Chapter 10
Reactive Power Optimization in AC Power Systems

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10.1 Introduction

Reactive Power is one of the most important features in power networks so that its appropriate production and distribution among consumers can affect performance, efficiency, and reliability of the power networks positively. Creating competitive mechanism via establishing a market to present different services and changing the current rules necessitate that in the new condition programming and controlling of reactive power as well as the voltage to be considered more accurately. The purpose of reactive power optimization in AC power systems is to recognize the best value for control variables in order to optimize the target function considering the possible constraints. With current developments of power grids, it has been growing in popularity among researchers to probe into how to use existent reactive power compensators in order to reduce active power losses and improve voltage profile.

The reactive power optimization is a complicated problem with a broad solution space, nonlinear and non-convex, in which there both continuous are and discrete variables. In general, reactive power optimization problem entails two separate branches as optimal placement of reactive power compensators and optimal oper-

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ation of existent reactive power compensators. Optimal placement of reactive power compensators problem tries to determine three parameters as the type of the compensator, the rate of the output power and installation location. However, optimal operation of existing reactive power compensators is about determining optimal reactive power output for the compensators that have already been installed.

Although producing reactive power involves no fee in the operation step, it influences the final cost via affecting the active power losses in power transmission system significantly. Optimal distribution of reactive power is a sub-problem of Optimal Power Flow (OPF) and the parameters that should be controlled are actually the control variables such as output reactive power or the terminal voltage of generators, output reactive power of all reactive power compensators, and tap-changing transformers' tap settings. Since flowing reactive power through power transmission system results in active power losses, the main goal of reactive power optimization is to reduce the active power losses via optimal controlling of the above mentioned parameters considering the security aspects, which must be fulfilled in order to have a reliable and stable power network with which consumers can be fed continuously.

In addition, some other goals are pursuit beside the main goal, like improving voltage profile which involves the security measures of power systems. Therefore, the target function of reactive power optimization problem is active power losses equation, in which some constraints should be included as voltage rate boundaries, equipment's power output limits, and transmission system margins for carrying power etc. In the reactive power optimization problem, there are both continuous and discrete types of variables, of which should be determined by an optimization algorithm that leads the power system to contain least possible active power losses. In one hand, discrete variables are transformers' tap settings and capacitors' output powers. In the other hand, continuous variables are reactive power outputs of generators as well as synchronous compensators'.

In the recent years, wide variety of optimization algorithms have been recommended to solve the reactive power optimization problem, containing traditional algorithms such as quadratic programing (QP) and sequential quadratic programming (SQP) etc., and heuristic ones which are inspired by nature such as particle swarm optimization (PSO), genetic algorithm (GA), evolutionary programming (EP) and their derivatives etc. Traditional algorithms have too many defects encountering problems that have nonlinear essences, the ones which consist of discrete variables and have got many local optimum solutions. Eventually, traditional gradient based algorithms lose their capability to be efficiently solve such challenging optimization problems. Overall, the traditional optimization algorithms become nonfunctional confronting practical reactive power optimization in large power systems having huge dimensions. On the other hand, optimization algorithms inspired by nature have proved their high performance optimizing problems with so many variables covering discrete and continuous ones while having many local optimum points.

This chapter is focusing on the reactive power optimization using artificial optimization algorithms and trying to give a comprehensive perspective of all formulations and constraints that are needed in order to implement reactive power optimization problem and use its practical implications effectively. First, brief fundamentals of reactive power optimization containing some relevancies which are of crucial importance is presented. Second, to enlighten how reactive power optimization works the classic method of reactive power optimization is presented, thanks to [1]. Third, basic principles and problem formulation of reactive power optimization using artificial intelligent algorithms is elaborated. Fourth, particle swarm optimization algorithm and pattern search method and how to use them in reactive power optimization problem have been expounded defining PSO's parameters way back into reactive power optimization involving ones. Finally, the offered algorithms simulation result on two case studies have been presented.

10.2 Fundamentals of Reactive Power Optimization

There are two types of reactive power flow in power transmission systems; one for fulfilling transmission system needs and the other for feeding loads that naturally consume reactive power in order to work properly, that cause according to Eq. (10.1) active power losses and following that, the fuel consumption of the power plant raises [2]. Taking the Eq. (10.1) into consideration, it is obvious that the imaginary part of the current equation simply affects the absolute value of that which influences total active power losses in the next place.

In addition, there is no any control possible on I_R as it is corresponding current to the active power which should be supplied to the loads via transmission systems. The idealistic condition recommends compensating all the reactive power needs of the loads at the same place, just beside the loads, in order not to impose reactive current into transmission systems. The closer reactive power compensator to loads, the less reactive power flowing in power transmission systems. It is not feasible however, because of technical and economical restrictions upon that, such as the relevancy of voltage stability to the reactive power in AC power systems, of which was broadly investigated in [3, 4].

Reactive power is absorbed by different equipment as synchronous condensers being operated with lagging power factor, shunt reactors, transmission lines and transformers' inductances, static reactive power compensators and loads etc., while being injected to transmission systems by generators, synchronous condensers operating with leading power factor, static capacitors, static compensators, and transmission lines' capacitances etc. [2]. There is another type of equipment that avails power system operators to have a control on reactive power flow through transmission system and it is transformers with tap changing facility under the load condition. There is a straight connection between two neighboring buses' voltages and reactive power flow in the corresponding transmission system which connects them together. In simple words, reactive power flows from the bus having higher voltage magnitude to the bus which has lower voltage magnitude in order to naturally raise the second bus's voltage close to the same level. Take a simple network shown in Fig. 10.1 into consideration, reactive power will flow from the slack bus

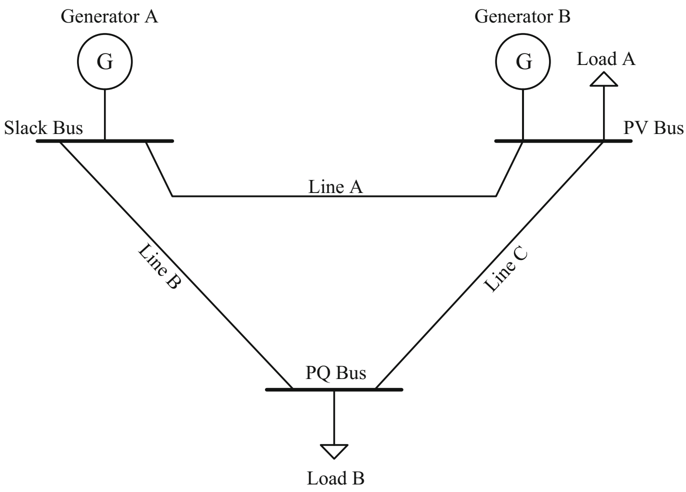


Fig. 10.1 Three-bus power system

to the PV bus if slack bus's voltage magnitude is higher than *PV* bus's. On the whole, there would not be any reactive power flow in transmission systems if the voltage magnitudes of all buses are equal.

$$\begin{cases} P_{loss} = R|I|^2 \\ I = I_R + jI_X \end{cases} \tag{10.1}$$

$$I_{ij} = \frac{V_i - V_j}{Z_{ij}} \tag{10.2}$$

$$P_{ij} + jQ_{ij} = V_i I_{ij}^* \tag{10.3}$$

(10.2) and (10.3)

$$\Rightarrow P_{ij} + jQ_{ij} = V_i \left[\frac{V_i - V_j}{Z_{ij}} \right]^* \tag{10.4}$$

$$\begin{cases} V_i^* = |V_i|(\cos\,\delta_i - j\,\sin\,\delta_i) \\ V_j^* = |V_j|(\cos\,\delta_j - j\,\sin\,\delta_j) \\ Z_{ij}^* = |Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij}) \end{cases}. \tag{10.5}$$

where, $|V_i|$ and $|V_j|$ are voltage magnitudes of *i*th and *j*th busbars respectively. $|Z_{ij}|$ is absolute value of the impedance of transmission line between *i*th and *j*th busbars. δ_i and δ_i are the voltage angles of *i*th and *j*th busbars respectively, while θ_{ij} is angle of impedance between *i*th and *j*th busbars.

Expanding Eq. (10.4) using the assumptions of Eq. (10.5) will be as following

$$P_{ij}+jQ_{ij} = |V_i|(\cos\,\delta_i + j\,\sin\,\delta_i)\left[\frac{|V_i|(\cos\,\delta_i - j\,\sin\,\delta_i) - |V_j|(\cos\,\delta_j - j\,\sin\,\delta_j)}{|Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij})}\right] \tag{10.6}$$

$$= |V_i|(\cos\,\delta_i + j\,\sin\,\delta_i)\left[\frac{|V_i|\cos\,\delta_i - j|V_i|\sin\,\delta_i - |V_j|\cos\,\delta_j + j|V_j|\sin\,\delta_j}{|Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij})}\right]$$

$$= \frac{\begin{pmatrix} |V_i|^2\cos^2\delta_i - j|V_i|^2\sin\,\delta_i\cos\,\delta_i - |V_i||V_j|\cos\,\delta_i\cos\,\delta_j + j|V_i||V_j|\sin\,\delta_j\cos\,\delta_i \\ + j|V_i|^2 sin\,\delta_i\cos\,\delta_i + |V_i|^2\sin^2\delta_i - j|V_i||V_j| sin\,\delta_i\cos\,\delta_j - |V_i||V_j|\sin\,\delta_j\sin\,\delta_i \end{pmatrix}}{|Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij})}$$

$$= \frac{\begin{pmatrix} \left(|V_i|^2\cos^2\delta_i - |V_i||V_j|\cos\,\delta_i\cos\,\delta_j + |V_i|^2\sin^2\delta_i - |V_i||V_j|\sin\,\delta_j\sin\,\delta_i\right) \\ + j\left(-|V_i|^2\sin\,\delta_i\cos\,\delta_i + |V_i||V_j|\sin\,\delta_j\cos\,\delta_i + |V_i|^2 sin\,\delta_i\cos\,\delta_i - |V_i||V_j| sin\,\delta_i\cos\,\delta_j\right) \end{pmatrix}}{|Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij})}$$

$$= \frac{\left(|V_i|^2 - |V_i||V_j|(\cos\,\delta_i\cos\,\delta_j + \sin\,\delta_j\sin\,\delta_i)\right) + j\left(|V_i||V_j|(\sin\,\delta_j\cos\,\delta_i - sin\,\delta_i\cos\,\delta_j)\right)}{|Z_{ij}|(\cos\,\theta_{ij} - j\,\sin\,\theta_{ij})}$$

$$= \frac{\begin{pmatrix} |Z_{ij}||V_i|^2\cos\,\theta_{ij} - |Z_{ij}||V_i||V_j|\cos(\delta_j - \delta_i)\cos\,\theta_{ij} + j|Z_{ij}||V_i||V_j|\sin(\delta_j - \delta_i)\cos\,\theta_{ij} \\ + j|Z_{ij}||V_i|^2\sin\,\theta_{ij} - j|Z_{ij}||V_i||V_j|\cos(\delta_j - \delta_i)\sin\,\theta_{ij} - |Z_{ij}||V_i||V_j|\sin(\delta_j - \delta_i)\sin\,\theta_{ij} \end{pmatrix}}{|Z_{ij}|^2}$$

$$P_{ij}+jQ_{ij} = \frac{\begin{pmatrix} |Z_{ij}||V_i|^2\cos\,\theta_{ij} - |Z_{ij}||V_i||V_j|\cos(\delta_j - \delta_i)\cos\,\theta_{ij} - |Z_{ij}||V_i||V_j|\sin(\delta_j - \delta_i)\sin\,\theta_{ij} \\ + j\left(|Z_{ij}||V_i||V_j|\sin(\delta_j - \delta_i)\cos\,\theta_{ij} + |Z_{ij}||V_i|^2\sin\,\theta_{ij} - |Z_{ij}||V_i||V_j|\cos(\delta_j - \delta_i)\sin\,\theta_{ij}\right) \end{pmatrix}}{|Z_{ij}|^2} \tag{10.7}$$

$$Q_{ij} = \frac{|V_i|^2\sin\,\theta_{ij} + |V_i||V_j| sin(\delta_j - \delta_i - \theta_{ij})}{|Z_{ij}|} \tag{10.8}$$

The term $(\delta_j - \delta_i)$ has a minute value and can be neglected in order to simplify Eq. (10.8), then

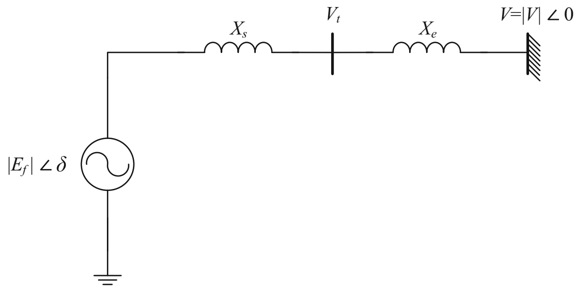


Fig. 10.2 The simplified electrical circuit of synchronous generator

$$Q_{ij} = \frac{|V_i|^2 \sin\,\theta_{ij} - |V_i||V_j| sin\,\theta_{ij}}{|Z_{ij}|} \tag{10.9}$$

$$Q_{ij} = \underbrace{\left(\frac{\sin\,\theta_{ij}}{|Z_{ij}|}\right)}_{\text{Constant}} |V_i| \underbrace{(|V_i| - |V_j|)}_{\text{Voltage Difference}} \tag{10.10}$$

Therefore, Eq. (10.10) shows that the value of the corresponding Q_{ij} flowing in transmission systems is directly dependent on the difference of voltage magnitudes between ith and jth busbars.

According to the electrical rules which synchronous machines work based on, the reactive power injected or absorbed by a synchronous generator can be controlled by its excitation system that affects the generator's terminal voltage straightforwardly [5]. Take Fig. 10.2 into consideration as a simplified electrical circuit of synchronous generators and power systems, then reactive power absorbing or producing by the generator will be as the following:

$$\begin{cases} V = |V|\angle 0 \\ E_f = |E_f|\angle\delta \\ X = X_e + X_s \end{cases} \tag{10.11}$$

$$I = \frac{|E_f|\angle\delta - |V|\angle 0}{X\angle 90} \tag{10.12}$$

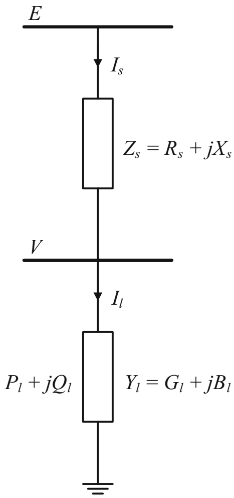
The apparent power transmitted from the generator to the infinite bus is:

$$S = P + jQ = E_f I^* \tag{10.13}$$

Expanding Eq. (10.13) using Eqs. (10.11) and (10.12) will be as following

$$S = |E_f|\angle\delta \frac{|E_f|\angle - \delta - |V|}{X\angle - 90} = \frac{|E_f|^2\angle 90 - |E_f||V|\angle 90 + \delta}{X} \tag{10.14}$$

Fig. 10.3 The single-phase Thevenin equivalent circuit



Then by segregating Eq. (10.14), the reactive power absorbed or injected to the infinite busbar by the synchronous generator will be as following

$$Q = \frac{|E_f|}{X}\left(|E_f| - |V|\cos\delta\right) \tag{10.15}$$

where V_t is the voltage of infinite busbar and δ is the angle between infinite busbar and the generator's terminal under no-load condition.

If the voltage magnitude of the infinite busbar to be considered as constant, it is obvious that consuming or feeding reactive power by the generator is entirely correlated with the generator's voltage magnitude which is controlled by the excitation system, while the active power being transmitted assumed to be constant over the handling period [5]. There is another issue as voltage regulation which is associated with the reactive power control in AC power systems. The voltage regulation is defined as a proportional (per-unit) change in the voltage magnitude of supply terminal in relation to defined change to the load current (e.g., from no-load to full-load).

Taking Fig. 10.3 as Thevenin equivalent of supply system into consideration, the voltage drop in the transmission system in the absence of compensator is shown in Fig. 10.4 as ΔV, which is as the following

$$\Delta V = E - V = Z_s I_l \tag{10.16}$$

$$\begin{cases} Z_s = R_s + jX_s \\ I_l = \dfrac{P_l - jQ_l}{V} \end{cases} \tag{10.17}$$

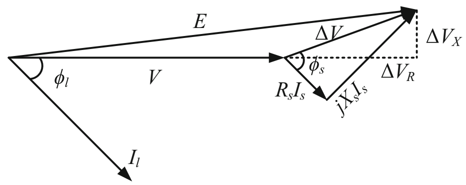


Fig. 10.4 The corresponding phasor diagram to Fig. 10.3, without compensation

So that

$$\Delta V = (R_s + jX_s)\left(\frac{P_l - jQ_l}{V}\right) \tag{10.18}$$

$$= \left(\frac{R_sP_l + X_sQ_l}{V}\right) + j\left(\frac{X_sP_l - R_sQ_l}{V}\right) \tag{10.19}$$

$$= \Delta V_R + j\Delta V_X \tag{10.20}$$

The voltage drop has two components as ΔV_R in the same phase with V and ΔV_X in quadrature with V which is illustrated in Fig. 10.4 elaborately. It is obvious that the magnitude and the phase of V are functions of the magnitude and the phase of load current so that the amount of voltage regulation depends straightly on the amount of real power as well as reactive power consuming of the load [2].

According to Eq. (10.10) reactive power flowing in transmission systems is a function of the difference of voltage magnitude between two neighboring busbars. Hence, the bigger the difference, the higher amount of reactive power flows toward busbar having lower voltage magnitude in order to raise it, and it causes additional active power losses in transmission systems. Therefore, it would be such a great asset if reactive power could be compensated at the same place as loads lie, which would help the power factor to reach near the unique value. However, a voltage drop that is relevant to active current flow in transmission systems would yet be consistent. Considering Eq. (10.18) while having loads' reactive power consumption compensated on-site, the equation will be as the following

$$\Delta V = (R_s + jX_s)\left(\frac{P_l}{V}\right) \tag{10.21}$$

$$= \left(\frac{R_sP_l}{V}\right) + j\left(\frac{X_sP_l}{V}\right) \tag{10.22}$$

$$= \Delta V_R + j\Delta V_X \tag{10.23}$$

In order to have a voltage regulation by the value of zero, the reactive power consumption by the load and the voltage drop being caused by active current flowing in the transmission system should be compensated. Referring to Fig. 10.5,

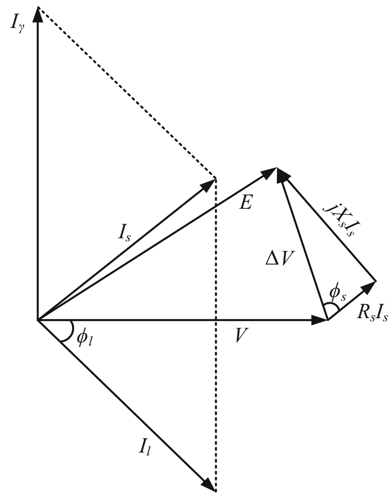


Fig. 10.5 The corresponding phasor diagram to Fig. 10.3, with compensation

it is possible to have $|E| = V$ via producing reactive power and injecting to the load busbar in presence of the load which will be greater than the load's absorption [2].

