarize the key points of this chapter as follows.

The role of inventory management is to strike a balance between inventory investment and customer service. Inventory is one of the most expensive assets of many companies representing as much as 50% of total invested capital. Operations and SC managers must balance inventory investment against levels of customer service. Inventory has different functions and types. Trade-off of "service level vs costs" is one of the most important issues in inventory management. Inventory creates costs, but at the same time it can be used to increase SC flexibility. The most important costs in inventory optimization are holding, ordering, and stock-out costs.

In making decisions in the scope of inventory management, the following two basic questions are put to the forefront for consideration:

- How much should I replenish?
- When should I replenish?

According to the inventory functions and types, inventory can be used to manage:

- economy of scale—this is cycle inventory;
- uncertainty—this is safety inventory.

Basic methods for item classification are the ABC and XYZ analysis. ABC analysis divides inventory into three classes A-B-C based on annual dollar volume. ABC analysis is used to establish policies that focus on the few critical parts and not the many trivial ones. XYZ analysis is used to classify items according their demand dynamics.

For cycle inventory, the basic methods are as follows:

- basic EOQ and EPQ
- quantity discount model
- reorder point (ROP).

The EOQ method helps us to understand the relationship between ordering and holding costs. EOQ finds the optimal order quantity for deterministic demand in one period mode. The EOQ model answers "how much".

In the quantity discount model, inventory costs can also be calculated on the basis of different unit prices. The EPQ model is fairly similar to the EOQ model, but is applied to production. This model is used when:

- inventory builds up over a period of time after an order is placed;
- units are produced and sold simultaneously.

The reorder point (ROP) tells a manager "when" to order. ROP is introduced to take into account the so-called lead time, i.e. the time between order placement and receipt.

However, in many practical cases, both demand and lead time can be fluctuating. We do not know their actual values, but can only estimate them on the basis of probability. For such cases, stochastic (probabilistic) models are needed. Uncertainty of demand makes it necessary to maintain certain customer service levels to avoid stock-outs. In the presence of demand uncertainty, safety inventory is introduced.

The essential difference between multi-period problems and single-period problems is that all the relevant parameters (e.g., demand, lead time, costs) can vary in different periods. In addition, inventory from previous periods can be used to cover demand in following periods. This is why the basic trade-off in multi-period models is how to balance inventory holding costs and ordering costs over time. Basic methods for multi-period problems are the Wagner–Whitin method (optimization) as well as Silver-Meal and unit cost methods (both methods are heuristics).

In many practical cases, EOQ and ROP cannot be applied because of business policies. For example, many companies in the electronics retail industry have to place their orders on Fridays on the basis of demand forecasts since this is required by OEMs. In other cases, deliveries may only be possible on certain dates (e.g., on Monday). In these cases, inventory control policies are applied.

In many cases, interesting trade-offs between the number of facilities, and inventory and transportation costs may arise when comparing different options. This is why integrated inventory-transportation models are considered. In order to take into account actual inventory in stock production capacities, the ATP/CTP concept is used, which allows the right delivery dates to be determined.

Many additional problems and methods of inventory management exist in practice, e.g. multi-echelon inventory management in the SC. Multi-echelon inventory management is a way to cut costs and increase customer service levels.

Single-echelon inventory management has some major problems. The SC carries excess inventory in the form of redundant safety stock. End customer service failures occur even when adequate inventory exists in the network. Customer-facing locations experience undesirable stock-outs, even while service between echelons is more than acceptable. External suppliers deliver unreliable performance when they have received unsatisfactory demand forecasts. Shortsighted internal allocation decisions are made for products with limited availability.

The objective of multi-echelon inventory management is to deliver the desired end customer service levels with minimum network inventory, with the inventory divided among the various echelons. However, multi-echelon techniques are more complex. The complexity of managing inventory increases significantly. All locations should be under the internal control of a single enterprise. Instead of simply replenishing the warehouse that sits between supplier and end customer, as in the single-echelon situation, we also need to contend with the problems of replenishing the multi-stage inventory system.

Additional problems and methods of inventory management which could also be considered for practical issues are as follows:

- different kinds of service level (alpha, beta, and gamma)
- dynamic safety stock calculations
- inventory-lot-size problems
- inventory-routing problems, especially in the context of vendor-managed inventory (VMI)
- production–inventory–transportation problems
- stochastic economic lot-sizing and scheduling problems with constraints on budget, capacity, etc.

These and a wide range of other problems of inventory management in the supply chain are truly fascinating and deserve much attention.

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Reference for Sect. 13.5

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