For sum up, all the components of power systems work together continuously as a united structure so that any single change in any part will influence all the other parameters directly, some of which are explained in the current section regarding that they are associated with reactive power's role in AC power systems. Therefore, all the relations should be evaluated carefully whenever a change happens in a specific parameter, which necessitates having a comprehensive control system to fulfill all the needs of power systems in order to have a reliable, stable, and a cost-effective energy produced and dispatched.

10.3 Reactive Power Optimization Using Classic Methods

The main aim of reactive power optimization is to minimize reactive power flowing in transmission systems including transmission lines, transformers etc., which leads to less real power losses. This will be approachable via specifying the best reactive power output value of the reactive power compensators and the other controllable parameters under a number of constraints which should be met in order to have a reliable, stable and cost-effective power system [1].

At first glance, it might come into mind that if loads' reactive power consumption need to be met just by the nearest reactive power source (in terms of electrical distance), the active power losses would be minimized, although it is not such a true interpretation. On one hand, while it is being talked about reactive power optimization, it should be considered that reactive power in AC power systems has plenty of associations with the other parameters of the system, most of which are crucial in terms of power system security and reliability. On the other hand, reactive power optimization is not a straight optimization, however, the power flow plays a key role in the reactive power optimization problem. For instance, the voltage magnitude is one of the most important parameters which influences and also is influenced by reactive power of the system.

According to Eq. (10.1), reactive power loss in AC power network can be considered as a function of load active and reactive power consumption as

$$P_L = P_L(P_1, P_2, \ldots, P_n, Q_1, Q_2, \ldots, Q_n) \tag{10.24}$$

Reactive power optimization is a constrained optimization problem that may be attacked formally using advanced calculus methods involving Lagrange function. In classic reactive power economic dispatch, the active power of all generators are already known and fixed during optimization procedure, except slack bus's, so that any change in active power losses associated to reactive power optimization could be distinguished easily. One of the well-known techniques to implement optimization in power network, of which consists of some constraints to be fulfilled, is the Lagrange method. In reactive power optimization, one of the main constraints is reactive power balance as Eq. (10.25), which is shown in the following [1]

$$\sum_{i=1}^{M} Q_{Gi} = Q_D + Q_L \tag{10.25}$$

where Q_G includes all reactive power sources, Q_D is reactive power demands of the loads, and Q_L is reactive power losses in the transmission system.

In order to establish the necessary conditions for an extreme value of the objective function, the constraint functions should be added to the objective function after the constraint functions has been multiplied by an undetermined multiplier. This is known as Lagrange function and is shown in Eq. (10.26).

$$L = F_T + \lambda \phi \tag{10.26}$$

where, F_T is target function which is aimed to be optimized, λ is an unspecified multiplier, and ϕ contains all the probable constraint functions that must be met during the optimization procedure.

Therefore, the Lagrange function formed to decrease active power losses, subjected to equality constraint of producing and consuming reactive power, constructed from Eqs. (10.24) and (10.25) will be as

$$L = P_L - \lambda \left(\sum_{i=1}^{M} Q_{Gi} - Q_D - Q_L \right) \tag{10.27}$$

Then, the necessary condition to have an extreme value for the Lagrange function is to set the first derivative of the function with respect to each independent variables (Q_G and λ) equal to zero [1].

$$\frac{\partial L}{\partial Q_{Gi}} = \frac{\partial P_L}{\partial Q_{Gi}} - \lambda\left(1 - \frac{\partial Q_L}{\partial Q_{Gi}}\right) = 0 \quad i = 1, 2, \ldots, M \tag{10.28}$$

$$\frac{\partial L}{\partial \lambda} = -\left(\sum_{i=1}^{M} Q_{Gi} - Q_D - Q_L\right) = 0 \tag{10.29}$$

From Eq. (10.28)

$$\frac{\partial P_L}{\partial Q_{Gi}} \times \frac{1}{\left(1 - \frac{\partial Q_L}{\partial Q_{Gi}}\right)} = \lambda \quad i = 1, 2, \ldots, N \tag{10.30}$$

Equation (10.30) is the formula of reactive power economic dispatch, where, $\frac{\partial P_L}{\partial Q_{Gi}}$ is incremental rate of active power losses with respect to ith reactive power source, $\frac{\partial Q_L}{\partial Q_{Gi}}$ is incremental reactive power losses with respect to ith reactive power source.

The terms $\frac{\partial P_L}{\partial Q_{Gi}}$ and $\frac{\partial Q_L}{\partial Q_{Gi}}$ can be calculated with impedance matrix method which is depicted below.

The real power losses in power transmission systems can be represented as [1]

$$P_L + jQ_L = V^T I^* = (ZI)^T \hat{I} = I^T Z^T I^* \tag{10.31}$$

$$I = I_P + jI_Q \tag{10.32}$$

$$Z = R + jX \tag{10.33}$$

where, I is the current vector in transmission lines, I^* is the conjugate current vector in transmission lines, Z is the impedance matrix of transmission lines, and V^T is the voltage vector of all busbars.

Substituting Eqs. (10.32) and (10.33) into Eq. (10.31), P_L and Q_L will be obtained as [1]

$$P_L = \sum_{j=1}^{n} \sum_{k=1}^{n} R_{jk}\left(I_{Pj} I_{Pk} + I_{Qj} I_{Qk}\right) \tag{10.34}$$

$$Q_L = \sum_{j=1}^{n} \sum_{k=1}^{n} X_{jk}\left(I_{Pj} I_{Pk} + I_{Qj} I_{Qk}\right) \tag{10.35}$$

The relation between injected power and current to the system is

$$P_i + jQ_i = (V_i \cos\theta_i + jV_i \sin\theta_i)(I_{Pi} - jI_{Qi}) \tag{10.36}$$

Then I_{Pi} and I_{Qi} will be as the following

$$I_{Pi} = \frac{P_i \cos\,\theta_i + jQ_i \sin\,\theta_i}{V_i} \tag{10.37}$$

$$I_{Qi} = \frac{P_i \sin\,\theta_i + jQ_i \cos\,\theta_i}{V_i} \tag{10.38}$$

Substituting Eqs. (10.37) and (10.38) into Eqs. (10.34) and (10.35) P_L and Q_L will be as [1]

$$P_L = \sum_{j=1}^{n} \sum_{k=1}^{n} \left[\alpha_{jk}\left(P_j P_k + Q_j Q_k\right) + \beta_{jk}\left(Q_j P_k - P_j Q_k\right)\right] \tag{10.39}$$

$$Q_L = \sum_{j=1}^{n} \sum_{k=1}^{n} \left[\delta_{jk}\left(P_j P_k + Q_j Q_k\right) + \gamma_{jk}\left(Q_j P_k - P_j Q_k\right)\right] \tag{10.40}$$

where

$$\alpha_{jk} = \frac{R_{jk}}{V_j V_k} \cos\left(\theta_j - \theta_k\right) \tag{10.41}$$

$$\beta_{jk} = \frac{R_{jk}}{V_j V_k} \sin\left(\theta_j - \theta_k\right) \tag{10.42}$$

$$\delta_{jk} = \frac{X_{jk}}{V_j V_k} \cos\left(\theta_j - \theta_k\right) \tag{10.43}$$

$$\alpha_{jk} = \frac{X_{jk}}{V_j V_k} \sin\left(\theta_j - \theta_k\right) \tag{10.44}$$

Then, taking the first derivative of Eq. (10.39) into consideration with respect to independent variables, the consequence will be as following [1]

$$\begin{aligned} \frac{\partial P_L}{\partial P_i} &= \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{\partial}{\partial P_i} \left[\alpha_{jk}\left(P_j P_k + Q_j Q_k\right) + \beta_{jk}\left(Q_j P_k - P_j Q_k\right)\right] \\ &= 2\sum_{k=1}^{n} \left(P_k \alpha_{ik} - Q_k \beta_{ik}\right) + \underbrace{\sum_{i=1}^{n} \sum_{k=1}^{n} \left[\left(P_j P_k + Q_j Q_k\right) \frac{\partial \alpha_{jk}}{\partial P_i} + \beta_{jk}\left(Q_j P_k - P_j Q_k\right) \frac{\beta_{jk}}{\partial P_i}\right]}_{\approx 0} \end{aligned} \tag{10.45}$$

The second term of the Eq. (10.45) is negligible, and then it will be simplified as [1]

$$\frac{\partial P_L}{\partial P_i} \approx 2\sum_{k=1}^{n}(P_k\alpha_{ik} - Q_k\beta_{ik}) \tag{10.46}$$

In a high-voltage power network, $(\theta_j - \theta_k)$ is infinitesimal, then sin $(\theta_j - \theta_k)$ 0. Therefore, β_{jk} can be ignored as well.

$$\frac{\partial P_L}{\partial P_i} \approx 2\sum_{k=1}^{n} P_k\alpha_{ik} \tag{10.47}$$

Similarly

$$\frac{\partial P_L}{\partial Q_i} \approx 2\sum_{k=1}^{n} Q_k\alpha_{ik} \tag{10.48}$$

$$\frac{\partial Q_L}{\partial P_i} \approx 2\sum_{k=1}^{n} P_k\delta_{ik} \tag{10.49}$$

$$\frac{\partial Q_L}{\partial Q_i} \approx 2\sum_{k=1}^{n} Q_k\delta_{ik} \tag{10.50}$$

Considering real and reactive power consumption of loads constant during the optimization procedure, two assumptions as the following can be made [1]

$$dP_i = d(P_{Gi} - P_{Di}) = dP_{Gi} \tag{10.51}$$

$$dQ_i = d(Q_{Gi} - Q_{Di}) = dQ_{Gi} \tag{10.52}$$

Then, Eq. (10.47) to Eq. (10.50) can be written as

$$\frac{\partial P_L}{\partial P_{Gi}} \approx 2\sum_{k=1}^{n} P_k\alpha_{ik} \tag{10.53}$$

$$\frac{\partial P_L}{\partial Q_{Gi}} \approx 2\sum_{k=1}^{n} Q_k\alpha_{ik} \tag{10.54}$$

$$\frac{\partial Q_L}{\partial P_{Gi}} \approx 2\sum_{k=1}^{n} P_k\delta_{ik} \tag{10.55}$$

$$\frac{\partial Q_L}{\partial Q_{Gi}} \approx 2\sum_{k=1}^{n} Q_k \delta_{ik} \tag{10.56}$$

Therefore, if the power system which is being investigated has enough amount of reactive power sources, the steps of classic reactive power optimization using Lagrange function would be as following [1]:

I. Power flow calculations should be carried out in order to have all generators' active power output, then fix them all in the current value as active power consumption of loads has been considered constant, the only exception is the slack generator. The output power of slack generators would not remain constant during reactive power optimization.
II. The value of λ should be computed for each reactive power source using Eqs. (10.54) and (10.56). For the sources having $\lambda < 0$, it means the active power losses of the system can be reduced by increasing the output amount of reactive power of the source. For the sources having $\lambda > 0$, it will be vice versa. Therefore, in order to decrease active power losses of the system, the amount of reactive power output should be increased for the sources having $\lambda < 0$ and also decreased for the sources having $\lambda > 0$. Each time, the source with minimum value of λ will be chosen to increase its output if $\lambda < 0$, and the source with maximum value of λ to decrease its output if $\lambda > 0$. Eventually, power flow calculations should be computed to have the result of optimization.
III. Using power flow results, the active power losses can be computed. Since the active power output of the reference unit was not fixed, whatever happening to the active power losses of the system can be sensed in the active power output of the slack generator. The reactive power process will be continued until active power losses cannot be reduced anymore.

It should be noted that limitations of reactive power sources were not considered in the procedure above. There is a limitation for each reactive power source as

$$Q_{Gi\,\min} \le Q_{Gi} \le Q_{Gi\,\max} \tag{10.57}$$

If they are supposed to be considered, the constraint in Eq. (10.57) should be checked in every iteration for each source. When it comes to choose an output amount for the power of the sources according to their λ value, if λ suggests to increase the output power of the ith source while it is exceeding either its above or its below margins, the amount of output reactive power of the corresponding source should be set to its margins accordingly. Thus, the source which its output power has been adjusted to either its maximum or its minimum values will not be considered any longer in the reactive power optimization procedure [1].

The above-mentioned structure of minimizing active power losses using reactive power economic dispatch was a simplified method depicted in order to represent the concept, and it is not much of a practical method to be used in a real system having large dimensions and plenty of constraints, as voltage profile, transmission lines bounds etc. There are a lot of conventional optimization methods which can be found in [6]. In the next section, reactive power optimization using artificial intelligence algorithms and the used model will be described elaborately, which is more applicable when it comes to a large system in which all the security constraints should be considered.

The reason why heuristic methods are highly concerned in reactive power optimization is a few complexities in such problems' nature, some of which known as non-convexity, having continuous and discrete variables, and having plenty of local and global optimums. Conventional methods and algorithms which were used to optimize such problems proved themselves rather unable to be applied on the problem satisfactorily, because conventional methods mostly use the gradient of the objective function, leading to a local minimum rather than a global one. Another problem is that conventional methods need the derivatives of objective functions which are not accessible in the ones consisting of discrete variables. Thus, there is a compelling need to some sort of algorithms, of which are able to redeem all defections of the conventional ones when applying to optimization problems in order to have as much performance as possible. Although pure mathematical methods like Lagrange function are very precise, the performance of intelligent algorithms has been proved in plenty of cases, and they happen to be more efficient than conventional methods in many of aspects.

10.4 Reactive Power Optimization Using Artificial Intelligent Algorithms

As it has been mentioned in the last part of previous section, the accuracy of conventional methods is their most redeeming feature, although they cause problems in terms of mathematical fulfilments. The conventional methods are not capable of finding the global optimum of the target function and they will get into trouble if the objective function consists of discrete variables. In addition, classic pure mathematical methods need the derivatives of target function, which will impose so many difficulties to the optimization procedure, such as the complexities that calculating derivatives of some functions possess. Heuristic algorithms use the target function itself during the optimization procedure instead of its derivatives and they are readily able to find the global optimum point. Fortunately, heuristic algorithms have solved many drawbacks that conventional algorithms had, and also they have proven their capabilities in the optimization respect. Therefore, it is worth to put some effort on using heuristic methods to optimize engineering problems like the reactive power optimization.

10.4.1 Basic Principles

There are two types of variables concerning reactive power optimization as control and state variables. The terminal voltage of generators, tap setting of transformers and reactive power output of reactive power sources are control variables that are changeable within their bounds. Control variables are the tools that heuristic algorithms use in order to optimize the target function. The voltage magnitudes and voltage angels of *PQ* buses are state variables, of which the active power losses can be computed using Eq. (10.58) and the corresponding values. The general procedure of optimization by heuristic algorithms is shown in Fig. 10.6, whereas the flowchart of reactive power optimization using intelligent algorithms is shown in Fig. 10.7.

$$F_{loss} = \sum_{k=1}^{N_L} g_k \left[V_{1,k}^2 + V_{2,k}^2 - 2V_{1,k}V_{2,k}\cos\left(\theta_{1,k} - \theta_{2,k}\right) \right] \tag{10.58}$$

where, g_k is conductance value of the transmission line between starting and ending buses, $V_{1,k}$ and $V_{2,k}$ are voltage magnitudes of starting and ending buses, and, $\theta_{1,k}$ and $\theta_{2,k}$ are voltage angles of starting and ending buses, respectively.

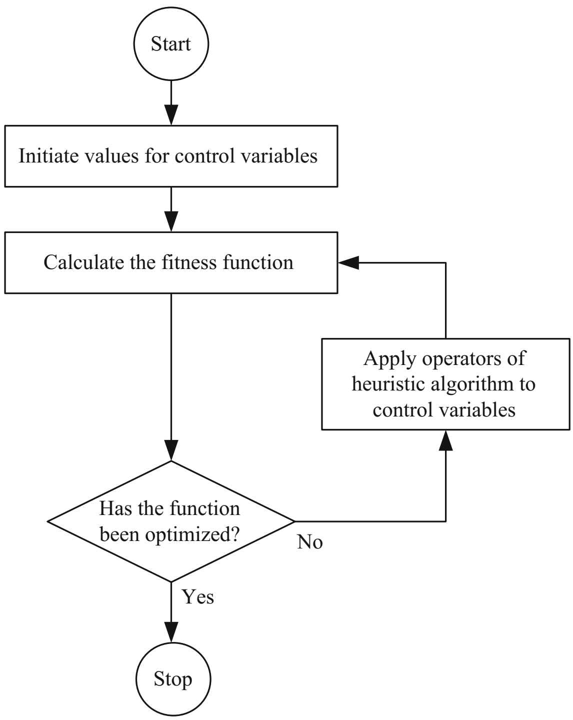


Fig. 10.6 General procedure of optimization by heuristic algorithms

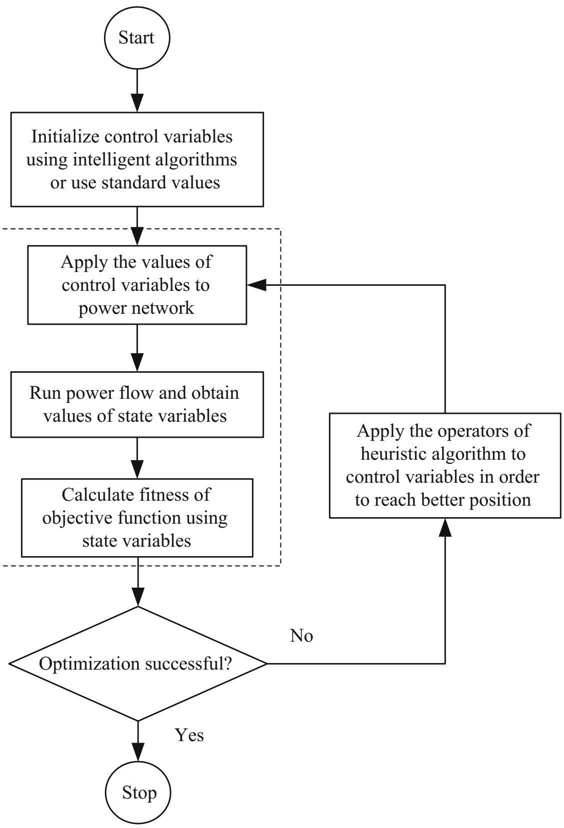


Fig. 10.7 General reactive power optimization trend using intelligent algorithms

As it can be inferred from Figs. 10.6 and 10.7, there is an important point in reactive power optimization problem to pay attention to, which is the calculation of objective function in order to examine whether it has improved or not. There is one extra step in reactive power optimization as the power flow calculation, shown with dashed rectangle in Fig. 10.7. In order to evaluate the objective function represented by Eq. (10.58) in which both state and control variables are involved, after initializing and/or determining control variables by heuristic algorithms, power flow should be run to make calculation of fitness function available. In other words, we have to run power flow calculations in order to achieve the values for the state variables which are mentioned before.

10.4.2 Problem Formulation of Reactive Power Optimization

There are several methods of formulating and modeling reactive power optimization problem according to what exactly is expected as its practical implications, for instance, what constraints should be considered and what objectives are aspired will determine the approach. In this section, one general model will be presented which is useful when it turns to be used in heuristic algorithms to optimize reactive power in AC power systems.

The objective function of the optimal reactive power flow (ORPF) includes technical and financial goals. The economic goal has mainly considered minimizing active power losses in transmission systems. The technical goals are to diminish voltage deviation of *PQ* buses (VD) from the ideal voltage setting and to increase voltage stability index (VSI). Therefore, objective functions for both the technical and monetary goals are considered in this chapter as following [7]

$$f(X) = \min(P_L)$$
$$\text{subject to:} \quad \begin{cases} (VD) \\ (VSI) \end{cases}$$

10.4.2.1 Active Power Losses Objective Function

Reducing active power losses is the most crucial aim of reactive power optimization problem, which influences final cost of dispatched energy. The active power losses can be computed by Eq. (10.59) as follows

$$F_{loss} = \sum_{k=1}^{N} g_k \left[V_{1,k}^2 + V_{2,k}^2 - 2V_{1,k}V_{2,k}\cos\left(\theta_{1,k} - \theta_{2,k}\right)\right] \tag{10.59}$$

where, g_k is conductance of the transmission line between starting and ending buses, $V_{1,k}$ and $V_{2,k}$ are voltage magnitudes of starting and ending buses, and $\theta_{1,k}$ and $\theta_{2,k}$ are voltage angles of starting and ending buses, respectively, and N is the number of transmission lines.

10.4.2.2 Voltage Deviation Constraint

Another significant goal of reactive power optimization in AC power systems is to shrink voltage deviation. Electrical equipment is designed for optimal operation at its nominal voltage. Any deviance from the nominal voltage can result in reducing the general effectiveness and decreasing longevity of electrical apparatus. Voltage deviance constraint is to enhance the voltage profile of power systems by minimizing the summation of voltage deviations at load buses. The voltage deviance constraint can be considered as the least possible amount of voltage deviation summation at each load bus. This function is defined as follows:

$$V_D = \sum_{j=1}^{M} \left| V_j - V_j^{ref} \right| \tag{10.60}$$

where, V_j is the actual voltage of jth load bus, V_j^{ref} is the ideal voltage of jth load bus, and M is number of load buses [8].

10.4.2.3 System Voltage Stability Index

A rather simple voltage stability index to define is V/V_0 ratio, where V is the voltage magnitude of all the PQ buses under load condition and V_0 is the voltage magnitude of all the PQ buses under no-load condition, both identified from load flow calculations or state estimation studies of the system. The ratio V/V_0 at each node offers a voltage stability diagram for the corresponding bus, providing power system operators with weak spots to be taken care of. There are wide range of indices for this purpose, while the current being chosen for the sake of simplicity.

$$VSI = \sum_{i=1}^{T} \left| 1 - \frac{V_i}{V_{i0}} \right| \tag{10.61}$$

where, V_i is the voltage magnitude of ith PQ bus under load condition, V_{i0} is the voltage magnitude of ith PQ bus under no-load condition, and T is number of load buses, respectively.

10.4.2.4 Constraints of Control and State Variables

The control variable constraints embrace tap changer settings of all transformers T, the output capacity of reactive power compensators C, and terminal voltage of all the generators V. The state variables consist of voltage magnitude of all PQ buses U, and reactive power output of all generators Q. Thus, the restriction expressions of control and state variables can be written as

$$V_{Gk,\min} < V_{Gk} < V_{Gk,\max} \tag{10.62}$$

$$T_{i,\min} < T_i < T_{i,\max} \tag{10.63}$$

$$C_{j,\min} < C_j < C_{j,\max} \tag{10.64}$$

$$Q_{Gk,\min} < Q_{Gk} < Q_{Gk,\max} \tag{10.65}$$

$$V_{l,\min} < U_l < V_{l,\max} \tag{10.66}$$

where, $V_{Gk,\min}$ ($V_{Gk,\max}$), $T_{i,\min}$ ($T_{i,\max}$), $C_{j,\min}$ ($C_{j,\max}$), $Q_{Gk,\min}$ ($Q_{Gk,\max}$) and $V_{l,\min}$ ($V_{l,\max}$) are lower (upper) boundary values of PV bus voltages, tap ratio of transformers, reactive power output of compensators, reactive power output of PV buses and voltage magnitude of load buses, respectively.

10.4.2.5 System Power Flow Constraint Equations

The reactive power optimization must fulfill the power flow balances, which are written as

$$P_{Gi} - P_{Li} - V_i \sum_{j=1}^{n} V_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) = 0 \tag{10.67}$$

$$Q_{Gi} - Q_{Li} - V_i \sum_{j=1}^{n} V_j \left(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right) = 0 \tag{10.68}$$

where, n is number of buses, P_{Gi} and Q_{Gi} are generator active and reactive power of the ith bus. P_{Li} and Q_{Li} are load active and reactive power of ith bus. V_i and V_j are voltage magnitudes of ith and jth buses (two neighboring busbars). G_{ij}, B_{ij}, and δ_{ij} are conductance parameters and voltage angle between ith and jth buses, respectively.

It should be noted that the optimization algorithm that determines the control variables in relation to the optimization procedure, satisfies the corresponding constraints as well. The constraints related to state variables will be met by the standard power flow procedure and the algorithm being used, such as Newton or Gauss Sidle. In addition, violations happened in the state variables due to adjusting control variables, can be controlled using a penalty function added to the final objective function of reactive power optimization.

10.4.2.6 General Form of Objective Functions Used in Intelligent Algorithms

Most of the intelligent optimization algorithms tend to have an unconstrained objective function rather than a constrained one. However, different sorts of constraints such as linear and non-linear are typically inseparable part of optimization procedure. Therefore, there is a compelling need to be able to convert a constrained fitness function to an unconstrained one in order to use it in intelligent algorithms. The solution is the penalty function which exactly transforms both objective function and its constraints to a unique unconstrained function.

Considering the optimization problem generally as Eq. (10.69) [6]

$$\begin{array}{c}\text{minimize } f(X) \\ \text{subject to: } \begin{cases} g_j(X) \leq 0 & j = 1, \ldots, m \\ h_i(X) = 0 & i = 1, \ldots, n \end{cases}\end{array} \tag{10.69}$$

It has been converted into an unconstrained optimization fitness function by constructing a function of the form:

$$\phi(X) = f(X) + \sum_{i=1}^{m} r_i \langle g_i(X) \rangle^2 + \sum_{j=1}^{p} R_j \big(h_j(X)\big)^2 \tag{10.70}$$

where X is the vector of control variables, r_i and R_j are penalty multipliers which are constant for all the constraints during the optimization procedure, $g_i(X)$ and $H_j(X)$ are inequality and equality constraints, respectively. $\langle g_i(X) \rangle$ is the bracket function which is defined as Eq. (10.71).

$$\langle g_i(X) \rangle = \begin{cases} g_i(X) & g_i(X) > 0 \\ 0 & g_i(X) \leq 0 \end{cases} \tag{10.71}$$

Equation (10.70) can be considered for maximization or minimization problems appropriate to which optimization algorithm is being used. If a specific algorithm is naturally good at minimizing functions and it is used to minimize a function, then it will be better to use $F(X) = \phi(X)$ as the fitness function, Otherwise it will be better to use the form shown in Eq. (10.72) to define the maximization problem back to minimization one. For instance, Genetic Algorithm is naturally good at maximizing objective functions, thus the fitness function shown in (10.72) will be used if the considered problem aims to minimize the objective function.

$$F(X) = \frac{1}{\phi(X)} \tag{10.72}$$

As it can be seen in the Eq. (10.70), $\phi(X)$ is a new fitness function consisting of constraints and the objective function itself. It is forced by an additional value, multiplying r_i and R_j, which are allocated big values in case the optimization procedure violates the minimum and maximum bounds of constraints. Therefore, the new objective function will have a huge value when the limits are violated, having been influenced by the multipliers. It is the exact method optimization algorithms use to perceive whether the bounds are violated or not. The same method is highly appreciated when it comes to using intelligent algorithms in reactive power optimization problem.

Thus, the penalty function to use in intelligent algorithms for reactive power optimization will be as follows

$$f(V,\theta) = \underbrace{\sum_{k=1}^{N} g_k\left[V_{1,k}^2 + V_{2,k}^2 - 2V_{1,k}V_{2,k}\cos\left(\theta_{1,k} - \theta_{2,k}\right)\right]}_{P_{loss}} + \underbrace{k_v \sum_{j=1}^{M}\left|V_j - V_j^{ref}\right|}_{\text{Voltage Deviation}} + \underbrace{k_s \sum_{i=1}^{T}\left|1 - \frac{V_i}{V_{i0}}\right|}_{\text{Voltage Stability}} \tag{10.73}$$

where, f is an unconstrained objective function, k_v and k_s are penalty multipliers which are used to obligate optimization procedure not to violate the corresponding bounds.

It should be noted that the constraints related to control variables are usually managed by optimization algorithms, while the constraints pertinent to power flow calculations represented by Eqs. (10.67) and (10.68) will be met by power flow. Moreover, the constraints containing state variables should be included in the penalty function to be fulfilled. There is another way to consider constraints of state variables and that is multi-objective optimization methods, in which each constraint is defined as an objective function separately and the procedure drives the main objective function as well as all the constraints simultaneously to be optimized and fulfilled. There is a major difference between using multi-objective optimization methods and single-objective ones, and that is, all the objectives including main fitness function are driven to be minimized while the multi-objective approaches are used. Using penalty function method (single-objective), however, tries to keep the constraints within their bounds.

10.5 Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) is one of the most successful optimization approaches, of which was inspired by nature. It has broadly been used in all sorts of optimization problems that possess large search spaces, without needing gradients of objective functions. Particle swarm optimization algorithm is based on the performance of a colony or swarm, a flock of birds, a school of fish and/or any kind of creatures that live in groups. The particle swarm optimization algorithm tries to simulate the behavior of these collective organisms which have evolved along centuries in order to enhance their performance of finding food, encountering danger and keeping themselves more competitive in a world that adaptability is a prerequisite feature to survive. The word ‘particle’ notes to an individual in a swarm which acts in a distributed way using its own intellect and also the cooperative intelligence of the crowd.

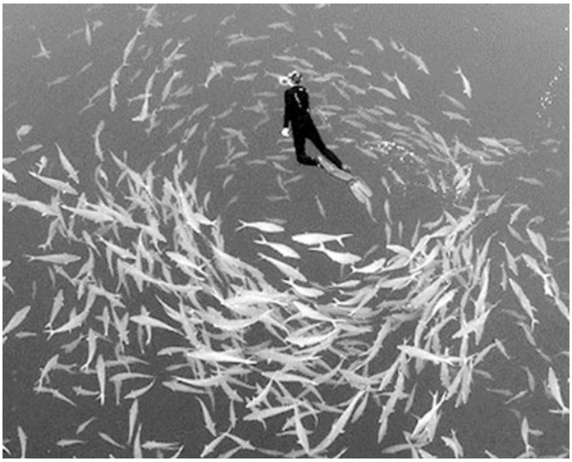


Fig. 10.8 A fish swarm using their collective intelligence

Taking Fig. 10.8 into consideration, since there is sight limitation under the water, each individual can just see a near distance radially so that can put all of them into trouble all the time facing sharks or any offensive actions. Nevertheless, they have been capable of keeping their species alive in harsh nature of the underwater. As it can be seen in the photo, individuals try to move close to each other keeping precautionary distance so that whenever there is a danger the nearest individual will sense it and act, and since all of them follow the same rules, the farthest one in the group will sense the danger and act appropriately. That was just one of the advantages taken by swarms in the nature, using individualistic intelligence as well as the collective intelligence. Optimization methods founded on swarm intelligence are called behaviorally enthused processes which are called evolution-based procedures. The PSO algorithm was originally proposed by Kennedy and Eberhart in 1995 [1].

On the whole, each particle tries to observe three rules instinctively in swarms and these rules are the bases of the optimization algorithm as well. Rules are as follows [1]:

1. It tries not to come too near and not to go too far from other individuals simultaneously.
2. It directs toward the middling track of other individuals.
3. It tries to fit the "average position" among other individuals with no extensive gaps in the flock.

Three rules mentioned above lead to the behavior of the swarm which is based on a mixture of three simple features as follows [1]:

1. Cohesion, which tries to keep the swarm altogether.
2. Separation, which causes the individuals not to came too close to each other.
3. Alignment, which causes the swarm to keep an eye on the general heading point of the flock.

In general, size of the swarm is assumed to be fixed, whereas each particle situated primarily at accidental positions in the multidimensional space of the optimization problem. Each individual has two sorts of data as position and velocity, both of which are stored and compared to each other continuously during the optimization procedure in order to work out the best position discovered by the particles. The velocity and position in the previous iteration are used to determine the new values of them in the next iteration. This process runs consecutively until it has found the best position possible or has reached one of the stopping criteria. Therefore, in simple words the procedure of particle swarm optimization algorithm is generally as follows [6]:

1. All the particles exchange their latest information with each other simultaneously to figure out which particle has found the best location so far.
2. All the particles considering the location and velocity of each one incline to the best point found in relation to their current parameters.
3. The past memory of each particle as well as its current position affects the next position where it will be.

Overall, the particle swarm optimization algorithm quests for the optimum position via a group of individuals similar to other AI-based exploratory optimization methods. The presented model simulates a partly random search method that is armed with individualistic and collective artificial intelligence, leading the process to a global optimum point of the objective function.

10.5.1 Computational Implementation of PSO for Reactive Power Optimization

Since the concept of particle swarm optimization algorithm has been elaborated in many articles and book chapters like [6, 9], this section focuses on the computational implementation of particle swarm algorithm in reactive power optimization problem instead of the algorithm itself. A reactive power optimization problem including the constraints using penalty function can be considered as:

$$\begin{aligned} f(V', \theta') = & \sum_{k=1}^{N} g_k \left[V_{1,k}'^2 + V_{2,k}'^2 - 2V_{1,k}' V_{2,k}' \cos\left(\theta_{1,k}' - \theta_{2,k}'\right) \right] \\ & + k_v \sum_{j=1}^{M} \left| V_j' - V_j'^{ref} \right| + k_s \sum_{i=1}^{T} \left| 1 - \frac{V_i'}{V_{i0}'} \right| \end{aligned} \tag{10.74}$$

As particle swarm optimization algorithm is naturally good at maximizing objective functions, then the appropriate fitness function in order to minimize the target function in Eq. (10.74) will be as follows

$$\text{maximize}\, F(V', \theta') = \frac{1}{f(V', \theta')} \tag{10.75}$$

Subject to control variables, such as terminal voltages of generator buses, tap settings of transformers, and reactive power output of compensators

$$X = [\, V \quad T \quad C \,] \tag{10.76}$$

$$V = [\, V_1 \quad V_2 \quad \ldots \quad V_n \,] \tag{10.77}$$

$$T = [\, T_1 \quad T_2 \quad \ldots \quad T_m \,] \tag{10.78}$$

$$C = [\, C_1 \quad C_2 \quad \ldots \quad C_d \,] \tag{10.79}$$

$$X^{(l)} = [\, V_{\min} \quad T_{\min} \quad C_{\min} \,] \tag{10.80}$$

$$X^{(u)} = [\, V_{\max} \quad T_{\max} \quad C_{\max} \,] \tag{10.81}$$

where, X is the vector of control variables, while $X^{(l)}$ and $X^{(u)}$ are lower and upper bounds of them, V is a vector containing terminal voltages of generator buses, T is a vector consisting of tap settings of transformers, and C is the vector of reactive power compensators. n, m and d are the number of generator buses, transforms having tap changing facility, and compensators, respectively.

The PSO procedure can be implemented through the following steps [1]:

1. Consider N as the size of the swarm which is mostly between 20 and 30. It is obvious that taking big numbers will raise the time of evaluation of the objective function and will influence the total calculation time. However, it should not be too small either as it can affect the performance of the PSO algorithm. In general, the size of the population is obtained using trial and error method for each optimization problem, although there as some approximate methods to apply in order to get appropriate numbers for them [6].
2. Produce the primary population of X in between $X^{(l)}$ and $X^{(u)}$ randomly as X_1, X_2, …, X_N. Henceforth, for the sake of convenience, the position of jth individual and its speed in ith iteration are signified as $X_j^{(i)}$ and $V_j^{(i)}$, respectively. Accordingly, the particles created initially are indicated as $X_1(0)$, $X_2(0)$, …, $X_N(0)$. The vectors $X_j(0)$ (j = 1, 2, …, N) are called particles or coordinate vectors of particles [6].
3. Apply the generated control variables to the power network and run power flow in order to obtain the values of state variables, because the objective function in Eq. (10.75) needs the values of both state (voltage magnitudes of PQ buses) and control variables (voltage values of PV buses). After doing power flow, the bus voltages and voltage angles will be considered as follows [6]:

$$Y_j(i) = \begin{bmatrix} V' & \theta' \end{bmatrix} \tag{10.82}$$

where, V' is the vector of voltage magnitudes and θ' is the vector of voltage angles of all the buses.

4. Work out the values of the objective function for each particle as $F[(Y_1(0)), F(Y_2(0)), \ldots, F(Y_N(0)]$.
5. Determine the velocity values for all the particles. The velocity will help leading the particles through reaching the optimum point. Primarily, the speed value for all the particles are presumed zero. Then, set the iteration number to one ($i = 1$).
6. In the ith iteration, the two following significant parameters should be calculated using the data of jth particle:

   (a) Work out the best value of the objective function for X_j among all the iterations and allocate it to $P_{best,j}$ (personal best) which is the best value of the $F[Y_j(i)]$ found by jth individual so far. Find the best value of the objective function for all the particles (X) among all the iterations so far and assign it to G_{best} (global best), which is the best value found for the objective function $F[Y_j(i)]$ $j = 1, 2, 3, \ldots, N$ [1].

   (b) Find the velocity of jth particle in ith iteration by means of the following equation:

$$V_j(i) = \theta V_j(i-1) + c_1 r_1 \left[P_{best,j} - X_j(i-1)\right] + c_2 r_2 \left[G_{best} - X_j(i-1)\right] \\ j = 1, 2, \ldots, N \tag{10.83}$$

   where, c_1 and c_2 are the perceptive (individual) and collective (group) learning coefficients, respectively. r_1 and r_2 are uniformly distributed random numbers in the range of 0 and 1. The factors c_1 and c_2 signify the comparative rank of the memory (location) of the particles to the memory (location) of the swarm. c_1 and c_2 are commonly considered to have the amount of 2 in a lot of implementations. The inertia weight θ is a constant which is used in order to decrease the velocities as time goes by (or iterations), facilitating the swarm to congregate more precisely and proficiently compared to the original PSO algorithm. Equation (10.83) represents a formulation for adjusting the velocity, which helps the accuracy increase. Equation (10.83) demonstrates that a greater amount of θ supports global exploration, while a smaller value encourages a local exploration better. Hence, a great value of θ marks the algorithm continually discover new areas deprived of much local examination, failing to find the true optimum point. To strike a balance between global and local search in order to speed up converging to the exact optimal location, an inertia coefficient whose value declines linearly in relation to the iteration number has been used as [6]

$$\theta(i) = \theta_{\max} - \left(\frac{\theta_{\max} - \theta_{\min}}{i_{\max}}\right) i \tag{10.84}$$

where, $\theta_{\max}$ and $\theta_{\min}$ are primary and final values of the inertia weight, respectively. $i_{\max}$ is the maximum quantity of iterations used in PSO. Values of $\theta_{\max} = 0.9$ and $\theta_{\min} = 0.4$ are commonly used [6].

(c) Find the coordinate of the *j*th particle in *i*th iteration using [6]

$$X_j(i) = X_j(i-1) + V_j(i); \quad j = 1, 2, \ldots, N \tag{10.85}$$

Evaluate the values of objective function for all the particles as $F[Y_1(i)]$, $F[Y_2(i)]$, …, $F[Y_N(i)]$, in which the matrix Y is acquired from the power flow calculations after applying new control variables to the power network [6].

7. Check if the algorithm has reached the optimal point or not, which is applicable via creating and checking a few stopping criteria for the optimization problem. If the points of all particles congregate to the similar set of values, the technique is considered to be converged. If the convergence criterion is not met, step 6 will be reiterated by updating the iteration number to $i = i + 1$, and calculating the new values for $P_{best,j}$ and G_{best}. The reiterative procedure will continue until almost all of the particles congregate to the same optimal point. The flow chart of PSO algorithm founded on above declared process is shown in Fig. 10.9 [6].

There are plenty of stopping criteria to use, of which can be chosen in relation to what the algorithm is expected to do or reach. The first one can be the iteration number so that the process will stop after a specified number of iterations whether the algorithm has reached the optimum point or not. The second one can be considered as the time duration of the process running, which completely depends on how heavy the process might be. The third one can be to use a pre-specified fitness value and compare the last overcome of the optimization procedure to it, and then if the appropriateness of the solution is met, the process can be stopped. The fourth one can be taken as whether there is any substantial progression in the result or not, and stop the process if not. Usually a combination of several criteria is used in practice appropriate to what the expectation is from the algorithm and the procedure. Mostly, the criteria that consider just the calculation cost such as the time duration and the number of iterations can be used with some other criteria that consider the fitness of the result.

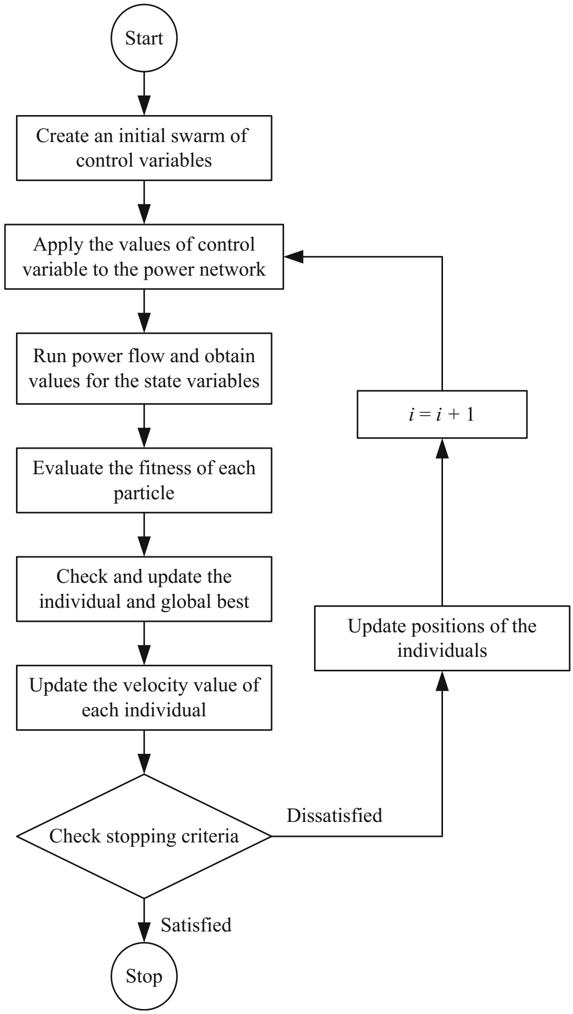


Fig. 10.9 The flowchart of reactive power optimization using PSO

10.6 Pattern Search Optimization Algorithm

Pattern search (PS) algorithm is one of the arithmetical optimization approaches that do not need the gradient of the objective function which is meant to be optimized. Therefore, PS can be used on discrete functions that are not differentiable as well. Such optimization means are also recognized as direct-search, derivative-free, or black-box methods. They are founded on producing search patterns which positively sweep the search space in order to detect the minimum points. The procedure of optimizing starts from a random starting point, though being highly dependent on what location they start from (initial point) is their biggest defect. This drawback can cause the optimization procedure to get stuck in one of the local minimums which is not favorable. However, because of their flexibility, they can be integrated with heuristic algorithms (global optimizers) for global search, which is a mode pattern search technique receives some of the properties of the imported global optimization technique (global optimization), without risking the convergence and being trapped in one of the local optimum points [10–12].

10.6.1 Mathematical Description of Pattern Search Optimization Algorithm

Pattern search algorithm can be counted as a direct search optimization algorithm in which the central notion is on the positive spanning sets. This section will present the PS algorithm in search/poll framework which is the best choice when it comes to cooperate with heuristic algorithms. One of the simplest positive spanning sets is formed by the vectors of the canonical foundation and also their negatives [10, 12]

$$D_{\oplus} = \{e_1, \ldots, e_n, -e_1, \ldots, -e_n\} \tag{10.86}$$

The set $D_{\oplus}$ is also a (highest) positive basis. The basic straight search technique based on this progressive spanning set is recognized as coordinate or compass search. Bearing in mind a progressive spanning set as D and the recent iteration $y(t)$, two groups of points are defined as the net M_t and the election set P_t. The net M_t is known as [10]

$$M_t = \left\{ y(t) + \alpha(t) D_z,\ z \in Z_{+}^{|D|} \right\} \tag{10.87}$$

where $\alpha(t) > 0$ is the mesh dimension factor and also identified as the step-length controller, and Z_{+} is the set containing nonnegative integer numbers. The mesh has to fulfil some integrality necessities for the technique to attain global convergence to static points from random initial points. The matrix D has to be in the formula of $G\hat{Z}$, where $G \in R^{n \times n}$ is a nonsingular producing matrix and $G \in R^{n \times n}$. The progressive foundation $D_{\oplus}$ meets the prerequisites when G is entitled the identity matrix. The pursuit step conducts a limited exploration in the net M_t. The poll step is performed only if the examination step fails to discover a position for which f is lesser than $f(y(t))$. The poll step assesses the function at the positions in the poll set P_t in order to discover a location where f is minor than $f(y(t))$ [10, 12]

$$P_t = \{ y(t) + \alpha(t) d,\ d \in D \} \tag{10.88}$$

It should be noted that P_t is a subdivision of M_t. If f is constantly differentiable at $y(t)$, the poll step is assured to succeed if $\alpha(t)$ is appropriately small, since the progressive spanning set D comprises at least one pattern of descent which sorts an acute angle with $-\nabla f(y(t))$. Consequently, if the poll step flops to discover a coordination better than the former one, the net size factor must be made smaller. The poll step which is the key tool of pattern search approach to explore the optimal point guarantees the global convergence.

In order to extrapolate pattern search optimization procedure for bound constrained problems, it is indispensable to use a practicable primary guess $y(0) \in \Omega$ and to keep feasibility of the iterates safe by declining any trial position that is out of the acceptable region. Rejecting unviable test locations can be achieved by applying a pattern search algorithm to the subsequent penalty function [10, 12].

$$\hat{f}(z) = \begin{cases} f(z) & \text{if} \quad z \in \Omega \\ +\infty & \text{otherwise} \end{cases} \tag{10.89}$$

There is no big dissimilarity between constrained and unconstrained pattern search optimization algorithm excluding it is applied to the minimization of $f(z)$ subject to simple bounds and to the refusal of impractical test points. It is also essential to embrace in the exploration directions D those patterns that warranty the existence of a practicable descent track at any nonstationary location of the bound constrained problem [10].

In order to have an elaborate depiction of the basic pattern search algorithm, it is necessary to state in what way to expand and contract the net size or step-length control factor $\alpha(t)$. The growths and reductions use the parameter $\phi(t)$ and $\theta(t)$, respectively, which must observe the subsequent rules:

$$\begin{array}{lll} \phi(t) = \bar{T}^{l_t}, & \text{for some } l_t \in \{0, \ldots, l_{\max}\} & \text{if} \quad t \text{ is successful} \\ \theta(t) = \bar{T}^{m_t}, & \text{for some } m_t \in \{m_{\min}, \ldots, -1\} & \text{if} \quad t \text{ is unsuccessful} \end{array} \tag{10.90}$$

where, $\bar{T} > 1$ is a positive rational, $l_{\max}$ is a nonnegative integer, and $m_{\min}$ is a negative integer, selected at the commencement of the procedure and unaffected with t. For instance, it can be considered $\theta(t) = 0.5$ for unproductive iterations and $\phi(t) = 1$ or $\phi(t) = 2$ for up-and-coming iterations [10, 12].

The process of basic pattern search method has been offered in follows [10]:

1. Select a positive rational $\bar{T}$ and the tolerance $\alpha_{tol} > 0$ as the stopping criterion. Pick the positive spanning set $D = D_{\oplus}$.
2. Set $t = 0$. Choose an primary practical guess $y(0)$. Pick $\alpha(0) > 0$.
3. [Search Step], Assess f at a limited number of points in M_t. If a position $z(t) \in M_t$ is discovered for which $\hat{f}(z(t)) < \hat{f}(y(t))$ then set $y(t + 1) = z(t)$, $\alpha(t + 1) = \phi(t)\alpha(t)$ (optionally increasing the net size factor), and tag both the exploration step and the present iteration as successful.
4. [Poll Step], Avoid the poll step if the examination step was successful.

   - If there exists $d(t) \in D$ so that $\hat{f}(y(t) + \alpha(t)d(t)) < \hat{f}(y(t))$, then:

     Set $y(t + 1) = y(t) + \alpha(t)d(t)$ (poll step and iteration successful).
     Set $\alpha(t + 1) = \phi(t)\alpha(t)$ (optionally increase the net size factor).

   - Otherwise, $\hat{f}(y(t) + \alpha(t)d(t)) \geq \hat{f}(y(t))$ for all $d(t) \in D$, and:

     Set $y(t + 1) = y(t)$ (iteration and poll step unsuccessful).
     Set $\alpha(t + 1) = \theta(t)\alpha(t)$ (contract the net size factor).
5. If $\alpha(t + 1) < \alpha_{tol}$ then break, where α_{tol} is the least value which is defined as the mesh dimension factor. Otherwise, increase t by one and go to Step 3.

10.6.2 Pattern Search Algorithm in Simple Words

The flowchart of pattern search algorithm is shown in Fig. 10.10 in the simplest way possible. At first an initial point as well as initial step size are produced randomly as $x_0 \in R^n$ (for one dimensional problem) and $\Delta_0 > 0$, respectively. Then, the fitness function will be calculated in neighboring points $x_0, x_0 + \Delta, x_0 - \Delta$. If $f(x_0 + \Delta) < f(x_0)$ then the point $x_0 + \Delta$ will be considered as the center point and Δ will be added to it, then new fitness values will be computed. This procedure will be continued until the fitness value does not get better, then the value of Δ will be decreased and added to center point, afterwards new fitness values will be calculated again. This procedure will go on until the stopping criteria stop the algorithm. A simple graphical example has shown in the Fig. 10.11 in order to illustrate how the algorithm works.

10.7 Particle Swarm Pattern Search Algorithm

The most considerable feature that distinguishes heuristic optimization techniques from traditional ones is their capability to find the global optimum point, not needing the gradient of the objective function as a great asset, so that the cost of the calculation (considering time as a resource) will be decreased. Nevertheless, their local minimization is not as efficient as their global optimization, or in another words, is more time consuming in comparison with to the extent the function gets minimized. Moreover, pattern search algorithm is such a great direct local minimizer, although in some cases it has proved its power finding even global optimum point. The more problem space gets non-convex, however, the more pattern search algorithm seems to fail finding the global optimum point, because of the so many local optimum points which exist. Therefore, the pattern search algorithm will be more practical finding local optimum points rather than global ones. In addition, pattern search algorithm is highly dependent on the starting point which is normally chosen randomly so that different starting points can lead the algorithm to different solutions not probably being a global optimum point.

The idea is to take the advantages of both types using heuristic algorithms as the global optimizers and direct search methods as local optimizers, in this case pattern search algorithm. Since pattern search algorithm needs a starting point and the final answer is directly dependent on it, considering the global minimum point found by intelligent algorithms like PSO that is mostly near the exact solution as a starting point to pattern search algorithm will fasten the optimization procedure finding the more accurate point. Likewise, if there is a time limitation to calculation, using the particle swarm pattern search algorithm will not be empty of favor [10].

There are some different strategies combining an intelligent optimization algorithm and a direct search method, each of them has its own advantages and disadvantages. For instance, one can be considered as a series one where a heuristic algorithm is chosen to implement a global optimization, after reaching near the global optimum point the direct search method is used in order to pinpoint the local one of the neighboring search space in which the best optimum point lies. The first

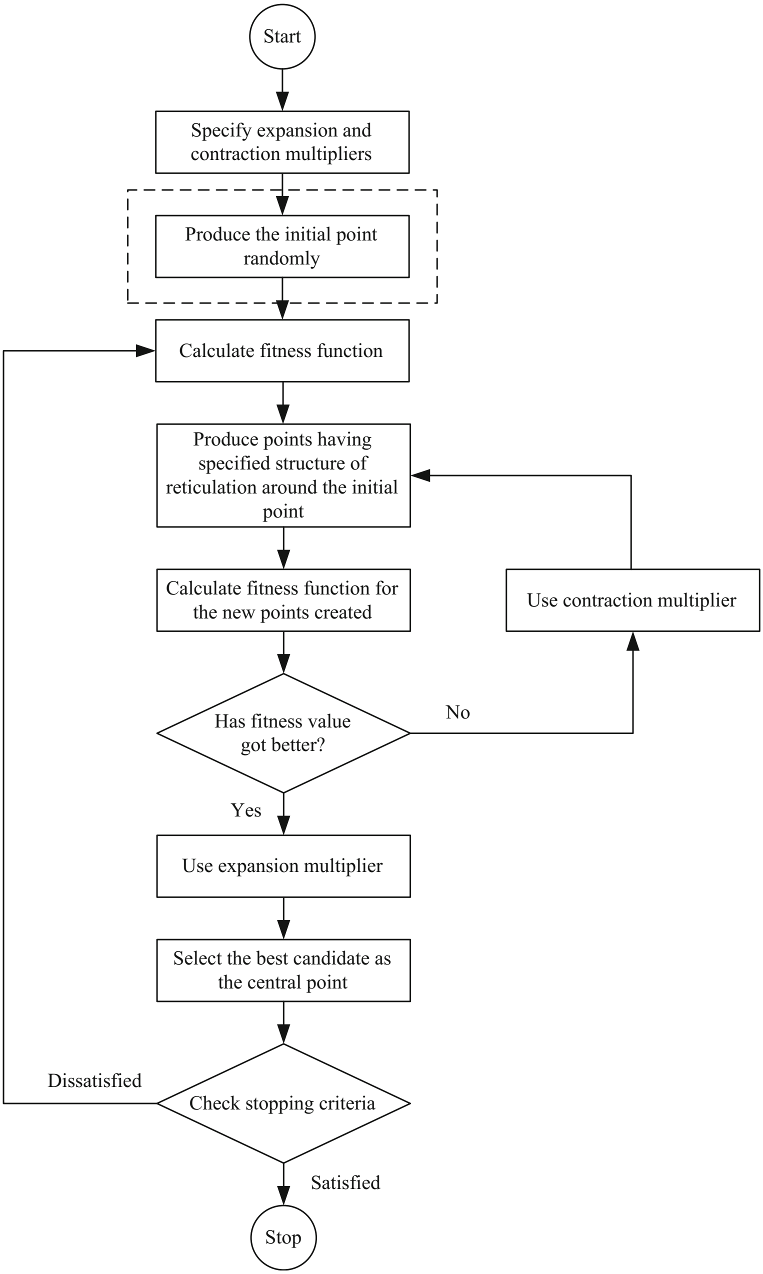


Fig. 10.10 Flowchart of pattern search optimization algorithm

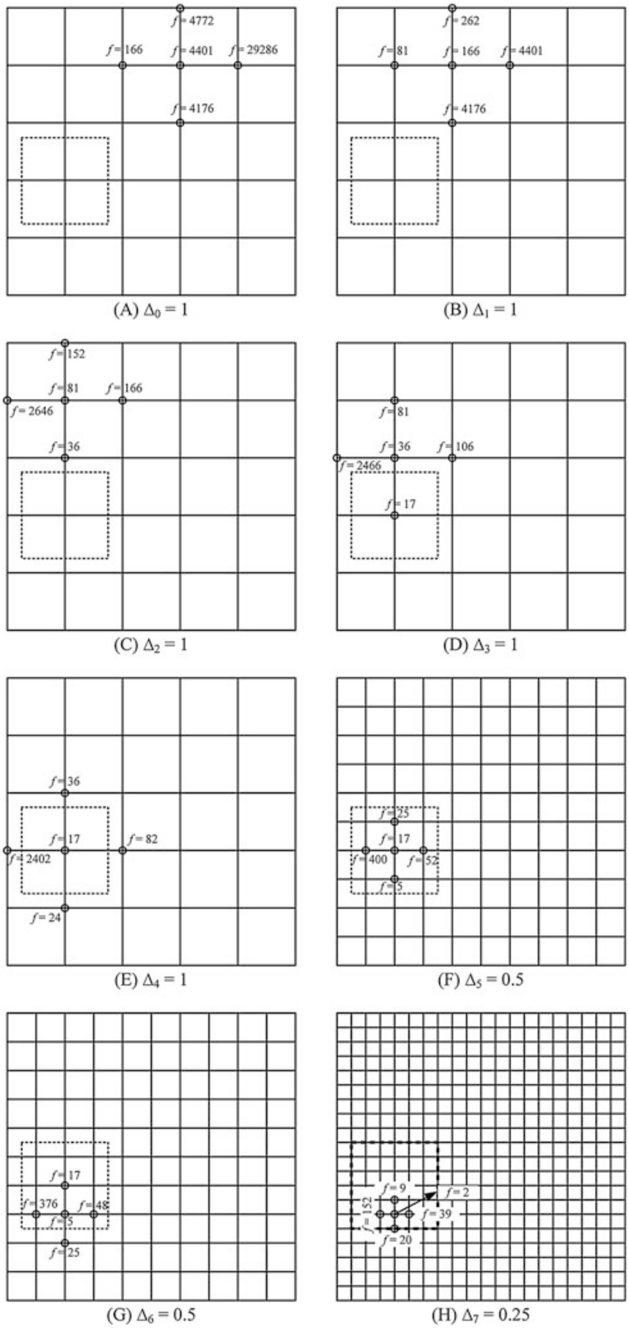


Fig. 10.11 Graphical show of how pattern search optimization algorithm works [12]

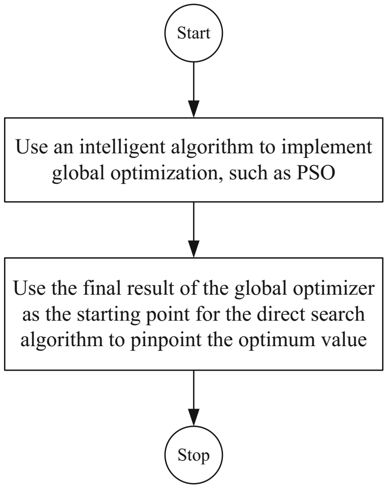


Fig. 10.12 Flowchart of the first strategy using heuristic algorithms incorporated with direct search method

strategy is the simplest and fastest one which can be taken into practice. The flowchart of the first strategy has been shown in Fig. 10.12. The second strategy can be like using a direct search method for each point found by particles (PSO) or genes (genetic algorithm) in order to find the exact optimum point using positions of particles of each iteration as starting points for direct search methods. This approach can be more time consuming than the first one, as the direct search process should be applied to each particle. Although, It sounds to be more accurate and also more reliable than waiting to an intelligent algorithm to find a global optimum point, in each iteration an exact optimum point (probably not global one at the beginning) will be found so that in online implications would be at work. The flowchart of second strategy has been shown in Fig. 10.13 [10, 12].

In order to avoid implementation difficulties, the first strategy has been chosen to be presented in this section. In addition, it can be turned to the other strategy readily considering a few changes in the procedure. Therefore, the first strategy as particle swarm pattern search algorithm can be coincided as follows:

The particle swarm pattern search algorithm procedure can be implemented through the following steps [6]:

1. Consider N as the size of the swarm which is mostly between 20 and 30. It is obvious that taking big numbers will raise the time of evaluation of the objective function and will influence the total calculation cost. However, it should not be too small as it can affect the performance of the PSO algorithm. In general, the number of population is obtained using trial and error method for each optimization problem, although there are some approximate methods to apply in order to get the appropriate numbers. Choose appropriate stopping criteria for PSO algorithm, considering the fact that optimization procedure will go on after getting to the neighborhood of global optimum point with pattern search algorithm which has its own stopping criteria [6].

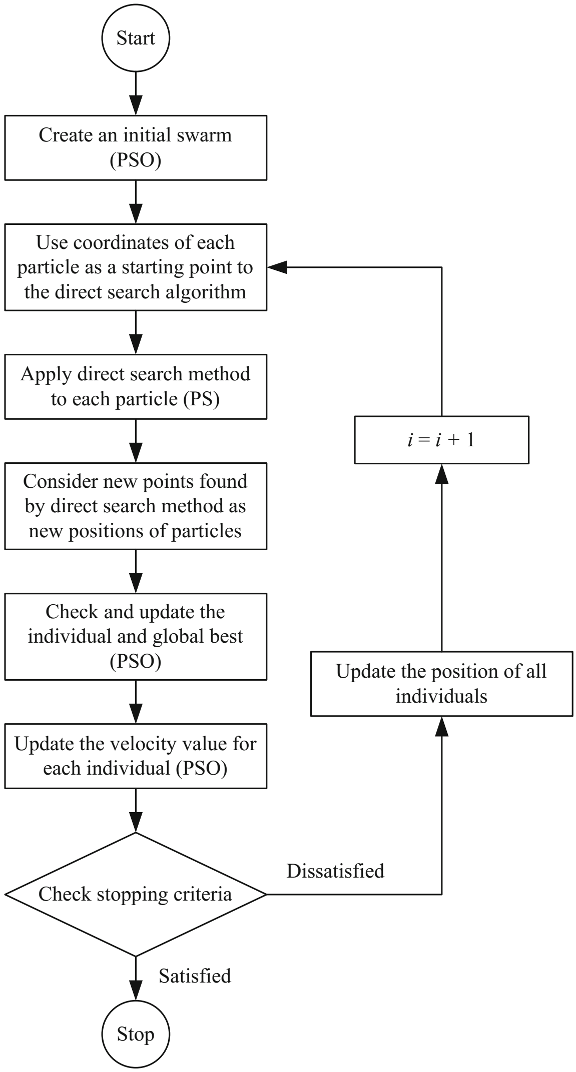


Fig. 10.13 The flowchart of the second strategy using heuristic algorithms incorporated with direct search methods

2. Produce the primary population of X in the assortments $X^{(l)}$ and $X^{(u)}$ randomly as $X_1, X_2, \ldots, X_N$. Henceforth, for the sake of convenience, the position of and velocity of jth individual in ith iteration are signified as $X_j^{(i)}$ and $V_j^{(i)}$, respectively. Accordingly, the particles created at the beginning are indicated as $X_1(0)$, $X_2(0), \ldots, X_N(0)$. The vectors $X_j(0)$ ($j = 1, 2, \ldots, N$) are called particles or vectors of coordinates of particles [6].
3. Apply the generated control variables to the power network and run power flow in order to obtain state variables as they are needed in the objective function represented by Eq. (10.75). Therefore, after doing power flow the bus voltages and voltage angels will be considered as follows [6]

$$Y_j(i) = \begin{bmatrix} V' & \theta' \end{bmatrix} \tag{10.91}$$

where, V' is the vector of voltage magnitudes of all buses and θ' is the bus voltage angles.

4. Calculate the objective function values for all the particles as $F(Y_1(0))$, $F(Y_2(0))$, …, $F(Y_N(0)]$.
5. Find the velocities of particles, which will help particles reach the optimum point. Primarily, the velocities of all particles are presumed zero. Set the iteration number to $i = 1$.
6. In the ith iteration, the two following significant parameters should be calculated using the data of jth particle:

   (a) Work out the best value of the objective function for X_j among all the iterations and allocate it to $P_{best,j}$ (personal best) which is the best value of the $F[Y_j(i)]$ found by jth individual so far. Find the best value of the objective function for all the particles (X) among all the iterations so far and assign it to G_{best} (global best), which is the best value found for the objective function $F[Y_j(i)]$ $j = 1, 2, 3, \ldots, N$ [1].

   (b) Find the velocity of jth particle in ith iteration by means of the following equation

$$V_j(i) = \theta V_j(i-1) + c_1 r_1 \left[P_{best,j} - X_j(i-1)\right] + c_2 r_2 \left[G_{best} - X_j(i-1)\right] \quad j = 1, 2, \ldots, N \tag{10.92}$$

where, c_1 and c_2 are the perceptive (individual) and collective (group) learning coefficients, respectively. r_1 and r_2 are uniformly distributed random numbers in the range of 0 and 1. The factors c_1 and c_2 signify the comparative rank of the memory (location) of the particles to the memory (location) of the swarm. c_1 and c_2 are commonly considered to have the amount of 2 in a lot of implementations. The inertia weight θ is a constant which is used in order to decrease the velocities as time goes by (or iterations).

$$\theta(i) = \theta_{\max} - \left(\frac{\theta_{\max} - \theta_{\min}}{i_{\max}}\right) i \tag{10.93}$$

where, $\theta_{\max}$ and $\theta_{\min}$ are primary and final values of the inertia weight, respectively. $i_{\max}$ is the maximum quantity of iterations used in PSO. Values of $\theta_{\max} = 0.9$ and $\theta_{\min} = 0.4$ are commonly used [1].

(c) Calculate the location or coordinate of the *j*th particle in *i*th iteration as [6]

$$X_j(i) = X_j(i-1) + V_j(i); \quad j = 1, 2, \ldots, N \tag{10.94}$$

Evaluate the values of objective function for all the particles as $F[Y_1(i)]$, $F[Y_2(i)]$, …, $F[Y_N(i)]$, in which the matrix Y is acquired from the power flow calculations after applying new control variables to the power network [1].

7. Check if the algorithm has reached the optimal point or not, which is applicable via creating and checking a few stopping criteria for the optimization problem. If the points of all particles congregate to the similar set of values, the technique is considered to be converged. If the convergence criterion is not met, step 6 will be reiterated by updating the iteration number to $i = i + 1$, and calculating the new values for $P_{best,j}$ and G_{best}. The reiterative procedure will continue until almost all of the particles congregate to the same optimal point. The flow chart of PSO algorithm founded on above declared process is shown in Fig. 10.9 [6].
8. Select a positive rational $\bar{T}$ and the tolerance $\alpha_{tol} > 0$ as the stopping criterion. Pick the positive spanning set $D = D_{\oplus}$.
9. Set $t = 0$. Choose a primary practical guess $y(0)$ and pick $\alpha(0) > 0$. The best position found by PSO algorithm will be considered as the initial point in this step.
10. Apply control variables to the power network and implement power flow to calculate the state values as

$$Y = \begin{bmatrix} V' & \theta' \end{bmatrix} \tag{10.95}$$

11. [Search Step], Assess f using Eq. (10.74) at a limited number of positions in M_t. If a coordinate $Z'(t) \in M_t$ is discovered for which $f(Y_{Z'(t)}) < f(Y_{Z(t)})$, [where, $(Y_{Z'(t)})$ is a matrix containing the values for the state variables after doing power flow for $Z'(t)$], then consider $Z(t) = Z'(t)$ and $\alpha(t+1) = \phi(t)\alpha(t)$ (optionally increasing the net size factor), and state successful both the exploration step and the present iteration.

$$Z = \begin{bmatrix} V & T & C \end{bmatrix} \tag{10.96}$$

$$V = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} \tag{10.97}$$

$$T = \begin{bmatrix} T_1 & T_2 & \ldots & T_m \end{bmatrix} \tag{10.98}$$

$$C = \begin{bmatrix} C_1 & C_2 & \ldots & C_d \end{bmatrix} \tag{10.99}$$

$$\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_n \end{bmatrix} \tag{10.100}$$

12. [Poll Step], Avoid the poll step if the exploration step was successful.
    - If there exists $[Z(t) + \alpha(t)d(t)] \in D$ so that $f(Y_{Z(t)+\alpha(t)d(t)}) < f(Y_{Z(t)})$, then

Consider $Z(t+1) = Z(t) + \alpha(t)d(t)$ (poll stage and iteration successful).
Adjust $\alpha(t+1) = \phi(t)\alpha(t)$ (optionally increase the net size factor).

- Otherwise, $f\left(Y_{Z(t)+\alpha(t)d(t)}\right) \geq f\left(Y_{Z(t)}\right)$ for all $d(t) \in D$, and

Consider $Z(t+1) = Z(t)$ (iteration and poll stage unsuccessful).
Adjust $\alpha(t+1) = \theta(t)\alpha(t)$ (contract the net size factor).

13. If $\alpha(t+1) < \alpha_{tol}$ then stop, where α_{tol} is the least value which is defined as the mesh dimension factor. Otherwise, increase t by one and go to step 10.
The flowchart of particle swarm pattern search algorithm which was mentioned above is shown in Fig. 10.14. In the next section practical implementation of reactive power optimization using the proposed particle swarm pattern search algorithm will be presented.

10.8 Simulation Results of Reactive Power Optimization

Two algorithms as particle swarm pattern search and genetic pattern search algorithms have been implemented on two standard systems as IEEE 6-bus and IEEE 14-bus and the results have been compared. In addition, the simulation results for IEEE 39-bus New England power network using particle swarm pattern search algorithm has been presented in this section. Besides, how to implement such optimization procedures using MATLAB and DIgSILENT has been presented in Chap. 11 step by step.

10.8.1 Case Study 1—IEEE 6-Bus Power Network

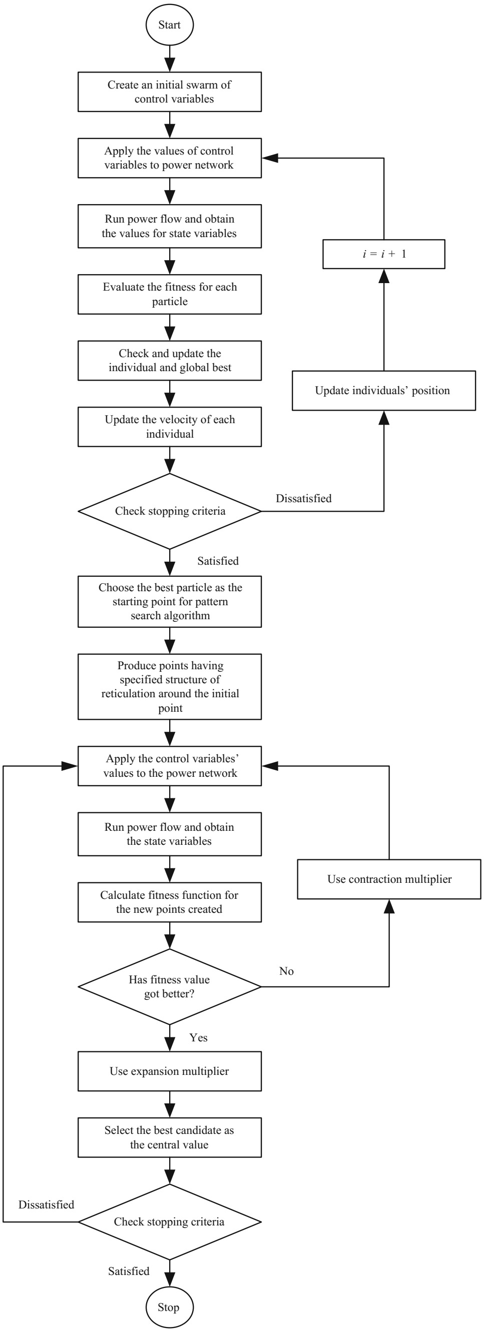
The corresponding data for IEEE 6-bus power grid has been presented in Appendix 1. The results for reactive power optimization on the related network will be offered in this section.

10.8.1.1 Reactive Power Optimization Using Particle Swarm Pattern Search Algorithm

The initial conditions and power flow results for 6-bus power network are presented in Table 10.19. In addition, the corresponding data after the optimization procedure on the same power grid are presented as below Figs. 10.15, 10.16 and 10.17.

Referring to Tables 10.1, 10.2 and 10.3, it is obvious that operational parameters have been improved considerably. Active power losses have decreased from

Fig. 10.14 Flowchart of particle swarm pattern search optimization algorithm



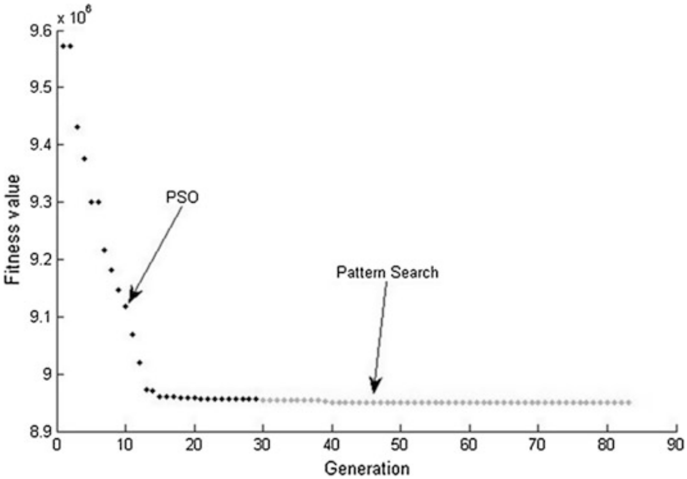


Fig. 10.15 Reactive power optimization trend for 6-bus power system using particle swarm pattern search algorithm

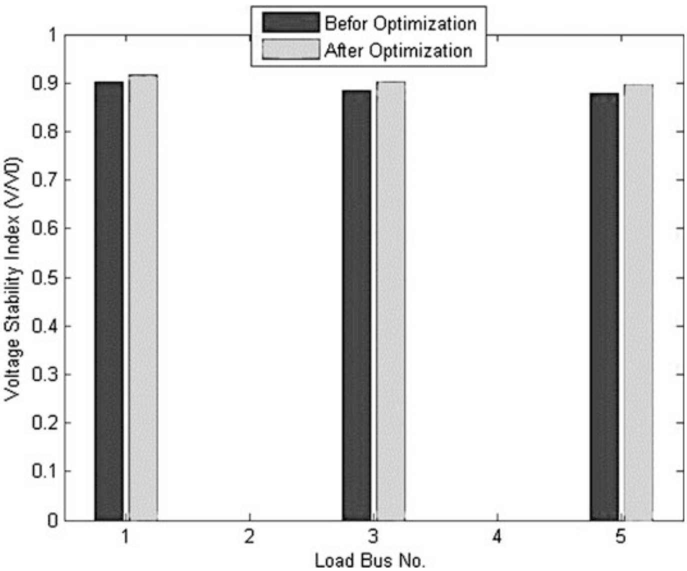


Fig. 10.16 Voltage stability index for 6-bus power system using particle swarm pattern search algorithm

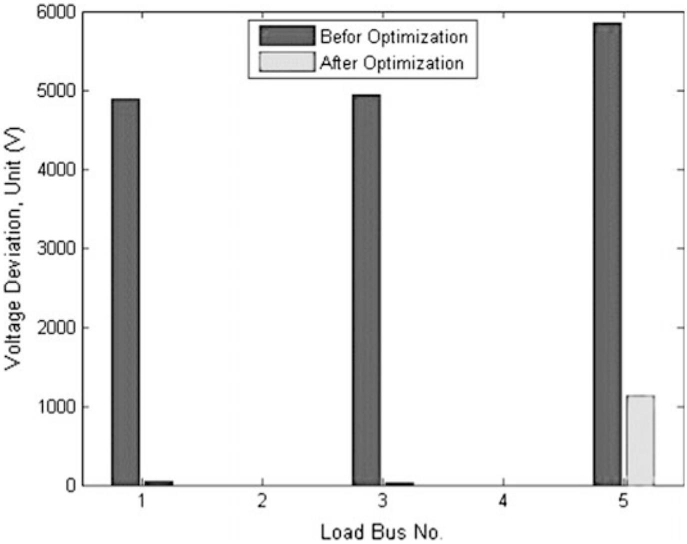


Fig. 10.17 Voltage deviation for 6-bus power system using particle swarm pattern search algorithm

Table 10.1 Power flow results after optimization for 6-bus power system using particle swarm pattern search algorithm

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	62.97	−11.69	55	13	0	0
2	72.45	−3.59	0	0	50	22.65
3	62.98	−10.97	50	5	0	5
4	63.86	−8.79	0	0	0	0
5	61.82	−11.23	0.3	18	0	5
6	69.30	0	0	0	93.96	33.13

Table 10.2 Active power losses for 6-bus power system using particle swarm pattern search algorithm

Active power losses (initial condition)	10,778,370 (W)
Active power losses (after optimization)	8,950,232 (W)
Reduction percentage	16.96121%

10,778,370 to 8,950,232 (W). Likewise, the total value of voltage stability index has increased from 2.665148 to 2.715127, and the total value of voltage deviation of load buses has been reduced considerably from 15.65904 to 1.185288 kV.

Table 10.3 Voltage deviation and voltage stability data for 6-bus power system using particle swarm pattern search algorithm

Bus no.	Voltage stability index value V/V_0 ideal value = 1		Voltage deviation of load buses	
	Initial status	After optimization	Initial status kV	After optimization kV
1	0.9009016	0.9161407	4.881641	0.03538300
3	0.8845708	0.9021702	4.934923	0.02064700
5	0.8796755	0.8968166	5.842471	1.129258
Total	2.665148	2.715127	15.65904	1.185288

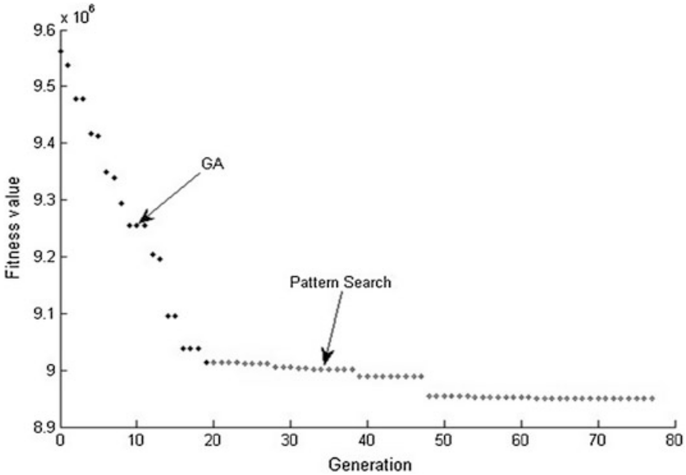


Fig. 10.18 Reactive power optimization trend for 6-bus power system using genetic pattern search algorithm

10.8.1.2 Reactive Power Optimization Using Genetic Pattern Search Algorithm

The initial conditions and power flow results for 6-bus power network are presented in Table 10.18. In addition, the corresponding data after the optimization procedure on the same power grid are presented as below Figs. 10.18, 10.19 and 10.20.

Considering Tables 10.4, 10.5 and 10.6, a marked improvement is obvious in the operational parameters. Active power losses have decreased from 10,778,370 to 8,950,550 (W). Similarly, the total value of voltage stability index has increased from 2.665148 to 2.712409, and the total value voltage deviation of load buses has reduced from 15.65904 to 2.127879 kV.

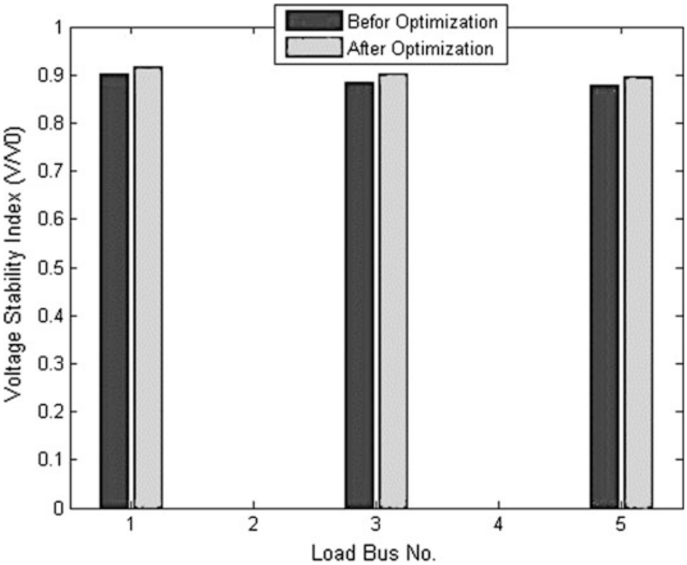


Fig. 10.19 Voltage stability index for 6-bus power system using genetic pattern search algorithm

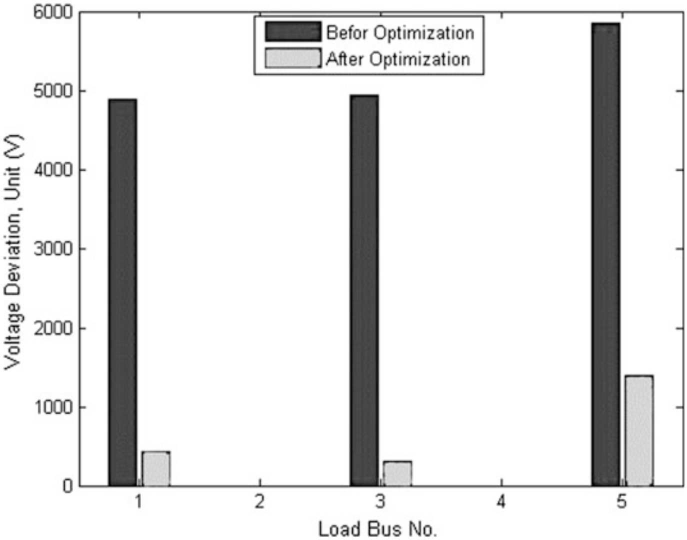


Fig. 10.20 Voltage deviation for 6-bus power system using genetic pattern search algorithm

Table 10.4 Power flow results after optimization for 6-bus power system using genetic pattern search algorithm

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	62.96	−11.69	55	13	0	0
2	72.45	−3.59	0	0	50	22.65
3	62.98	−10.97	50	5	0	5
4	63.86	−8.79	0	0	0	0
5	61.87	−11.23	0.3	18	0	5
6	69.30	0	0	0	93.96	33.13

Table 10.5 Active power losses data for 6-bus power system using genetic pattern search algorithm

Active power losses (initial condition)	10,778,370 (W)
Active power losses (after optimization)	8,950,550 (W)
Reduction percentage	16.95826%

Table 10.6 Voltage deviation and voltage stability data for 6-bus power system using genetic pattern search algorithm

Bus no.	Voltage stability index value V/V_0 Ideal value = 1		Voltage deviation of load buses	
	Initial status	After optimization	Initial status kV	After optimization kV
1	0.9009016	0.9152744	4.881641	0.4280500
3	0.8845708	0.9011948	4.934923	0.3137400
5	0.8796755	0.8959399	5.842471	1.386089
Total	2.665148	2.712409	15.65904	2.127879

10.8.2 Case Study 2—IEEE 14-Bus Power Network

The corresponding data to IEEE 14-bus power grid are accessible in Appendix 2. The results for reactive power optimization on the related network will be presented in this section.

10.8.2.1 Reactive Power Optimization Using Particle Swarm Pattern Search Algorithm

The initial circumstances and power flow results for 14-bus power network are presented in Table 10.22. In addition, the corresponding data after the optimization procedure on the same power grid are presented as below Figs. 10.21, 10.22 and 10.23.

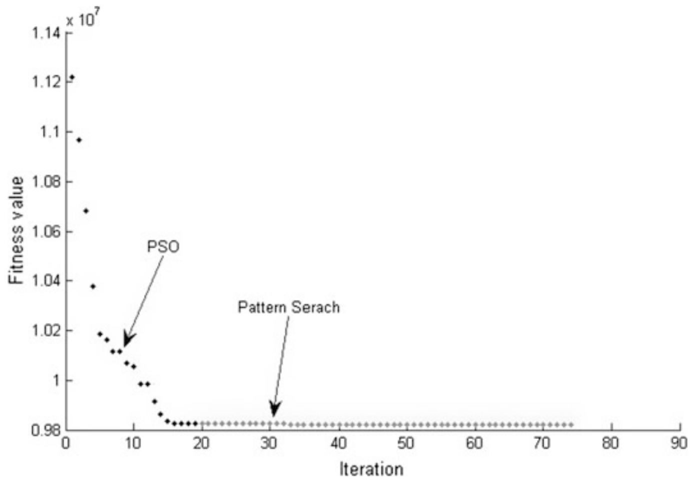


Fig. 10.21 Reactive power optimization trend for 14-bus power system using particle swarm pattern search algorithm

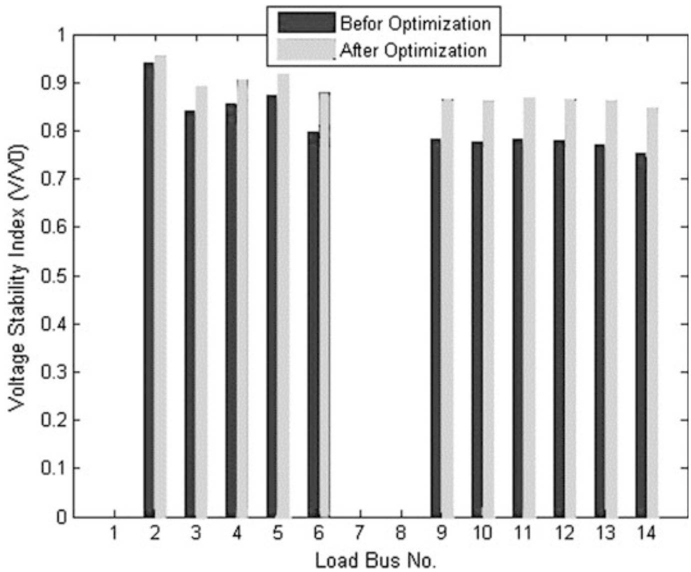


Fig. 10.22 Voltage stability index for 14-bus power system using particle swarm pattern search algorithm

Regarding Tables 10.7, 10.8 and 10.9, active power losses have decreased from 16,710,700 to 9,823,200 (W). Likewise, the total value of voltage stability index has increased from 8.953467 to 9.708002 and total value of voltage deviation of load buses has reduced considerably from 219.7092 to 56.17259 kV.

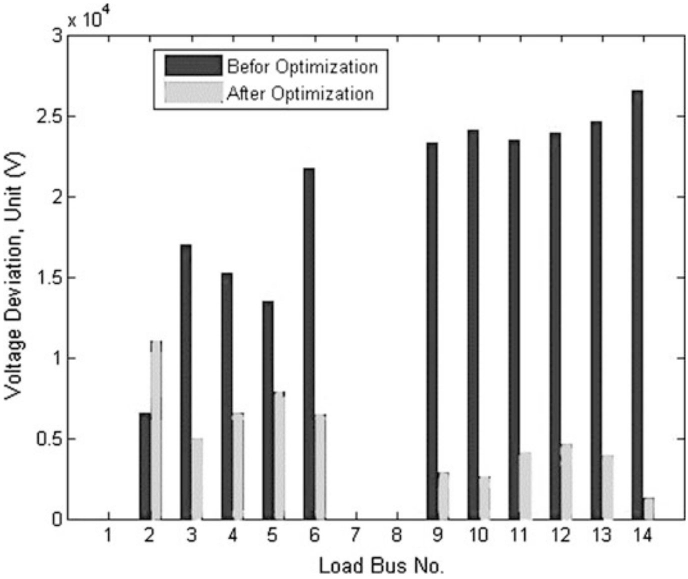


Fig. 10.23 Voltage deviation for 14-bus power system using particle swarm pattern search algorithm

Table 10.7 Power flow results after optimization for 14-bus power system using particle swarm pattern search algorithm

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	125.376906	0	0	0	231.36	45.67
2	121.000004	−3.957964	21.7	12.7	40	16.73
3	114.908254	−10.725941	94.2	19	0	7.436791
4	116.456311	−8.576776	47.8	0	0	0
5	117.737189	−7.335156	7.6	1.6	0	0
6	116.381916	−13.008229	11.2	7.5	0	2.399997
7	114.224630	−11.632678	0	0	0	0
8	114.805564	−11.632674	0	0	0	3.129134
9	112.753677	−13.290581	29.5	16.6	0	0
10	112.559232	−13.530266	9	5.8	0	0
11	114.056146	−13.391523	3.5	1.8	0	0
12	114.573162	−13.866247	6.1	1.6	0	0
13	113.897577	−13.900069	13.5	5.8	0	0
14	111.226337	−14.602206	14.9	5	0	0

Table 10.8 Active power losses data for 14-bus power system using particle swarm pattern search algorithm

Active power losses (initial condition)	16710700 (W)
Active power losses (after optimization)	9823200 (W)
Reduction percentage	41.21612%

Table 10.9 Voltage deviation and voltage stability data for 14-bus power system using particle swarm pattern search algorithm

Bus no.	Voltage stability index value V/V_0 ideal value = 1		Voltage deviation of load buses	
	Initial status	After optimization	Initial status kV	After optimization kV
2	9.409176	0.9552817	6.499059	11
3	8.398882	0.8916201	16.96615	4.975350
4	8.563136	0.9047917	15.24372	6.534758
5	8.728775	0.9160301	13.45913	7.815445
6	7.977896	0.8778006	21.72454	6.421239
9	7.835140	0.8636542	23.25914	2.831756
10	7.761442	0.8598163	24.08241	2.630989
11	7.822556	0.8658360	23.42380	4.112191
12	7.780087	0.8651542	23.91052	4.615991
13	7.718829	0.8609605	24.58441	3.943326
14	7.538748	0.8470567	26.55627	1.291542
Total	8.953467	9.708002	219.7092	56.17259

10.8.2.2 Reactive Power Optimization Using Genetic Pattern Search Algorithm

The initial circumstances and power flow results for 14-bus power network are presented in Table 10.22. In addition, the corresponding data after the optimization procedure on the same power grid are presented as below Figs. 10.24, 10.25 and 10.26.

Referring to Tables 10.10, 10.11 and 10.12, active power losses have decreased remarkably from 16,710,700 to 9,862,619 (W). Similarly, the total value of voltage stability index increased from 8.953467 to 9.655891 and the total value of voltage deviation of load buses has reduced considerably from 219.7092 to 57.82206 kV.

10.8.3 Case Study 3—IEEE 39-Bus New England Power Network

The corresponding data to IEEE 39-bus New England power grid are accessible in Appendix 3. The results for reactive power optimization on the related network will be presented in this section.

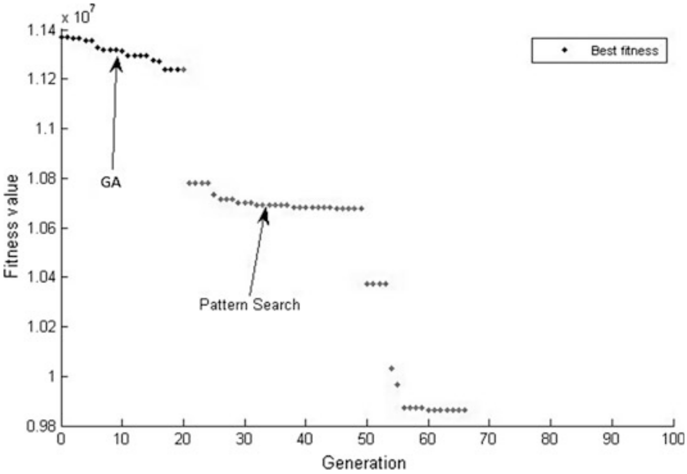


Fig. 10.24 Reactive power optimization trend for 14-bus power system using genetic pattern search algorithm

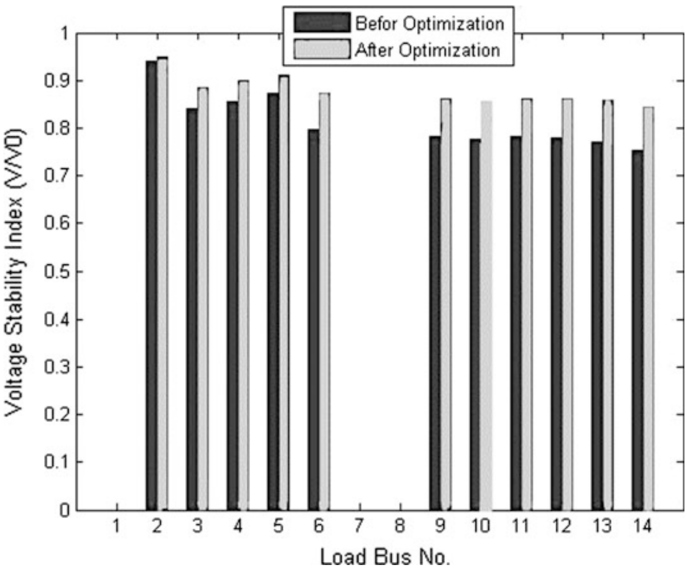


Fig. 10.25 Voltage stability index for 14-bus power system using genetic pattern search algorithm

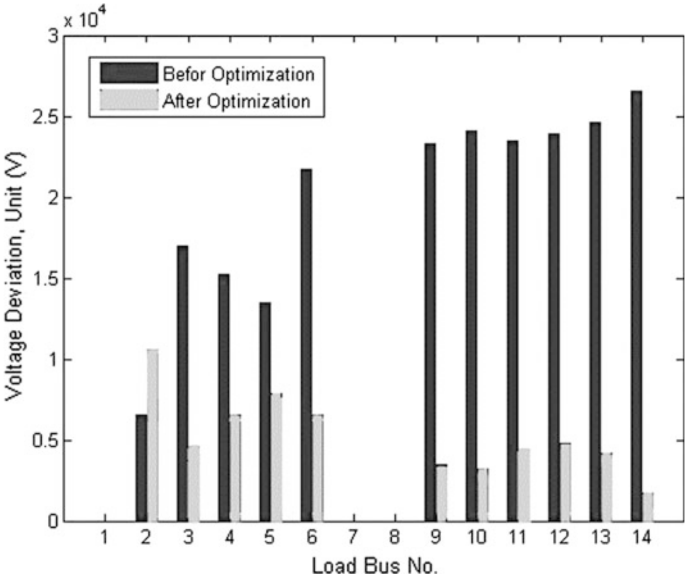


Fig. 10.26 Voltage deviation for 14-bus power system using genetic pattern search algorithm

Table 10.10 Power flow results after optimization for 14-bus power system using genetic pattern search algorithm

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	125.803591	0	0	0	231.42	58.56
2	120.793094	−3.853065	21.7	12.7	40	−0.15
3	115.351591	−10.701274	94.2	19	0	12.74691
4	116.493973	−8.503816	47.8	0	0	0
5	117.785497	−7.265805	7.6	1.6	0	0
6	116.251125	−12.931138	11.2	7.5	0	23.72837
7	114.143428	−11.564184	0	0	0	0
8	114.561700	−11.564181	0	0	0	2.373186
9	112.686674	−13.225312	29.5	16.6	0	0
10	112.480325	−13.463648	9	5.8	0	0
11	113.951417	−13.320288	3.5	1.8	0	0
12	114.445424	−13.791402	6.1	1.6	0	0
13	113.773793	−13.826640	13.5	5.8	0	0
14	111.132858	−14.535142	14.9	5	0	0

Table 10.11 Active power losses data for 14-bus power system using genetic pattern search algorithm

Active power losses (Initial condition)	16710700 (W)
Active power losses (After optimization)	9862619 (W)
Reduction percentage	40.98023%

Table 10.12 Voltage deviation and voltage stability data for 14-bus power system using genetic pattern search algorithm

Bus no.	Voltage stability index value V/V_0 ideal value = 1		Voltage deviation of load buses	
	Initial status	After optimization	Initial status kV	After optimization kV
2	9.409176	0.9476370	6.499059	10.59916
3	8.398882	0.8850283	16.96615	4.580411
4	8.563136	0.8994890	15.24372	6.543957
5	8.728775	0.9111127	13.45913	7.844678
6	7.977896	0.8734883	21.72454	6.543719
9	7.835140	0.8599726	23.25914	3.448204
10	7.761442	0.8561004	24.08241	3.164167
11	7.822556	0.8618414	23.42380	4.445315
12	7.780087	0.8609751	23.91052	4.777684
13	7.718829	0.8568609	24.58441	4.141013
14	7.538748	0.8433849	26.55627	1.733747
Total	8.953467	9.655891	219.7092	57.82206

10.8.3.1 Reactive Power Optimization Using Particle Swarm Pattern Search Algorithm

The initial circumstances and power flow results for 39-bus New England power network are presented in Table 10.13. In addition, the corresponding data after the optimization procedure on the same power grid are presented as below Fig. 10.27.

One of the best criteria that can be taken into consideration as the calculation cost is Number of Function Evaluation (NFE) when it comes to comparing the performance of two or more intelligent algorithms, especially the ones which do not use the gradient of objective functions but the function itself. As it can be inferred from the name, each time the algorithm refers to the objective function to work its value out the amount of NFE increases by one. Eventually, the best optimum point found by every algorithm can be compared to one another in relation to their NFE. In addition, NFE itself can be considered as a stopping criterion as well based on the requirements of the corresponding study.

As mentioned before, one of the benefits of using MATLAB and DIgSILENT together is to take the advantages of built-in toolboxes and functions in both. After the optimization procedure small signal analysis (Eigenvalue or Modal Analysis) has been carried out for the grid in PowerFactory without experiencing

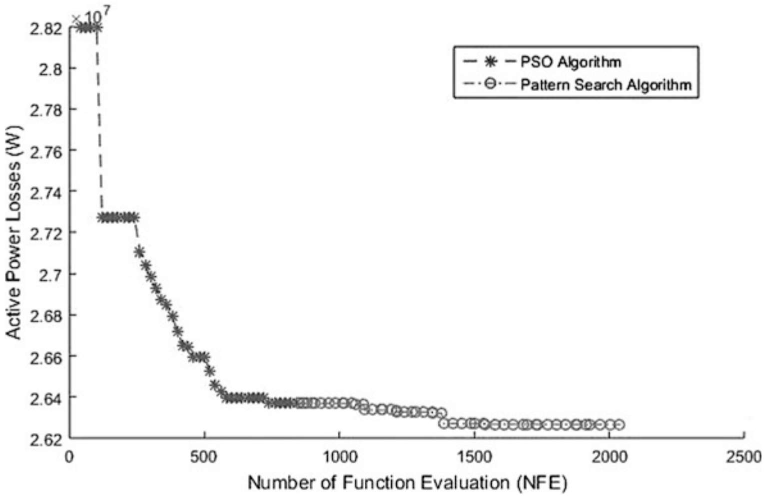


Fig. 10.27 Reactive power optimization trend for 39-bus New England power system using particle swarm pattern search algorithm

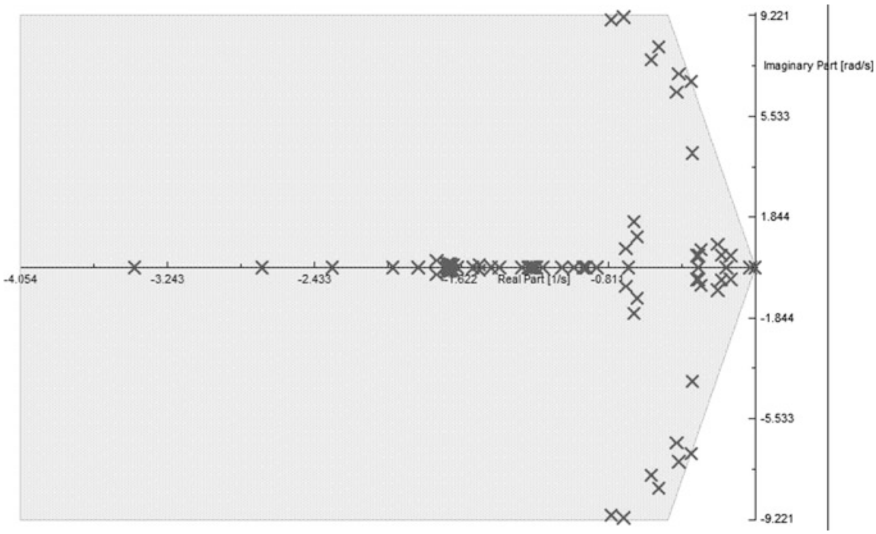


Fig. 10.28 Eigenvalues of 39-bus New England power system without power system stabilizers after optimization

programming difficulties and the result has been shown in Fig. 10.28. The steps how to calculate eigenvalues has been presented in Chap. 11.

According to Tables 10.14, 10.15 and 10.16, active power losses have decreased from 28,194,720 to 26,266,740 (W) via adjusting the terminal voltages of

Table 10.13 Active power losses data of IEEE 39-bus New England power system using particle swarm pattern search algorithm

Bus name	Voltage magnitudes p.u.	Voltage angles deg.	Generators	
			P (MW)	Q (Mvar)
Bus01	1.0474	−8.44	–	–
Bus02	1.0487	−5.75	–	–
Bus03	1.0302	−8.60	–	–
Bus04	1.0039	−9.61	–	–
Bus05	1.0053	−8.61	–	–
Bus06	1.0077	−7.95	–	–
Bus07	0.9970	−10.12	–	–
Bus08	0.9960	−10.62	–	–
Bus09	1.0282	−10.32	–	–
Bus10	1.0172	−5.43	–	–
Bus11	1.0127	−6.28	–	–
Bus12	1.0002	−6.24	–	–
Bus13	1.0143	−6.10	–	–
Bus14	1.0117	−7.66	–	–
Bus15	1.0154	−7.74	–	–
Bus16	1.0318	−6.19	–	–
Bus17	1.0336	−7.30	–	–
Bus18	1.0309	−8.22	–	–
Bus19	1.0499	−1.02	–	–
Bus20	0.9912	−2.01	–	–
Bus21	1.0318	−3.78	–	–
Bus22	1.0498	0.67	–	–
Bus23	1.0448	0.47	–	–
Bus24	1.0373	−6.07	–	–
Bus25	1.0576	−4.36	–	–
Bus26	1.0521	−5.53	–	–
Bus27	1.0377	−7.50	–	–
Bus28	1.0501	−2.01	–	–
Bus29	1.0499	0.74	–	–
Bus30	1.0475	−3.33	250.00	146.16
Bus31	0.9820	0.00	520.81	198.25
Bus32	0.9831	2.57	650.00	205.14
Bus33	0.9972	4.19	632.00	109.91
Bus34	1.0123	3.17	508.00	165.76
Bus35	1.0493	5.63	650.00	212.41
Bus36	1.0635	8.32	560.00	101.18
Bus37	1.0278	2.42	540.00	0.44
Bus38	1.0265	7.81	830.00	22.84
Bus39	1.0300	−10.05	1000.00	88.28

Table 10.14 Power flow results after optimization for 39-bus New England power system using particle swarm pattern search algorithm

Active power losses (Initial condition)	28194720 (W)
Active power losses (After optimization)	26266740 (W)
Reduction percentage	6.84%

Table 10.15 The tap setting of transformers of IEEE 39-bus New England power system after optimization

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (Mvar)	P_G (MW)	Q_G (Mvar)
1	364.810659	−6.452421	–	–	–	–
2	370.798179	−3.981883	–	–	–	–
3	372.902266	−6.776672	322.0	2.4	–	–
4	374.141517	−7.781993	500.0	184.0	–	–
5	374.772457	−6.949538	–	–	–	–
6	375.577162	−6.385371	–	–	–	–
7	372.536129	−8.253005	233.8	84.0	–	–
8	371.586660	−8.667435	522.0	176.0	–	–
9	366.270135	−8.296282	–	–	–	–
10	377.511546	−4.188488	–	–	–	–
11	376.869300	−4.942417	–	–	–	–
12	150.734523	−4.933962	7.5	88.0	–	–
13	377.168446	−4.770793	–	–	–	–
14	376.168387	−6.099718	–	–	–	–
15	375.663420	−6.121481	320.0	153.0	–	–
16	376.505858	−4.680447	329.0	32.3	–	–
17	375.344286	−5.652905	–	–	–	–
18	374.016041	−6.467971	158.0	30.0	–	–
19	379.190499	0.018998	–	–	–	–
20	237.505552	−0.769076	628.0	103.0	–	–
21	375.538973	−2.509637	274.0	115.0	–	–
22	378.827835	1.548230	–	–	–	–
23	377.526406	1.366011	247.5	84.6	–	–
24	377.641053	−4.568311	308.6	−92.2	–	–
25	375.556902	−2.870753	224.0	47.2	–	–
26	376.153057	−3.965808	139.0	17.0	–	–
27	374.094746	−5.811787	281.0	75.5	–	–
28	371.978374	−0.616475	206.0	27.6	–	–
29	370.455789	2.031981	283.5	26.9	–	–
30	17.130497	−1.599642	–	–	250.0	−54.17687004

(continued)

Table 10.15 (continued)

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (Mvar)	P_G (MW)	Q_G (Mvar)
31	17.926045	0.000000	9.2	4.6	517.55180622	179.62643204
32	17.584661	2.436932	–	–	650.0	82.93753618
33	17.681268	4.575524	–	–	632.0	328.35318619
34	16.557073	4.375941	–	–	508.0	−164.8434324
35	17.899414	6.135797	–	–	650.0	128.77506507
36	18.144109	8.626973	–	–	560.0	46.86633642
37	17.536826	3.495544	–	–	540.0	19.78359724
38	17.563459	8.560209	–	–	830.0	−58.73820034
39	355.349990	−7.996433	1104.0	250.0	1000	−332.33380789

Table 10.16 The data for transmission lines and transformers of IEEE 6-bus standard power system

From bus	To bus	Transformers tap magnitude (p.u.)	Transformers tap tap position
12	11	1.0000	0
12	13	1.0000	0
6	31	1.0350	1
10	32	1.0350	1
19	33	1.0700	2
20	34	1.0000	0
22	35	1.0250	1
23	36	1.0000	–
25	37	1.0250	1
2	30	1.0250	1
29	38	1.0000	0
19	20	1.0300	1

generators and the tap settings of transformers considering the voltage limitations for all the busbars.

10.9 Summary

Investigating active power losses caused in transmission system is of crucial importance in designing power systems as well as their operation and development, of which so many methods have been carried out practically on. Plenty of approaches have been recommended in order to decrease active power losses in power grid in which reactive power optimization is one of the most influential ones. Reactive power optimization investigates the operational condition of reactive power sources in AC power systems in order to have a minimum reactive power current flowing in transmission systems to lessen active power losses associated with it. There are three main parameters on which power system operators have control, as the voltage magnitude of *PV* busbars, reactive power output of compensators and tap setting of transformers which have under-load tap changer facility. It seems quite obvious that the number of controlling parameters can be so many in a real power network so as there are eight dimensions in a 14-bus power grid. In addition, since there is a tough association between plenty of parameters in electric power systems, reactive power optimization is a very nonlinear and non-convex problem comprising discrete and continuous variables simultaneously, which has a lot of local optimum points so that traditional optimization algorithms, most of which are based on the gradient of objective function, lose their performance.

Heuristic optimization methods have proven their performance in such complex optimization problems and it will be efficient to be able to use them in reactive power optimization problem as well. Two heuristic algorithms in combination with direct search optimization methods have been applied to three standard power grids and the results have been presented. Genetic and particle swarm optimization algorithms are great methods in terms of global optimization, whereas pattern search optimization method is remarkable in the respect of local optimization. Therefore, using a combination of a global optimizer and a local one will have so many benefits in its favor. There are plenty of articles, book chapters, books etc. in this respect so that each ones' advantages can be taken.

This chapter tries to focus on the fundamentals and implementation of reactive power optimization problem using MATLAB and DIgSILENT and creating an effectual link between them. It will avail engineers of using the professional tools of PowerFactory DIgSILENT in the respect of electrical engineering as stability analysis, power flow calculations etc. Besides, MATLAB also has got a lot of toolboxes and flexibility while optimizing problems, using artificial intelligence. The method has been presented for a simple standard power grid step by step using particle swarm pattern search algorithm in Chap. 11, while both MATLAB and DIgSILENT files for both approaches have been presented in book attachment as particle swarm pattern search and genetic pattern search for IEEE 6- and 14-bus power grids. In addition, reactive power optimization using built-in particle swarm and pattern search algorithm on IEEE 39-bus New England power system has been depicted in Chap. 11.

Appendices

Appendix 1: IEEE 6-Bus Standard Power System

The single-line diagram of IEEE 6-bus power network has been presented in Fig. 10.29 and the corresponding data are given in Tables 10.17, 10.18 and 10.19.

Appendix 2: IEEE 14-Bus Standard Power System

The single-line diagram of IEEE 14-bus power network has been presented in Fig. 10.30 and the corresponding data are given in Tables 10.20, 10.21 and 10.22.

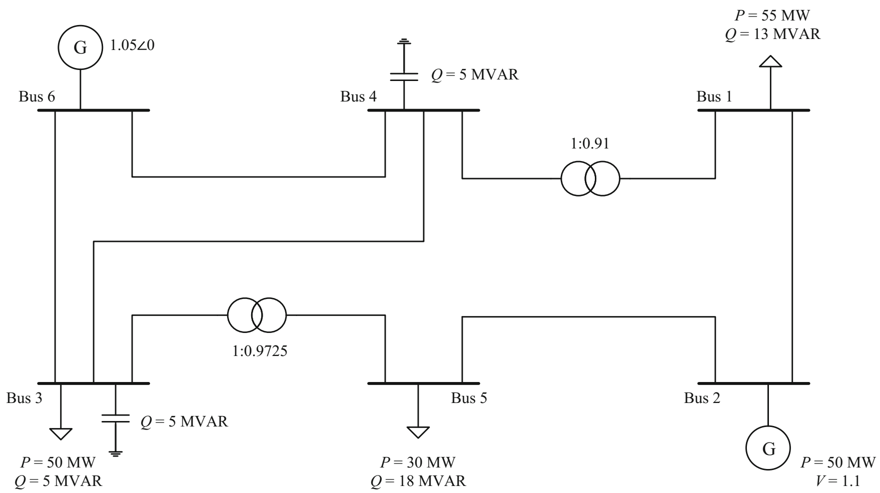


Fig. 10.29 Single-line diagram of IEEE 6-bus standard power system

Table 10.17 The corresponding limitations for control and state variables of IEEE 6-bus standard power system

Line no.	Starting busbar	Ending busbar	Line impedances		Line admittances		Transformer tap settings
			R (Ω)	X (Ω)	G (S)	B (S)	
1	6	3	4.88187	20.55942	0.0109	−0.0460	–
2	6	4	3.1752	14.6853	0.0141	−0.0651	–
3	4	3	3.84993	16.15383	0.0140	−0.0586	–
4	5	2	11.19258	25.4016	0.0145	−0.0330	–
5	2	1	28.69587	41.6745	0.0112	−0.0163	–
6	3	5	0	11.907	0	−0.0840	0.9725
7	4	1	0	5.27877	0	−0.1894	0.9100

Table 10.18 Initial statues and power flow results for IEEE 6-bus standard power system

	Transformer tap settings	Voltage magnitudes of PV busbars		Output capacity of reactive power sources		Load bus voltage magnitudes	Reactive power output of PV busbars
	T_{35}, T_{41}	V_6	V_2	Q_3	Q_4	V_1, V_3, V_5	Q_2, Q_6
Min	0.910	63	69.3	0.0	0.0	56.7	−20
Max	1.110	69.3	72.45	5.0	5.0	69.3	100

Table 10.19 The data for transmission lines and transformers of IEEE 14-bus standard power system

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (degree)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	58.12	−13.24	55	13	0	0
2	69.3	−4.78	0	0	50	27.63
3	58.07	−12.44	50	5	0	0
4	59.11	−9.85	0	0	0	0
5	57.16	−12.87	0.3	18	0	0
6	66.15	0	0	0	95.79	44.37

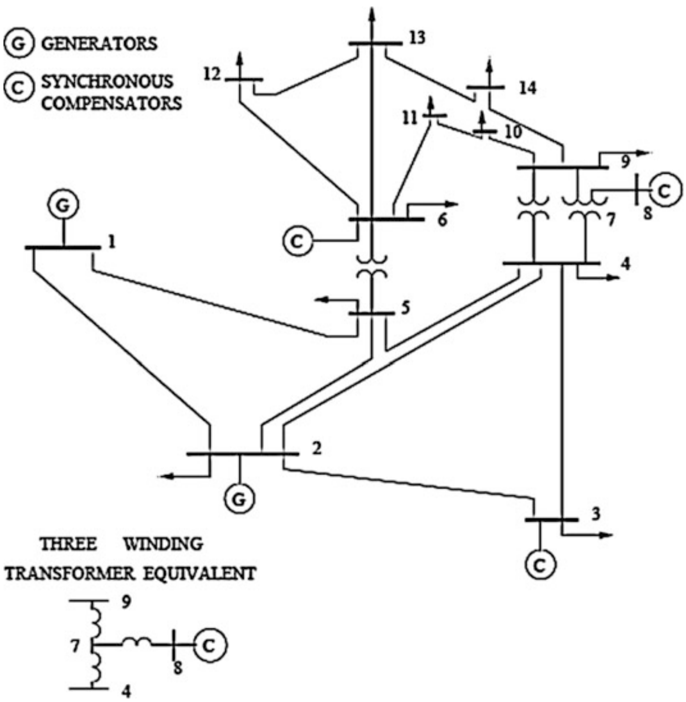


Fig. 10.30 Single-line diagram of IEEE 14-bus standard power system

Table 10.20 The corresponding limitations to control variables of IEEE 14-bus standard power system

Line no.	Starting busbar	Ending busbar	Line impedances		Line admittances		Line capacitances	Transformer tap settings
			R (Ω)	*X* (Ω)	*G* (S)	*B* (S)	*C* (F)	
1	1	2	2.34498	7.15957	0.041315	−0.12614	4.3636e–4	–
2	1	5	6.53763	26.98784	0.0085	−0.0350	4.066e–4	–
3	2	3	5.68579	23.95437	0.0094	−0.0395	3.61983e–4	–
4	2	4	7.03131	21.33472	0.0139	−0.0423	3.0909e–4	–
5	2	5	6.89095	21.03948	0.0141	−0.0429	2.80991e–4	–
6	3	4	8.10821	20.69463	0.0164	−0.0419	2.8595e–4	–
7	4	5	1.61535	5.09531	0.0565	−0.1783	1.05785e–4	–
8	4	7	0	25.30352	0	−0.0395	0	0.978
9	4	9	0	67.29778	0	−0.0149	0	0.969
10	5	6	0	30.49442	0	−0.0328	0	0.932
11	6	11	11.49258	24.0669	0.0162	−0.0338	0	–
12	6	12	14.87211	30.95301	0.0126	−0.0262	0	–
13	6	13	8.00415	15.76267	0.0256	−0.0504	0	–
14	7	8	0	21.31415	0	−0.0469	0	–
15	7	9	0	13.31121	0	−0.0751	0	–
16	9	10	3.84901	10.2245	0.0322	−0.0857	0	–
17	9	14	15.38031	32.71598	0.0118	−0.0250	0	–
18	10	11	9.92805	23.24047	0.0155	−0.0364	0	–
19	12	13	26.73132	24.18548	0.0206	−0.0186	0	–
20	13	14	20.68253	42.11042	0.0094	−0.0191	0	–

Table 10.21 Initial statues and power flow results for IEEE 14-bus standard power system

	Transformer tap settings	Voltage magnitudes of PV busbars		Output capacity of reactive power sources		
	T_{47}, T_{49}, T_{56}	V_1	V_2	Q_3	Q_6	Q_8
Min	0.910	110	110	0.0	−6	−6
Max	1.110	126.5	121	40	24	24

Table 10.22 Data of lines of IEEE 39-bus New England power system (100 MVA, 60 Hz) [13]

Bus no.	Voltage magnitudes and angles		Load consumption		Injection power	
	V (kV)	θ (°)	P_l (MW)	Q_l (MVar)	P_G (MW)	Q_G (MVar)
1	110	0	0	0	242.05	102.32
2	102.39	−6.54	21.7	12.7	40	50
3	91.83	−16.11	94.2	19	0	0
4	93.67	−12.60	47.8	0	0	0
5	95.53	−10.60	7.6	1.6	0	0
6	87.10	−19.66	11.2	7.5	0	0
7	88.14	−17.72	0	0	0	0
8	88.14	−17.72	0	0	0	0
9	85.51	−20.66	29.5	16.6	0	0
10	84.68	−21.02	9	5.8	0	0
11	85.36	−20.58	3.5	1.8	0	0
12	84.88	−21.21	6.1	1.6	0	0
13	84.19	−21.35	13.5	5.8	0	0
14	82.17	−22.85	14.9	5	0	0

Appendix 3: IEEE 39-Bus New England Power System

The Single-line diagram of IEEE 39-bus New England power system has been shown in Fig. 10.31 and the corresponding data are given in Tables 10.23, 10.24, 10.25, 10.26, 10.27 and 10.28. The nominal frequency of the New England transmission system is 60 Hz and the main voltage level is 345 kV (nominal voltage). For nodes at a different voltage level, following nominal voltages have been assumed for the PowerFactory model: Bus 12–138 kV, Bus 20–230 kV, Bus 30 and Bus 38–16.5 kV [13].

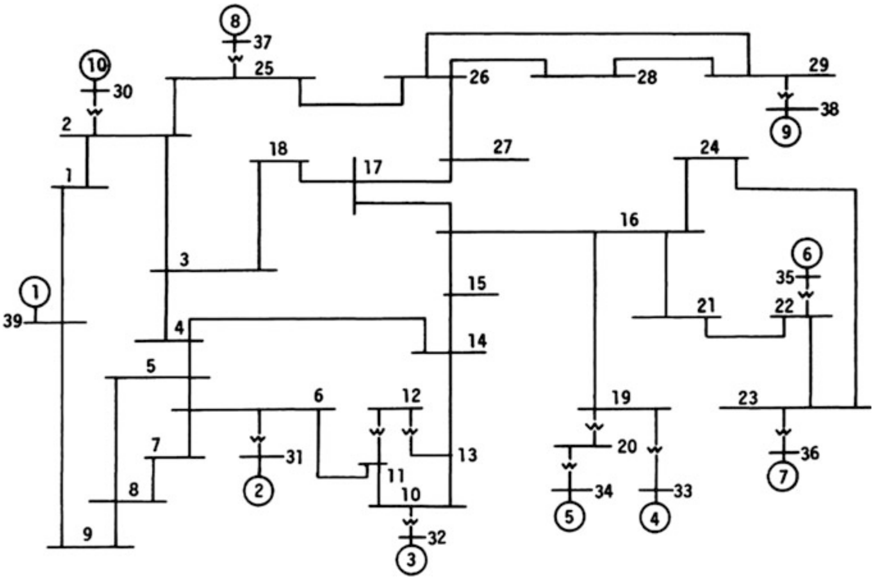


Fig. 10.31 Single-line diagram of IEEE 39-bus New England power system [13]

Table 10.23 Load demands of IEEE 39-bus New England power system [13]

Line	From bus	To bus	*R* in p.u.	*X* in p.u.	*B* in p.u.
Line 01–02	Bus 01	Bus 02	0.0035	0.0411	0.6987
Line 01–39	Bus 01	Bus 39	0.0010	0.0250	0.7500
Line 02–03	Bus 02	Bus 03	0.0013	0.0151	0.2572
Line 02–25	Bus 02	Bus 25	0.0070	0.0086	0.1460
Line 03–04	Bus 03	Bus 04	0.0013	0.0213	0.2214
Line 03–18	Bus 03	Bus 18	0.0011	0.0133	0.2138
Line 04–05	Bus 04	Bus 05	0.0008	0.0128	0.1342
Line 04–14	Bus 04	Bus 14	0.0008	0.0129	0.1382
Line 05–06	Bus 05	Bus 06	0.0002	0.0026	0.0434
Line 05–08	Bus 05	Bus 08	0.0008	0.0112	0.1476
Line 06–07	Bus 06	Bus 07	0.0006	0.0092	0.1130
Line 06–11	Bus 06	Bus 11	0.0007	0.0082	0.1389
Line 07–08	Bus 07	Bus 08	0.0004	0.0046	0.0780
Line 08–09	Bus 08	Bus 09	0.0023	0.0363	0.3804
Line 09–39	Bus 09	Bus 39	0.0010	0.0250	1.2000
Line 10–11	Bus 10	Bus 11	0.0004	0.0043	0.0729
Line 10–13	Bus 10	Bus 13	0.0004	0.0043	0.0729
Line 13–14	Bus 13	Bus 14	0.0009	0.0101	0.1723

(continued)

Table 10.23 (continued)

Line	From bus	To bus	R in p.u.	X in p.u.	B in p.u.
Line 14–15	Bus 14	Bus 15	0.0018	0.0217	0.3660
Line 15–16	Bus 15	Bus 16	0.0009	0.0094	0.1710
Line 16–17	Bus 16	Bus 17	0.0007	0.0089	0.1342
Line 16–19	Bus 16	Bus 19	0.0016	0.0195	0.3040
Line 16–21	Bus 16	Bus 21	0.0008	0.0135	0.2548
Line 16–24	Bus 16	Bus 24	0.0003	0.0059	0.0680
Line 17–18	Bus 17	Bus 18	0.0007	0.0082	0.1319
Line 17–27	Bus 17	Bus 27	0.0013	0.0173	0.3216
Line 21–22	Bus 21	Bus 22	0.0008	0.0140	0.2565
Line 22–23	Bus 22	Bus 23	0.0006	0.0096	0.1846
Line 23–24	Bus 23	Bus 24	0.0022	0.0350	0.3610
Line 25–26	Bus 25	Bus 26	0.0032	0.0323	0.5130
Line 26–27	Bus 26	Bus 27	0.0014	0.0147	0.2396
Line 26–28	Bus 26	Bus 28	0.0043	0.0474	0.7802
Line 26–29	Bus 26	Bus 29	0.0057	0.0625	1.0290
Line 28–29	Bus 28	Bus 29	0.0014	0.0151	0.2490

Table 10.24 Generator dispatch of IEEE 39-bus New England power system [13]

No.	Load	Bus	P (MW)	Q (Mvar)
1	Load 03	Bus 03	322.0	2.4
2	Load 04	Bus 04	500.0	184.0
3	Load 07	Bus 07	233.8	84.0
4	Load 08	Bus 08	522.0	176.0
5	Load 12	Bus 12	7.5	88.0
6	Load 15	Bus 15	320.0	153.0
7	Load 16	Bus 16	329.0	32.3
8	Load 18	Bus 18	158.0	30.0
9	Load 20	Bus 20	628.0	103.0
10	Load 21	Bus 21	274.0	115.0
11	Load 23	Bus 23	247.5	84.6
12	Load 24	Bus 24	308.6	−92.2
13	Load 25	Bus 25	224.0	47.2
14	Load 26	Bus 26	139.0	17.0
15	Load 27	Bus 27	281.0	75.5
16	Load 28	Bus 28	206.0	27.6
17	Load 29	Bus 29	283.5	26.9
18	Load 31	Bus 31	9.2	4.6
19	Load 39	Bus 39	1104.0	250.0

Table 10.25 Data of transformers (100 MVA) of IEEE 39-bus New England power system [13]

Generator	Bus	Bus type	*P* in MW	*V* in p.u.
G 01	Bus 39	PV	1000.0	1.0300
G 02	Bus 31	Slack	N.A.	0.9820
G 03	Bus 32	PV	650.0	0.9831
G 04	Bus 33	PV	632.0	0.9972
G 05	Bus 34	PV	508.0	1.0123
G 06	Bus 35	PV	650.0	1.0493
G 07	Bus 36	PV	560.0	1.0635
G 08	Bus 37	PV	540.0	1.0278
G 09	Bus 38	PV	830.0	1.0265
G 10	Bus 30	PV	250.0	1.0475

Table 10.26 Data of generators (100 MVA) of IEEE 39-bus New England power system [13]

From bus	To bus	*R* (p.u.)	*X* (p.u.)	Transformers tap magnitude (p.u.)	Transformers tap angle (degree)	Lower and upper limits of taps
12	11	0.0016	0.0435	1.0060	0.00	$1 \pm (1 \times 0.006)$
12	13	0.0016	0.0435	1.0060	0.00	$1 \pm (1 \times 0.006)$
6	31	0.0000	0.0250	1.0700	0.00	$1 \pm (2 \times 0.035)$
10	32	0.0000	0.0200	1.0700	0.00	$1 \pm (2 \times 0.035)$
19	33	0.0007	0.0142	1.0700	0.00	$1 \pm (2 \times 0.035)$
20	34	0.0009	0.0180	1.0090	0.00	$1 \pm (1 \times 0.009)$
22	35	0.0000	0.0143	1.0250	0.00	$1 \pm (1 \times 0.025)$
23	36	0.0005	0.0272	1.0000	0.00	–
25	37	0.0006	0.0232	1.0250	0.00	$1 \pm (1 \times 0.025)$
2	30	0.0000	0.0181	1.0250	0.00	$1 \pm (1 \times 0.025)$
29	38	0.0008	0.0156	1.0250	0.00	$1 \pm (1 \times 0.025)$
19	20	0.0007	0.0138	1.0600	0.00	$1 \pm (2 \times 0.03)$

Table 10.27 Data of AVRs of IEEE 39-bus New England power system [13]

Unit no.	H (s)	R_a (p.u.)	x'_d (p.u.)	x'_q (p.u.)	x_d (p.u.)	x_q (p.u.)	T'_{d0} (s)	T'_{q0} (s)	x_l (p.u.)	x'' (p.u.)	T''_{d0} (s)	T''_{q0} (s)
1	500.0	0.0000	0.0060	0.0080	0.0200	0.0190	7.000	0.700	0.0030	0.0040	0.050	0.035
2	30.3	0.0000	0.0697	0.1700	0.2950	0.2820	6.560	1.5000	0.0350	0.0500	0.050	0.035
3	35.8	0.0000	0.0531	0.0876	0.2495	0.2370	5.700	1.5000	0.0304	0.0450	0.050	0.035
4	28.6	0.0000	0.0436	0.1660	0.2620	0.2580	5.690	1.5000	0.0295	0.0350	0.050	0.035
5	26.0	0.0000	0.1320	0.1660	0.6700	0.6200	5.400	0.4400	0.0540	0.0890	0.050	0.035
6	34.8	0.0000	0.0500	0.0814	0.2540	0.2410	7.300	0.4000	0.0224	0.0400	0.050	0.035
7	26.4	0.0000	0.0490	0.1860	0.2950	0.2920	5.660	1.5000	0.0322	0.0440	0.050	0.035
8	24.3	0.0000	0.0570	0.0911	0.2900	0.2800	6.700	0.4100	0.0280	0.0450	0.050	0.035
9	34.5	0.0000	0.0570	0.0587	0.2106	0.2050	4.790	1.9600	0.0298	0.0450	0.050	0.035
10	42.0	0.0000	0.0310	0.0500	0.1000	0.0690	10.200	0.0000	0.0125	0.0250	0.050	0.035

Table 10.28 Initial statues and power flow results for IEEE 39-bus New England power system [13]

Unit no.	$K_a = K_A$	$T_a = T_A$	$V_{r\min} = V_{R\min}$	$V_{r\max} = V_{R\max}$	$K_e = K_E$	$T_e = T_E$	$K_f = K_F$	$T_f = T_F$	$S_{e1} = C_1$	$S_{e2} = C_2$	$E_1 = E_{X1}$	$E_2 = E_{X2}$
2	6.2	0.05	−1.0	1.0	−0.6330	0.405	0.0570	0.500	0.660	0.880	3.036437	4.048583
3	5.0	0.06	−1.0	1.0	−0.0198	0.500	0.0800	1.000	0.130	0.340	2.342286	3.123048
4	5.0	0.06	−1.0	1.0	−0.0525	0.500	0.0800	1.000	0.080	0.314	2.868069	3.824092
5	40.0	0.02	−10.0	10.0	1.0000	0.785	0.0300	1.000	0.070	0.910	3.926702	5.235602
6	5.0	0.02	−1.0	1.0	−0.0419	0.471	0.0754	1.246	0.064	0.251	3.586801	4.782401
7	40.0	0.02	−6.5	6.5	1.0000	0.730	0.0300	1.000	0.530	0.740	2.801724	3.735632
8	5.0	0.02	−1.0	1.0	−0.0470	0.528	0.0854	1.260	0.072	0.282	3.191489	4.255319
9	40.0	0.02	−10.5	10.5	1.0000	1.400	0.0300	1.000	0.620	0.850	4.256757	5.675676
10	5.0	0.06	−1.0	1.0	−0.0485	0.250	0.0400	1.000	0.080	0.260	3.546099	4.728132

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